

Math 32A. Quiz 2b January 26, 2006

**Solutions:**

A plane  $P$  in 3-space has equation  $x + y + 2z = 4$ .

1. Find the distance from the point  $(0, 0, 0)$  to the plane.

Solution: The plane contains the point  $(1, 1, 1)$  and has normal vector  $\vec{n} = \langle 1, 1, 2 \rangle$ .

Let  $\vec{b} = \langle 1, 1, 1 \rangle$  be the vector from  $(0, 0, 0)$  to  $(1, 1, 1)$ . Then the distance is

$$\frac{|\vec{n} \cdot \vec{b}|}{|\vec{n}|} = 4/\sqrt{6}.$$

2. Find the point  $(x, y, z)$  on the plane  $P$  that is closest to (i.e. smallest distance from)  $(0, 0, 0)$ .

Solution: Let  $\vec{b} = \langle 1, 1, 1 \rangle$  be as in Problem 1. Then

$$\vec{b} = \frac{\vec{b} \cdot \vec{n}}{|\vec{n}|^2} \vec{n} + \left( \vec{b} - \frac{\vec{b} \cdot \vec{n}}{|\vec{n}|^2} \vec{n} \right) = \vec{u} + \vec{v},$$

where  $\vec{v} \in P$  and  $\vec{u}$  is the position vector of a point in  $P$ . Hence the nearest point is the terminal point of  $\vec{u}$ , which is  $(4/\sqrt{6}) \langle 1, 1, 2 \rangle$ .