nance of soda, sulphuric acid, and chlorine, which distinguishes it from the Ottawa. It is an interesting geographical feature of these two rivers, that they each pass through a series of great lakes in which the waters are enabled to deposit their mechanical impurities, and thus are rendered remarkably clear and transparent.

The presence of large amounts of silica in river-waters is a fact but recently established. Until the late analyses by H. St.-Claire Deville of the rivers of France*, the silica in water had generally been overlooked wholly or in great part; and, as he suggests, had, from the mode of analysis, been confounded with gypsum. (The purity of the silica in all my determinations was established by the blowpipe.) The importance in an agricultural point of view of this large amount of dissolved silica, where river-waters are employed for the irrigation of the land, is very great: and geologically, the fact is not less significant, as it marks a decomposition of the siliceous rocks by the action of waters holding in solution carbonic acid, and the organic acids arising from the decay of vegetable matters, which, dissolving the alkalies, the lime and magnesia, from the native silicates, liberate the silicic acid in a soluble form. Silica is never wanting in natural waters, whether neutral or alkaline, although proportionally less abundant in neutral waters which contain large amounts of earthy ingredients. The alumina, whose presence is not less constant, although in much smaller quantity, appears equally to belong to the soluble constituents of the waters. The amount of dissolved silica annually carried to the sea by the rivers must be very great; yet sea-water, according to Forchammer, does not contain any considerable quantity in solution; it doubtless goes to form the shields of Infusoria, and may play an important part in the consolidation of the ocean sediments and the silicification of organic remains.

Montreal, March 1, 1857.

XXXVII. *On a Problem in the Partition of Numbers.* *By* A. Cayley, *Esq.*†

It is required to find the number of partitions into a given number of parts, such that the first part is unity, and that no part is greater than twice the preceding part.

Commencing to form the partitions in question, these are

$$\begin{array}{c|cc|cccccc|l} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & \&c.; \\ & 1 & 2 & 1 & 1 & 2 & 2 & 2 & 2 & \\ & & & 1 & 2 & 1 & 2 & 3 & 4 & \end{array}$$

* *Annales de Chimie et de Physique*, 1848, vol. xxiii. p. 32.

† Communicated by the Author.

and if we were to proceed to the 4-partitions, each 3-partition ending in 1 would give rise to two such partitions; each 3-partition ending in 2 to four such partitions; each 3-partition ending in 3 to six such partitions; and each 3-partition ending in 4 to eight such partitions. We form in this manner the Table—

Number of	Ending in 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Totals.
1-partitions.	1																1
2-partitions.	1	1															2
3-partitions.	2	2	1	1													6
4-partitions.	6	6	4	4	2	2	1	1									26
5-partitions.	26	26	20	20	14	14	10	10	6	6	4	4	2	2	1	1	166
&c.																	

And we are thus led to the series

$$\begin{array}{l} 1 \\ 1,\ 2 \\ 1,\ 2,\ 4,\ 6 \\ 1,\ 2,\ 4,\ 6,\ 10,\ 14,\ 20,\ 26 \\ \&c.; \end{array}$$

where, considering 0 as the first term of each series, the first differences of any series are the terms twice repeated of the next preceding series: thus the differences of the fourth series are 1, 1, 2, 2, 4, 4, 6, 6. It is moreover clear that the first half of each series is precisely the series which immediately precedes it. We need, in fact, only consider a single infinite series, 1, 2, 4, 6, &c. It is to be remarked, moreover, that in the column of totals, the total of any line is precisely the first number in the next succeeding line.

Consider in general a series A, B, C, D, E, &c., and a series A′, B′, C′, D′, E′, &c. derived from it as follows:—

$$\begin{aligned} A' &= 1A \\ B' &= 2A \\ C' &= 2A+B \\ D' &= 2A+2B \\ E' &= 2A+2B+C \\ F' &= 2A+2B+2C \\ &\&c.; \end{aligned}$$

viz. the first differences of the series 0, A′, B′, C′, D′, E′, &c. are A, A, B, B, C, C, &c. Then multiplying by 1, x, x^2, &c. and adding, we have

$$A' + B'x + C'x^2 + \&c. = (1 + 2x + 2x^2 + \ldots)(A + Bx^2 + Cx^4 + \&c.)$$
$$= \frac{1+x}{1-x}(A + Bx^2 + Cx^4 + \&c.).$$

And if we form in a similar manner A'', B'', C'', D'', &c. from A', B', C', D', &c. and so on, we have

$$A'' + B''x + C''x^2 + \&c. = \frac{1+x}{1-x}(A' + B'x^2 + C'x^4 + \&c.)$$
$$= \frac{1+x}{1-x}\,\frac{1+x^2}{1-x^2}(A + Bx^4 + Cx^8 + \&c.),$$

and so on. Write $A=1$, and suppose that the process is repeated an indefinite number of times, we have

$$1 + \mathfrak{B}x + \mathfrak{C}x^2 + \mathfrak{D}x^3 + \&c. = \frac{1+x\,.\,1+x^2\,.\,1+x^4\,.\,\&c.}{1-x\,.\,1-x^2\,.\,1-x^4\,.\,\&c.}.$$

And the coefficients 1, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, &c. are precisely those of the infinite series 1, 2, 4, 6, &c. We have more simply

$$1 + \mathfrak{B}x + \mathfrak{C}x^2 + \mathfrak{D}x^3 + \&c. = \frac{1}{(1-x)^2(1-x^2)(1-x^4)(1-x^8)\ \&c.},$$

which gives rise to the following very simple algorithm for the calculation of the coefficients:—

1,	2,	3,	4,	5,	6,	7,	8,	9,	10,	11,	12,	13,	14,	15,	16
0,	0;	1,	2,	4,	6,	9,	12,	16,	20,	25,	30,	36,	42,	49,	56
1,	2,	4,	6,	9,	12,	16,	20,	25,	30,	36,	42,	49,	56,	64,	72
0,	0,	0,	0;	1,	2,	4,	6,	10,	14,	20,	26,	35,	44,	56,	68
1,	2,	4,	6,	10,	14,	20,	26,	35,	44,	56,	68,	84,	100,	120,	140
0,	0,	0,	0,	0,	0,	0,	0;	1,	2,	4,	6,	10,	14,	20,	26

$$1 \,\big|\, 2 \,\big|\, 4,\ 6 \,\big|\, 10,\ 14,\ 20,\ 26 \,\big|\, 36,\ 46,\ 60,\ 74,\ 94,\ 114,\ 140,\ 166 \,\big|$$

&c.

The last line is marked off into periods of (reckoning from the beginning) 1, 2, 4, 8, &c.; and by what has preceded, the series which gives the number of 1-partitions, 2-partitions, 3-partitions, &c. is found by summing to the end of each period and doubling the results; we thus, in fact, obtain (1), 2, 6, 26, 166, 1626, &c.: and the same series is also given by means of the last terms of the several periods.

The preceding expression for $1 + \mathfrak{B}x + \mathfrak{C}x^2 + \&c.$ shows that $\mathfrak{B}$, $\mathfrak{C}$, &c. are the number of partitions of 1, 2, 3, 4, 5, 6, &c. respectively into the parts 1, 1′, 2, 4, 8, &c.: and we are thus led to—

Theorem. The number of x-partitions (first part unity, no part greater than twice the preceding one) is equal to the number of partitions of $2^{x-1}-1$ into the parts $1, 1', 2, 4, \dots 2^{x-2}$. Or, again, it is equal to twice the sum of the number of partitions of $0, 1, 2, \dots 2^{x-2}-1$ respectively into the parts $1, 1', 2, 4, \dots 2^{x-3}$ (where the number of partitions of 0 counts for 1).

For example, the partitions of 0, 1, 2, 3, &c. with the parts $1, 1', 2, \dots$ are

(.)

$1,\ 1'$

$1+1,\ 1+1',\ 1'+1',\ 2$

$1+1+1,\ 1+1+1',\ 1+1'+1',\ 1'+1'+1',\ 2+1,\ 2+1',$

the numbers of which are 1, 2, 4, 6. Hence, by the first part of the theorem, the number of 3-partitions is 6, and by the second part of the theorem, the number of 4-partitions is

$$2(1+2+4+6)=26.$$

2 Stone Buildings,
March 17, 1857.

XXXVIII. *On the Connexion of Catalytic Phænomena with Allotropy. By* C. S. Schönbein*.

THE number of the phænomena hitherto made known which have been named catalytic, or actions by contact, has already become tolerably large, and will daily increase. Both Berzelius, who was the first to direct attention to these enigmatical phænomena, and Mitscherlich, who has also devoted much time to their investigation, have carefully abstained from expressing even an opinion as to their ultimate cause. For if the one used the word "*Catalysis,*" and the other the expression "*Action by contact,*" neither, if I have rightly understood them, considered these terms to imply any explanation. A peculiar class of facts was to be briefly distinguished; and if these names have been misused in science, these illustrious inquirers are certainly not to blame.

I am of opinion that the time is now come when many of the catalytic phænomena may be better understood than hitherto; that is, may be referred to another series of facts which have been made known within the last few years. I allude to the remarkable capacity which many simple bodies possess of undergoing, under the influence of imponderable and ponderable agents, essential changes in the complex whole of their properties. This kind of material change Berzelius has distinguished

* Translated by Dr. E. Atkinson from Poggendorff's *Annalen*, vol. c. p. 1.