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2011–2012 WINTER MEETING
OF THE ASSOCIATION FOR SYMBOLIC LOGIC

John B. Hynes Veterans Memorial Convention Center
Boston Marriott Hotel, and Boston Sheraton Hotel
Boston, MA

January 6–7, 2012

The 2011–2012 Winter Meeting of the Association for Symbolic Logic was held on January 6–7, 2012 at the Hynes Veterans Memorial Convention Center in conjunction with the Joint Mathematics Meetings. The members of the Program Committee were Natasha Dobrinen, Peter Fejer, and Philip Scowcroft (Chair). The program consisted of seven invited fifty-minute lectures and twelve contributed talks. The ASL hosted a welcoming reception on January 6th at the Boston Sheraton Hotel.

On January 4–5 there was a joint AMS-ASL Special Session, **The Life and Legacy of Alan Turing**, organized by Damir Dzhafarov, Jeff Hirst, and Carl Mummert.

The fifty-minute invited addresses at the ASL meeting were as follows:

John Baldwin (University of Illinois at Chicago), *Set theory and infinitary model theory*.

Johanna N. Y. Franklin (University of Connecticut), *Randomness and ergodic theory*.

C. Ward Henson (University of Illinois at Urbana-Champaign), *Continuous first-order logic and Gurarii's universal homogeneous separable Banach space*.

Julia F. Knight (University of Notre Dame), *Structures associated with real closed fields and real closed exponential fields*.

Roman Kossak (CUNY Graduate Center), *Twenty questions*.

Dima Sinapova (University of California at Irvine), *Prikry type forcings and singular combinatorics*.

W. Hugh Woodin (University of California at Berkeley), *The end of the inner model program: ultimate L or not ultimate L*.

Abstracts of the invited talks and of contributed talks given (in person or by title) by members of the Association for Symbolic Logic follow.

For the Program Committee
PHILIP SCOWCROFT

Abstracts of invited talks

► JOHN BALDWIN, *Set theory and infinitary model theory*.
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The fundamental notions of first order stability theory are absolute. We explore the role of this fact in the development of model theory as an independent subject since the 1970's. We then discuss questions and results about the absoluteness of fundamental notions of infinitary model theory. As in the first order case, amalgamation and ω-stability are absolute notions, although of higher complexity. But while $\aleph_1$-categoricity is absolute for first order model theory, for $L_{\omega_1,\omega}$, absoluteness is an open question.

► JOHANNA N. Y. FRANKLIN, *Randomness and ergodic theory*.
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There has been a great deal of interest recently in the connection between algorithmic randomness and ergodic theory. Random points are regular with respect to measure, and many theorems in ergodic theory state that, given an ergodic transformation T on a probability space, the orbits of almost all points in the space under T will have a certain regularity property. When we put these theorems into the framework of recursion theory, we find that we can relate them to randomness: a point in a recursive probability space is random if and only if it satisfies an ergodic theorem with certain recursion-theoretic restrictions. I will survey the current state of the field, including a result of mine that is joint with Noam Greenberg, Joseph S. Miller, and Keng Meng Ng.

► C. WARD HENSON, *Continuous first order logic and Gurarij's universal homogeneous separable Banach space*.
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A continuous, [0,1]-valued generalization of first order logic has been developed during the last decade; see [2] and [1] for basic details. The structures associated to this logic are based on bounded metric spaces and equipped with uniformly continuous operations and predicates. All formulas take values in [0,1]. Model theory based on this logic has been effectively applied to a wide variety of structures from analysis, probability, metric geometry, etc. This talk will provide a brief introduction to this logic as well as pointers to some of these applications, and then it will concentrate on the model theory of Gurarij's Banach space as a new example. This space was constructed in the 1960s using a metric version of a Fraïssé construction; it is isometrically universal (for separable Banach spaces) and homogeneous in an almost-isometric sense relative to its finite dimensional subspaces. It is the analogue (for Banach spaces) of such structures as the random graph, and Urysohn's metric space. General results in Banach space theory from the 1960s show that its dual space is of the form $L^1(\mu)$ for some measure μ, so it falls into the important class of "classical Banach spaces," a fact that is far from obvious based on the original construction. Wolfgang Lusky showed in the 1970s that the Gurarij space is isometrically unique, a surprising result. He also showed that the set of smooth points of norm 1 is an orbit of its automorphism group, which implies that this action on the unit sphere has at least two orbits, and that all orbits are dense. In this talk it will be shown how these results can be seen from the point of view of continuous model theory. In particular, the class of separable Gurarij spaces can be realized as the class of separable models of a certain continuous theory T (of unit balls of Banach spaces). Analysis of the type spaces of T over finite sets of parameters can be successfully carried out, and used to prove the separable categoricity of T (= Lusky's uniqueness result) as well as to prove new results about the action of the automorphism group of Gurarij's space.

Part of this work is joint with Itaï Ben Yaacov.

[1] ITAÏ BEN YAACOV, ALEXANDER BERENSTEIN, C. WARD HENSON, AND ALEXANDER USVYATSOV, *Model theory for metric structures*, ***Model theory with applications to algebra and analysis***, vol. II, (Z. Chatzidakis, D. Macpherson, A. Pillay, and A. Wilkie, editors), Lecture Notes Series of the London Mathematical Society, no. 350, Cambridge University Press, 2008, pp. 315–427.

[2] ITAÏ BEN YAACOV AND ALEXANDER USVYATSOV, *Continuous first order logic and local stability*, ***Transactions of the American Mathematical Society***, vol. 362 (2010), pp. 5213–5259.

► JULIA F. KNIGHT, *Structures associated with real closed fields and real closed exponential fields*.
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Real closed fields and real closed exponential fields have been studied extensively by model theorists. These structures are also interesting from the point of view of computability. Tarski [8] gave an elimination of quantifiers for the theory of the ordered field of reals. His goal was to show that the theory is decidable. The elimination of quantifiers also shows that the theory is o-minimal. It is still unknown whether the theory of the reals with exponentiation is decidable. Macintyre and Wilkie [5] have shown that the theory of the reals with exponentiation is decidable provided that the real version of Schanuel's Conjecture holds.

There are further interesting structures associated with real closed fields, with and without exponentiation: "value group sections," "residue field sections," and "integer parts." I will describe some results of Mourgues and Ressayre [6], [7], and further work with Paola D'Aquino, Karen Lange, Sergei Starchenko, and Salma Kuhlmann [3], [2], [1], [4].

[1] P. D'AQUINO, J. F. KNIGHT, S. KUHLMANN, AND K. LANGE, *Real closed exponential fields*, in preparation.

[2] P. D'AQUINO, J. F. KNIGHT, AND K. LANGE, *Limit computable integer parts*, ***Archive for Mathematical Logic***, to appear.

[3] P. D'AQUINO, J. F. KNIGHT, AND S. STARCHENKO, *Real closed fields and models of Peano Arithmetic*, ***The Journal of Symbolic Logic***, vol. 75 (2010), pp. 1–11.

[4] J. F. KNIGHT AND K. LANGE, *Countable real closed fields and developments of bounded length*, preprint, being revised.

[5] A. J. MACINTYRE AND A. J. WILKIE, *On the decidability of the real exponential field*, ***Kreiseliana***, A. K. Peters, 2005, pp. 441–467.

[6] M. H. MOURGUES AND J.-P. RESSAYRE, *Every real closed field has an integer part*, ***The Journal of Symbolic Logic***, vol. 58 (1993), pp. 641–647.

[7] J.-P. RESSAYRE, *Integer parts of real closed exponential fields*, ***Arithmetic, proof theory, and computational complexity***, (P. Clote and J. Krajicek, editors), 1993, pp. 278–288.

[8] A. TARSKI, ***A decision method for elementary algebra and geometry***, Rand Corporation Publication, 1948.

► ROMAN KOSSAK, *Twenty questions*.
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In the last chapter of [4], Jim Schmerl and I put together twenty open problems concerning models of PA. Since the book was published, two problems were solved by Ali Enayat [1], [2], and partial solutions to three other problems were given by Victoria Gitman [3], and Saharon Shelah [5]. The problems can be roughly divided into three groups: general model theoretic/set theoretic questions about existence of models with special second order properties; permutation group theory questions concerning automorphisms and automorphism groups of recursively saturated models; questions concerning complete types and lattices of elementary substructures specific to models of PA. There is also an important question that stands alone: the Scott set problem. I will talk about selected problems from each group, and I will comment on recent progress.

[1] Ali Enayat, *Automorphisms of models of arithmetic: A unified view*, ***Annals of Pure and Applied Logic***, vol. 14 (2007), no. 1, pp. 16–36.

[2] ———, *A standard model of Peano Arithmetic with no conservative elementary extension*, ***Annals of Pure and Applied Logic***, vol. 156, (2008), no. 2–3, pp. 308–318.

[3] Victoria Gitman, *Scott's problem for proper Scott sets*, ***The Journal of Symbolic Logic***, vol. 73 (2008), no. 3, pp. 845–860.

[4] Roman Kossak and James H. Schmerl, ***The structure of models of Peano Arithmetic***, Oxford Logic Guides 50, Clarendon Press, Oxford, 2006.

[5] Saharon Shelah, *Can the order determine the addition for models of PA?*, to appear.

► DIMA SINAPOVA, *Prikry type forcings and singular combinatorics*.
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Cardinal arithmetic is of primary importance in set theory. We will discuss relative consistency results about singular cardinal arithmetic in the context of forcing and large cardinals. We will focus on the relationship between the Singular Cardinal Hypothesis, Jensen's square principle, scales, and the tree property. In particular, we will explore constructions involving Prikry type forcings to obtain consistency results.

► W. HUGH WOODIN, *The end of the Inner Model Program: ultimate L or not ultimate L*.
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The Inner Model Program is the search for generalizations of Gödel's L which are compatible with the existence of large cardinals. There have been a series of notable successes in this program and arguably these successes provide some of our deepest insights into the nature of strong axioms of infinity.

Supercompact cardinals quickly emerged 40 years ago as the principal target for the Inner Model Program and are just beyond the level of large cardinals that the current theory can handle (assuming iteration hypotheses). In an unexpected turn of events it is now known that (subject to very general constraints) the solution to the inner model problem for exactly one supercompact cardinal must yield the "final" inner model, ultimate L.

Thus the inner model program will either end in ultimate triumph or it will end in ultimate failure with the latter resulting from an "anti" inner model theorem. Which future does the subject face?

Abstracts of contributed talks

► KATALIN BIMBÓ AND J. MICHAEL DUNN, *The decision problem of $T_{\to}$*.
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The decision problem of $T_{\to}$ (the implicational fragment of ticket entailment) was open for over 50 years after its axiom system was specified in [1]. In [3], we introduced a sequent calculus for $R^{\boldsymbol{t}}_{\to}$, which we denoted by $LT^{\circledt}_{\to}$ to emphasize its connection to the sequent calculus $LT^{\boldsymbol{t}}_{\to}$ (from [2]). We claimed there that using the sequent calculus $LT^{\circledt}_{\to}$ we can prove that $T_{\to}$ is decidable.

This talk highlights the main steps in our proof showing that $T_{\rightarrow}$ is decidable. $R^{\mathbf{t}}_{\rightarrow}$ is a conservative extension of $R_{\rightarrow}$ (the implicational fragment of the logic of relevant implication with truth). The decidability of $R_{\rightarrow}$ was enunciated in [5]; see §3.6 in [4] for the details of the decision procedure. It is easy to prove that $R^{\mathbf{t}}_{\rightarrow}$ is decidable—given that $R_{\rightarrow}$ is decidable. Both decision procedures yield a finite set of cut-free proofs for a provable formula. We give an effective transformation for each proof in $[LR^{\mathbf{t}}_{\rightarrow}]$ (one of the sequent calculi for $R^{\mathbf{t}}_{\rightarrow}$ in [3]) into a finite set of proofs in $LT^{\circledast}_{\rightarrow}$. If a formula is provable in $R^{\mathbf{t}}_{\rightarrow}$, then it is provable in $LT^{\circledast}_{\rightarrow}$. Because of how the transformation procedure is defined, all the cut-free (and irredundant) proofs of a provable formula are among the constructed ones. A proof in $LT^{\circledast}_{\rightarrow}$ that does not contain applications of $\mathsf{K}_{\mathbf{t}}\vdash$ or $\mathsf{T}_{\mathbf{t}}\vdash$ is a proof in $LT^{\mathbf{t}}_{\rightarrow}$. The decidability of $T^{\mathbf{t}}_{\rightarrow}$ implies the decidability of $T_{\rightarrow}$.

[1] Alan R. Anderson, *Entailment shorn of modality,* (abstract), ***The Journal of Symbolic Logic***, vol. 25 (1960), p. 388.

[2] Katalin Bimbó, *Relevance logics*, ***Philosophy of logic*** (Dale Jacquette, editor), Handbook of the Philosophy of Science (D. Gabbay, P. Thagard and J. Woods, eds.), vol. 5, North-Holland, Amsterdam, 2007, pp. 723–789.

[3] Katalin Bimbó and J. Michael Dunn, *New consecution calculi for $R^{\mathbf{t}}_{\rightarrow}$*, (18 pages, submitted)

[4] J. Michael Dunn, *Relevance logic and entailment*, ***Handbook of philosophical logic*** (D. Gabbay and F. Guenthner, editors), vol. 3, D. Reidel, Dordrecht, 1st ed., 1986, pp. 117–229.

[5] Saul A. Kripke, *The problem of entailment,* (abstract), ***The Journal of Symbolic Logic***, vol. 24 (1959), p. 324.

► JUSTIN BRODY, *Superstable generic graphs with rich geometries*.
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For rational r with $0 < r < 1$, an amalgamation class of graphs $(\mathbf{K}_r, \preccurlyeq_r)$ is presented in [1] which results in a generic whose theory is essentially undecidable. The primary cause of the of the theory's undecidability is the richness of the geometry associated with the construction. This richness is reflected in the presence of intrinsic transcendental extensions—non-algebraic extensions which form part of a finite set's closure. We give a sufficient condition for such classes to produce superstable generics, and show how a modification of Hrushovski's collapse can be employed to derive a class from $(\mathbf{K}_r, \preccurlyeq_r)$ whose generic is superstable.

[1] J. Brody and M. C. Laskowski, *On rational limits of Shelah–Spencer graphs*, ***The Journal of Symbolic Logic***, vol. 77 (2012), no. 2, pp. 580–592.

► DONALD BROWER, *Indiscernible sequences and simplicity*.
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Simple theories have many characterizations owing to the fact that they are the theories where non-forking is an independence relation. Because of this, many of the equivalences are in terms of the behavior of forking or the related notions of dividing and the tree property. This talk will present a new characterization of simple theories in terms of indiscernible sequences and with no mention of forking or dividing. In particular, simple theories are exactly the ones where pairs of indiscernible sequences which should have a common extension have one.

► WESLEY CALVERT, JENNIFER CHUBB REIMANN, AND RUSSELL MILLER, *The distance function on a computable graph*.
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It is not hard to see that the distance function on a computable graph may not be computable. Indeed, one can simply wait until a function gives an answer for a distance, and then add a shorter path. The exact spectrum of the distance function is more interesting. The present talk will give examples of graphs in which this spectrum may have several interesting values, characterized by functions approximable from above and by strong reducibilities.

► JENNIFER CHUBB REIMANN AND RUSSELL MILLER, *Approximable functions and strong reducibilities*.

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We investigate algorithmic properties of functions that can be approximated by computable functions by adapting strong reducibilities (refinements of Turing reducibility) for *sets* to corresponding reducibilities for *functions*. In addition, we describe an Ershov-type hierarchy for these functions and study them in that context.

Our main focus is on properties of those functions having non-increasing computable approximations. Natural examples of such functions include distance functions on computable graphs and functions computing the Kolmogorov complexity of finite strings.

► SARAH COTTER AND SERGEI STARCHENKO, *Forking in VC-minimal theories*.

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VC-minimality, first defined by Adler, is a generalization of o-minimality; other examples of VC-minimal theories include strongly minimal and C-minimal theories. We consider a specific class of VC-minimal theories, those with unpackable generating families, and show that in such theories, forking of formulae over a model is equivalent to containment in a global definable type.

► SEAN COX, *Martin's Maximum and tower forcing*.

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A *tower of ideals of height* δ is a sequence of ideals with certain coherence properties; δ is called the *height* of the tower. If $\vec{I}$ is a tower then there is a natural poset $\mathbb{P}_{\vec{I}}$ associated with

$\vec{I}$, and if G is generic for $\mathbb{P}_{\vec{I}}$ then in $V[G]$ there is a generic ultrapower $j_G : V \to_G ult(V, G)$; the critical point of j_G corresponds to the completeness of the ideals in the sequence $\vec{I}$. $\vec{I}$ is called *precipitous* iff $ult(V, G)$ is always wellfounded, and *presaturated* iff δ (= the height of the tower) remains a regular cardinal in $V[G]$.

Woodin proved that there are several natural towers of height δ which are definable over V_δ and are presaturated whenever δ is a Woodin cardinal. On the other hand, Burke [1] showed that, if κ is supercompact and $\delta > \kappa$ is inaccessible, then there is a tower of height δ with critical point κ which is *not* precipitous. We prove that under strong forcing axioms like Martin's Maximum and PFA^+, there are towers with critical point ω_2 which are not presaturated (though they can be precipitous). Moreover, these towers are very different from the tower $\vec{I}_{IA}$ with critical point ω_2 which was shown in [2] to be non-presaturated under Martin's Maximum.

This is joint work with Matteo Viale.

[1] Douglas R. Burke, *Precipitous towers of normal filters*, ***The Journal of Symbolic Logic***, vol. 62 (1997), no. 3, pp. 741–754.

[2] Matthew Foreman and Menachem Magidor, *Large cardinals and definable counterexamples to the continuum hypothesis*, ***Annals of Pure and Applied Logic***, vol. 76 (1995), no. 1, pp. 47–97.

► FRANÇOIS G. DORAIS, *Classical consequences of continuous choice principles from intuitionistic analysis*.

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The sequential form of a statement of the form

$$\forall \xi (B(\xi) \to \exists \zeta A(\xi, \zeta))$$

is the statement

$$\forall \langle \xi_n \rangle_{n=0}^{\infty} (\forall n B(\xi_n) \to \exists \langle \zeta_n \rangle_{n=0}^{\infty} \forall n A(\xi_n, \zeta_n)).$$

There are many classically true statements of the first form whose proofs lack uniformity and therefore the corresponding sequential form is not provable in weak classical systems, such as those used in reverse mathematics. The main culprit for this lack of uniformity is, of course, the law of excluded middle.

Continuing along the lines of Hirst and Mummert [1], I will show that if a statement of the first form above that satisfies certain syntactic requirements is provable in certain weak intuitionistic systems, then the proof is necessarily sufficiently uniform that the corresponding sequential form is provable in a corresponding weak classical system.

I will present two results of this form, relating systems of classical second-order arithmetic and systems of intuitionistic analysis. The proof of the first result relies on Kleene's realizability with functions while the proof of the second result relies on a Lifshitz variant thereof. It is remarkable that, in both cases, the intuitionistic systems involved include continuous choice principles that are not classically valid. Moreover, for the second result, both the intuitionistic and classical systems involved incorporate the weak König lemma.

[1] J. L. Hirst and C. Mummert, *Reverse mathematics and uniformity in proofs without excluded middle*, ***Notre Dame Journal of Formal Logic***, vol. 52 (2011), no. 2, pp. 149–162.

► FRANÇOIS DORAIS, JEFFRY HIRST, AND PAUL SHAFER, *Reverse mathematics and dichotomy*.

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The dichotomy principle asserts that for every real number α, either $\alpha \geq 0$ or $\alpha \leq 0$. RCA_0 can prove the dichotomy principle, but the following theorem indicates that the proof is not uniform.

THEOREM 1. RCA_0 *proves that the following are equivalent*:

(1) WKL_0.
(2) *If* $\langle \alpha_i \rangle_{i \in \mathbb{N}}$ *is a sequence of reals, then there is a set* I *such that for all* $i \in \mathbb{N}$, $i \in I$ *implies* $\alpha_i \leq 0$ *and* $i \notin I$ *implies* $\alpha_i \geq 0$.

The concept of a *persistant real* can be used to find a restriction of statement (2) that is provable in RCA_0, and also leads to a restriction of the dichotomy principle which is deducible in a weak subsystem of constructive analysis.

► TIMOTHY H. MCNICHOLL, *Computing boundary extensions of conformal maps*.
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Suppose γ denotes a planar Jordan curve and that D denotes its interior. Let $\mathbb{D}$ denote the disk of radius 1 whose center is the origin. By the Riemann Mapping Theorem, there is a conformal map ϕ of D onto $\mathbb{D}$. This map is known as a *Riemann map*. The Carathéodory Theorem states that ϕ extends to a homeomorphism of $\overline{D}$ onto $\overline{\mathbb{D}}$. This extension is also denoted by ϕ and is commonly referred to as the *boundary extension* of ϕ. These maps play an important role in the solution of Dirichlét problems. We show that there is a Jordan curve γ that is computably compact (in the sense that there is an algorithm that will plot γ on a computer screen with any given resolution) and a computable conformal map of its interior onto $\mathbb{D}$ whose boundary extension is incomputable. We also show that to compute boundary extensions of Riemann maps, it is necessary and sufficient to have, in addition to covering information about γ, local connectivity information about γ.

[1] T. H. MCNICHOLL, *An effective Carathéodory Theorem*, ***Theory of Computing Systems***, vol. 50 (2012), no. 4, pp. 579–588.

► PAUL SHAFER, *Complexity in the degrees of unsolvability of mass problems*.
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A *mass problem* is a set $\mathcal{A} \subseteq \omega^\omega$ thought of as representing an abstract mathematical problem, namely the problem of finding a member of $\mathcal{A}$. The Medvedev and Muchnik reducibilities allow us to express the "problem $\mathcal{B}$ is at least as hard as problem $\mathcal{A}$" relation in computability theoretic terms: if mass problem $\mathcal{A}$ Medvedev (or Muchnik) reduces to mass problem $\mathcal{B}$, then solutions to $\mathcal{B}$ can compute solutions to $\mathcal{A}$, so solving $\mathcal{B}$ is at least as hard as solving $\mathcal{A}$. The Medvedev and Muchnik reducibilities partition $\mathcal{P}(\omega^\omega)$ into degrees of unsolvability. In this talk, we describe the recent complexity results concerning these degree structures and their substructures. The results express that the "problem $\mathcal{B}$ is at least as hard as problem $\mathcal{A}$" relation is as complicated as possible, even when restricted to several natural subclasses of mass problems. For example:

- The first-order theories of the Medvedev and Muchnik degrees are recursively isomorphic to the third-order theory of arithmetic [1, 2].
- The first-order theories of the closed Medvedev and Muchnik degrees are recursively isomorphic to the second-order theory of arithmetic [2].
- The first-order theory of the effectively closed Medvedev degrees is recursively isomorphic to the first-order theory of arithmetic [3].
- A Turing degree computes a presentation of the effectively closed Medvedev degrees if and only if that Turing degree computes $\mathbf{0}'''$ [3].

[1] Andrew E. M. Lewis, André Nies, and Andrea Sorbi, *The first order theories of the Medvedev and Muchnik lattices*, ***CiE 2009: Mathematical theory and computational practice*** (Klaus Ambos-Spies, Benedikt Löwe and Wolfgang Merkle, editors), vol. 5635, Springer, 2009, pp. 324–331.

[2] Paul Shafer, *Coding true arithmetic in the Medvedev and Muchnik degrees*, ***The Journal of Symbolic Logic***, vol. 76 (2011), no. 1, pp. 267–288.

[3] ———, *Coding true arithmetic in the Medvedev degrees of Π^0_1 classes*, ***Annals of Pure and Applied Logic***, vol. 163 (2012), no. 3, 321–337.

► DAN E. WILLARD, *An umbrella formalism for examining self-justifying systems*.
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Let $Add(x, y, z)$ and $Mult(x, y, z)$ denote atomic predicates indicating $x + y = z$ and $x * y = z$. Let L^- denote a revised language for encoding arithmetic that replaces the addition and multiplication function symbols with the $Add(x, y, z)$ and $Mult(x, y, z)$ predicates. During the last ten years, we have published a series of several papers, about generalizations and boundary-case exceptions for the Second Incompleteness Theorem under such a language L^-, including six papers in the ***The Journal of Symbolic Logic*** and ***Annals of Pure and Applied Logic***. Among our results were that an axiom system [2] could recognize its semantic tableaux consistency while simultaneously proving all Peano Arithmetics Π_1 styled theorems in L^-'s revised language and recognizing the validity of Equation (1)'s axiom that addition is a total function.

$$\forall x\, \forall y\, \exists z\, Add(x, y, z). \tag{1}$$

At the same time, [1, 3, 4] showed that analogs of this boundary-case exception to the Second Incompleteness Theorem for semantic tableaux deduction were essentially infeasible when the germane axiom system either included the analog of Equation (1)'s totality axiom for multiplication or incorporated one of three encodings for $I\Sigma_0$'s axiom system. (Part of the reason such generalizations for the Second Incompleteness Theorem were of interest was because [4] showed that some unusual encodings for $I\Sigma_0$ were compatible with certain types of boundary-case exceptions to the Second Incompleteness Theorem under Herbrandized but not semantic tableaux deduction.)

We have recently produced a new 63-page technical report [5] that provides an umbrella formalism that simplifies, extends and unifies several of the preceding results from the prior mentioned articles. The Appendix F of [5] also explains the epistemological significance and inherent limitations of self-justifying axiom systems. The current talk will offer a brief summary of the material in [5]'s 63-page technical report.

[1] D. Willard, *How to extend the semantic tableaux and cut-free versions of the Second Incompleteness Theorem almost to Robinson's arithmetic*, ***The Journal of Symbolic Logic***, vol. 67 (2002), pp. 465–49.

[2] ———, *An exploration of the partial respects in which an axiom system recognizing solely addition as a total function can verify its own consistency*, ***The Journal of Symbolic Logic***, vol. 70 (2005), pp. 1171–1209.

[3] ———, *Passive induction and a solution to a Paris–Wilkie question*, ***Annals of Pure and Applied Logic***, vol. 146 (2007), pp. 124–149.

[4] ———, *Some specially formulated axiomizations for $I\Sigma_0$ manage to evade the Herbrandized version of the Second Incompleteness Theorem*, ***Information and Computation***, vol. 207 (2009), pp. 1078–1093.

[5] ———, *A detailed examination of methods for unifying, simplifying and extending several results about self-justifying logics*, http://arxiv.org/abs/1108.6330.

Abstracts of papers submitted by title

► JOHN CORCORAN, *From proposition to judgment*.
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This lecture compares seven concepts: *proposition, sentence, fact, statement, information, belief, and judgment*.

Only the *proposition* admits of belief, disbelief, and doubt. Only the proposition is true or false. The proposition is distinguished from the *sentence*, the string of words or signs through which it is expressed; from the *fact*, the actual complex situation whose existence renders the proposition true or false; from the *statement*, by which a person affirms the proposition; from the *information*, which the proposition contains and which is conveyed by the statement; from the *belief*, the private state made public by the statement; and from the *judgment*, the private act which produces the belief.

Of the seven, only the proposition is composed exclusively of meanings and only the proposition implies, contradicts, or is a logical consequence. Of the remaining six, only the sentence is exhaustively described by giving the order of occurrence of its component characters, and only the fact is composed of whatever entities the proposition is about. Of the remaining four—statement, information, belief, and judgment—only the statement is a voluntary public act, only the information is contained by logical equivalents of the proposition, only the belief comes into existence through judging and persists after the completed judging, and only the judgment is a private act. Of the seven, only three are produced by a person; every statement, belief, or judgment is the result of someone's doing.

This lecture sharpens and interrelates [1] and [2].

[1] John Corcoran, *Information-theoretic logic*, ***Truth in Perspective***, (Concha Martinez et al., editors), Ashgate, Aldershot UK, 1998, pp. 143–176.

[2] ———, *Sentence, proposition, judgment, statement, and fact*, ***The many sides of logic***, (W. Carnielli et al., editors), College Publications, London, 2009, pp. 71–103.

► JOHN CORCORAN AND IDRIS SAMAWI HAMID, *Meta-language, object-language*.
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We survey use, misuse, and conspicuous non-use of two words, 'meta-language' and 'object-language'—and their cognates and synonyms. Logicians considered include Aristotle, Augustine, al-Farabi, Ockham, Boole, al-Ahsa'i, De Morgan, Peirce, Frege, Russell, Hilbert, Wittgenstein, Carnap, Tarski, Church, and Quine.

In primary senses, 'meta-language' and 'object-language' are *relational* nouns like 'predecessor' and 'successor' as opposed to *sortal-common* nouns like 'language' and 'number'. Primarily, 'meta-language' and 'object-language' do not signify two species of language—as Indic and Celtic. Likewise 'predecessor' and 'successor' do not signify two species of number—as cardinal and ordinal. Primary occurrences are in three-place *relational-verb phrases* expressing three-place relations.

language X is a meta-language of language Y for context Z

language X is an object-language of language Y for context Z

Both are used to express converse correlatives like 'predecessor' and 'successor'.

X is a meta-language of Y for Z
if-and-only-if
Y is an object-language of X for Z.

X is a predecessor of Y for ordering Z
if-and-only-if
Y is a successor of X for ordering Z.

Definitions:

X is a meta-language of Y for context Z iff X is used in context Z to discuss Y
X is an object-language of Y for context Z iff Y is used in context Z to discuss X

Fixing context, the *three-place* relational-verb phrases become elliptical *two-place* relational-verb phrases, which express converse two-place relations.

X is a meta-language of Y

X is an object-language of Y

Every language that is a meta-language of itself is an object-language of itself.

Secondarily, 'meta-language' and 'object-language' are used as *relation-based common nouns* signifying species of language. A meta-language (object-language) is a language that is a meta-language (object-language) *of* some language *for* some context.

► JOHN CORCORAN AND LEONARD JACUZZO, *Multiplying negative integers*.
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This study—in history of logic and foundations of mathematics—surveys, classifies, compares, and criticizes historical treatments of multiplication of negative integers by mathematicians and logicians. We consider interpretations and answers for questions such as *why the product of two negative integers is positive*. We analyze proofs and stipulations of propositions such as *the product of two negative integers equals the product of the corresponding positive integers*. Where proofs were proposed, what theories of proof were presupposed? Where stipulations were made, what theories of definition were presupposed?

Such issues require discerning how people understood negative integers, more generally, how they understood integers. Did people think integers were quantities like natural numbers, or were symbols, or were one-place functions, or were performable operations—like incrementing and decrementing a fixed quantity, crediting and debiting an account, or moving forward and backward—or were fictitious, "ideal" entities like points at infinity postulated for computational purposes?

We also ask how people understood—and expressed the need for—multiplication of negative integers and when and how the non-mental, non-performable abstract two-place function called integer multiplication was distinguished from the corresponding humanly performed mental operation. We ask when and how people differentiated *signed numerals* such as '$+5$' and '-5'—which are character strings—from *signed numbers* such as $+5$ and -5, which are positive and negative *integers*. We investigate related notational issues such as the various roles of the minus sign and whether in '-5' the minus sign is merely a part of the canonical name of -5 or whether it is a one-place function symbol. Historical figures treated include Al-Khwarizmi, Frend, Boole, De Morgan, Dedekind, Huntington, Birkhoff, Mac Lane, and Whitehead.

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"Because all mathematically precise results in logic (aside from those concerned with infinitary languages) involve the syntactical interrelations among finite strings of characters over a finite alphabet [. . .], mathematical logic itself may be said to presuppose the theory of strings [. . .]"—Corcoran, Frank, and Maloney, 1974.

This statement—made in the just-quoted opening sentence of [2]—has not been disputed in the 30-odd years since its publication. But it has not been established by rigorous documentation.

We survey representative publications in the main branches of mathematical logic—proof theory, model theory, computability theory, and set theory—to locate and describe specific applications in logic of string theory—also known as *concatenation theory*, *pure syntax*, and *theoretical syntax*. As a secondary goal, we document explicit references to the first presentation in English of an axiomatic foundation of string theory by Alfred Tarski in his now-famous truth-definition paper [3, pp. 173–174]. Immediately following his axiom system, Tarski referred to it as "a basis for the construction of the metalanguage" [3, p. 174].

Alonzo Church was one of a surprisingly small number of logicians to recognize the foundational importance for logic of string theory. He wrote [1, p. 59]: "Like any branch of mathematics, theoretical syntax may, and ultimately must, be studied by the axiomatic method".

Logicians considered include Carnap, Church, Davis, Feferman, Henkin, Hilbert, Lewis, Mendelson, Post, Quine, Rosenbloom, Shapiro, Smullyan, and Turing.

[1] ALONZO CHURCH, ***Introduction to mathematical logic***, Princeton University Press, Princeton, 1956.

[2] JOHN CORCORAN, WILLIAM FRANK, AND MICHAEL MALONEY, *String theory*, ***The Journal of Symbolic Logic***, vol. 39 (1974), pp. 625–637.

[3] ALFRED TARSKI, *The concept of truth in formalized languages*, ***Logic, semantics, metamathematics***, Hackett, Indianapolis, 1983, pp. 152–278.

► JOHN CORCORAN AND JUSTIN LEGAULT, *Aristotle and Tarski on consequence*.
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We compare Aristotle's [logical] consequence relation [1] with Tarski's [2].

There are striking similarities. Each concerns truth-value-bearing linguistic entities satisfying analogues of "excluded middle" and "non-contradiction"—not logical forms, propositional functions, or other entities lacking truth-values. Both relations are two-place and non-homogeneous: the first argument (or subject)—the "conclusion"—is an *individual*; the second argument (or object)—the "premises"—is a *class*. Both admit single-membered and infinite-membered premise classes. Moreover, both are *formal*: if a given conclusion is a consequence of given premises, the same holds in all formally similar cases. Thus, both admit of "refutation by counterargument": the conclusion of a given premise-conclusion argument is *not* a consequence of its premise class if there are *counterarguments*—premise-conclusion arguments in the same form having a false conclusion and all true premises.

There are striking differences. In fact, the two are *incommensurable*: Aristotle's conclusions are potential speech acts; Tarski's are character-strings—sentences of an interpreted formalized language. If a given "conclusion" is a consequence of given "premises" in one of the two senses, then applying the other sense of consequence in the same case is incoherent. Further, Aristotle's relation is primitive. No discussion in [1] considers defining it in more fundamental terms, but Aristotle gave one criterion determining if it holds and another determining if it doesn't hold. In contrast, the main point of Tarski's [2] is defining "the common concept of consequence", but no discussion of criteria occurs.

Aristotle's logical consequence should not be confused with his more restrictive syllogistic concludence—which relates the conclusion of a syllogism to its premise set.

[1] ROBIN SMITH, ***Aristotle's prior analytics***, Hackett, Indianapolis, 1989.

[2] ALFRED TARSKI, ***Logic, semantics, metamathematics***, Hackett, Indianapolis, 1983, pp. 409–420.

► JOHN CORCORAN AND HASSAN MASOUD, *Aristotle's* "whenever three terms"*.*
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Whenever three terms are related so that the last is wholly contained in the middle and the middle either is wholly contained in or excluded from the first, the extremities are necessarily related by a complete deduction.—Aristotle, *PrAn*, 25b32.

Commentators say that 25b32 presents Aristotle's two syllogistic deduction rules Barbara and Celarent. Let A, B, C be terms.

$$\frac{\begin{array}{c}\text{C is-contained-in B}\\ \text{B is-contained-in A}\end{array}}{\text{C is-contained-in A}}$$

$$\frac{\begin{array}{c}\text{C is-contained-in B}\\ \text{B is-excluded-from A}\end{array}}{\text{C is-excluded-from A}}$$

But without context 25b32 doesn't convey that information. 25b32 doesn't indicate whether the conclusion's subject is the first or last, or which conclusion is affirmative or negative. Moreover, 25b32 suggests the absurdity that actual containment-exclusion relations among terms determine whether they occur in deductions: *every* two terms are extremities of complete deductions. Further, placement of 'either' admits interpreting 25b32 as concerning disjunctive arguments. One such follows.

$$\frac{\begin{array}{c}\text{C is-contained-in B}\\ \text{B is-contained-in or excluded-from A}\end{array}}{\text{C is-contained-in or excluded-from A}}$$

Paraphrasing further highlights 25b32's deficiencies.

Whenever [two premises say that] three terms are related so that the first is wholly contained in the second and [either] that the second is wholly contained in the third or [that it] is wholly excluded from the third, the extremities are necessarily related by a complete deduction [whose conclusion either says that the first is contained in the third or that the first is excluded from the third].—25b32, paraphrase.

Supplementing [1], we enumerate further deficiencies, we describe the context, and we discuss how and to what extent that context warrants traditional interpretations.

[1] John Corcoran, *Aristotle's demonstrative logic*, ***History and Philosophy of Logic***, vol. 30 (2009), pp. 1–20.