(Q-1) Let the triangle ABC be inscribed in a circle, let P denote the centroid of the triangle, and let O denote the circumcenter. Suppose that A, B, C have coordinates $(0,0)$, $(a,0)$, and (b,c) respectively.

(a) Express the coordinates of P and O in terms of a, b, c.

(b) Extend the line segments AP, BP, and CP to the circle in points D, E, and F, respectively. Show that

$$\frac{AP}{PD} + \frac{BP}{PE} + \frac{CP}{PF} = 3.$$

(Hint: One way to proceed is the following. Let x denote OP, and let R be the radius of the circumcircle. Then first show that

$$\frac{AP}{PD} + \frac{BP}{PE} + \frac{CP}{PF} = \frac{AP^2 + BP^2 + CP^2}{R^2 - x^2}.$$

Then express each of the terms on the right hand side in terms of a, b, c (using results from the previous part).)

(Q-2) Find the relation that must hold between the parameters a, b, c so that the line $x/a + y/b = 1$ will be tangent to the circle $x^2 + y^2 = c^2$.

(Q-3) A parabola with equation $y^2 = ax$ is cut in four points by the circle $(x-h)^2 + (y-k)^2 = r^2$. Determine the product of the distances of the four points of intersection from the axis of the parabola.

(Q-4) The sides AD, AB, CB, CD of the quadrilateral $ABCD$ are divided by points E, F, G, H so that $AE : ED = AF : FB = CG : GB = CH : HD$. Prove that $EFGH$ is a parallelogram.

(Q-5) On the sides of an arbitrary parallelogram, squares are constructed lying exterior to it. Prove that their centers M_1, M_2, M_3, M_4 are themselves vertices of a square.

(Q-6) On the sides of an arbitrary quadrilateral $ABCD$, equilateral triangles ABM_1, BCM_2, CDM_3, DAM_4 are constructed so that the first and third are exterior on the quadrilateral, while the second and fourth are on the same side of BC and DA as the quadrilateral itself. Prove that the quadrilateral $M_1M_2M_3M_4$ is a parallelogram.

(Q-7) In a tetrahedron, two pairs of opposite edges are orthogonal. Prove that the third pair of opposite edges must also be orthogonal.

(Q-8) Given a point P on the circumference of a unit circle and the vertices $A_1, A_2, \ldots, A_n$ of a regular polygon of n sides, prove that $PA_1^4 + PA_2^4 + \cdots + PA_n^4$ is constant (i.e., independent of the position of P on the circumference).

(Q-9) Let G be the centroid of a triangle ABC. Prove that

$$3(GA^2 + GB^2 + GC^2) = AB^2 + BC^2 + CA^2.$$

(Q-10) Let $ABCDEF$ be a hexagon in a circle of radius r. Show that if $AB = CD = EF = r$, then the midpoints of BC, DE, and FA are the vertices of an equilateral triangle.