FINAL (MATH 184, WINTER 2018)

Your Name: ____

UCLA id: ____

The rules: You are allowed to use only this paper and pen/pencil. No calculators. No books, no notebooks, no scratch paper, no web access. You MUST write your name and UCLA ID number, both in INK. You MUST simplify completely and BOX all answers. You MUST write out your logical reasoning and/or proof in full. You have exactly 180 minutes.

Points:

1	
2	
3	
4	
5	
6	
7	
8	

Total: **(out of 160)**

Problem 1. (20 points, 5 points each part)

Find a closed form for the EGF for $\{a_n\}$, where

1) a_n is the number of set partitions of $[n]$ into parts, all of them even (and nonempty)
2) $a_n = D_n$, where D_n is the number of *derangements*,
   i.e. permutations $\sigma \in S_n$ with no fixed points.
3) $a_n = 2^n + n^2 - n$
4) $a_n = n!(n^2 - 1)$

Problem 2. (20 points, 5 points each)

Let σ be a random permutation in S_{10}. Let $X = \sigma(1)+\sigma(2)+\sigma(3)$, $Y = \sigma(10)^2+\sigma(9)^2+\sigma(8)^2$, Compute:

a) $E[X]$

b) $E[X+Y]$

c) $E[X^2]$

d) $Var(X)$

Problem 3. (20 points, 10 points each)

a) Compute the number of permutations $\sigma \in S_{13}$ with four cycles, all of distinct lengths.

b) Compute the number of 4-subsets $A \subset \{1, \ldots, 20\}$, such that the product of elements in A is equal to $2^5 \cdot 5 \cdot 7 \cdot 11$.

Problem 4. (20 points)

Compute the number of spanning trees in the following graphs. You can use any method.

a) complete graph K_9 on [9] minus one edge $(1, 2)$,

b) complete graph K_9 on [9] minus two edges $(1, 2)$ and $(3, 4)$.

Problem 5. (20 points, 10 points each)

Let a_n be the number of partitions of n into parts $1, 2, 4$.

$a)$ Compute the GF for $\{a_n\}$.

$b)$ Find a closed formula for a_n.

Problem 6. (20 points, 5 points each)

Compute the number of binary trees T on 15 vertices which satisfy the following properties:

a) the root of T has degree 2, i.e. root has both children,

b) there is exactly one leaf (vertex with no children),

c) T has no vertices with 1 child,

d) T has exactly 1 right edge.

(these are four separate problems).

Note: you can write the result in terms of Catalan numbers $C_n = \frac{1}{n+1}\binom{2n}{n}$.

Problem 7. (20 points)

For the following functions decide which pairs satisfy $f(n) = o\big(g(n)\big)$

$$\sqrt{n!}, \qquad e^{n\sqrt{n}(1+\sin n)}, \qquad n^{\sqrt{n}\log n/(\log\log n)^n}, \qquad (e^n/\log n)^{\log n}, \qquad e^{\sqrt{n}(\log n)^n}$$

Problem 8. (20 points)

In the n hunters vs. n rabbits problem, suppose $n = 100$, and 20 hunters hit the randomly chosen rabbit 80% of the time, while 80 hunters hit the randomly chosen rabbit 20% of the time. Find the expected number of surviving rabbits.

Scratch page