HOME ASSIGNMENT 3 (MATH 184, SPRING 2022)

Read: **Bona** (Second ed.), sections 1.4.1.1, 4.2, 4.4, 5.3.1

Solve: Exc 47 in §1.10, 26, 27 in §2.10, 13, 14, 28, 43 in §4.10, 17, 18, 19 in §5.10, and the following problems:

I. Find and prove a closed formula for the number of centrally-symmetric triangulations of $2n$-gon.

II. Define 3-ary trees to be similar to binary trees, but with each vertex having subtrees of three type: Left, Center and Right. Denote by R_n the number of 3-ary trees with n vertices. For example, $R_2 = 3$, $R_3 = 12$, $R_4 = 55$. Find a closed formula for R_n.

III. Let W_n be the number of grid walks $(0,0) \to (n,2n)$ which are on or above the $y = 2x$ line. Prove that $W_n = R_n$.

IV. Prove bijectively the following product formula:

$$\prod_{\lambda=(1^{m_1}2^{m_2}\dots)\vdash n} m_1!\, m_2! \cdots = \prod_{\lambda=(1^{m_1}2^{m_2}\dots)\vdash n} 1^{m_1}\, 2^{m_2} \cdots$$

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This HA is due Wednesday May 25, at 2:59:59 pm. (right before class). Please read the collaboration policy on the course web page. Make sure you write your name in the beginning and your collaborators' names at the end.

P.S. Each item above has the same weight.