Sample problems for final exam

Solutions will not be provided.

(1) Sample problems for the midterm.
(2) State Cauchy's theorem and Cauchy's integral formula.
(3) State Liouville's theorem, the Casorati-Weierstrass theorem, and "Riemann's theorem on removable singularities".
(4) Why does a holomorphic function $f(z)$ on $\{1 < |z| < 2\}$ admit a Laurent series centered at $z = 0$?
(5) Define the following:
  (a) Uniform convergence.
  (b) Isolated singularity, pole, essential singularity.
  (c) Residue $\mathrm{Res}(f(z), z_0)$ of an isolated singularity z_0.
(6) Evaluate the integrals $\displaystyle\int_{|z|=1} \frac{dz}{z^2(z^2-4)e^z}$ and $\displaystyle\int_{|z|=2} \frac{z^2}{z-1} dz$:
  (a) using the Cauchy integral formula;
  (b) using residues.
(7) Problem IV.5: 2
(8) Show that the series $\sum_{n=0}^{\infty} a_n z^n$, the differentiated series $\sum n a_n z^{n-1}$, and the integrated series $\sum_n \frac{a_n}{n+1} z^{n+1}$ all have the same radius of convergence.
(9) Problems V.3: 2(a)(d), 3
(10) What is the radius of convergence of the power series for the following functions about the points indicated:
  (a) $\frac{\sin z}{\cos^2 z}$ about 0? about $5i$?
  (b) $\frac{z}{\sin^3 z}$ about πi?
  (c) $\frac{z-1}{z^4-1}$ about $z = 5 + 2i$?
  (d) $\mathrm{Log}(z)$ about $z = 1 + 5i$?
(11) For each of $f(z) = \frac{e^{1/z}}{\sin z}$ and $f(z) = \frac{z^2+1}{(z^2-1)^m}$, m positive integer,
  (a) Find all the singularities of f.
  (b) Which singularities are isolated?
  (c) Classify all singularities as removable, essential, or pole, and find the order of each pole.
(12) Problem VI.4: 2(c)
(13) Problems VII.1: 3(d)(e)(f)
(14) Problems VII.2: 2,8
(15) Problems VII.3: 4,5