1	2	3	4	5	6	Total

7	8	9	10	11	12	Total

Last name: First name:

(12) 1. Suppose X is normally distributed with mean 2 and variance 9.

(a) Find $P(X \geq 7)$.

(b) Find a number c so that $P(X \leq c) = .10$.

(20) 2. A point (X, Y) is chosen uniformly from the square $[-1, 1]^2$.

(a) Compute $P(Y > X^2)$.

(b) Find the distribution function (CDF) of $Z = |X| + |Y|$.

(c) Find the density function (PDF) of Z.

(15) 3. Find the PMF of $X+Y$ where X and Y are independent geometric random variables with parameters p and q respectively, when

(a) $p = q$.

(b) $p \neq q$.

(20) 4. Suppose X is a continuous random variable with PDF

$$f(x) = \begin{cases} 6x(1-x) & \text{if } 0 \leq x \leq 1; \\ 0 & \text{otherwise.} \end{cases}$$

Then X divides the unit interval $[0,1]$ into two parts. Let $Y \geq 1$ be the ratio of the longer part to the shorter part.

(a) Find $P(Y \leq 2)$.

(b) Find the PDF of Y.

(c) Find EY.

(18) 5. Short answer questions. No explanation necessary.

(a) Suppose X has the $N(-1, 9)$ distribution. What is the distribution of $2X + 5$?

(b) Suppose X satisfies $EX = -5$ and $EX^2 = 25$. What can you say about X?

(c) What value of a minimizes $E(X - a)^2$?

(d) If (X_1, X_2, X_3, X_4) has the multinomial distribution with parameters n and $p_1 = \frac{1}{6}, p_2 = \frac{1}{3}, p_3 = \frac{1}{4}, p_4 = \frac{1}{4}$, what is the distribution of $X_1 + X_3$?

(e) Suppose the events A and B are both disjoint and independent. What can you conclude about A and B?

(f) If (X, Y) is uniformly distributed on a two-dimensional connected region G, and X and Y are independent, what can you conclude about G?

(15) 6. Suppose that X has the $N(0, 1)$ distribution.

(a) Find the density function of $|X|$.

(b) Compute Ee^X.

(20) 7. Consider a collection of n married couples. A sample of size $m \leq 2n$ is taken from the $2n$ people without replacement. Let X be the number of married couples in the sample.

(a) Find the PMF of X.

(b) Use indicator random variables to compute EX.

(20) 8. Let $X_1, X_2, \ldots$ be i.i.d. Bernoulli random variables with parameter p (i.e., they are independent and satisfy $P(X_i = 1) = p, P(X_i = 0) = 1 - p$). Define $Y_i = X_i X_{i+1} X_{i+2}$ and $S_n = Y_1 + Y_2 + \cdots + Y_n$.

(a) Find EY_i.

(b) Find EY_iY_j for **all** pairs $i, j \geq 1$.

(c) Find ES_n.

(d) Find $\text{Var}(S_n)$.

(20) 9. The joint probability density function of (X, Y) is given by

$$f(x,y) = \begin{cases} c(x+y)^2 & \text{if } 0 \le x \le 1 \text{ and } 0 \le y \le 1; \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the value of c.

(b) Are X and Y independent? Explain.

(c) Let $U = X + Y$ and $V = X - Y$. Find the joint density function $g(u, v)$ of (U, V). Be sure to specify the values of (u, v) for which $g(u, v) > 0$.

(15) 10. Suppose $X_1, X_2, \ldots, X_n$ are independent continuous random variables, each having distribution function $F(x)$ and density function $f(x)$. Let $Y = \max(X_1, \ldots, X_n)$.

(a) Find the distribution function of Y.

(b) Find the density function of Y.

(15) 11. Box I contains 2 white balls and 2 black balls, box II contains 2 white balls and 1 black ball, and box III contains 1 white ball and 3 black balls. One of the three boxes is chosen at random, and one ball is drawn from it.

(a) What is the probability that the ball drawn is white?

(b) Given that the ball drawn is white, what is the probability that the first box was chosen?

(10) 12. Two fair dice are thrown, and let S be sum of the outcomes. Consider the events

$$A = \{S = 2, 3, \text{ or } 4\}, \quad B = \{S = 3, 5, \text{ or } 7\}, \quad C = \{S = 4, 7, 8, \text{ or } 9\}.$$

(a) Are A and B independent? Explain.

(b) Are A, B, C independent? Explain.