HOMEWORK 2

13.3

(8.)

$$a \cdot b = (4j - 3k)(2i + 4j + 6k) = (0 \cdot 2)i + (4 \cdot 4)j + ((-3) \cdot 6)k$$

$$= 16j - 18k$$

(11.) $u \cdot v = |u||v| \cos\measuredangle(u,v) = 1 \cdot 1 \cdot \cos\frac{\pi}{3} = \frac{1}{2}$

(12.) $u \cdot v = |u||v| \cos\measuredangle(u,v) = 1 \cdot \frac{\sqrt{2}}{2} \cos\frac{\pi}{4} = \frac{1}{2}$

$u \cdot w = |u||v| \cos\measuredangle(u,v) = 1 \cdot 1 \cdot \cos\frac{\pi}{2} = 0$

(25.)

$\vec{PQ}\ (1,3,-2)$

$\vec{PR}\ (5,1,-3)$

$\vec{QR} = (4,-2,-1)$

$\vec{PQ} \cdot \vec{PR} = \langle 1,3,-2 \rangle \cdot \langle 5,1,-3 \rangle$

$= 14$

$\vec{PQ} \cdot \vec{QR} = \langle 1,3,-2 \rangle \cdot \langle 4,-2,-1 \rangle$

$= 4 - 6 + 2 = 0$

HENCE, $\vec{PQ} \perp \vec{QR}$ so ΔPQR is RIGHT-ANGED.

(36.)

$$\text{comp}_a b = \frac{a \cdot b}{|a|} = \frac{\langle 1,2 \rangle \cdot \langle -4,1 \rangle}{|\langle 1,2 \rangle|} = \frac{-2}{\sqrt{5}}$$

$$\text{proj}_a b = \frac{a \cdot b}{|a|^2} a = \frac{-2}{5} \langle 1,2 \rangle = \langle -\frac{2}{5}, -\frac{4}{5} \rangle$$

(50.)

$$(r-a) \cdot (r-b) = 0$$

$$\langle x-a_1, y-a_2, z-a_3 \rangle \cdot \langle x-b_1, y-b_2, z-b_3 \rangle = 0$$

$$(x-a_1)(x-b_1) + (y-a_2)(y-b_2) + (z-a_3)(z-b_3) = 0$$

$$x^2 - (a_1+b_1)x + a_1b_1 + y^2 - (a_2+b_2)y + a_2b_2 + z^2 - (a_3+b_3)z + a$$

$$\left(x-\frac{a_1+b_1}{2}\right)^2+\left(y-\frac{a_2+b_2}{2}\right)^2+\left(z-\frac{a_3+b_3}{2}\right)^2=$$

$$=\left(\frac{a_1+b_1}{2}\right)^2+\left(\frac{a_2+b_2}{2}\right)^2+\left(\frac{a_3+b_3}{2}\right)^2-a_1b_1-a_2b_2-a_3b_3$$

$$\left(x-\frac{a_1+b_1}{2}\right)^2+\left(y-\frac{a_2+b_2}{2}\right)^2+\left(z-\frac{a_3+b_3}{2}\right)^2=\frac{1}{4}(a_1-b_1)^2+\frac{1}{4}(a_2-b_2)^2+\frac{1}{4}(a_3-b_3)^2$$

CENTER $C\left(\frac{a_1+b_1}{2},\frac{a_2+b_2}{2},\frac{a_3+b_3}{2}\right)$

RADIUS $r=\sqrt{\frac{1}{4}(a_1-b_1)^2+\frac{1}{4}(a_2-b_2)^2+\frac{1}{4}(a_3-b_3)^2}$

$\boxed{13.4}$

(1) $a\times b=\begin{vmatrix} i & j & k\\ 1 & 2 & 0\\ 0 & 3 & 1\end{vmatrix}=i\begin{vmatrix}2&0\\3&1\end{vmatrix}-j\begin{vmatrix}1&0\\0&1\end{vmatrix}+k\begin{vmatrix}1&2\\0&3\end{vmatrix}=$

$=2i-j+3k$

$a\cdot(a\times b)=\langle 1,2,0\rangle\cdot\langle 2,-1,3\rangle=0$

$b\cdot(a\times b)=\langle 0,3,1\rangle\cdot\langle 2,-1,3\rangle=0$

(4) $a\times b=\begin{vmatrix} i & j & k\\ 1 & -1 & 1\\ 1 & 1 & 1\end{vmatrix}=i\begin{vmatrix}-1&1\\1&1\end{vmatrix}-j\begin{vmatrix}1&1\\1&1\end{vmatrix}+k\begin{vmatrix}1&-1\\1&1\end{vmatrix}=$

$=-2i+2k$

$a\cdot(a\times b)=\langle 1,-1,1\rangle\cdot\langle -2,0,2\rangle=0$

$b\cdot(a\times b)=\langle 1,1,1\rangle\cdot\langle -2,0,2\rangle=0$

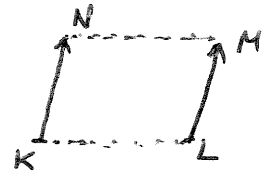
(16.) $(i+j+k)\times(2i+k)=\begin{vmatrix} i & j & k\\ 1 & 1 & 1\\ 2 & 0 & 1\end{vmatrix}=i\begin{vmatrix}1&1\\0&1\end{vmatrix}-j\begin{vmatrix}1&1\\2&1\end{vmatrix}+k\begin{vmatrix}1&1\\2&0\end{vmatrix}$

$=i+j-2k$

DESIRED VECTORS ARE $\pm\frac{i+j-2k}{|i+j-2k|}=\pm\frac{i+j-2k}{\sqrt{6}}$

(24)

NOTICE THAT



$$\overrightarrow{KN} = \langle 2,5,0 \rangle = \overrightarrow{LM}$$

$$\text{AREA} = |\overrightarrow{KN} \times \overrightarrow{KL}| = \left\| \begin{vmatrix} i & j & k \\ 2 & 5 & 0 \\ 0 & 1 & 3 \end{vmatrix} \right\| = |15i - 6j + 2k|$$

$$= \sqrt{15^2 + (-6)^2 + 2^2} = \sqrt{265}$$

(26.)

(a)

$$\overrightarrow{PQ} = \langle -3, 2, -1 \rangle$$

$$\overrightarrow{PR} = \langle 1, -1, 1 \rangle$$

$$\text{normal vector} = \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} i & j & k \\ -3 & 2 & -1 \\ 1 & -1 & 1 \end{vmatrix} = i \begin{vmatrix} 2 & -1 \\ -1 & 1 \end{vmatrix} - j \begin{vmatrix} -3 & -1 \\ 1 & 1 \end{vmatrix} + k \begin{vmatrix} -3 & 2 \\ 1 & -1 \end{vmatrix}$$

$$= i + 2j + k$$

(b)

$$\text{AREA} = \frac{1}{2} |\overrightarrow{PQ} \times \overrightarrow{PR}| = \frac{1}{2} |i + 2j + k| = \frac{1}{2}\sqrt{6}$$

(32.)

$$\overrightarrow{PQ} = \langle 2, 3, 3 \rangle$$

$$\overrightarrow{PR} = \langle -1, -1, -1 \rangle$$

$$\overrightarrow{PS} = \langle 6, -2, 2 \rangle$$

$$\text{VOLUME} = |\overrightarrow{PQ} \cdot (\overrightarrow{PR} \times \overrightarrow{PS})| =$$

$$= \left\| \begin{vmatrix} 2 & 3 & 3 \\ -1 & -1 & -1 \\ 6 & -2 & 2 \end{vmatrix} \right\| =$$

$$= \left| 2 \begin{vmatrix} -1 & -1 \\ -2 & 2 \end{vmatrix} - 3 \begin{vmatrix} -1 & -1 \\ 6 & 2 \end{vmatrix} + 3 \begin{vmatrix} -1 & -1 \\ 6 & -2 \end{vmatrix} \right|$$

$$= |2(-4) - 3 \cdot 4 + 3 \cdot 8| = 4$$

35.

$$|\tau| = |r \times F| = |r||F| \sin 80° =$$
$$= 0.18 \cdot 60 \cdot \sin 80° = 10.8 \sin 80°$$
$$\approx 10.6\,J$$