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REVIEWS

The Association for Symbolic Logic publishes analytical reviews of selected books and articles in the field of symbolic logic. The reviews were published in ***The Journal of Symbolic Logic*** from the founding of the JOURNAL in 1936 until the end of 1999. The Association moved the reviews to this BULLETIN, beginning in 2000.

The Reviews Section is edited by Steve Awodey (Managing Editor), John Burgess, Mark Colyvan, Anuj Dawar, Steffen Lempp, Rahim Moosa, Ernest Schimmerling, Alex Simpson, Kai Wehmeier, and Matthias Wille. Authors and publishers are requested to send, for review, copies of books to *ASL, Box 742, Vassar College, 124 Raymond Avenue, Poughkeepsie, NY 12604, USA.*

In a review, a reference "JSL XLIII 148," for example, refers either to the publication reviewed on page 148 of volume 43 of the JOURNAL, or to the review itself (which contains full bibliographical information for the reviewed publication). Analogously, a reference "BSL VII 376" refers to the review beginning on page 376 in volume 7 of this BULLETIN, or to the publication there reviewed. "JSL LV 347" refers to one of the reviews or one of the publications reviewed or listed on page 347 of volume 55 of the JOURNAL, with reliance on the context to show which one is meant. The reference "JSL LIII 318(3)" is to the third item on page 318 of volume 53 of the JOURNAL, that is, to van Heijenoort's *Frege and vagueness*, and "JSL LX 684(8)" refers to the eighth item on page 684 of volume 60 of the JOURNAL, that is, to Tarski's *Truth and proof*.

References such as 49***5*** or 280***1*** are to entries so numbered in *A bibliography of symbolic logic* (the JOURNAL, vol. 1, pp. 121–218).

BERNARD LINSKY. ***The Evolution of* Principia Mathematica*: Bertrand Russell's manuscripts and notes for the second edition.*** Cambridge University Press, Cambridge, 2011, vii + 407 pp.

The second edition of ***Principia Mathematica*** (hereafter PM^2) includes a new introduction and three new appendices. Most commentators who have reviewed this new material have found it to be either unclear or simply false. In this book Linsky argues for a "more charitable assessment" (p. 6) of PM^2. The first half of the book presents a detailed reconstruction of how Russell came to present the new material of PM^2. A central question for this part of the book is why Russell might have made the changes that he did. Linsky adopts a generally sympathetic approach to Russell. For example, he reviews the "contributions to mathematical logic since the publication of the first edition" (PM^2, p. xlv) that Russell added at the end of his introduction and argues that "Russell almost certainly knew all the works, except perhaps for Ramsey's very late additions" (p. 48). Linksy also considers Russell's new more extensional conception of propositional functions and the attempt to get by without the axiom of reducibility. He shows that these new ideas arose internally in the course of Russell's philosophical development and engagement with the views of Wittgenstein's ***Tractatus***.

The second half of the book provides a transcription of the surviving manuscripts from this period. In addition to the corrected manuscripts of the new introduction and the three appendices, Linsky provides three new manuscripts. One manuscript is the list of definitions for Carnap that Russell composed in 1922. Another manuscript is called "Hierarchy of

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propositions and functions". It is an early version of the new material that is found in PM^2. A third manuscript "Amended list of propositions: notes" contains various attempts to generalize the formal results obtained in appendix B. By presenting his reconstruction alongside these archival materials Linksy hopes to rekindle interest in the philosophical and technical issues raised by PM^2.

Most of the critical attention paid to PM^2 has focused on the flaws of appendix B and Linsky devotes chapter six to a thorough discussion of these issues. In the first edition Whitehead and Russell had largely followed Frege's technique for proving mathematical induction. Whitehead and Russell obtain mathematical induction by first proving some general theorems about relations R and their ancestrals R_*. xR_*y obtains just in case x is in the field of R and for any class μ, if μ is R-hereditary and $x \in \mu$, then $y \in \mu$. A class μ is R-hereditary when, for any z and w, if zRw and $z \in \mu$, then $w \in \mu$. Whitehead and Russell go on to define the natural numbers as similar classes of classes of individuals. The immediate successor relation on these classes and its ancestral are then well-suited to permit a proof of mathematical induction.

On the no class theory of the first edition, classes are eliminated in terms of certain propositional functions of lowest order ($*20 \cdot 01$). The axiom of reducibility is critical to ensuring the elimination because without it there is no assurance that for every higher-order propositional function there is a coextensive propositional function of lowest order. Without this assurance, it is not possible to deploy class expressions without actually finding the appropriate lowest order propositional function. So when Russell tries to get by without the axiom of reducibility a number of the proofs of the first edition need to be reconsidered. In particular, the proof of mathematical induction must be recast. Without reducibility Russell is forced to distinguish between classes and relations based on the order of the propositional function that is used to introduce the class or relation. For example, we can no longer talk simply of the R_* of relation R, but must posit many distinct ancestrals R_{*n}, where the n indicates the order of the μ classes that we quantify over in the definition of R_{*n}. Without reducibility, if $m > n$ it may very well be the case that for some x and y, $xR_{*n}y$ but not $xR_{*m}y$ (PM^2, p. 650). Russell claims to have shown that if $m > 5$, then $R_{*m} = R_{*5}$ (p. 146). That is, once we get to a high enough order, there is no difference in the extension of the ancestral of R. If this was true, and the appropriate general theorems from the first edition could be established for that order, then the proof for mathematical induction could be obtained even without reducibility.

Linsky provides a thorough review of the debates, going back to Gödel, about Appendix B and Russell's supposed proof. The two most detailed discussions have been offered by Landini [*The* Definability *of the Set of Natural Numbers in the 1925* ***Principia Mathematica***, ***Journal of Philosophical Logic***, vol. 25 (1996), pp. 597–615; ***Wittgenstein's Apprenticeship with Russell***, Oxford: Oxford University Press, 2007, esp. ch. 6] and Hazen [*A "Constructive" Proper Extension of Ramified Type Theory* (*The Logic of* ***Principia Mathematica****, Second Edition, Appendix B*), in G. Link (ed.), ***One Hundred Years of Russell's Paradox***, Berlin: de Gruyter, 2004, pp. 448–480]. Landini recovers Russell's result by proposing a strong axiom of extensionality that allows one to transfer properties of a propositional function of one order to a propositional function of another order (1996, p. 604). Hazen rejects this interpretation of Russell because this axiom allows one to prove the axiom of reducibility (2004, p. 476). (In his 2007 book, Landini responds by weakening the axiom of extensionality. See p. 214.) Hazen presents a different system BMT and provides evidence that it fails to prove full mathematical induction (2004, p. 476). Linsky sides largely with Hazen when he endorses Hazen's complaint that Landini's extensionality axiom is too strong to be the "less objectionable" substitute for reducibility that Russell hoped for (p. 168).

It is precisely at this point that the reader of Linsky's commentary is left with additional questions. Linsky does a masterful job of summarizing the issues and the extant interpretations. But he does not link the details of the manuscripts he has found to the interpretative debate between Landini and Hazen or present a new interpretation. There are a few tantalizing hints about how the manuscripts might be used to resolve these sorts of interpretive questions. Linsky rejects Myhill's complaint that Russell deployed "slipshod notation" by tracing the consistent use of the order indices throughout the different manuscripts (p. 162). Linsky also uses the manuscripts to identify the first occurrence of the error that Gödel spotted in one of Russell's proofs (pp. 156, 285). These are valuable first steps in deploying the manuscript evidence to help provide a better interpretation of what Russell was up to. But we must wait for future work by Linsky and others to see where the evidence points. One hopes that the previously unpublished manuscript material will show Russell operating in a way that supports a viable interpretation of the appendix B proof. However, it is difficult to know if these hopes can be realized. Russell scholars interested in these aspects of Russell owe Linsky a great deal for putting these manuscripts before us and fueling a new round of interpretive investigations, but as Linsky would be the first to admit, a lot of interpretive work remains to be done.

CHRISTOPHER PINCOCK

Department of Philosophy, The Ohio State University, Columbus, OH 43210-1365, USA. chrispincock@gmail.com.

DAVIDE SANGIORGI. ***Introduction to bisimulation and coinduction.*** Cambridge University Press, 2012, 247 pp.

Advanced topics in bisimulation and coinduction, edited by Davide Sangiorgi and Jan Rutten, Cambridge Tracts in Theoretical Computer Science, vol. 52. Cambridge University Press, 2012, 326 pp.

Bisimulation and coinduction, as described from different perspectives in the two books under review, form a mathematical framework particularly useful to reason about (computational) objects whose behaviour may be infinite, *i.e.*, non-terminating. Whereas the concept of bisimulation provides a very robust way of defining equivalence relations for such kinds of computational objects—usually referred to as processes, coinduction naturally arises as the dual of the well-known reasoning technique of induction.

The first book, ***Introduction to Bisimulation and Coinduction***, by Prof. Davide Sangiorgi, is a self-contained, very well-written monograph surveying and explaining in a pleasant pedagogical way the two topics of interest. The book is full of examples and exercises, which should help to increase the confidence of the reader in mastering the subject. Even though it is not the main topic of interest, the book also offers a simple and short introduction to process algebras, in particular to Milner's CCS process calculus, and through it also an introduction to fundamental concepts, issues, and reasoning techniques in Concurrency Theory. Reading the book requires rather basic prior mathematical background, which should make it accessible to beginning graduate (and even some undergraduate) students of Informatics, Mathematics, Engineering, or related sciences.

The first book starts by motivating the two topics of interest from different angles, giving the reader plenty of justification as to why bisimulation and coinduction are worth studying, and more importantly, why they may be the right reasoning concepts and techniques in certain settings. After these motivating and introductory sections, the book explicitly displays the strong relationship of bisimulation and coinduction with the theory of concurrent processes/systems. In these chapters bisimulation is shown to be the core concept at the heart of some of the best-known behavioural equivalences for concurrent systems. This study is complemented by a careful treatment of coinduction as the associated reasoning technique in such scenarios. Even though CCS is the process language/calculus most used throughout the book, other languages such as the λ-calculus are used in examples at various points; in doing so, the wide applicability of the concept of bisimulation is shown by providing evidence that not only is bisimulation a fundamental concept in concurrency but also in the study of behavioural equivalences for other kinds of computing systems, *e.g.*, sequential ones.

The second book, ***Advanced Topics in Bisimulation and Coinduction***, edited by Prof. Davide Sangiorgi and Prof. Jan Rutten, provides a compilation of different topics where either bisimulation or coinduction has found applicability. The material in most chapters of the second book is kept to an introductory level. The chapters are self-contained and disconnected of each other, so that they can be read in a "call-by-need" style—each chapter even has its own bibliography section. The chapters provide introductions to several other topics such as, *e.g.*, coalgebras, modal logic, the μ-calculus, the λ-calculus, and probabilistic systems amongst others. As for the first book, reading this collection of chapters requires very little mathematical background. This book also contains plenty of exercises, making it very attractive from a pedagogical viewpoint—let alone the simplicity with which the topics are presented. One small complaint is a lack of uniformity in the presentation of the book as a whole. Although it is a good idea that each chapter is self-contained (because the reader only needs to learn what is in the chapter to understand it), this leads to having some of the core concepts introduced several times throughout the book, sometimes with a slightly different style.

The book on advanced topics starts with a motivating chapter by one of the editors, Davide Sangiorgi, where bisimulation and coinduction are put in context; the chapter, for the most part, talks about the origins of bisimulation and coinduction in three different areas: Computer Science, Modal Logic, and Set Theory. Special recognition is given to the seminal contributions of, *e.g.*, Robin Milner, David Park, Johan van Benthem, Peter Aczel, and Jon Barwise amongst others. The second chapter, by Bart Jacobs and Jan Rutten, focuses on the key role that bisimulation equivalences play when studying the mathematical dualities between algebras and coalgebras as well as between the induction and coinduction proof methods; this chapter requires some acquaintance with Category Theory but all fundamental ideas are introduced nevertheless.

The third chapter—by Luca Aceto, Anna Ingolfsdottir, and Jiří Srba—describes in great detail some of the algorithms for computing bisimulation relations; this chapter places the reader in a position where practical implementations are not too far away, making the two concepts, bisimulation and coinduction, concrete enough from an algorithmic viewpoint. Chapter 4, by Colin Stirling, then shows how (and why) bisimulation and coinduction arise from a logical perspective, primarily from the work of Johan van Benthem in relating Modal and First-Order logic as well as the work of Robin Milner and Matthew Hennessy in looking for a logical characterisation of the equivalence (the congruence) induced by the equational theory of CCS processes, *i.e.*, of the CCS process algebra. This chapter also describes how an extension of Modal logic by fixed-point operators due to Dexter Kozen induces an equivalence that is the same as the one induced by the bisimulation-invariant fragment of Monadic Second-Order logic. The fifth chapter, by Andrew Pitts, presents one of the most interesting applications of the bisimulation and coinduction proof methods, namely the definition of congruences for core programming languages; the chapter presents Howe's construction for deriving contextual equivalences for higher-order languages (based on the λ-calculus) and therefore clear evidence that bisimulation and coinduction are not restricted to applications in concurrency.

Chapter 6, by Damien Pous and Davide Sangiorgi, studies improvements of the bisimulation and coinduction techniques, especially when used to analyse systems with some internal behaviour which one wishes to abstract away. This second book finishes with a chapter by Prakash Panangaden on applications to probabilistic systems; the chapter, unlike all previous ones, encourages the reader to think about bisimulation and coinduction in a context where non-determinism is replaced by probabilistic information and discrete systems are replaced by systems with continuous state spaces. The chapter provides interesting and powerful tools for studying real-life problems since hybrid systems (those involving discrete and continuous requirements) are everywhere.

Overall, the two books deserve a positive review. They survey some of the most important contributions on bisimulation and coinduction, while lending themselves suitable for pedagogical purposes. It is clear to this reviewer that a lot of effort has been put into making the two books uniformly accessible to students and researchers with different backgrounds.

JULIAN GUTIERREZ

Department of Computer Science, University of Oxford, UK.
Julian.Gutierrez@cs.ox.ac.uk.

The Vienna Circle in Hungary, edited by András Máté, Miklós Rédei and Friedrich Stadler, Springer, Wien–New York, 2011, 300 pp.

In the academic year 2007/2008 the Institute Vienna Circle and the Department of Logic at Eőtvős University organized four bilingual (English and German) workshops, and a concluding conference on the connections between Hungary and the Vienna Circle. This book contains the essays of the project. It complements a series of books investigating the international connections and reception of the Vienna Circle, such as The Vienna Circle in the Nordic Countries and Cambridge and Vienna among others [Juha Manninen and Friedrich Stadler (editors), ***The Vienna Circle in the Nordic Countries***, Springer, Dordrecht, 2010, 326 pp.; Maria Carla Galavotti (editor), ***Cambridge and Vienna: Frank P. Ramsey and the Vienna Circle***, Springer, Dordrecht, 2005, 256 pp.; and Jan Wolenski and Eckehart Köhler (editors), ***Alfred Tarski and the Vienna Circle: Austro-Polish Connections in Logical Empiricism***, Springer, London, 1998, 356 pp.]. Besides, this is the third Hungarian themed volume published by the Institute Vienna Circle. The two earlier ones were concerned with the work of John von Neumann and Imre Lakatos [Miklós Rédei and Michael Stölzner (editors), ***John von Neumann and the Foundations of Quantum Physics***, Kluwer, Dordrecht, 2000, 371 pp.; and George Kampis, Ladislav Kvasz and Michael Stölzner (editors), ***Appraising Lakatos: Mathematics, Methodology and the Man***, Kluwer, Dordrecht, 2002, 382 pp.]. This volume contains twelve essays (nine written in English and three in German) which are, disregarding the introductory essay, divided into three sections, namely, History of Philosophy, Physics and Logic.

The introduction, *Austria-Hungary in Philosophy and Science: A Search for the Evidence*, is written by Miklós Rédei and Friedrich Stadler. In their short essay they sketch the reception of Ernst Mach and the Vienna Circle in Hungary and their influence on Hungarian scientists and philosophers from the turn of the nineteenth and twentieth century up to this day. They touch upon topics, among many others, such as the pursuit of the Galilei Circle and the work of Karl Polányi; the debate of von Neumann and Gödel on the significance of the second incompleteness theorem; or the interaction between the 'Third Vienna Circle' and Imre Lakatos and Karl Popper. As far as contemporary work, they also address the investigations of the 'Budapest School' on probabilistic causality and the logical analysis of special and general relativity theory done by the members of the Algebraic Logic Research Group of Rényi Institute of Mathematics (see this volume).

The History of Philosophy section contains four essays. Two of these are concerned with the life and work of Béla Juhos, the only Hungarian member of the Vienna Circle. Gergely Ambrus, in *Juhos' Antiphysicalism and his Views on the Psychophysical Problem*, concentrates on the early stage of his career. In the 1930s Juhos, Moritz Schlick's student, was mainly interested in the psychophysical (mind-body) problem and the protocol sentence debate. They both argued for the existence of incorrigible Konstatierung- or affirmation sentences. Ambrus discusses Juhos' arguments against the physicalist account of protocol and mental sentences in detail and places them in a broader context.

Béla Juhos and Victor Kraft were the only members of the Vienna Circle who remained in Vienna during and after the Second World War. Wolfgang Reiter's biographical study, *Wer war Béla Juhos? Eine biographische Annäherung*, focuses on Juhos' career after 1945. Besides this, Reiter gives a short insight into Juhos' books on the philosophical interpretation of physics which appeared in the 60's (Béla Juhos und Hubert Schleichert, ***Die Erkenntnislogischen Grundlagen der Klassischen Physik***, Duncker und Humblot, Berlin, 1963, 119 pp. and Béla Juhos, ***Die Erkenntnislogischen Grundlagen der Modernen Physik***, Duncker und Humblot, Berlin, 1967, 247 pp.).

Gábor Zemplén, in his essay *Early 20th Century Conventionalism in Hungary—A Case for Győző Zemplén's reappreciation*, reviews the reception of Poincaré's conventionalism in Hungary through the life and philosophical work of Győző Zemplén. According to Gábor Zemplén, the famous physician Győző Zemplén, who died an early death during the First World War, can be considered as the first exponent of Conventionalism in Hungary. (More information on Győző Zemplén can be found in László Kovács (editor), ***Zemplén: The Scientist and the Teacher***, Berzsenyi College, Szombathely, 2004, 47 pp., `http://sci-ed.org/documents/anett-zempl.pdf`.)

The research physician and physician Anton (Antal) Fischer went to medical school in Vienna from 1919 to 1926. As a young researcher he worked in Aachen and France and from 1946 to 1971 he was a medic at the Budapest Medical University. He published many papers and books on medical issues. What makes him interesting for the history of the philosophy of science is his book on epistemology, ***Philosophischen Grundlagen der wissenschaftlichen Erkenntnis***, which was published by Springer in 1947 and 1967. In *Anton Fischers philosophische Grundlagen der wissentschaftlichen Erkenntnis*, Károly Kókai exhibits the two editions of Fischer's book, which approaches epistemology through medical and biological examples. Kókai indicates its connection to the Vienna Circle and places it in a wider context. (More on Fischer's life and work can be found in Gábor Boros (editor), ***Anton Fischer (1901–1976): Sein Leben und sein Verhältnis zur Philosophie des Wiener Kreises***, Mitteilungen der Deutsch–Ungarischen Gesellschaft für Philosophie 3, 2009, 80 pp., `http://www.doki.net/tarsasag/labor/upload/labor/document/Boros_NFMT3_Fischer_HT.pdf`)

András Máté's essay, *Die Rezeption der neuen Logik in Ungarn*, focuses on the philosophical work of László Kalmár, Rózsa Péter and Imre Ruzsa. Kalmár and Péter, lifelong colleagues, were the ones who introduced logic to the Hungarian university curricula. Moreover, Kalmár is usually considered as the founder of computer science in Hungary. Although Péter did not explicate a philosophical standpoint, her remarks on the philosophy of mathematics are discernible through her popularizing book ***Playing with Infinity*** (Translated by Zoltan P. Dienes, Bell, London, 1961, 268 pp.). On the other hand, Kalmár expounded his peculiar fallibilist and empiricist philosophy of mathematics in several places. One of Kalmár's most important philosophical papers, *The Development of Mathematical Rigor from Intuition to Axiomatic Method*, appears in this volume, the first time in English. Originally it appeared in Hungarian in 1942. In this paper Kalmár claims that mathematics is not infallible and that its exactness and reliability is a result of a long development and it is not absolute. (His other philosophical essays available in English are *An Argument Against the Plausibility of Church's Thesis* in Arend Heyting (editor), ***Constructivity in Mathematics***, North–Holland, Amsterdam, 1959, pp. 72–80 and *Foundations of Mathematics—Whither Now?* in Imre Lakatos (editor), ***Problems in the Philosophy of Mathematics***, North–Holland, Amsterdam, 1967, pp. 187–207.) Imre Ruzsa's work concerned philosophical, modal and intensional logic and foundations of mathematics. In 1971 he founded the first Logic Department in Hungary at Eőtvős University, Budapest. In the remaining part of the essay, Máté gives an overview of the logic scene in Hungary between the two world wars.

The volume contains three papers from the members of the Algebraic Logic Research Group of Rényi Institute of Mathematics. In *Circle and Logical Analysis of Relativity Theory*, Hajnal Andréka, Judit Madarász, István Németi, Péter Németi and Gergely Székely survey their work on the foundations of special and general relativity theory and its connections to the Vienna Circle. Their goal is to build up relativity theory in a first order axiomatic framework as a hierarchy of theories. These investigations, born in the spirit of 'reverse mathematics', make transparent the logical structure of relativity theory, that is, they show which predictions follow from which axioms.

Judit Madarász and Gergely Székely proceed in a similar way. In *Comparing Relativistic and Newtonian Dynamics in First-Order Logic*, they axiomatize these theories in a way that they differ only in one axiom. The surprising result of the paper is that the seemingly harmless assumption that 'a system of point masses can be substituted by one point mass living on the center-line of the system point masses and having mass equal to the sum of the mass of the system' follows from Newtonian dynamics but it is inconsistent with relativistic dynamics.

In *On Why-Questions in Physics*, Gergely Székely sets the first steps toward the formalization of the method of 'reverse mathematics' applied by their research group. First, Székely defines when an axiomatic system can be considered as an answer to a why-question. Then he makes suggestions how to compare different answers and provides several examples from axiomatic relativity theory.

Ladislav Kvasz introduced a method to reconstruct the development of geometry and algebra. His method, based on Wittgenstein's Picture Theory of Meaning, interprets the development of scientific theories as changes of the pictorial form (*History of Geometry and the Development of the Form of Its Language*, ***Synthese***, vol. 116 (1998), no. 2, pp. 141–186). In the essay appearing in this volume, *Classical Mechanics between History and Philosophy*, he investigates the development of classical mechanics from Newton to Lagrange.

According to the prevalent view, although the Einsteinian Special Relativity and Lorentz's rival theory are empirically equivalent theories, they are two different theories about space-time. László E. Szabó claims that this difference is illusory and is due only to the fact that these theories call something else space-time. In *Lorentzian Theories vs. Einsteinian Special Relativity—A Logico-empiricist Reconstruction*, his aim is to show that these theories "are completely identical (...), as theories about space and time and as theories about the behavior of moving physical objects" (p. 192)

The connections between Hungary and the Vienna Circle, or, in a broader context, between Hungarian and Austrian science and philosophy, have not received enough scholarly attention. This cooperation between the Institute Vienna Circle and the Department of Logic at Eőtvős University made the first step and showed that this is a topic worth of further research. This book is a good starting point and could be used as a source book of research topics.

MÁTÉ SZABÓ
Department of Philosophy, Carnegie Mellon University, USA. mszabo@andrew.cmu.edu.

Finite and algorithmic model theory, edited by Javier Esparza, Christian Michaux, and Charles Steinhorn, London Mathematical Society Lecture Note Series. Cambridge University Press, 2011, 356 pp.

The "and" in the book's title indicates that it will range over two inter-related areas: the study of logics on finite structures, and the murkier "algorithmic model theory"—a newish term for the study of structures that have presentations that are not only effective, but lead to specialized decision procedures. Neither topic subsumes the other, since the models may be infinite, while the model theory of finite structures can be considered from a non-algorithmic perspective. The volume does not aim for a general theory of either topic—the algorithmic model theory covered here is quite different from computable model theory or feasible model theory, and the finite model theory chapters are not meant to be comprehensive or tightly-connected. Instead the book looks at a few threads within each area, many of which involve interaction between model theory and theoretical computer science.

Algorithmic model theory is represented in the first two chapters. The opening chapter of Bárány, Grädel, and Rubin is a broad view of "algorithmically tame" structures: ones that have presentations that guarantee decidable satisfaction problems for a variety of logics. They give a sort of taxonomy of such structures, depending on whether they are presented by a generating set of operations on relations, by closing under interpretations, or directly as a language accepted by a given class of machines. One particular case of algorithmic tameness via graph-theoretic restrictions originates in the work of Rabin, showing that the theory of the complete infinite binary tree is decidable. Extensions of this work have identified decidable classes of infinite graphs via connections to the tree case—e.g., interpretability in the infinite tree. On the side of machine presentations, particular attention is given here to automatic structures—those whose relations are presented as finite state machines. Automatic structures have an obvious motivation from the computational point-of-view, since an automatic presentation leads directly to effective decision procedures. The chapter gives an in-depth view of a number of variants of automatic-structures: over trees vs. over words; on finite words/trees vs. ω-words/trees.

Although the first chapter covers an enormous range of approaches to algorithmic model theory, it focuses on structures that are computably presented, hence countable. The second algorithmic model theory chapter, by Kuijpers and Van den Bussche, deals with structures given by regions definable within the real ordered field. The logics for manipulating these structures quantify not within the structure, but over the reals with an additional predicate for each relation of the structure. The motivation is from spatial information systems in computer science: the input structures represent geometric objects to be manipulated by the system, while the logics represent queries that a user wishes to ask about them. Logicians need not worry that the chapter will be focused too tightly on the application space: the questions dealt with are about expressiveness of logics—for example, which topologically-invariant properties can be expressed in first-order logic. The main theorems apply tools from the model theory of the reals—e.g., o-minimality and local conic structure—to characterize first-order invariant queries.

The remaining four chapters are concerned with finite model theory, with these splitting into two chapters departing chiefly from concerns in model theory, and the last two motivated by questions in computer science.

The first "model-theoretic finite model theory" chapter is Vera Koponen's, dealing with one of the more intriguing aspects of finite model theory, the importance of the fragment L_n of first-order logic consisting of formulas with at most n variables. While there are no non-trivial first-order theories of finite structures, complete finite L_n theories can be quite complex, and one can attempt to classify them in the spirit of stability theory. Koponen's chapter presents sufficient conditions for determining that an L_n theory has arbitrary large finite models. This program was initiated for general L_n theories, aiming for an analog of the Lowenheim–Skolem theorems for L_n, but stability-theoretic restrictions on the theories turn out to give a much cleaner theory. Although one needs a new notion of stability specific to the L_n setting, the result are closely connected with smoothly approximable first-order theories in classical model theory. The connection between L_n and FO issues is discussed here in depth.

The chapter of Macpherson and Steinhorn shifts from particular complete theories to well-behaved classes of finite structures, with no assumption that the class is elementary. The first few sections introduce asymptotic classes of finite structure. These are inspired by the theory of finite fields, and in particular the results of Chatzadakis, Macintyre and Van den Dries giving estimates of the size of definable sets that are uniform in the field. Asymptotic classes take the existence of these estimates as "axioms", and explore their connection to traditional stability-theoretic criteria. The last part of the chapter looks at Robust Classes, directed sequences of structures in which first-order formulas stabilize. Robust Classes allow a notion of limit theory, analogous to Fraïssé limits. In both cases, the chapter considers the notions primarily via example. The connection between Robust Classes and Asymptotic Classes is mainly that they both explore notions of behaviour of finite classes in the limit. There are also key examples in common, such as the classes of finite structures arising from a smoothly approximable structure.

This leads us to the two "Computer Science Finite Model Theory" chapters. The longer of the two is Stephan Kreutzer's, which returns to the issue of restricted classes of structures, but this time focusing not on decidability problems for infinite structures, but on the complexity of satisfaction problems for finite structures: in CS vocabulary "model checking problems". The research program here was initiated by Bruno Courcelle, who showed that the satisfaction of a Monadic Second Order Logic formula in a finite structure could be checked in linear time, provided that the structure was restricted to be "close to a tree"—e.g., captured via the notion of bounded tree-width. The bulk of Kreutzer's chapter deals with a wide-ranging line of research deriving from this. If we fix a class of structures, for which logics is there a collapse in the complexity of satisfaction? Conversely, if we start with a logic, can we identify the maximal class of structures for which the data complexity of satisfaction is tractable. An example result (Theorem 5.85) states that well-behaved classes with tree-width unbounded by a particular power of log(n) cannot have tractable MSO model-checking checking, under reasonable complexity-theoretic hypotheses.

Martin Otto's chapter looks at restricted classes of structures as well, but looking at the structure of models and expressiveness of logics. The chapter is loosely constructed around a set of themes, the chief of which is model-theoretic games. The classical Ehrenfeucht–Fraïssé game is defined, and is then compared with a number of weakenings, including k-pebble games, bisimulations, and guarded bisimulations. Two aspects of the role of games in finite model theory are emphasised. The first is preservation under games as an analog of preservation properties in classical model theory. Indeed, some of the key results of the section are characterizations of sublogics that are preserved by various game equivalences. The second is game equivalence of an arbitrary finite structure to a member of a restricted class of (possibly infinite) structures. For example, arbitrary Kripke structures are bisimilar to trees. The "unfolding" construction for bisimilarity that shows this is the simplest example, but there are more complex constructions needed for refined equivalences such as guarded bisimilarity. The move from finite structures to well-behaved infinite structures such as trees is complemented by an ability to move back to finite graphs by giving up full game equivalence, replacing it with a finite approximation.

So much for the content—now you will want to know, who should read this book? Obviously, people interested in connections of model theory to computer science will find it essential. During the 1990's there was an enormous amount of activity connecting model theory and computer science. This was accompanied (and spurred) by a few essential texts, particularly the DIMACS volume on Descriptive Complexity and Finite Models, the textbook of Ebbinghaus and Flum on finite structures, and the later textbook of Libkin. Since then research activity has continued at a fairly rapid pace, but the sparsity of follow-up volumes has made figuring out the state of research very painful. The appearance of this text is thus an important event, one of the few places to catch a panorama of research activity in the area over the last 10 years.

The only comparison point for this book would be the volume ***Finite Model Theory and its Applications*** (FMTA), by Grädel, Kolaitis, Libkin, Marx, Vardi, Venema, and Weinstein. The chapters of FMTA are more expository and broad, with each chapter overviewing a whole line of work. Most of the chapters here are more demanding than those of FMTA, while mentioning more specialized and recent results. Although the articles here are from the workshop in Algorithmic and Finite Model Theory held in 2006, many (although not all) have been updated to reflect current work, making it the closest thing on the market to the state-of-the-art.

My only warning for the reader is: sharp curves ahead. These papers came out of a workshop that brought together researchers with radically different backgrounds and perspectives. The diversity no doubt made for an electric workshop, but it leads to a volume where each chapter is "something completely different" on every level. The articles differ dramatically in length—e.g., almost 100 pages for the article of Kreutzer to 31 for that of Koponen. They differ greatly in level of detail, with some chapters explaining techniques as well as results, while others staying strictly at the level of definitions and theorems, serving mainly as pointers into the papers. The most significant difference is in the intended function of the chapters. The opening chapter of Bárány, Grädel and Rubin, as well as the chapter of Kreutzer, are good introductions to areas where there is large body of work done spanning a couple decades. The Koponen and Macpherson/Steinhorn chapters deal with topics still in their infancy, discovered and explored almost exclusively by the authors. The Kuijpers/Van den Bussche chapter is somewhere in between, going into some detail on a few aspects of spatial querying, while deferring others to earlier surveys or textbooks. There are chapters here that everyone will want to dip into for results, but few would read from beginning to end; there are others (notably Otto's) that form a tight narrative and can only be read from the opening sentence to the last period without skipping a word. Put in a more positive way: the book has selections to fit any appetite.

In addition to recommending this volume for those already working in one of the two title topics, I would rate it as a "buy" for all logicians who want to get a sense of how the questions of classical model theory mutate when we move away from arbitrary infinite structures to a more restricted domain. One measure of the achievement of a workshop volume is how much it makes you wish you had been at the event—by that criterion I cannot think of a more successful workshop proceedings than this one.

Michael Benedikt

Parks Road, Oxford OX1 3QD, UK. michael.benedikt@cs.ox.ac.uk.

André Nies. *Lowness properties and randomness*. ***Advances in Mathematics***, vol. 197 (2005), no. 1, pp. 274–305.

Bjørn Kjos-Hanssen, André Nies, and Frank Stephan. *Lowness for the class of Schnorr random reals*. ***SIAM Journal on Computing***, vol. 35 (2005), no. 3, pp. 647–657.

Noam Greenberg and Joseph S. Miller. *Lowness for Kurtz randomness*. ***The Journal of Symbolic Logic***, vol. 74 (2009), no. 2, pp. 665–678.

Laurent Bienvenu and Joseph S. Miller. *Randomness and lowness notions via open covers*. ***Annals of Pure and Applied Logic***, vol. 163 (2012), no. 5, pp. 506–518.

Johanna N. Y. Franklin, Frank Stephan, and Liang Yu. *Relativizations of randomness and genericity notions*. ***The Bulletin of the London Mathematical Society***, vol. 43 (2011), no. 4, pp. 721–733.

George Barmpalias, Joseph S. Miller, and André Nies. *Randomness notions and partial relativization*. ***Israel Journal of Mathematics***, vol. 191 (2012), no. 2, pp. 791–816.

Lowness and highness are notions of relative computational strength. A real is said to be low for a relativizable class if the class generated using this real as an oracle is no different from the class generated using no oracle at all, and a real is said to be high for a relativizable class if using it as an oracle is equivalent to using the strongest possible oracle for some particular function. The most familiar use of these terms is based on the Turing jump. In this case, the relativizable class is the Δ^0_2 sets and the strongest oracle is, of course, $0'$, so a real A is said to be low if $A' \equiv_T 0'$ and high if $A' \equiv_T 0''$. Over the past decade, these concepts have been studied extensively in the context of randomness. This allows us to relate randomness and computational strength in a new way: rather than ask which Turing degrees contain random reals, we may ask which Turing degrees contain reals that either make no difference for a randomness notion when used as oracles (in the case of lowness) or correspond to the strongest possible oracle for a randomness notion (in the case of highness). The papers considered in this review address the question of lowness and highness for the classes of weakly 2-random reals (W2R), Martin-Löf random reals (MLR), computably random reals (CR), Schnorr random reals (SR), and Kurtz random reals (KurtzR). It should be noted that each class listed here is a strict subset of all the classes that appear after it.

These notions can be extended to pairs of classes as follows. Given two relativizable classes R and P and a real A, we say that A is in Low(R,P) if $\mathsf{R} \subseteq \mathsf{P}^A$. (We write Low(R,R) simply as Low(R).) We can see that P^A will always be a subclass of P. A real A is in Low(R,P) if R is contained in P^A as well as in P. For instance, $\mathsf{MLR} \subseteq \mathsf{SR}$, and A is in Low(MLR,SR) if $\mathsf{MLR} \subseteq \mathsf{SR}^A$. Note that the class Low(R,P) is only nontrivial when R is a subset of P. On the other hand, we say that a real A is in High(R,P) if its use as an oracle with respect to R generates a class of reals contained entirely within P; i.e., if $\mathsf{R}^A \subseteq \mathsf{P}$. In this case, we only get nontrivial classes if R properly contains P.

The case of Martin-Löf randomness is a particularly interesting one. Low(MLR) is unusual in that its only known characterizations involve concepts related to randomness. In fact, every other class Low(R,P) for which this is known to be the case is identical to Low(MLR). Nies's *Lowness properties and randomness*, ***Advances in Mathematics***, vol. 197 (2005), pp. 274–305 deals with this topic as well as computable randomness. The primary result in this paper is the theorem in §6 stating that a real is low for Martin-Löf randomness if it is K-trivial; that is, if its initial-segment complexity is no more than that of a computable real with respect to a universal prefix-free Turing machine. The construction proceeds via a tree with runs of procedures that each commit, in various ways, to the claim that an arbitrary K-trivial real is low for Martin-Löf randomness. One of these runs, called a golden run, will allow the enumeration of a Kraft–Chaitin set witnessing that the real in question is low for Martin-Löf randomness.

This paper contains several additional results describing the class Low(MLR) and its elements, mostly in the context of K-triviality. However, there is also a proof in §5 that Low(MLR,CR) = Low(MLR). This is proven in an entirely different way: Nies simply constructs a martingale functional L such that if L^A only succeeds on reals that are not Martin-Löf random, then A must be low for Martin-Löf randomness.

Kjos-Hanssen, Nies, and Stephan's *Lowness for the class of Schnorr random reals*, ***SIAM Journal on Computing***, vol. 35 (2005), pp. 647–657 discusses Schnorr randomness and contains characterizations of Low(SR), Low(CR,SR), and Low(MLR,SR). Their methods of proof are entirely different from those in Nies's paper. Each of the classes discussed in this paper can be characterized in terms of traceability. The proofs in §4 that Low(MLR,SR) coincides with c.e. traceability and that Low(CR,SR) coincides with computable traceability are both highly measure-theoretic. The first of these results is obtained using techniques similar to those in Terwijn and Zambella's paper *Computational randomness and lowness*, ***The Journal of Symbolic Logic***, vol. 66 (2001), pp. 1199–1205. To show that any $A \in$ Low(MLR,SR) is c.e. traceable, the authors construct a Schnorr test relative to an arbitrary function $g \leq_T A$ and use the fact that $A \in$ Low(MLR,SR) to construct an c.e. trace for g given a universal Martin-Löf test. To show that any c.e. traceable real A is in Low(MLR,SR), they consider an arbitrary Schnorr test relative to A and use a trace to construct a Martin-Löf test whose intersection contains that of the Schnorr test.

The situation for the weakest of these randomness notions, Kurtz randomness, was finally resolved in Greenberg and Miller's *Lowness for Kurtz randomness*, ***The Journal of Symbolic Logic***, vol. 74 (2009), pp. 665–678. Stephan and Yu had previously proven in *Lowness* *for weakly 1-generic and Kurtz-random*, ***Theory and applications of models of computation***, Lecture Notes in Computer Science, vol. 3959 (2006), pp. 756–764 that every hyperimmune-free degree that is not diagonally noncomputable is low for Kurtz randomness. A degree is diagonally noncomputable, or DNC, if it can compute a function that is not equal to $\varphi_e(e)$ for every e such that $\varphi_e(e)$ converges, so the non-DNC reals are relatively computationally weak. In this paper, Greenberg and Miller completed the characterization of Low(KurtzR) and fully described Low(MLR,KurtzR), Low(CR,KurtzR), and Low(SR,KurtzR). The proofs in this paper are highly computational and involve the definition of a new sort of tree, a svelte tree, which is developed in §§2–4. These trees are then used in §5 to show that no real that is low for Kurtz randomness is DNC. Since every hyperimmune degree contains a Kurtz random real, this is enough to show that the reals that are low for Kurtz randomness are precisely those that are hyperimmune-free and not DNC. Finally, the last section contains characterizations of the last three classes mentioned above. Low(MLR,KurtzR) is shown to be the non-DNC reals, and Low(CR,KurtzR) and Low(SR,KurtzR) are shown to be the non-DNC reals that are not high.

An entirely new approach to lowness for randomness was taken by Bienvenu and Miller in *Randomness and lowness via open covers*, ***Annals of Pure and Applied Logic***, vol. 163 (2012), pp. 506–518. While in the other papers discussed here, the same techniques are generally used to characterize Low(R_1,P) and Low(R_2,P), Bienvenu and Miller use the same technique to characterize Low(W2R,P) when P is MLR, CR, or SR. They begin in §2 by characterizing MLR, CR, and SR by identifying for each an open subset of 2^ω that contains every tail of every real that is not random in the appropriate sense. They then prove a technical lemma in §3 showing that, given a sufficiently nice collection $\mathcal{C}$ of bounded open sets and a prefix-free subset W of $2^{<\omega}$ such that $[W]$ cannot be covered by any open set in $\mathcal{C}$, some element of W^ω passes all tests that can be constructed from the elements of $\mathcal{C}$ and, furthermore, that the collections of open subsets mentioned above satisfy the hypotheses of this lemma. This allows them to describe lowness notions using only properties of open sets. Bienvenu and Miller then use a partial relativization of Martin-Löf randomness in §4 to show that Low(W2R,P) is identical to Low(MLR,P) when P is MLR, CR, or SR. They end the paper with a short proof of Nies's characterization of Low(MLR,CR), new characterizations of lowness notions in terms of prefix-free machines, and a brief discussion of highness for pairs of randomness notions.

Highness for pairs of randomness notions was first introduced by Franklin, Stephan, and Yu in *Relativizations of randomness and genericity notions*, ***The Bulletin of the London Mathematical Society***, vol. 43 (2011), pp. 721–733. The results in §2 concern High(SR,CR), High(SR,MLR), and High(SR,W2R). Each of these classes coincides with the class of reals that do not Turing compute the halting problem. The proof of each is immediate from the following result: for every real $A \not\geq_T 0'$ and every real B, there is a real R such that $R \geq_T B$, R is not computably random, and R is Schnorr random relative to A. This can also be used to characterize the bases for Schnorr randomness. The authors prove this result by constructing R by finite extensions, producing a computable martingale that does not succeed on R, and showing that no martingale computable from A can succeed on R quickly enough for R to be non-Schnorr random relative to A. The authors also provide a partial characterization of High(CR,MLR) in §4: this class contains every PA-complete real, and every real in this class computes some Martin-Löf random real; however, not every Martin-Löf random real is in High(CR,MLR). Highness for pairs of genericity notions is also explored.

Finally, Barmpalias, Miller, and Nies's paper *Randomness notions and partial relativization*, ***Israel Journal of Mathematics***, vol. 191 (2012), pp. 791–816 also discusses highness for pairs of randomness notions. The primary focus of this paper is randomness notions between Martin-Löf randomness and Martin-Löf randomness relative to $\mathbf{0}'$, or 2-randomness. Such notions include Schnorr randomness relativized to $\mathbf{0}'$, weak 2-randomness, and the combination of Kurtz randomness relativized to $\mathbf{0}'$ and Martin-Löf randomness. Section 3 contains characterizations of the reals that are high for several pairs of these randomness notions, including High(MLR,$\mathsf{W2R}$). This class is simply the class of reals A such that $\mathbf{0}'$ does not compute a function that is DNC in A. Once again, the proof is measure-theoretic. It should be noted here that their techniques can be used to give shorter proofs of most of the results in Greenberg and Miller's paper. The rest of the paper is devoted to a discussion of a reducibility based on weak 2-randomness.

Johanna N. Y. Franklin

Department of Mathematics, 196 Auditorium Road, University of Connecticut, U-3009, Storrs, CT 06269-3009, USA. johanna.franklin@uconn.edu.