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Ribbon tile invariants. (English. English summary)

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Ribbon tiles are connected polyominos which have at most one lattice square per diagonal $x+y=$const. This paper concerns tiling problems with sets of ribbon tiles: What regions can be tiled with (translated) copies of tiles from a fixed set of ribbon tiles? A number of invariants are constructed to answer this problem, in particular, invariants which define linear relations between the number of tiles of each type necessary for a tiling. These invariants are shown to be stronger than the classical coloring invariants which are of the form "a tile covers one square of each color, therefore a tileable region has the same number of squares of each color".

The proof of existence of these tile-counting invariants involves showing that one can get from any tiling of a region to another tiling of the same region by a sequence of local rearrangements; each local rearrangement gives a linear tile-counting relation. The proof uses the so-called "rim-hook bijection" (from the theory of integer partitions), which is a bijection between collections of Young tableaux and "rim-hook tableaux", which resemble Young tableaux with ribbon tiles instead of squares. *Richard Kenyon* (F-PARIS11-M)

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