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2012–2013 WINTER MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

San Diego Convention Center
San Diego Marriott Marquis and Marina
San Diego, CA

January 11–12, 2013

The 2012–2013 Winter Meeting of the Association for Symbolic Logic was held on January 11–12, 2013 at the San Diego Convention Center in conjunction with the Joint Mathematics Meetings. The members of the Program Committee were Matthias Aschenbrenner, Kenny Easwaran, and Martin Zeman (Chair). The program consisted of seven invited fifty-minute lectures and seventeen contributed talks. The ASL hosted a welcoming reception on January 11th in the Leucadia Room on the first floor of the San Diego Marriott Marquis and Marina.

An AMS/ASL co-sponsored Special Session, *Effective Algebra and Model Theory*, organized by Sam Buss, Mia Minnes and Jeff Remmel, was held on Wednesday and Thursday, January 9 and 10, in the San Diego Convention Center.

The fifty-minute invited addresses at the ASL meeting were as follows:

Bradd Hart (McMaster University), *John von Neumann, model theorist*.

Philipp Hieronymi (University of Illinois at Urbana-Champaign), *Interpreting the projective hierarchy in expansions of the real line*.

Peter Koellner (Harvard University), *Reason and evidence in mathematics*.

Colin McLarty (Case Western Reserve University), *Grothendieck's cohomology founded on finite order arithmetic*.

Justin Moore (Cornell University), *Nonassociative Ramsey Theory and the amenability of Thompson's group*.

Itay Neeman (University of California Los Angeles), *Three-type side conditions and forcing axioms*.

Christian Rosendal (University of Illinois at Chicago), *Global and local boundedness properties in Polish groups*.

Abstracts of the invited talks and of contributed talks given (in person or by title) by members of the Association for Symbolic Logic follow.

For the Program Committee
Martin Zeman

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Abstracts of invited talks

► BRADD HART, *John von Neumann, model theorist*.
Mathematics and Statistics, McMaster University, 1280 Main St., Hamilton ON L8S 4K1, Canada.
E-mail: hartb@mcmaster.ca.

I will survey some of the recent work on the model theory of operator algebras but will pay particular attention to some of the early work of von Neumann that turns out to be very relevant.

► PHILIPP HIERONYMI, *Interpreting the projective hierarchy in expansions of the real line*.
Department of Mathematics, University of Illinois at Urbana-Champaign, 1409 W. Green Street, Urbana, IL 61801, USA.
E-mail: P@hieronymi.de.
URL Address: http://www.math.uiuc.edu/~phierony/.

In [1] it was shown that every expansion of the real *field* that defines a closed and discrete set $D \subseteq \mathbb{R}$ and a function $f \colon D^n \to \mathbb{R}$ such that $f(D)$ is somewhere dense, also defines $\mathbb{Z}$ (and hence the whole projective hierarchy). In this talk, I will present a generalization of this statement to expansions of the real line without the field structure present. In particular, we give a criterion when an expansion of the ordered set of real numbers defines the image of $(\mathbb{R}, +, \cdot, \mathbb{N})$ under a semialgebraic injection. This allows us to answer several questions raised by Chris Miller about expansions of the additive group of real numbers.

THEOREM. [2, Theorem B] Let $\alpha \in \mathbb{R}$ be a non-quadratic irrational number and let $f \colon \mathbb{R} \to \mathbb{R}$ be the function that maps x to αx. Then $(\mathbb{R}, <, +, \mathbb{N}, f)$ defines multiplication on $\mathbb{R}$.

THEOREM. [2, Theorem D] $(\mathbb{R}, <, +, \sin, \mathbb{N})$ defines multiplication on $\mathbb{R}$.

This is joint work with Michael Tychonievich.

[1] P. HIERONYMI, *Defining the set of integers in expansions of the real field by a closed discrete set*, ***Proceedings of the American Mathematical Society***, vol. 138 (2010), pp. 2163–2168.

[2] P. HIERONYMI AND M. TYCHONIEVICH, *Interpreting the projective hierarchy in expansions of the real line*, ***Proceedings of the American Mathematical Society***, to appear.

► PETER KOELLNER, *Reason and evidence in mathematics*.
Department of Philosophy, Harvard University, 320 Emerson Hall, Cambridge, MA, 02138, USA.
E-mail: koellner@fas.harvard.edu.

In this talk I will address some of the general, structural features that arise in the case for new axioms. According to a traditional view axioms are "self-evident". One difficulty with this view is that in foundational discussions there is often disagreement as to which statements are "self-evident". Nevertheless, there is generally large agreement as to whether a given statement is "more evident" than another. For this reason, the relative notion is more suitable for articulating foundational disputes in mathematics. In the first part of the talk I will elucidate this relative notion by distinguishing it from kindred notions and by providing several examples. This will lead to what I will call the *evidentiary hierarchy*. In the second part of the talk I will use this framework to make a number of points concerning the case for new axioms. First, I will present what appears to be a strong argument that the case for new axioms is ultimately doomed to fail since we can expect that as the statements get stronger the best candidates for new axioms will become less and less evident. Second, I will argue that (at least in certain known cases) this problem can be overcome by appealing to reasons that involve deep mathematics. Finally, I will use this framework to place Harvey Friedman's program in philosophical perspective.

► COLIN MCLARTY, *Grothendieck's cohomology founded on finite order arithmetic*.
Philosophy and Mathematics, Case Western Reserve University, Cleveland, OH 44106, USA.
E-mail: colin.mclarty@case.edu.

Despite rumors to the contrary, Grothendieck and others published proofs using Grothendieck universes, which are sets so large that ZF does not prove they exist. Number theorists to this day cite those published proofs. More conservative references avoid universes but still use far stronger set theory than the number theory requires, such as large amounts of replacement. We describe the issues and show how to formalize the entire Grothendieck apparatus at the strength of finite order arithmetic (simple type theory with infinity) and describes progress towards yet weaker foundations.

► JUSTIN TATCH MOORE, *Nonassociative Ramsey Theory and the amenability of Thompson's group*.
Department of Mathematics, Cornell University, Ithaca, NY 14853-4201, USA.
E-mail: justin@math.cornell.edu.

In 1973, Richard Thompson considered the question of whether his newly defined group F was amenable. The motivation for this problem came from his observation—later rediscovered by Brin and Squire—that F did not contain a free group on two generators, thus making it a candidate for a counterexample to the von Neumann–Day problem. While the von Neumann–Day problem was solved by Olshanskii in the class of finitely generated groups and Olshanskii and Sapir in the class of finitely presented groups, the question of F's amenability was sufficiently basic so as to become of interest in its own right.

In this talk, I will analyze this problem from a Ramsey-theoretic perspective. In particular, the problem is related to generalizations of Ellis's Lemma and Hindman's Theorem to the setting of nonassociative binary systems. The amenability of F is itself equivalent to the existence of certain finite Ramsey numbers. I will also discuss the growth rate of the Følner function for F (if it exists).

► ITAY NEEMAN, *Three-type side conditions and forcing axioms*.
Department of Mathematics, University of California Los Angeles, Los Angeles, CA 90095-1555, USA.
E-mail: ineeman@math.ucla.edu.

The *proper forcing axiom* (PFA) has for several decades served as an axiomatic center point for consistency proofs. The associated class of *proper* posets is among the most studied and most useful in the theory of forcing. The posets preserve the first uncountable cardinal $\aleph_1$, and the axiom states that for every proper poset, and every collection of $\aleph_1$ maximal antichains in the poset, there is a filter meeting all antichains in the collection.

In this talk we discuss higher analogues of the axiom, that allow meeting collections of $\aleph_2$ maximal antichains, in specific classes of posets that preserve both $\aleph_1$ and $\aleph_2$.

These analogues rely on particular posets that use finite sequences of models of three types to ensure preservation of cardinals. Posets of this kind but with models of only one type (namely countable) have a long history of use in ensuring properness, see for example Todorčević [4], and more recently Friedman [1] and Mitchell [2]. Generalizations to incorporate models of more types led to a finite support proof of the consistency of PFA in Neeman [3], and to the current work on higher analogues.

[1] Sy-David Friedman, *Forcing with finite conditions*, ***Set theory***, Trends in Mathematics, Birkhäuser, Basel, 2006, pp. 285–295.

[2] William J. Mitchell, *Adding closed unbounded subsets of ω_2 with finite forcing*, ***Notre Dame Journal of Formal Logic***, vol. 46 (2005), no. 3, pp. 357–371.

[3] Itay Neeman, *Forcing with sequences of models of two types*, ***Notre Dame Journal of Formal Logic***, to appear.

[4] Stevo Todorčević, ***Partition problems in topology***, Contemporary Mathematics, volume 84, American Mathematical Society, Providence, RI, 1989.

► CHRISTIAN ROSENDAL, *Global and local boundedness properties in Polish groups*.
Mathematics, Statistics and Computer Science (M/C 249), University of Illinois at Chicago, 851 S. Morgan St., Chicago, IL 60607-7045, USA.
E-mail: `rosendal.math@gmail.com`.

We present a comprehensive theory of boundedness properties for Polish groups developed with a main focus on Roelcke precompactness (precompactness of the lower uniformity) and Property (OB) (boundedness of all isometric actions on separable metric spaces). In particular, these properties are characterised by the orbit structure of isometric or continuous affine representations on separable Banach spaces. We further study local versions of boundedness properties and the microscopic structure of Polish groups and show how the latter relates to the local dynamics of isometric and affine actions.

Abstracts of contributed talks

► PAUL BAGINSKI, *Stable, $\aleph_0$-categorical nonassociative rings*.
Department of Mathematics and Statistics, Smith College, 44 College Lane, Northampton, MA 01063, USA.
E-mail: `pbaginsk@smith.edu`.

The model-theoretic properties of stability and $\aleph_0$-categoricity have been studied extensively in the context of groups and rings. Individually, each property induces a rich framework upon these algebraic objects; the joint presence of these properties severely restricts the algebraic structures one may encounter. Baur, Cherlin, and Macintyre [2] and, independently, Felgner [3] showed in the 1970s that a stable, $\aleph_0$-categorical group is nilpotent by finite. Baur, Cherlin, and Macintyre further conjectured that a stable, $\aleph_0$-categorical group is abelian by finite, a conjecture which remains open.

Contemporaneously, Baldwin and Rose [1] considered stable, $\aleph_0$-categorical associative rings and showed that they are nilpotent by finite. They further conjectured that such rings are null by finite, i.e., up to a finite ring extension, multiplication is trivial. This conjecture is equivalent to the conjecture on groups.

Rose [4] generalized many of the results on stable rings and $\aleph_0$-categorical rings to the nonassociative context of alternative rings. However, he was unable to show that a stable, $\aleph_0$-categorical alternative ring is nilpotent by finite. We have succeeded in proving this conjecture and we will outline the proof. We will also discuss the obstacles in generalizing this result to other nonassociative rings.

[1] John T. Baldwin and Bruce Rose, *$\aleph_0$-categoricity and stability of rings*, ***Journal of Algebra***, vol. 45 (1977), no. 1, pp. 1–16.

[2] Walter Baur, Gregory Cherlin and Angus Macintyre, *Totally categorical groups and rings*, ***Journal of Algebra***, vol. 57 (1979), no. 2, pp. 407–440.

[3] Ulrich Felgner, *$\aleph_0$-categorical stable groups*, ***Mathematische Zeitschrift***, vol. 160 (1978), no. 1, pp. 27–49.

[4] Bruce I. Rose, *Model theory of alternative rings*, ***Notre Dame Journal of Formal Logic***, vol. 19 (1978), no. 2, pp. 215–243.

► WILLIAM C. CALHOUN, *Reducibilities associated with Kolmogorov complexity*.
Department of Mathematics, Computer Science and Statistics, Bloomsburg University, Bloomsburg, PA 17815, USA.
E-mail: `wcalhoun@bloomu.edu`.

Turing reducibility is a fundamental concept in computability theory. Other reducibilities have been introduced related to Kolmogorov complexity. For instance, K-reducibility compares the complexities of initial segments of two sets: $A \leq_K B$ if $K(A \upharpoonright n) \leq K(B \upharpoonright n) + O(1)$, where K denotes prefix-free Kolomogorov complexity. On the other

hand, LK-reducibility compares the powers of the sets as oracles, (in particular, the power to reduce the complexity of strings): $A \leq_{\mathrm{LK}} B$ if $\mathrm{K}^B(y) \leq \mathrm{K}^A(y) + O(1)$. Similar reducibilities can be defined by substituting other variants of Kolmogorov complexity, such as monotone complexity or a priory complexity, for K. We will discuss differences and similarities among these various reducibilities.

► SAMUEL COSKEY, *Constructing automorphisms of corona algebras*.
Mathematics Department, Boise State University, 1910 University Dr., Boise, ID 83725, USA.
E-mail: scoskey@nylogic.org.
URL Address: boolesrings.org/scoskey.

In 2007 Phillips and Weaver showed that assuming CH, there exists an outer automorphism of the Calkin algebra. Here, the *Calkin algebra* is the algebra of bounded operators on a separable Hilbert space modulo the compact operators; it can be thought of as the non-commutative analog of $\mathcal{P}(\mathbb{N})/\mathrm{Fin}$. The Calkin algebra is just a special case of a quotient construction called the corona construction: for $A \subset B(\mathcal{H})$ the *corona* of A is $M(A)/A$, where $M(A)$ denotes the multiplier algebra of A. The corona construction can be thought of as the non-commutative analog of the Čech–Stone remainder.

A series of recent results show that assuming CH, various classes of corona algebras have outer automorphisms. In all cases, the techniques have come from logic: either using a set-theoretic construction or a model-theoretic arguments. In this talk we will very briefly sketch the case when A is separable and either simple or stable. At the heart of the argument is the construction of a tree of partial automorphisms of height $\aleph_1$.

This is joint work with Ilijas Farah.

[1] Samuel Coskey and Ilijas Farah, *Automorphisms of corona algebras, and group cohomology*, submitted.

► SCOTT CRAMER, *Inverse limit reflection and the structure of $L(V_{\lambda+1})$*.
Department of Mathematics, University of California, Berkeley, 970 Evans Hall #3840, Berkeley, CA 94720-3840, USA.
E-mail: scramer@math.berkeley.edu.

The structure $L(V_{\lambda+1})$ was first studied by Woodin to prove the consistency of $AD^{L(R)}$ from large cardinals. He later showed that many of the same structural properties of $L(R)$ under determinacy hold for $L(V_{\lambda+1})$ under large cardinals. Laver first introduced the tool of inverse limits in this context to tackle the problem of reflecting large cardinals at this level. In this talk we will extend Laver's results on inverse limit reflection, and then we will use this technique to analyze further the structure of $L(V_{\lambda+1})$ and its relationship to models of determinacy.

► RACHEL EPSTEIN, *Truth-table degrees of Turing complete sets and random strings*.
Department of Mathematics, Harvard University, 1 Oxford St, Cambridge, MA 02138, USA.
E-mail: repstein@math.harvard.edu.
URL Address: http://math.harvard.edu/~repstein/.

A Turing functional Φ is called a *truth-table reduction* if for every set A, there is a set B such that $\Phi^A = B$. If $\Phi^A = B$ via a truth-table reduction, then we say B is *tt-reducible* to A, $B \leq_{\mathrm{tt}} A$. Two sets A_1 and A_2 form a *minimal pair* in the tt-degrees if the only sets tt-reducible to both A_1 and A_2 are the computable sets. We show that there is a pair of Turing complete c.e. sets that form a minimal pair in the tt-degrees.

This problem was inspired by the sets of random strings. Intuitively, a finite binary string σ is random if there is no description of σ shorter than the length of σ, $|\sigma|$. More formally, σ is *random* if its prefix-free Kolmogorov complexity $K(\sigma)$ is at least $|\sigma|$. However, the set of random strings depends on which universal prefix-free machine U we are using when we write $K(\sigma)$, which is shorthand for $K_U(\sigma)$. $K_U(\sigma)$ is the minimal length of all strings τ such that $U(\tau) = \sigma$. The set of all strings random with respect to the machine U is denoted R_{K_U}. For any universal prefix-free machine U, R_{K_U} is Turing complete, but not necessarily tt-complete. We show that no two R_{K_U} and R_{K_V} form a minimal pair in the tt-degrees.

This is joint work with Mingzhong Cai, Rod Downey, Steffen Lempp, and Joe Miller.

► MATTHEW JURA, OSCAR LEVIN, AND TYLER MARKKANEN, *Domatic partitions of computable graphs*.

Department of Mathematics, Manhattan College, 4513 Manhattan College Parkway, Riverdale, NY 10471, USA.
E-mail: Matthew.Jura@manhattan.edu.
E-mail: Tyler.Markkanen@manhattan.edu.
School of Mathematical Sciences, University of Northern Colorado, Campus Box 122, Greeley, CO 80639, USA.
E-mail: Oscar.Levin@unco.edu.

Given a graph G, we say that a subset D of the vertex set $V(G)$ is a *dominating set* if it is near all the vertices, in that every vertex outside of D is adjacent to a vertex in D. A *domatic k-partition* of G is a partition of $V(G)$ into k dominating sets. In this talk, we will consider issues of computability related to domatic partitions of computable graphs. Our investigation will center on answering two types of questions for the case when $k = 3$. First, if domatic 3-partitions exist in a computable graph, how complicated can they be? Second, a decision problem: given a graph, how difficult is it to decide whether it has a domatic 3-partition? We will completely classify this decision problem for highly computable graphs, finite-degree computable graphs, and computable graphs in general.

► WEI LI, *Friedberg numbering in fragments of Peano Arithmetic and α-recursion theory*.

Department of Mathematics, National University of Singapore, 10, Lower Kent Ridge Road, Singapore.
E-mail: wei.li@nus.edu.sg.

We discuss the existence of a Friedberg numbering in fragments of Peano Arithmetic and initial segments of Gödel's constructible hierarchy L_α, where α is Σ_1 admissible. We prove that

(1) Over $P^- + B\Sigma_2$, the existence of a Friedberg numbering is equivalent to $I\Sigma_2$, and

(2) For L_α, there is a Friedberg numbering if and only if the tame Σ_2 projectum of α equals the Σ_2 cofinality of α.

[1] C. T. Chong and K. J. Mourad, *The degree of a Σ_n cut*, ***Annals of Pure and Applied Logic***, vol. 48, no. 3, pp. 227–235.

[2] C. T. Chong and Yue Yang, *Recursion theory on weak fragments of Peano arithmetic: a study of definable cuts*, ***Proceedings of the sixth Asian Logic Conference***, (C. T. Chong, Q. Feng, D. Ding, Q. Huang and M. Yasugi, editors), World Scientific, 1998, pp. 47–65.

[3] R. M. Friedberg, *Three theorems on recursive enuemration I. Decomposition. II. Maximal set. III. Enumeration without duplication*, ***The Journal of Symbolic Logic***, vol. 23, no. 3, pp. 309–316.

[4] Martin Kummer, *An easy priority-free proof of a theorem of Friedberg*, ***Theoretical Computer Science***, vol. 74, no. 1, pp. 249–251.

[5] M. Mytilianios, *Finite injury and Σ_1-induction*, ***The Journal of Symbolic Logic***, vol. 54, no. 1, pp. 38–49.

[6] M. Mytilinaios and T. Slaman, *Σ_2-collection and the infinite injury priority method*, ***The Journal of Symbolic Logic***, vol. 53, no. 1, pp. 212–221.

[7] J. B. Paris and L. A. S. Kirby, *Σ_n-collection schemas in arithmetic*, ***Logic Colloquium '77***, (A. Macintyre, L. Pacholski and J. Paris, editors), vol. 96, North-Holland, 1978.

[8] Gerald E. Sacks, ***Higher Recursion Theory***, Perspectives in Mathematical Logic, vol. 2, Springer, 2010.

[9] Theodore A. Slaman and W. Hugh Woodin, *Σ_1 collection and the finite injury priority method*, ***Mathematical Logic and its Applications***, vol. 1388, pp. 178–188.

► RICHARD MCKINLEY, *Canonical proof objects for classical sequent proofs*.
Logic and Theory Group, Institute for Applied Mathematics and Computer Science, University of Bern, Neubrückstrasse 10, 3012 Bern, Switzerland.
E-mail: mckinley@iam.unibe.ch.

Proof nets provide abstract counterparts to sequent proofs modulo rule permutations; the idea being that if two proofs have the same underlying proof-net, they are in essence the same proof. Providing a convincing proof-net counterpart to proofs in the classical sequent calculus is thus an important step in understanding classical proofs. By convincing, we mean that (a) there should be a canonical function from sequent proofs to proof nets, (b) it should be possible to check the correctness of a net in polynomial time, (c) every correct net should be obtainable from a sequent calculus proof, and (d) there should be a cut-elimination procedure which preserves correctness. The major barrier to a canonical notion of proof net for classical logic has been the weakening rule; in this talk, we discuss two recent approaches to overcoming this barrier: one via Herbrand's theorem, and the other by considering a nonstandard weakening-free sequent calculus. We briefly discuss cut-elimination, the notion of a subproof in proof nets, and the common underlying higher-order structure of the two approaches.

[1] Richard McKinley, *Canonical proof nets for classical logic*, ***Archive for Pure and Applied Logic***, vol. 164 (2013), no. 6, pp. 702–732.

[2] Richard McKinley, *Proof nets for Herbrand's Theorem*, ***ACM Transactions on Computational Logic***, vol. 14 (2013), no. 1, art. 5, 31 pp.

► MAARTEN MCKUBRE-JORDENS AND DOUGLAS BRIDGES, *Solving the Dirichlet problem constructively*.
Department of Mathematics and Statistics, University of Canterbury, Christchurch, New Zealand.
E-mail: maarten.jordens@canterbury.ac.nz.

Potential theory has, thus far, been a missing chapter in the development of constructive mathematics. Here we take some steps to remedy that. First, we present a constructive proof of the existence of the weak solution to the Dirichlet problem when the domain is internally approximable by certain compact subsets. The extra hypotheses we pose are constructively strictly stronger than the usual classical requirement, yet are classically trivially satisfied. We have the following:

Theorem 1. *Under the hypotheses mentioned above, the Dirichlet problem*

$$\Delta u = 0 \text{ on } \Omega, \quad u(x) = f \text{ for all } x \in \partial\Omega, \tag{1}$$

where Ω is a domain in $\mathbb{R}^n$ and f a uniformly continuous function on $\partial\Omega$, has a constructive solution.

Second, we present Brouwerian examples showing that the existence of solutions in the general case of the Dirichlet problem is an essentially non-constructive (and hence non-algorithmic) proposition. Recall the *limited principle of omniscience*:

> Given any binary sequence, either all its terms are zero, or there exists a term equal to 1. (LPO)

It is known that LPO is not provable in Bishop-style constructive mathematics, since it is true classically but false intuitionistically and recursively, and is hence uncontroversially regarded as non-constructive. We prove the following:

Theorem 2. *The existence of solutions to the Dirichlet problem* (1) *for arbitrary domains is recursively equivalent to LPO.*

This immediately yields the following corollary:

Corollary 3. *There is no universal algorithmic procedure for computing solutions to the Dirichlet problem.*

Further, since the Dirichlet problem is a special case of the Navier-Stokes equations for fluid flow, we have:

COROLLARY 4. *There is no universal algorithmic procedure for computing solutions to the Navier-Stokes equations for fluid flow.*

Of these, Corollary 3 is perhaps the most surprising. Indeed, the domains we present as counterexamples satisfy almost all conditions that one might reasonably expect to establish the existence of solutions in potential theory.

► ŽIKICA PEROVIĆ AND BOBAN VELIČKOVIĆ, *Maharam algebras*.
MiraCosta College, 1 Barnard Drive, Oceanside, CA 92056, USA.
E-mail: zperovic@miracosta.edu.
Equipe de Logique, Université de Paris 7, 2 Place Jussieu, 75251 Paris, France.
E-mail: boban@logique.jussieu.fr.

Maharam algebras are complete Boolean algebras carrying a positive continuous submeasure. They were introduced and studied by Maharam, in relation to Von Neumann's problem on the characterization of measure algebras. The question whether every Maharam algebra is a measure algebra has been the main open problem in this area for around 60 years. It was finally resolved by Talagrand [2] who provided the first example of a Maharam algebra which is not a measure algebra. Using the original Talagrand's example and Schreier families, we provide a class of new examples of Maharam algebras whose exhaustivity ranks are arbitrary large countable ordinals, thus answering a question of Fremlin [1].

[1] D. H. Fremlin, ***Measure theory***, vol. 5, Torres–Fremlin, 2008.
[2] M. TALAGRAND, *Maharam's problem*, ***Annals of Mathematics***, vol. 168 (2008), no. 3, pp. 981–1009.

► JASON RUTE, *The computability of martingale convergence*.
Department of Mathematical Sciences, Carnegie Mellon University, 5000 Forbes Ave, Pittsburgh, PA 15213, USA.
E-mail: jrute@cmu.edu.
URL Address: www.math.cmu.edu/~jrute.

Martingales are an important tool in probability and analysis. This talk will be about the computability of martingale convergence.

A martingale, which can be thought of a fair betting strategy, is a sequence $(f_n)_{n\in\mathbb{N}}$ of integrable functions satisfying the property $\mathbb{E}[f_{n+1} \mid f_0, \dots, f_n] = f_n$. Doob showed that a martingale $(f_n)_{n\in\mathbb{N}}$ converges almost-surely if $\sup_n \|f_n\|_{L^1}$ is bounded.

Firstly, we may ask the usual questions from computable and constructive analysis. Given a martingale $(f_n)_{n\in\mathbb{N}}$: (1) Is the limit of $(f_n)_{n\in\mathbb{N}}$ computable? (2) If the limit is computable, is the rate of convergence computable? (3) If neither is computable, is there a different computable witness for convergence. I will address all of these questions, showing that the answers relate to the computability of various parameters.

Secondly, we may also use the field of algorithmic randomness to explore the almost-everywhere nature of martingale convergence. I will present a variety of theorems which characterize the exact points for which computable martingales converge under various assumptions. These classes of points are exactly the well-known notions of Schnorr randomness, computable randomness, and Martin-Löf randomness.

The connections between computable analysis and algorithmic randomness are very close. Further, these results have implications for differentiability, the law of large numbers, and de Finetti's theorem. They also parallel results about the ergodic theorem.

Studying topics such as this are important since they help us to understand how the field of probability theory can, on one hand, be incredibly applicable and computable, but on the other hand, be based heavily on non-constructive set-theoretic foundations.

► LYNN SCOW, *Tree indiscernibles and a Ramsey class of trees*.
Department of Mathematics, Statistics, and Computer Science, University of Illinois at Chicago, 851 S. Morgan St., Chicago IL 60607, USA.
E-mail: lynn@math.uic.edu.

In [1], it was shown that for a certain class of structures $\mathcal{I}$, $\mathcal{I}$-indexed indiscernibles have the modeling property just in case age($\mathcal{I}$) is a Ramsey class. We expand this known class from ordered structures $\mathcal{I}$ in a finite relational language to ordered locally finite structures $\mathcal{I}$ with quantifier-free types isolated by quantifier-free formulas. As a corollary we produce a new Ramsey class of trees from known modeling property results about tree-indexed indiscernibles.

[1] L. Scow, *Characterization of NIP theories by ordered graph-indiscernibles*, ***Annals of Pure and Applied Logic***, vol. 163 (2012), pp. 1624–1641.

► IOANNIS SOULDATOS, *On automorphisms groups of uncountable structures of countable cofinality*.
University of Detroit Mercy, 4001 W. McNichols Ave., Detroit, MI 48221, USA.
E-mail: souldaio@udmercy.edu.

In [1], Su Gao proves the following:

Theorem 1. *Let $\mathcal{M}$ be a countable structure. The following are equivalent*:

(I) *There is no uncountable model of the Scott sentence of $\mathcal{M}$.*
(II) *$Aut(\mathcal{M})$ admits a compatible left-invariant complete metric.*
(III) *There is no $\mathcal{L}_{\omega_1,\omega}$- elementary embedding from $\mathcal{M}$ to itself which is not onto.*

We extend this result to structures of size κ, where $\kappa > \aleph_0$ and $cf(\kappa) = \omega$. This is research in progress. Open problems will be discussed.

[1] Su Gao, *On automorphism groups of countable structures*, ***The Journal of Symbolic Logic***, vol. 63 (1998), no. 3, pp. 891–896.

► BRETT TOWNSEND, *Dimension theory in some dense regular groups*.
Department of Mathematics and Computer Science, Wesleyan University, 265 Church Street, Middletown, CT 06457, USA.
E-mail: btownsend@wesleyan.edu.

A dense regular ordered abelian group, all of whose prime invariants are equal to 1 is elementarily equivalent to $\mathbb{R}$, and hence, is o-minimal. This talk will address progress which has been made towards understanding the structure of such groups with arbitrary finite prime invariants. In particular, we will present a cell-decomposition theorem similar to the well known result for o-minimal structures [1], and from that derive a coherent dimension theory for such groups.

[1] L. van den Dries, ***Tame topology and O-minimal structures***, London Mathematical Society Lecture Note Series, vol. 248, Cambridge University Press, 1998.

► STEVEN VANDENDRIESSCHE, *Maxima for the $\leq_{tc}$ ordering with respect to $\sim^c_\alpha$*.
Department of Mathematics, University of Notre Dame, Notre Dame, IN 56556, USA.
E-mail: svandend@nd.edu.

The Turing computable embedding was introduced in [1] as an effective reduction of the isomorphism relation on a class of countable structures K to that on K'. When such an embedding exists, we write $K \leq_{tc} K'$, indicating that the isomorphism problem for K' is at least as difficult as that for K. The relation $\leq_{tc}$ is a preordering on classes of countable structures, and it is known that there is a maximum degree in this order. Motivated by the importance of computable infinitary sentences in this work, we define the equivalence relation $\sim^c_\alpha$ on structures, where $\mathcal{A} \sim^c_\alpha \mathcal{B}$ if and only if $\mathcal{A}$ and $\mathcal{B}$ satisfy the same computable infinitary Σ^0_α sentences. For any $\alpha < \omega_1^{ck}$, we give a subclass of the countable reduced abelian p-groups which is a maximum for the $\leq_{tc}$ ordering with respect to $\sim^c_\alpha$. Further, these embeddings are highly uniform, and we describe work toward constructing an operator witnessing that the abelian p-groups are a maximum for $\leq_{tc}$ with respect to $\sim^c_{\omega_1^{ck}}$. Finally, we show that if ZFC has an ω-model, then it is consistent that the abelian p-groups are a maximum with respect to $\sim^c_{\omega_1^{ck}}$.

[1] Julia F. Knight, Sara Miller, and M. Vanden Boom, *Turing computable embeddings*, ***The Journal of Symbolic Logic***, vol. 72 (2007), no. 3, pp. 901–918.

► DAN E. WILLARD, *On the significance of axiom systems that can recognize their own consistency*.
Computer Science & Mathematics Departments, University at Albany, Albany, NY 12222, USA.
E-mail: dew@cs.albany.edu.

We have published a series of articles about generalizations and boundary-case exceptions to the Second Incompleteness Theorem, including six papers in the ***JSL*** and ***APAL***. (Citations to these articles and a formalism that extends their techniques can be found in [4].) In these articles, an axiom system β is called **Self Justifying** iff (i) β can recognize its own consistency by containing such a statement as either a built-in axiom or as a derived theorem, and (ii) β is formally consistent.

It is well known that Gödel's Second Theorem assures that sufficiently strong axiom systems are unable to satisfy both these conditions. This talk will discuss what *partial* results about self-justifying systems can be obtained, despite these limitations.

We will explain that self-justifying axiom systems are a topic of interest, provided one partially scales down one's expectation about what this subject can accomplish. On the one hand, it has been known since the time of the Gödel and Hilbert-Bernays discoveries that self-justifying systems of full-scale strength cannot be constructed. On the other hand, it is difficult to explain how human beings can motivate themselves to cogitate, if they did not retain some type of ability to recognize their own consistency, in at least the context of some weak axiom system employing a weak deduction method. In this context, [1, 2, 3, 4] showed there are some limited extents to which axiom systems can prove analogs of all Peano Arithmetic Π_1 theorems and recognize the validity of weak definitions of their own consistency.

[1] D. Willard, *An exploration of the partial respects in which an axiom system recognizing solely addition as a total function can verify its own consistency*, ***The Journal of Symbolic Logic***, vol. 70 (2005), pp. 1171–1209.

[2] ———, *A generalization of the Second Incompleteness Theorem and some exceptions to it*, ***Annals of Pure and Applied Logic***, vol. 141 (2006), pp. 472–496.

[3] ———, *Some specially formulated axiomizations for $I\Sigma_0$ manage to evade the Herbrandized version of the Second Incompleteness Theorem*, ***Information and Computation***, vol. 207 (2009), pp. 1078–1093.

[4] ———, *A detailed examination of methods for unifying, simplifying and extending several results about self-justifying logics*, http://arxiv.org/abs/1108.6330.

► SEBASTIAN WYMAN AND DOUGLAS CENZER, *Symbolic dynamics in the arithmetic hierarchy*.
Department of Mathematics, University of Florida, 358 Little Hall, PO Box 118105, Gainesville, FL 32611-8105, USA.
E-mail: swyman@ufl.edu.

Recently, Cenzer, Dashti, and King showed that the subshifts arising from the symbolic dynamics of computable functions on the cantor space are exactly the decidable Π^0_1 subshifts and those arising by avoiding a c.e. set of words are exactly the Π^0_1 subshifts. We define conservatively approximable functions and use them to find functions whose symbolic dynamics give rise to exactly the Π^0_1 subshifts. We also give conditions on a computable set of avoidable words which give rise to exactly the decidable subshifts.

Abstracts of papers submitted by title

► ACHILLES A. BEROS, *Anomalous vacillatory learning*.
Department of Mathematics, University of Wisconsin–Madison, 480 Lincoln Dr., Madison, WI 53706-1388, USA.
E-mail: aberos@math.wisc.edu.

In 1986, Osherson, Stob and Weinstein asked whether two variants of anomalous vacillatory learning, TxtFex^*_* and $\mathrm{TxtFext}^*_*$, are distinct [2]. These two learning criteria place bounds neither on the number of hypotheses between which the learner is allowed to vacillate nor on the number of errors permitted, merely requiring that both are finite. The criteria differ in that the more restrictive one, $\mathrm{TxtFext}^*_*$-learning, requires that all hypotheses output infinitely often must describe the same finite variant of the correct set, while TxtFex^*_* permits the learner to vacillate between finitely many different finite variants of the correct set. We show that $\mathrm{TxtFex}^*_* \neq \mathrm{TxtFext}^*_*$, thereby answering the question posed by Osherson, *et al.* We prove this in a strong way by exhibiting a family in $\mathrm{TxtFex}^*_2 \setminus \mathrm{TxtFext}^*_*$.

[1] Mark Fulk, Sanjay Jain and Daniel Osherson, *Open problems in "systems that learn"*, ***Journal of Computer and System Sciences***, vol. 49 (1994), pp. 589–604.

[2] Daniel Osherson, Michael Stob and Scott Weinstein, ***Systems that learn: An introduction to learning theory for cognitive and computer scientists***, MIT Press, 1986.

► KONSTANTINOS A. BEROS, *Universal subgroups of Polish groups*.
Department of Mathematics, University of Wisconsin–Madison, 480 Lincoln Dr., Madison, WI, 53706, USA.
E-mail: kberos@math.wisc.edu.

Given a topological group Γ and a family $\mathcal{C}$ of subgroups of Γ, define a *universal* $\mathcal{C}$ *subgroup of* Γ to be a subgroup $K \in \mathcal{C}$ such that every subgroup $H \in \mathcal{C}$ is a continuous homomorphic pre-image of K.

My recent work has shown that, for any Polish group Γ, the countable power Γ^ω has a universal analytic subgroup. In addition, if Γ is locally compact, then Γ^ω also has universal compactly generated and K_σ subgroups. For structural reasons, the classes of compactly generated, K_σ and analytic subgroups are natural ones to work with in this context and share key properties.

The Baer–Specker group provides a practical setting in which to demonstrate the techniques used in the proofs of the results above. I will prove at least one of my results in this special case.

► JOHN CORCORAN, *"Deriving" Euclid's Interchange Rule from Leibniz's Law*.
Philosophy, University at Buffalo, Buffalo, NY 14260-4150, USA.
E-mail: corcoran@buffalo.edu.

Tarski relates Euclid's Interchange Rule to Leibniz's Law [3, p. 56].

Euclid's Interchange Rule EIR—often misleadingly called "substitution of equals for equals"—produces instances of the following [premise-conclusion] argument schema [1, pp. 233–238], where '=' means strict identity not mere equality.

$$\begin{array}{c} t = s \\ P \\ \hline Q \end{array}$$

P and Q are sentences, t and s constant terms, and Q results from "partly or completely interchanging" t and s in P. This means replacing any number of occurrences of t in P by s and/or conversely [3, p. 56].

Tarski's *Leibniz's Law* LL [3, p. 55] is the second-order sentence:

$$x = y \text{ iff } x \text{ has every property } y \text{ has and conversely.}$$

Tarski [3, p. 56] says Euclid's Interchange Rule is a "consequence of Leibniz's Law".

(1) Tarski isn't using *consequence* as in his 1936 consequence-definition paper [2, pp. 409–420]. *Rules* can't be consequences of *sentences*: only sentences are such consequences.

(2) Tarski isn't saying that EIR instances are LL consequences: arguments can't be such consequences.

(3) Tarski isn't saying that each EIR instance's corresponding conditional is an LL consequence: those conditionals are empty tautologies and thus consequences not only of LL but also of any arbitrary sentence. Such a remark would be misleadingly trivial.

We interpret Tarski as saying that LL can be used to derive conclusions of instances of EIR from their respective premises—although he gives no clues how this can be done or indeed how properties are related to interchanges. Using formalized second-order logic with property abstracts, we reconstruct hypothetical Tarskian derivations.

[1] John Corcoran, *Schemata*, this Bulletin, vol. 12 (2006), pp. 219–240.
[2] Alfred Tarski, ***Logic, semantics, metamathematics***, Hackett, Indianapolis, 1983.
[3] ———, ***Introduction to logic***, Dover, New York, 1995.

► JOHN CORCORAN AND WILLIAM DUNCAN, *Tarski's truth schemata*.
Philosophy, University at Buffalo, Buffalo, NY 14260-4150, USA.
E-mail: corcoran@buffalo.edu.

We discuss *truth schemata* in Tarski's four English-language truth-definition articles. Tarski used five different *templates* [1, p. 220]:

$T1$: (1935) x is a true sentence if and only if p
$T2$: (1935) $x \in Tr$ if and only if p
$T3$: (1936) the sentence x is true if and only if p
$T4$: (1944) X is true if, and only if, p
$T5$: (1969) "p" is true if and only if p

All except T4 start with a lowercase letter. None end with a period [1, p. 226]. None contain the operator expression 'it is true that'. None contain 'true' on the right.

Only T1 contains a common-noun phrase: 'true sentence'. Only T2 contains a proper name: '*Tr*'—a metalanguage name of the class of true object-language sentences. Only T3 contains 'the'. Only T4 contains commas. Only T5 has quotes. Only T5 has a single placeholder: '*p*'. The others have two placeholders. Only T1 and T3 contain 'sentence'. Only T3, T4, and T5 contain a predicate adjective: 'true'.

The *side conditions* [1, p. 223] vary. T1 requires '*p*' be replaced by an English sentence *any* name of which replaces '*x*'. T2 requires for '*x*' a structural-descriptive name of an object-language sentence and for '*p*' a translation into the metalanguage of the sentence named. T3's side condition is similar to T2's. T4's side condition is similar to T1's. In T5 both occurrences of '*p*' are replaced by the same English sentence. It is only T5 whose instances all contain on the left the double-quote name of the sentence on the right.

We revisit lectures abstracted in this Bulletin, vol. 11 (2005), p. 460 and pp. 554–555.

[1] John Corcoran, *Schemata*, this Bulletin, vol. 12 (2006), pp. 219–240.

► JOHN CORCORAN AND WILLIAM FRANK, *Tarski's quoted-letters*.
Philosophy, University at Buffalo, Buffalo, NY 14260-4150, USA.
E-mail: corcoran@buffalo.edu.

Quoted-letters are three-character strings: letters flanked by quotation-marks. Quoted-letters have multiple uses. We survey Tarski's quoted-letter uses.

Tarski insisted that no string, for example the letter p (sic), serves as an *autonym*—denoting itself ([2, p. 344]; [3, p. 104]). But quoted-letters can serve as *letter-names*. Tarski wrote [1, pp. 159–160]:

The name " 'p' " denotes one of the letters of the alphabet.

Tarski regarded letter-names *as unitaries*—"syntactically simple expressions" having no independently meaningful parts. Presumably, quoted-letters are arbitrarily assigned to denote their letters when they could have been assigned something else or used non-denotatively [1, pp. 159–160]; [3, p. 104].

Tarski also used quoted-letters as abbreviated sentence-names. He held that quoted-letters containing letters abbreviating sentences denote—not letters—but sentences ([2, p. 347]; [3, p. 108]).

's' is the sentence printed in this paper [. . .].

"s" is undoubtedly a sentence in English.

In the truth-definition paper [1, pp. 152–278], "functions" contain free variables. The *quotation-function* ' 'x' ' converts to expression-names: ' 'a' ', ' 'b' ', etc. Tarski noted that the quoted-letter ' 'x' ' is "ambiguous" (sic): sometimes it *is* a *function*—not a name; sometimes it is a name of the 24th letter—not a function [1, p. 162].

Another usage is in the truth schema [3, pp. 105, 114].

"p" is true if and only if p

Here the quoted-letter is neither a function nor a name but merely contains its letter as a place-holder (schematic letter) co-ordinate with a place-holder not part of a quoted name.

[1] ALFRED TARSKI, ***Logic, semantics, metamathematics***, Hackett, 1956/1983.

[2] ———, *Semantic conception of truth*, ***Philosophy and Phenomenological Research***, vol. 4 (1944), pp. 341–375.

[3] ———, *Truth and proof*, ***Scientific American***, June 1969, Hughes reprint.

▶ JOHN CORCORAN AND HASSAN MASOUD, *Verifying and falsifying*.
Philosophy, University at Buffalo, Buffalo, NY 14260-4150, USA.
E-mail: corcoran@buffalo.edu.
Philosophy, University of Alberta, Edmonton, AB T6G2E7, Canada.
E-mail: hassan.masoud@ualberta.ca.

We study the verbs *verify* and *falsify*—and cognates such as *verifiable* and *falsifiable*—in logic-related fields.

In some contexts, *verifying* a proposition is ascertaining its truth; *falsifying* a proposition is ascertaining its falsity [1, pp. 33f.].

People verify existential propositions by finding known *proexamples* [2]. One verifies "some integer is neither positive nor negative" using known facts about zero.

People falsify universal propositions by finding known *counterexamples* [2]. One falsifies "every integer is either positive or negative" using known facts about zero.

In such contexts, *verifiable* propositions are true but not necessarily known true; *falsifiable* propositions are false but not necessarily known false.

In a wider and potentially confusing sense, *verifiable* simply means "checkable" or "decidable"—subject to routine checking methods. Whether true or false, arithmetic equations of the form $x + y = z$—with x, y, and z numerals—are all verifiable.

Analogous wide usage of *falsifiable* would render it synonymous with *verifiable* and thus redundant, and perhaps more confusing. Accordingly, especially in Popper's [3] and elsewhere, *falsifiable* has a different—but equally vague—meaning: "checkable if false". Thus, there are many arithmetic propositions that—although *not* verifiable in the wide sense—*are* falsifiable in Popper's sense, e.g., Goldbach's Hypothesis.

We also treat expressions applying *verifying* and *falsifying* to documents, evidence, records, etc. There *verify* and *falsify* are not analogous to each other: verifying is *determining whether* a document is fake; falsifying is *making* a document fake or *making* a fake document.

[1] JOHN CORCORAN, *Argumentations and logic*, ***Argumentation***, vol. 3 (1989), pp. 17–43.
[2] ———, *Counterexamples and proexamples*, this BULLETIN, vol. 11 (2005), p. 460.
[3] KARL POPPER, ***Conjectures and refutations***, Routledge, 1963.

► JOHN CORCORAN AND SRIRAM NAMBIAR, *Tarski's expression 'sentential function'*.
Philosophy, University at Buffalo, Buffalo, NY 14260-4150, USA.
E-mail: corcoran@buffalo.edu.

Tarski's 'sentential function' ['SF'] derives from ***PM***'s 'propositional function' ['PF'] [3, pp. 14 ff.]. We relate 'SF'—and indirectly 'PF'—to 'function' in the modern "mapping" sense.

In Tarski's 1923 dissertation, "functions" are mathematical mappings: not linguistic expressions [1, pp. 5, 23]. In Tarski's 1935 truth-definition paper, "functions" are expressions—*some but not all* having free variables [1, pp. 152–278]. In §2 of his 1941 ***Introduction***, "functions" are expressions—*all* having free variables [2, pp. 5–7].

Below *mapping* and *function* have contrasting senses: *mapping* the mathematical sense, *function* the linguistic sense. Mappings never contain variables or other linguistic characters; functions contain linguistic characters always, sometimes free variables. Following [2, p. 3], variables have numbers as *values* [3, p. 4] and number-names—including Arabic–Hindi numerals, fraction-names, 'e', 'π', etc.—as *substituents*.

In the 1935 and 1941 works, SFs containing free variables are expressions lacking truth-values that transform into "sentences" having truth-values—by substituting suitable names (substituents) for free variables: '$x < 3$' goes to 'e < 3' (true) and '$\pi < 3$' (false) [2, p. 5].

Each such SF having one free variable determines three mappings: (1) one carrying names—viz. the variable's substituents—to sentences; (2) one carrying things—viz. the variable's values—to truth-values; (3) one carrying individual concepts—viz. the variable's substituents's senses—to propositions. Also each such SF determines three sets of objects: (1) those satisfying it, its *truth* set; (2) those satisfying its negation, its *falsity set*; (3) those satisfying it or its negation, its *range* of *significance* [3, p. 199].

[1] ALFRED TARSKI, ***Logic, semantics, metamathematics***, Hackett, 1983.
[2] ———, ***Introduction to logic***, Dover, 1995.
[3] ALFRED WHITEHEAD AND BERTRAND RUSSELL, ***Principia Mathematica***, Cambridge University Press, 1910.

► JOHN CORCORAN AND LEONARDO WEBER, *Tarski's Convention T: condition beta*.
Philosophy, University at Buffalo, Buffalo, NY 14260-4150, USA.
E-mail: corcoran@buffalo.edu.
Philosophy, Federal University of Paraíba, João Pessoa, Paraíba, 58051, Brazil.
E-mail: webercastor@yahoo.com.br.

Tarski's Convention T—which presents his notion of *adequate definition of truth* (sic)—contains two conditions, alpha and beta, that are separately necessary and jointly sufficient [1, pp. 187–188]. Alpha requires all instances of a "T Schema" be provable. Beta requires in effect the provability of 'every truth is a sentence'. Beta recognizes a fact repeatedly emphasized by Tarski: that sentences—sentential functions devoid of free occurrences of variables—exhaust the range of significance of the predicate *is true*. In Tarski's sense of *true*, attribution of *true* to a given thing presupposes the thing be a sentence ([1, pp. 186–188, 195–197]; [2, pp. 4, 11]). Beta's necessity is highlighted by the fact that alpha is "satisfied" by the recursively definable concept *being satisfied by every sequence*, which Tarski rejects as inadequate [1, p. 189] and which he supplements with *being a sentence* in the famous truth-definition [1, p. 195].

Surprisingly, in the English translation, the sentence just before the preamble to Convention T implies that alpha might be sufficient. Even more surprising is the sentence just after Convention T: it says beta "is not essential".

We look at the broader context, the Polish original, the German translation from which the English was derived, and other sources to determine what Tarski intended by the two troubling sentences which, taken literally, are contrary to the spirit, if not the letter, of several other passages in [1], [2], and elsewhere.

[1] ALFRED TARSKI, *The concept of truth in formalized languages*, ***Logic, semantics, metamathematics, papers from 1923 to 1938***, Hackett, 1983, pp. 152–278.

[2] ———, ***Introduction to logic***, Dover, New York, 1995.

► JOHN CORCORAN AND SUSAN WOOD, *Boolean operation-class calculus*.
Philosophy, University at Buffalo, Buffalo, NY 14260-4150, USA.
E-mail: corcoran@buffalo.edu.

In 1847 Boole described "laws of the mental processes" that "render logic possible" [1, pp. 4–6]. He considered "mental acts"—operations applying to classes and yielding respective subclasses:

> Now the several mental operations [. . .] are subject to peculiar laws. [Some concern] *repetition* of a given operation or the *succession* of different ones. [. . .] These will [. . .] be ranked as necessary truths [. . .] *probably they are noticed for the first time in this Essay*.

Our *Boolean operation-class calculus* [BOC] represents—probably for the first time in this lecture—Boole's 1847 laws, but in modernized form. BOC has two sorts of variables: f, g . . . ranging over *operations* and x, y . . . ranging over *classes*—subclasses of the Universe 1. Besides "Operation" O and "Class" C, BOC takes the usual Boolean primitives:

$$0, 1, -, \times, +$$

BOC is to Boole's 1847 operation-class calculus as modernized Boolean algebra [3, pp. 320–341] is to Boole's 1847 algebra [2, pp. 616–618].

To introduce BOC, we present some of its basic principles. Read $\forall \mathrm{C}x$ "for every Class x"; $\forall \mathrm{O}f$ "for every Operation f".

Class-selection: $\forall \mathrm{O}f\ \forall \mathrm{C}x\ \{f(1) = (x \times 1) \rightarrow \forall \mathrm{C}y\ [f(y) = (x \times y)]\}$,
Class-operation: $\forall \mathrm{C}x\ \exists \mathrm{O}f\ [f(1) = (x \times 1)]$,
Unity: $\forall \mathrm{C}x\ [(x \times 1) = x]$,
Operation-distributivity: $\forall \mathrm{O}f\ \forall \mathrm{C}x\ \forall \mathrm{C}y\ \{f([x + y]) = [f(x) + f(y)]\}$,
Repetition-idempotence: $\forall \mathrm{O}f\ \forall \mathrm{C}x\ \{f(f(x)) = f(x)\}$,
Succession-commutativity: $\forall \mathrm{C}x\ \forall \mathrm{O}f\ \forall \mathrm{O}g\ [g(f(x)) = f(g(x))]$,
Complementarity: $\forall \mathrm{C}x\ \{[x + -x] = 1\ \ \&\ \ [x \times -x] = 0\}$.

[1] GEORGE BOOLE, ***Mathematical analysis of logic***, Macmillan, 1847.

[2] JOHN CORCORAN AND SUSAN WOOD, *Boole's criteria of validity and invalidity*, ***Notre Dame Journal of Formal Logic***, vol. 21 (1980), pp. 609–639.

[3] ALFRED TARSKI, ***Logic, semantics, metamathematics***, Hackett, 1983.