HW 3

(Q-1) (a) Prove that any two successive Fibonacci numbers F_n, F_{n+1}, $n > 0$, are relatively prime.
(b) Given that $T_1 = 2$ and $T_{n+1} = T_n^2 - T_n + 1$, $n > 0$, prove that T_n and T_m are relatively prime whenever $n \neq m$.

(Q-2) Prove $(a + b)/(c + d)$ is irreducible if $ad - bc = 1$.

(Q-3) Prove that $\gcd(a_1, \ldots, a_m) \gcd(b_1, \ldots, b_n) = \gcd(a_1 b_1, \ldots, a_m b_n)$, where the parentheses on the right include all mn products $a_i b_j$, $i = 1, \ldots, m$, $j = 1, \ldots, n$.

(Q-4) When Mr. Smith cashed a check for x dollars and y cents, he received instead y dollars and x cents, and found that he had two cents more than twice the proper amount. For how much was the check written?

(Q-5) Prove that any subset of 55 numbers chosen from the set $\{1, 2, \ldots, 100\}$ must contain numbers differing by 10, 12, and 13, but need not contain a pair differing by 11.

(Q-6) Show that $4^{3x+1} + 2^{3x+1} + 1$ is divisible by 7.

(Q-7) (a) Prove that the sequence (in base-10 notation)

$$11, 111, 1111, 11111, \ldots$$

contains no squares.
(b) Prove that the difference of the squares of any two odd numbers is divisible by 8.

(Q-8) Prove that $(21n - 3)/4$ and $(15n + 2)/4$ cannot both be integers for the same positive integer n.

(Q-9) Complete the proof of Larson 3.2.10. ("A lattice point $(x, y) \in \mathbb{Z}^2$ is visible if $\gcd(x, y) = 1$. Prove or disprove: Given a positive integer n, there exists a lattice point (a, b) whose distance from every visible point is $\geq n$.")

(Q-10) Prove that there does not exist an integer which is doubled when the initial digit is transferred to the end.