(Q-1) The number 3 can be expressed as a sum of one or more positive integers, taking order into account, in four ways, namely, 3, $1+2$, $2+1$, and $1+1+1$. Show that any positive integer n can be so expressed in 2^{n-1} ways.

(Q-2) In how many ways can 10 be expressed as a sum of 5 *nonnegative* integers, when order is taken into account? (Hint: Find an equivalent problem in which the phrase "5 nonnegative integers" is replaced by "5 positive integers".)

(Q-3) Given n objects arranged in a row. A subset of these objects is called *unfriendly* if no two of its elements are consecutive. Show that the number of unfriendly subsets with k elements is $\binom{n-k+1}{k}$. (Hint: Adopt an idea similar to that used in Larson 1.3.6.)

(Q-4) Let $a_1, a_2, \ldots, a_n$ be a permutation of the set $S_n = \{1, 2, \ldots, n\}$. An element i of S_n is called a fixed point of this permutation if $a_i = i$. A *derangement* of S_n is a permutation of S_n having no fixed points. Let g_n be the number of derangements of S_n. Show that

$$g_1 = 0, \qquad g_2 = 1,$$

and

$$g_n = (n-1)(g_{n-1} + g_{n-2}), \qquad \text{for } n > 2.$$

(Hint: a derangement either interchanges the first element with another element, or it doesn't.)

(Q-5) Sum: $1 - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \cdots + (-1)^n \binom{n}{n}$.

(Q-6) Sum: $1 \times 2\binom{n}{2} + 2 \times 3\binom{n}{3} + \cdots + (n-1)n\binom{n}{n}$.

(Q-7) Sum: $\binom{n}{1} + 2^2\binom{n}{2} + 3^2\binom{n}{3} + \cdots + n^2\binom{n}{n}$.

(Q-8) Sum: $\binom{n}{1} - 2^2\binom{n}{2} + 3^2\binom{n}{3} - \cdots + (-1)^{n+1}n^2\binom{n}{n}$.

(Q-9) Prove $\binom{r}{0}\binom{s}{n} + \binom{r}{1}\binom{s}{n+1} + \binom{r}{2}\binom{s}{n+2} + \cdots + \binom{r}{r}\binom{s}{n+r} = \binom{r+s}{s-n}$.

(Q-10) Prove $\binom{n}{0}^2 + \binom{n}{1}^2 + \binom{n}{2}^2 + \cdots + \binom{n}{n}^2 = \binom{2n}{n}$.