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THE PHILOSOPHY OF LOGIC

PENELOPE MADDY

Abstract. This talk surveys a range of positions on the fundamental metaphysical and epistemological questions about elementary logic, for example, as a starting point: what is the subject matter of logic—what makes its truths true? how do we come to know the truths of logic? A taxonomy is approached by beginning from well-known schools of thought in the philosophy of mathematics—Logicism, Intuitionism, Formalism, Realism—and sketching roughly corresponding views in the philosophy of logic. Kant, Mill, Frege, Wittgenstein, Carnap, Ayer, Quine, and Putnam are among the philosophers considered along the way.

Let me begin by saying that it was an honor and a privilege to serve a term as President of the ASL. The Association is in many ways the most effective, productive, ambitious professional organization I know of, and its most active members—the people I was fortunate to work with during my time in office—are not only dedicated and selfless champions of logic and logicians around the world, but also imaginative and innovative leaders, always open to new directions and new undertakings. I'm immensely grateful to my fellow officers, and the many editors and committee members, for all their good efforts, and also to various old hands of the Association who helped me so much with their advice and wise counsel.

When the Nominating Committee approached me about standing for the presidency, they made clear that my charge would be to reinforce, or even to some extent to reinstate, the traditional role of philosophy in the Association. Back in the 1930s, when the ASL was founded, key players like Alonzo Church and C. I. Lewis comfortably spanned what we now think of as a stark disciplinary divide between mathematics and philosophy, not to mention those who stood at the origins of what we now think of as computer science. As mathematical logic flourished in subsequent decades, it came to dominate the Association's official organ, the ***JSL***, so the new and growing areas of philosophical logic began to move elsewhere. Meanwhile, more

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purely philosophical and historical studies were eventually left to find their place in ***The Bulletin of Symbolic Logic***, though only when they could fit the ***BSL***'s defining profile, as work of interest to and accessible to a wide range of the logic community.

Given this history, it's gratifying that the newly founded ***The Review of Symbolic Logic*** now provides a more comprehensive home for the full range of work in philosophical and non-classical logics, in formal epistemology, in philosophy and methodology of mathematics, in history and philosophy of logic, and in applications of logic to computer science and such fields as linguistics and cognitive science. When the new journal was still in the discussion phase, I was pleasantly surprised to find ASL members even from the remotest parts of mathematics insisting on the importance of philosophy and related fields to the ongoing health and vitality of the Association; more than one expressed the firm conviction that its multi-disciplinary reach is the life's-blood of the organization, what gives it its unique purpose and character. With the ***RSL***, that strength and shared purpose is re-affirmed. I'm told that the ***Review*** is already receiving more good papers, across its whole range of subjects, than could be fitted into the initial contractual page count, and that the ASL and Cambridge University Press have recently agreed to boost that number for the 2011 volume.

Participating in these happy developments gave me ample opportunity to reflect on the evolving role in the Association of issues on the non-technical, what might be called the 'prose' side of philosophy. Back in the 30s, questions about the nature of mathematics—its subject matter, its proper methods, its foundations—were closely intertwined with technical logical questions about formalization, proof theory, set-theoretic reduction, and so on. Gradually, as technical logic became mathematical logic, the philosophical questions came to be approached as pure metaphysics and epistemology, without much attention to mathematical detail, or to anything beyond arithmetic and elementary geometry, for that matter. More recently the philosophical study of mathematics has returned to a careful examination of the practice of mathematics; though this study is often less technical than in the 30s, more broadly methodological, it does come back into contact with mathematical logic itself, now as a full-fledged branch of mathematics, and a branch particularly ripe for methodological analysis, given its foundational role. In these and other ways, the philosophy of mathematics and mathematical logic have remained inter-twined over the years, if the nature of their interconnection has sometimes shifted along the way.

In the course of these ponderings, I was struck by what still seems to me a surprising anomaly: though the philosophy of mathematics has been an ongoing, well-recognized component of the ASL's disciplinary mix, the 'philosophy of x' that one might expect to be most prominent in the publications and on the programs of the Association for Symbolic *Logic*—that is, the philosophy of *logic*—has been much less in evidence. What I have in mind here isn't philosophical logic—a technical discipline centered on formal systems of particular interest to philosophers—or even philosophical questions about particular logical topics—the significance of Gödel's theorems, the nature of logical constants, the status of higher-order logics, and so on. What I have in mind, rather, is the sort of fundamental philosophical questions that run parallel to the questions typically asked by philosophers of mathematics: not 'what is the subject matter of mathematics—what makes its truths true?' but 'what is the subject matter of logic—what makes logical truths true?'; not 'how do we come to know the truths of mathematics?' but 'how do we come to know the truths of logic?'[1]

Among the membership of the ASL, a passing familiarity with the three great schools in the philosophy of mathematics at the turn of the 20th century—Logicism, Intuitionism, and Formalism—is fairly common coin; there's also a general awareness that Logicism, the idea that mathematics is reducible to logic, soon gave way to Set-Theory-ism, to the idea that mathematics is reducible to set theory, a view now often identified with Realism or Platonism about mathematical objects.[2] But I couldn't help wondering if the various schools of thought on the nature of logical truth are comparably well-known. Perhaps not.

Of course, there are historical and sociological explanations for this, beginning with the way mathematical logic grew out of early explorations in the foundations of mathematics by such seminal figures as Cantor and Frege. But given that this is, after all, the Association for Symbolic *Logic*, I thought I might presume upon your patience today, and instead of talking yet again about the philosophy of set theory,[3] I might try to lay out a few of the central positions in the philosophy of elementary logic, something like the counterparts to the familiar Logicism, Intuitionism, Formalism, and Realism or Platonism in the philosophy of mathematics.[4] This taxonomy will naturally reflect my personal predilections, and I reserve the right to give my own views the final word,[5] but I hope these flaws can be forgiven in a retiring address. In any case, you stand forewarned.

[1]As we'll see, one way of responding to questions like these is to reject their presuppositions—for example, to deny that logic has a subject matter—but I hope these simple formulations serve to open the discussion and to illustrate the general type of question at issue. Also, though I speak of logical truth, no contrast with logical inference is intended: the two are treated here as more-or-less interchangeable (the inference from p to q is logically valid iff $p \rightarrow q$ is a logical truth). Finally, let me remark that no analysis of what counts as a logical truth is presupposed: the plan is just to inquire into the status of particular truths that as a matter of fact most people would classify as 'logical'.

[2]I personally wouldn't endorse this identification (see [2011]).

[3]For this topic, see [2011].

[4]Viewing various disparate approaches to the nature of logical truth through this particular organizing lens inevitably involves some selective attention and even some occasional fudging (e.g., see footnote 6), but I hope the exercise is of some interest nonetheless.

[5]But only a brief word. For a more complete presentation, see [2007], Part III.

Let's start by getting some markers out onto the table. First we can agree that the logical counterpart to Logicism isn't worth much—to say that logic is reducible to logic in whatever sense doesn't count as news—but we can ask for a counterpart to Realism in the philosophy of mathematics: a view according to which logical truths represent objective, mind-independent facts. But what exactly are these logical facts facts about?, what is the subject matter of the science of logic?, what's the counterpart to sets, functions, and Hilbert spaces? If the answers are to run parallel with Platonism in the philosophy of mathematics, the subject matter should be some realm of abstracta, and the truths about these abstracta should be necessary, that is, not just contingent truths about the way this particular world happens to be, but necessary truths about the way any possible world must be. And sure enough, on at least one reading of Frege,[6] he offers us a position of this general stripe.

Consider a simple inference: if a coin either came up heads or came up tails, and it didn't come up heads, then it must have come up tails.[7] Frege tells us that a sentence like 'the coin came up heads' expresses a thought:

> The thought, in itself imperceptible by the senses, gets clothed in the perceptible garb of a sentence, and thereby we are enabled to grasp it. We say a sentence *expresses* a thought. (Frege [1918], p. 328)

A thought, in turn, is an objective abstract object:

> Thoughts are neither things in the external world nor ideas. . . . A third realm must be recognized. Anything belonging to this realm has it in common with ideas that it cannot be perceived by the senses, but has it in common with things that it does not need an owner so as to belong to the contents of his consciousness. Thus for example the thought we have expressed in the Pythagorean

[6]I hope the specialists will forgive me for focusing, for rhetorical purposes, on only one of the various hotly-debated approaches to Frege: the resolutely Platonistic line of interpretation championed, e.g., in Burge [1992]. Even on this reading, some fudging is required on necessity: according to Burge, though Frege doesn't employ the full modal notion of necessity, his insistence on the generality of logic, combined with the breadth of his abstract ontology, 'have some of the force and effect of necessary truths that purport to quantify over possible objects or possible worlds' (Burge [2000], p. 370). I include necessity—understood in terms of the intuitive heuristic 'true in all possible worlds'—as part of my characterization of Platonism because it's typically present in versions of the view in the philosophy of mathematics, and because it makes for the clearest contrast with other forms of Realism to be considered later.

[7]As suggested at the end of footnote 1, I take this to be an uncontroversial case of what's commonly thought of as a logical truth or a logically valid inference. (Relevance logicians reject disjunctive syllogism as a general rule, but this particular case should satisfy any additional requirements of relevance.) By focusing on a simple example, I hope to side-step debates over matters of classification—what counts a logical consequence or a logical constant?—and to highlight instead the question of grounding—what makes this claim true, this inference reliable?

> Theorem is timelessly true, true independently of whether anyone takes it to be true. (Frege [1918], pp. 336–337)

Frege argues that thoughts pick out other denizens of the third realm—the truth values, True or False—much as (the senses of) names pick out their referents (see, e.g., Frege [1892], pp. 156–159); furthermore, words like 'or' or 'not' do similar duty for maps that take truth-values, or pairs of truth-values, to truth-values (see, e.g., Frege [1891], p. 147, [1893], p. 217). So the story for our little inference would go: the simple sentences pick out truth-values; the logical words pick out truth-value-mappings; the properties of those mappings guarantee that if the premises are true, the conclusion must be true. This is an objective, timeless fact about the relations between various items in the third realm.[8]

Now it's worth asking why we should believe in the abstracta posited to form the subject matter of this account. One consideration that apparently moves Frege here is the fact that we manage to communicate:

> A thought does not belong specially to the person who thinks it ... everyone who grasps it encounters it in the same way, as the same thought. Otherwise two people would never attach the same thought to the same sentence, but each would have his own thought; and if, say, one person put $2\times 2 = 4$ forward as true whilst another denied it, there would be no contradiction, because what was asserted by one would be different from what was rejected by the other. (Frege [1897], p. 233)

Interesting that Frege seems most concerned that 'there would simply be no common ground to fight on' (op. cit., p. 234), but fight or no fight, the fundamental worry is that if thoughts were subjective, I couldn't communicate my thought to you, because your thought would inevitably be different from mine.[9]

How we manage to communicate in language is obviously a big topic, but for what it's worth, it seems to me unlikely that our best scientific explanation of successful human language use will ultimately take this form. Another nagging worry is the well-known epistemological problem of how we human

[8] I've over-simplified the Fregean metaphysics here. It might be more accurate to say that for Frege, the inference is valid because a certain second-level concept—(((Fx or Gx) and not-Fx) implies Gx)— with three unsaturated slots (one for objects, two for first-level concepts) falls under the third-level concept 'always instantiated, whatever goes in the unsaturated slots'. This isn't a claim about the coin or the properties of coming up heads or coming up tails; indeed, it would seem to belong among those whose 'justification is carried in their own contents' (Burge, [1998], pp. 337–338). In any case, the main point for our purposes is that the laws of logic 'are general laws which concern—"what is"—in the third realm' (Burge [1992], p. 313). (Burge is alluding to Frege [1918], p. 325: 'it is not a matter of what happens but of what is'.) Thanks to Jeremy Heis for his help on this point.

[9] Cf. Burge [1992], p. 305: Frege 'claims, more than once, that the assumption of the relevant entities explains the intersubjective objectivity of science and communication'.

cognizers come to know about abstracta at all,[10] but this worry is premature: until the third realm earns its scientific stripes, we needn't concern ourselves with the question of access; and if it were to earn those stripes, the particulars of how it did so should form the basis of our epistemological theorizing. In any case, this is our Platonistic option: the subject matter of logic is abstract; its truths are necessary.[11]

The second famous school in the philosophy of mathematics is Intuitionism, which initially grew out of the Kantian idea that mathematical objects are constructed in pure intuition. For Kant, our cognition of the world involves two distinct faculties: the sensibility, which shapes the raw 'matter' of experience into spatiotemporal form; and the understanding, which brings the sensibility's empirical intuitions under pure concepts, like the concept of an individual object, or the concept of causal connection.[12] The sensibility's forms of intuition and the understanding's pure categories are present a priori—that is, we bring them to experience without the aid of any previous experience—and as a result, we can know a priori that the world we experience must be composed, for example, of spatiotemporal objects standing in causal relations.

Kant held that mathematics is the study of these forms of intuition—space and time—independent of any 'matter' of experience. In geometry, for example, we might construct a triangle, a three-sided figure, in our pure intuition, add a line through the apex parallel to the base, and reason from there that the three interior angles must add to two right angles. No amount of pondering the mere concept of a three-sided planar figure will yield this result, but the forms of intuition help shape the path of the construction, and those forms of intuition Kant took to be Euclidean. Alas, by the late 19th century, even before non-Euclidean ideas made their way into physical geometry, mathematical geometry had outgrown this Kantian picture: points at infinity and points marked by complex coordinates outstripped any simple spatial intuition.

The Intuitionists, beginning with Brouwer, saw this as refutation of Kant's a priori intuition of space, but they hoped to found mathematics instead on our a priori intuition of time. Understood in orthodox Kantian terms, this would mean that mathematics is true of the world, and we can know this a priori, because the world we experience, the only world available to us, is partly shaped by our temporal form of intuition. This is one portion of Kant's transcendental Idealism.

[10] I have in mind here the logical counterpart to the famous Benacerraf problem (Benecerraf [1973]).

[11] Again, this isn't exactly Frege's own position. See footnote 6.

[12] A terminological nicety: Kant does describe the pure category here as 'cause and effect' (A80/B106), but until it's schematized (see below), no spatiotemporality is involved. As our current notion of causal connection would seem to involve both spatial and temporal elements, this usage can be misleading, so from now on I'll refer to the pure category as 'ground-consequent', reserving 'cause and effect' for the schematized category.

To call this Idealism 'transcendental' is to distinguish it from the claim that spatiotemporality or causality are products of our particular human psychology, the sort of thing studied by empirical psychologists; for Kant, ordinary empirical science studies the world we experience, and that world has an objective spatiotemporal and causal structure, entirely independent of human psychology. From the ordinary empirical perspective, there's a clear distinction between appearance and reality, for example, between a rainbow and a rain-drop:

> We would certainly call a rainbow a mere appearance in a sun-shower, but would call this rain the thing in itself, and this is correct . . . in a merely [empirical] sense, as that which in universal experience and all different positions relative to the senses is always determined thus and not otherwise in intuition. (A45/B63)

But when we engage in Kant's critical philosophizing, when we inquire into our a priori knowledge of the world, we gain a new understanding of the empirical object:

> If we consider this empirical object in general and . . . ask whether it . . . represents an object in itself, then the question of the relation of the representation to the object is transcendental [as opposed to empirical], and not only these drops are mere appearances, but even their round form, indeed even the space through which they fall are nothing in themselves, but only mere modifications or foundations of our sensible intuition. (A45-6/B63)

Thus Kant combines empirical Realism—from an empirical point of view, the raindrops are objectively real and the rainbow subjectively ideal—with transcendental Idealism—from a transcendental point of view, the raindrops are partly formed by the structure of our cognition and the real thing as it is in itself is entirely unknown to us.

Now where does logical truth fit into this picture?[13] Here Kant's interest is in discursive knowers, that is, beings who cognize the world using concepts, which essentially means any beings other than God. For discursive cognition to get off the ground, a discursive knower must have some concepts in place independently of experience, a priori, and these concepts, the twelve pure categories, correspond exactly to the twelve logical forms that a discursive judgment can take. So, for example, in order to cognize an individual object, a discursive knower needs the pure category of an object-with-properties, and this corresponds in turn to the logical form subject-predicate; to cognize one state of affairs as depending on another, he needs the category ground-consequent, which corresponds to the logical form if–then.

[13] Here again I opt for one among many interpretations, this time my own reading of Kant on logic. For a more complete discussion, see [2007], § III.2.

When these pure categories are schematized, that is, brought into accord with our peculiarly human forms of intuition, they become spatiotemporal-object and cause-and-effect, but the forms of sensibility of other discursive knowers needn't be the same as ours; still, the logical forms and pure categories govern the cognition of any discursive knower, whatever his particular form of intuition. Just as the world we humans experience must be spatiotemporal, the world any discursive knower experiences must be structured as the forms of judgment and pure categories dictate: for example, as objects-with-properties standing in ground-consequent relations. Viewed transcendentally, the logical laws will be true in such a world because of the structuring the discursive knower brings to it,[14] but viewed empirically, they are simply objective features of the world itself. In this way, much like the case of mathematics, logical truth is empirically real, but transcendentally ideal.

So we have here an account of the nature of logical truth that runs parallel to the Kantian account of the nature of mathematics, except that logic is more general: it applies to the world as experienced by a more general class of knowers. Speaking empirically, as ordinary scientists, we say if the coin came up either heads or tails, and it didn't come up tails, it must have come up heads, and that this is true independently of us, because of the way the world is. Speaking transcendentally, we say that the whole structure of objects like coins, with properties like landing heads or tails, and so on—all this is imposed by our discursive forms of judgment and pure categories. Viewed this way, logic is the study of our form of cognition, much as the intuitionist's mathematics is the study of our mental constructions. This is one form of Idealism in the philosophy of logic.

Notice, though, that this particular idealistic view of logic carries with it a strong Idealism about the ordinary physical world as well.[15] Might there be a way of regarding logical truth as subjective, as somehow dependent on us, without sweeping the whole world up into the same basket? One straightforward way of doing this would be to regard logic as the study of the ways humans actually infer, as a branch of empirical psychology. Views of this sort were held, for example, by two distinct schools of thought in 19th-century Germany: the so-called 'scientific materialists', who traced our patterns of inference to physiology and from there to chemistry and physics,[16] and the group of neo-Kantians who interpreted (or reinterpreted) Kant's transcendental psychology in terms of ordinary empirical psychology.[17] Nowadays I

[14]Strictly speaking, logic depends only on the forms of judgment, but a world structured by the forms of judgment *is* a world structured by the pure categories.

[15]Insofar as it follows Kant (or Husserl? See van Atten [2004], chapter 6), Intuitionism would also appear to involve a broader Idealism.

[16]See Sluga [1980], chapter 1, or Gregory [1977].

[17]See, e.g., Anderson [2005] for discussion.

think most philosophers are persuaded by a simple consideration raised by Frege:[18]

> logic is concerned ... not with the question of how people think, but with the question of how they must think if they are not to miss the truth. (Frege [1897], p. 250)

The view of logic being rejected here is usually called Psychologism.

Another way of taking logical truth to be dependent on us, one more popular today, is the idea that logical truth arises from the way we use language. In the words of A. J. Ayer:

> [the] validity [of a logical law] depends solely on the definitions of the symbols it contains ... (Ayer [1936], p. 78)

In drawing a logical conclusion, 'I am simply calling attention to the implications of a certain linguistic usage', and thus

> there is a sense in which [logical inferences] do give us new knowledge. They call attention to linguistic usages, of which we might otherwise not be conscious. (Ayer [1936], pp. 79–80).

Another positivist, Hans Hahn, puts it this way:

> If we remind ourselves of the way in which the designations 'red', 'not red', 'blue', 'red or blue', etc. are applied to objects, we can ... assert ... : everything to which both designations 'red or blue' and 'not red' are applied, is also designated as 'blue'—which is usually put more briefly: if a thing is red or blue and not red, then it is blue. (Hahn [1933], p. 156)

This depends only on 'the sense in which the logical words "not" and "or" are used' (op. cit.). The same would obviously go for our example of the flipped coin. This might qualify as a sort of linguistic Idealism: as a community, we come together to establish and evolve patterns of shared usage, and along the way, we generate some logical truths.

Faced with this suggestion, it's hard not to worry that explanation has stopped too soon. What's to keep the Fregean from agreeing that logic reflects our linguistic practices, then going on to insist that those practices themselves arise from the properties of abstracta in the third realm?—whereupon the view collapses into Platonism. Or, on the other hand, an observer sympathetic to Psychologism might say instead that those linguistic practices are as they are strictly due to the particulars of human cognitive

[18]Or, as Boole ([1854], § II.6, p. 408) put it almost half a century earlier: 'The mathematical laws of reasoning are, properly speaking, the laws of *right* reasoning only.' Despite calling his book ***The Laws of Thought***, and despite having introduced it as an investigation of 'the fundamental laws of those operations of the mind by which reasoning is preformed' (Boole [1854], p. 1), Boole recognizes in the end that what's at issue isn't how we in fact reason, but how we should reason.

structure. Ayer explicitly considers a concern somewhere in this neighborhood:

> It has ... been suggested that my treatment of [logical] propositions makes them into a sub-class of empirical propositions. For I sometimes seem to imply that they describe the way in which certain symbols are used, and it is undoubtedly an empirical fact that people use symbols in the ways that they do. (Ayer [1946], p. 16)

This thought prompts Ayer to make room for some further explanatory grounding for logical truths:

> I allow that the usefulness of [logical] propositions is founded both on the empirical fact that certain symbols are used in the way that they are and *on the empirical fact that the symbols in question are successfully applied to our experience*. (Ayer [1946], p. 17, emphasis mine)

Here Ayer traces the ground of logical truth to empirical facts about language usage *and* to the underlying facts that make such usage effective; along these lines, we might call attention to the various physical facts that constrain the evolution of our language. But he steps back from the apparent consequence that logical truth rests on more than language by pointing out that logical statements don't 'describe these facts in the sense in which empirical propositions may describe the facts that verify them' (op. cit.).

To sort this out, consider a simple case. Suppose I now declare to you that for the duration of this lecture, I'm going to use the name 'George' to refer to my left hand, and furthermore that I'm also going to use the name 'Edward' for the same purpose. I then assert that 'George is the same as Edward'. I assume we'd agree that this assertion is true, but what makes it true? Nothing much about my left hand really matters here, except for its being an individuated physical object suitable for naming; apart from that, what makes the thing true is those stipulations I just made, despite the fact that 'George is the same as Edward' doesn't describe those stipulations. So the question we've been asking—what makes logical truths true?—isn't the same as the question Ayer is asking—what do logical truths describe? If our question—what makes logical truths true?—is to be answered along Ayer's linguistic lines, it's hard to see that only facts directly dependent on us will be involved, that ordinary physical facts about the world we live in won't also serve to shape our language. Come to that, we might ask our Psychologist about the role of such ordinary physical facts in the shaping of the psychological patterns that he takes to underlie logic. In both cases, we might begin to doubt that the full story would continue to count as a version of Idealism.

I think the divide between the question I'm asking and the question Ayer is asking can be clarified by looking at Ayer's answer to his question, and at the historical source of that answer. His question is: what do the statements of logic describe? His answer is: 'they do not ... describe any facts at all' (Ayer [1946], p. 17):

> "Either some ants are parasitic or none are" provides no information whatsoever about the behavior of ants, or, indeed, about any matter of fact. And this applies to all [logical] propositions. They none of them provide any information about any matter of fact. In other words, they are entirely devoid of factual content. (Ayer [1936], p. 79)

In a loose way, we might consider this a counterpart to Formalism in the philosophy of mathematics, the kind of Formalism that treats mathematical claims as meaningless, that denies mathematics is in the business of uncovering contentful truths. In both cases, we would meet the challenge of explaining the subject matter of our inquiry—of logic or of mathematics—by denying that it has any subject matter at all.

This idea for the case of logic came to Ayer—and to the logical positivists for whom his book served as a manifesto—from Wittgenstein, more specifically from Wittgenstein's ***Tractatus Logico-Philosophicus***.[19] This remarkable book can be thought of as beginning from a question that runs parallel to the one that inspired Kant: Kant asks 'What must the world be like that we can know it as we do?'; Wittgenstein asks 'What must the world be like that we can represent it, linguistically, as we do?' Kant thinks part of our knowledge of the world is a priori—geometry, for example; to explain this, as we've seen, he posits the sensibility's forms of intuition and the understanding's pure categories. Wittgenstein thinks that we must be able to understand the meanings of our sentences, what it would take to make them true or false, without knowing anything about how the world actually is; on analogy with Kant, he thinks part of our ability to represent the world is a priori, just as Kant thinks part of our knowledge of the world is a priori.

To explain his quota of a priority, Wittgenstein posits that a batch of simple objects exists necessarily, in all possible worlds, that each simple object has a logical form, which determines the ways in which it can and can't combine with other simple objects into atomic facts, and that all the facts of our actual world are fully determined by which of these atomic facts do and don't obtain here. We're able to represent the world, he continues, because our language consists of simple names with the same logical form as the simple objects—that is, with exactly the same possibilities of combination—so we can represent the world as being thus-and-so by combining our simple

[19] Again specialists will note that the ***Tractatus*** has spawned a bewildering array of interpretations. The one sketched here is a fairly standard 'metaphysical' reading (see Stern [2003] for an overview of the various schools of thought).

names thus-and-so, regardless of whether or not the world really is thus-and-so. The actual world doesn't play any role in determining the meanings of our atomic sentences; its only job is to settle whether they are true or false.

Since the atomic facts are entirely independent of each other—all combinations of individual atomic facts are equally possible—we can describe a complete state of the world by what's essentially a truth-table row, a stipulation of which atomic facts obtain and which don't. The meaning of a complex statement, then, can be understood as a specification of which truth table rows make it true and which make it false; to put it in Ayer's terms, the complex statement describes the world as being in one of the states that gets the value True and not in any of the states that get the value False. So what's the meaning of a logical truth? It draws no such division in the space of logical possibilities; it tells us nothing about how the world is or isn't. And now we've arrived at Ayer's view.

But for Wittgenstein this is hardly the end of the story. The key here is his important distinction between saying and showing:

> A proposition *shows* its sense. The proposition *shows* how things stand, *if* it is true. And it *says*, that they do so stand. (Wittgenstein [1922], 4.022)

In other words, a statement shows us what it would take for it to be true or for it to be false; this we grasp independently of how the world happens to be, and in particular, independently of whether or not the given statement is in fact true or false. Furthermore, this meaning that is shown to us is not what the statements says, not what Ayer is asking about. When we reach the case of logical truths and logical falsehoods, they say nothing:

> Among the possible groups of truth-conditions there are two extreme cases. In the one case the proposition is true for all truth-possibilities of elementary propositions. . . . . In the second case the proposition is false for all truth possibilities. . . . The proposition shows what it says, the tautology and the contradiction that they say nothing. (Wittgenstein [1922], 4.46–4.461)

But by focusing exclusively on what logical claims *say*, Ayer ignores the (at least) equally important matter of what they show:

> The fact that the propositions of logic are tautologies *shows* the formal—logical—properties of language, of the world. (Wittgenstein [1922], 6.12)

What logical truths show us is the features of that logical structure shared by our language and the world, the very shared structure that makes representation possible.

So Wittgenstein's answer to my question, as opposed to Ayer's, is that logic is grounded in the underlying structure of language and the world: the structure the world has by virtue of its being one of the possible ways the necessarily existing objects can combine into atomic facts, the structure language must have if it is to represent how those objects happen to be combined. This is a bold and substantive thesis, one that prompts us to ask whether this structure is shared because language copies the world or because the world copies language. The latter would follow up on our Kantian analogy: the logical structure of our language (the way our simple names can combine, and so on) would partly constitute the world-as-represented, much as Kant's forms and categories partly constitute the world-as-experienced; Wittgenstein's account of linguistic representation would replace Kant's transcendental psychology. Here we have a new form of Idealism, a linguistic Idealism quite different from Ayer's.[20]

The other option is that the world comes first, that language copies the world, or more specifically, that language copies the structure that our particular world shares with all possible worlds. This is a kind of Realism: logic reflects an objective structure. Furthermore, logical truths are necessary. Whether or not this adds up to Platonism, as we're using the term here, depends on whether or not the objective structure in question belongs to something abstract. If Wittgenstein's objects were ordinary things, the answer would be no, but they aren't: the existence of an ordinary physical object is a contingent fact, one among the various ways the necessarily existing simple objects can combine. The issue turns on what these simple objects are, but alas Wittgenstein gives us no clue, so best to leave the matter here: on this second reading, Wittgenstein gives us a Realism about logical truth with many of the markers of Platonism.

We'll consider some clearly non-Platonistic forms of Realism in a moment, but first let's look at a more formalistic, more syntactic type of position, the position that in fact provided the historical bridge between Wittgenstein and Ayer. Wittgenstein's idea that logic is somehow empty held great appeal for Rudolf Carnap:

> For me personally, Wittgenstein was perhaps the philosopher who, besides Russell and Frege, had the greatest influence on my thinking. The most important insight I gained from his work was the conception that ... Logical statements are true under all conceivable circumstances; thus their truth is independent of the contingent facts of the world. On the other hand, it follows that these statements do not say anything about the world and thus have no factual content. (Carnap [1963], p. 25)

Here Carnap might seem to be missing the way logical truths can show features of the necessary structure of the world, but this aspect of the ***Tractarian*** view depends on a Wittgensteinian thesis that Carnap rejects:

[20]On analogy with 'transcendental idealism', Stenius [1960], p. 220, suggests the term 'transcendental lingualism'.

> Wittgenstein ... overlooked the fact that there is a multiplicity of possible languages. Wittgenstein ... speaks continually of 'the' language. (Carnap [1934], p. 245)

'Overlooked' may be a bit mild here: the notion of exactly one logical form that language and the world share is central to Wittgenstein's answer to his guiding question, 'how are we able to represent the world?'

In any case, this isn't Carnap's question; his goal is quite different:

> *Philosophy is to be replaced by the logic of science*—that is to say, by the logical analysis of the concepts and sentences of the sciences, for *the logic of science is nothing other than the logical syntax of the language of science*. (Carnap [1934], p. xiii)

In pursuit of this goal, Carnap argues that a wide range of languages can be described in his formal syntax, and that the choice between these languages is entirely unconstrained:

> *In logic, there are no morals*. Everyone is at liberty to build up his own logic, i.e., his own form of language, as he wishes. All that is required of him is that, if he wishes to discuss it, he must state his methods clearly, and give syntactical rules instead of philosophical arguments. (Carnap [1934], p. 52)

This is the famous Principle of Tolerance: we may chose one logic over another for pragmatic reasons, but that logic is no more true or false than any other; issues of truth or falsity are only properly raised after a language, with its logic, has been adopted, and they are then resolved by the means specified in the conventions of that chosen language.[21] Here Carnap gives us something akin to those stipulations I floated earlier about names for my left hand: a language, for Carnap, has its own formation rules (logical axioms or fundamental assumptions of some scientific theory) and transformation rules (rules of inference, or more generally, rules saying which statements in the language count as evidence for which other statements in the language); nothing factual, nothing beyond convenience, constrains our choice of these rules; it's a purely conventional matter.

We can now see how Ayer's position arises from Wittgenstein's logic-is-empty by incorporating Carnap's idea that many languages are possible:

> It is perfectly conceivable that we should have employed different linguistic conventions from those which we actually do employ. But whatever these conventions might be, the tautologies in which we recorded them would always be necessary. (Ayer [1936], p. 84)

[21] Here again I beg forgiveness from the specialists: this simple reading of Carnap, encouraged by Carnap [1950], adopted by Quine, has been vigorously challenged in recent years. Many now deny that Carnap is offering an account of the ground of logical truth, so these more subtle versions of Carnap are beside the point for our purposes. (See [2007], § I.5 for discussion and references.)

The difference between Carnap and Ayer is that Carnap has in mind our deciding to adopt a new language, perhaps a formal language, while Ayer is imagining our natural language as the result of many conventional decisions now mostly lost to history. The fact that the actual evolution of natural language appears to be guided by physical and psychological factors other than explicit, arbitrary choices was what led us to our concerns about Ayer's position. Carnap's focus on explicit, even formal conventions promises to side-step this problem.[22]

Unfortunately, this Conventionalism about logical truth faces another daunting obstacle, one apparently first described by Lewis Carroll in his story, 'What the Tortoise said to Achilles' (Carroll [1895]).[23] Quine later reworked the objection in more sober philosophical terms:

> The linguistic doctrine of logical truth is sometimes expressed by saying that such truths are true by linguistic convention. ... it is impossible in principle ... to get even the most elementary part of logic exclusively by the explicit application of conventions stated in advance. The difficulty is the vicious regress, familiar from Lewis Carroll, which I have elaborated elsewhere [in Quine [1936]). Briefly the point is that the logical truths, being infinite in number, must be given by general conventions rather than singly; and logic is needed then to begin with ... in order to apply the general conventions to individual cases. (Quine [1954], p. 115)

There was no such problem when I stipulated the two names for my left hand, nor would there be if I stipulated the truth of all the axioms of ZFC, but when we try to stipulate the truth of logic itself, we find our explicit conventions must be general, and then that these general conventions are without their intended force unless logic is in fact already available to oversee the derivation of particular logical truths from the generalities. So it appears that the suggestion that 'logic is empty' either masks a deeper, more complex account of what makes logic true (Ayer) or appeals to a straightforward notion of explicit convention that turns out not to be viable for the case of logic (Carnap).

Let's pause a moment to take stock. We've looked at array of ideas on the nature of logical truth, all analogous, more or less, to familiar positions in the philosophy of mathematics. Frege presents a Platonistic view: logical truths report the objective features of abstracta in the third realm. Along the lines of Intuitionism or of Idealism more broadly, Kant gives us his

[22] In practice, it's not clear this promise would be realized. If we choose these conventions over those on pragmatic grounds, the question inevitably arises: why are these conventions more effective than those? The answer to this question might well return us to the very physical and psychological factors that emerge for Ayer. I set this aside to highlight the new concern in the next paragraph.

[23] I give a brief reconstruction in [2007], p. 204.

characteristic combination of transcendental Idealism—logic is true because of the structure of discursive cognition—and empirical Realism—logic is true because of the objective structure of the world. The Psychologists take a more straightforwardly idealistic stance: logical laws are simply the most general patterns of our thinking. Ayer attempts another form of Idealism—logical laws arise from the way we use language—combined with a more formalistic insistence that logical statements are empty. The source of that formalistic idea is Wittgenstein's ***Tractatus***, but there it comes combined with an ambitious account of the ground of logical truth that can be understood either as a form of linguistic Idealism parallel to Kant's transcendental Idealism or as a form of Realism that leans toward Platonism. A purer analogue to Formalism is Carnap's Conventionalism: logical truth arises from explicit syntactic conventions.

The last idea I'd like to explore is the one set aside a moment ago, namely, a non-Platonistic form of Realism. The closest analogue in the philosophy of mathematics to what I have in mind here is a not-very-popular position usually called Empiricism and associated in the first instance with John Stuart Mill:

> Three pebbles in two separate parcels, and three pebbles in one parcel, do not make the same impression on our senses; and the assertion that the very same pebbles may by an alteration of place and arrangement be made to produce either the one set of sensations or the other ... is a truth know to us by early and constant experience—an inductive truth ... The fundamental truths of [the science of Numbers] all rest on the evidence of sense; they are proved by showing to our eyes and our fingers that any given number of objects, ten balls, for example, may by separation and rearrangement exhibit to our senses all the different sets of numbers the sum of which is equal to ten. (Mill [1843], § II.VI.2, pp. 168–9)

Thus arithmetic is a straightforward part of our empirical theory of the world. Mill takes arithmetic to be 'necessary', but by this he means 'no more than certainty'; he denies any 'claim to the character of necessity in any sense beyond this, as implying an evidence independent of and superior to observation and experience' (Mill [1843], § II.VI.1, pp. 164–5). This is Realism—arithmetic reports objective truths—but not Platonism—its subject matter isn't abstract, but the ordinary physical world, and its truths are ordinary contingencies.

This view of arithmetic was ridiculed by Frege as 'Mill's piles of pebbles and gingersnaps' (Frege [1884], § 27, p. 38):

> What a mercy that ... not everything in the world is nailed down; for if it were, we should not be able to bring off this separation, and $2 + 1$ would not be 3! (Frege [1884], § 7, p. 9)

And of course it becomes considerably less plausible as an account of modern infinitary mathematics. But what about elementary logical truth? Though Mill isn't entirely consistent on these matters, he apparently doesn't embrace an analogous position for logic; instead, he treats examples like our inference about the coins as 'merely apparent':[24]

> we were not proving the proposition, but only appealing to another mode of wording it. (Mill [1843], § II.I.2, p. 104)

Still, hints of a simple inductive Realism can be found in Bolzano:[25]

> The only reason why we are so certain that the rules [of syllogistic logic] are valid is because they have been confirmed in thousands of arguments in which we have applied them. (Bolzano [1837], § 315.4, p. 354)

and hints of a non-abstract subject matter turn up in Russell:[26]

> logic is concerned with the real world just as truly as zoology, though with its more abstract and general features. (Russell [1919], p. 169)

In any case, whatever its lineage, the non-Platonic Realist's notion that logic reflects contingent features of the physical world surely deserves a place in our taxonomy.

Unfortunately, the simple inductivism of the pseudo-Millian version is problematic. To see this, consider how we're supposed to know that our coin must have come up tails: we know it came up either heads or tails, we know it didn't come up heads, and we know from experience that disjunctive syllogisms are valid. Two problems arise for this appeal to disjunctive syllogism (DS): how do we know that the various previous cases we've observed count as evidence for DS unless we know the logical relation between a generalization and its instances?, and how do we know that DS is relevant to the present case unless we recognize that same relation? For that matter, how would we know that the coin having come up heads would be a violation of DS unless we knew the law of non-contradiction (LNC)? The general point is that our simple methods of inductive confirmation presuppose many logical relations between statements, so they can't be used to give us knowledge of those logical relations in the first place.

Quine presents a more sophisticated, but still empirical picture of logical knowledge, based on his holism:

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[24]See, e.g., Jackson [1941]. (I discuss this and other readings of Mill in [FBB].)

[25]But then Bolzano might well deny that these inductively supported truths are ordinary contingent truths about the physical world. (See [FBB] for discussion and references.)

[26]In fact, this may well not be what Russell has in mind; he might simply be insisting that the world includes the abstracta (like propositional functions) that constitute the subject matter of logic just as robustly as it does the animals that constitute the subject matter of zoology. (My thanks to Peter Hylton for help with this point.) On the other hand, see Klement [2010].

> The totality of our so-called knowledge or beliefs, from the most casual matters of geography and history to the profoundest laws of atomic physics or even of pure mathematics and logic, is a man-made fabric which impinges on experience only along the edges. (Quine [1951], p. 42)

Quine's thought here is that a typical statement only has consequences for experience when taken in combination with a fairly large collection of auxiliaries, so our claims 'face the tribunal of experience not individually but only as a corporate body' (Quine [1951], p. 41). It might seem that a simple perceptual claim would escape this generalization, but Quine insists that:

> Even a statement very close to the periphery can be held true in the face of recalcitrant experience by pleading hallucination or by amending certain statements of the kind called logical laws. (Quine [1951], p. 43)

Here the laws of logic are part of the web like everything else:

> Having reevaluated one statement we must reevaluate some others, which may be statements logically connected with the first or may be the statements of logical connections themselves. (Quine [1951], p. 42)

As an example of a possible revision of logic, he notes that:

> Revision even of the ... law of the excluded middle has been proposed as a means of simplifying quantum mechanics; and what difference is there in principle between such a shift and the shift whereby Kepler superseded Ptolemy, or Einstein Newton, or Darwin Aristotle? (Quine [1951], p. 43)

Logical truths, then, are known empirically and fallibly along with the rest. We tend to think of logic as fixed and certain only because we're reluctant to revise it, given the wide-spread changes this would demand, and would do so only as a last resort.

For all its sophistication, this Quinean empiricism faces a difficulty not unlike the one described a moment ago for the pseudo-Millian view: it's fairly easy to imagine how the web of belief might evolve to leave out the law of the excluded middle (LEM)—the Intuitionists do without it, after all, as do we ordinary folk when we judge that Joe isn't quite bald but then again isn't quite not-bald, either—but it isn't easy to imagine how the web would fare without, say, the law of non-contradiction. Without LNC, what would even count as a recalcitrant experience? The problem is that the machinery of web maintenance involves at least some logic, so it's hard to see how that much logic can be counted on a par with the rest of the web.[27] Still, the idea that the progress of science could lead us to a different logic just as it

[27] Given that it was Quine who lodged the Lewis Carroll objection so forcefully against Carnap, there's some irony in the fact that both our pseudo-Mill and Quine are subject to a

once led us to a different geometry, that we could be led to a revise our logic on empirical grounds—this idea doesn't depend on a strict or problematic holism.

Just such a picture is presented by Putnam with a focus on the case of quantum mechanics.[28] Experimental evidence there confirms a wide range of oddities: electrons seem not to behave as individuated objects with continuous trajectories; they apparently don't enjoy a definite position and a definite momentum, a definite vertical spin and a definite horizontal spin, at the same time; their behaviors can be intertwined in ways that violate the usual laws of causality; and so on. Quine's passing remark about LEM apparently refers to an early proposal of Reichenbach that a three-valued logic be employed:[29] for example, the claim that a given particle is in a certain position could sometimes take the third truth value, could sometimes be neither true nor false, but indeterminate. This proposal is now generally regarded as 'a nonstarter',[30] but Putnam has a more sophisticated quantum logic in mind.

To see how this works, let u and d be 'the electron has vertical spin up' and 'the electron has vertical spin down', and let l and r be the corresponding claims for horizontal spin left and right. Then it's straightforwardly true that—(u or d) and (l or r)—but it doesn't follow that one of the four possibilities—(u and l), (u and r), (d and l), (d and r)—must hold, because each of these stipulates a determinate vertical spin and a determinate horizontal spin at the same time, just what quantum indeterminacy denies. The quantum logic advocated by Putnam invalidates this distributive law and thus removes the bad inference. A neat trick.

Putnam's hope is that this alternative logic will remove the quantum anomalies and at the same time reinstate our ordinary ways of thinking about objects and their properties and relations at the micro-level; in this way, the distributive law would be empirically falsified and a non-standard quantum logic empirically confirmed.[31] Alas, this happy outcome is not achieved.

more general form of the same problem: some logic has to be presupposed for their accounts of logical truth to be implemented.

[28]See Putnam [1968].

[29]See Reichenbach [1944].

[30]Gibbins [1987], p. 127. For more on quantum logic, with references, see [2007], §§ III.4, III.6.

[31]Putnam doesn't hold that all logical laws are subject to direct empirical dis-confirmation: he distinguishes between those whose falsity we can conceive—like LEM or the distributive law—and those whose falsity we can't conceive—like LNC. This doesn't mean that LNC couldn't turn out to be false, but that it 'cannot be overthrown merely by observatons, but only by thinking of a whole body of alternative theory as well' (Putnam [1995], p. 272). Putnam ([1968], pp. 189–190) also seems to toy with the idea that some changes of logic would be changes of meaning while others would be changes of theory. Of course any such appeal to meaning raises the question we've harped on before: what inclines us toward these meanings rather than those?

Though the absence of one distributive law blocks the derivation of one quantum conundrum, Peter Gibbins has shown that another distributive law is retained and that it permits a very similar confounding derivation.[32] Furthermore, the only bit of common sense attained is that the electron is determinately either spin up or spin down, despite being neither determinately up nor determinately down. But this we knew. Our ordinary ways of thinking demand an objective fact of the matter as to which state the electron is in.

Of course we shouldn't insist that a viable philosophy of logic must also solve the problems of quantum mechanics—that's a lot to ask—so the failure of this particular quantum logic needn't undercut the appeal of a non-Platonistic Realism. We've seen that Quine and Putnam, like our pseudo-Mill, take logic to be one part of our ordinary empirical theory of the physical world. It's striking, though, that apart from Russell's 'more abstract and general features', little is said about what aspects of the world are reflected in the logical parts of our theory. This isn't so surprising for Quine, because his holism dictates that no individual claim answers to the world by itself, but only as part of a 'corporate body', and perhaps Putnam is inclined to follow Quine on this. Those of us who reject this holism, who hold that our individual scientific claims describe different parts of the world, would like a better sense of those 'abstract and general features'. I'll close here with a quick review of my own attempt to do this and of the resulting account of logical truth.[33]

To see how this goes, recall Kant's combination of transcendental Idealism with empirical Realism: viewed transcendentally, logical structure is imposed on the world by our discursive modes of thought; viewed empirically, the world simply displays those structures as a matter of objective fact. Leaving the transcendental bit aside for the moment, what's striking here is that Kant is quite specific about what those objective structures are: they're listed in his Table of Judgments and the corresponding Table of Pure Categories. Just how Kant gets from the bare notion of discursive cognition—that is, cognition through concepts—to the conclusion that all such cognition must take these twelve particular forms is a matter much-debated by Kant scholars and almost universally admitted not to be one of the great man's finest hours; and of course the Tables rely on accounts of logical form now superseded by Fregean innovations. But let's see what we can salvage here.

In Kant's version, empirical Realism holds that the world is made up of individual objects enjoying various properties and that various states of these objects stand in dependency relations to other states of these objects. These features of the world reflect the forms of judgment ('subject-predicate' and 'if–then') and the corresponding pure categories ('object-with-properties'

[32] Gibbins [1987], pp. 147–151.

[33] Again, a more complete account appears in [2007], Part III.

and 'ground-consequent').[34] Fregean updating replaces 'subject-predicate' with 'function-and-arguments' and 'object-with-properties' with 'objects-standing-in-relations'. So post-Fregean empirical Realism would hold that the world consists of objects enjoying various interrelations (including the degenerate case of an object enjoying a property) and that various states of these objects (now more complex relational states) stand in dependency relations, where this structuring now reflects the new post-Fregean form ('function and arguments') and the corresponding pure category ('objects-in-relations').

But what about the transcendental part of Kant's story? If we try to import it more-or-less intact into our ordinary empirical context, it becomes a strong version of Psychologism: the logical structure of the world is a reflection of our cognitive mechanisms, not an objective feature of the world itself. But if the world does in fact have this structure, as our updated empirical Realism asserts, then it wouldn't be surprising that our cognitive mechanisms have evolved to detect it, and in fact, this is just what contemporary developmental psychology suggests.[35] What makes this Psychologism unattractive is its second clause, the denial that these psychological structures reflect objective features of the world itself. If we drop this problematic add-on, a first pass at our updated, non-Platonistic Realism would go like this: logical truths are true because the world is made up of objects enjoying various interrelations with dependencies between them, and we tend to believe some of the simpler of these truths because human cognition has been tuned by evolution to detect these very features.

So far so good, but is it true? Is the world so neatly structured? At first blush, it seems the answer is obviously yes: this sort of structuring is so obvious that we hardly notice it, but there it was, for example, in the background of my stipulations about names for my left hand (in the assumption that it's an individual object, suitable for naming); for that matter, it even comes built into our intuitive notion of a 'possible world'. Unfortunately, if we've learned anything from our brush with quantum mechanics, it's that the 'objects' of the micro-world don't behave according to these simple patterns, don't act as our most fundamental conceptualizations lead us to expect. If logical truth is in fact grounded in such structuring, then our logic should break down in the micro-world, which, as we've seen,

[34]These are only two of the twelve forms of judgment and corresponding pure categories, but I propose to ignore the rest. Also, the schematized categories aren't included here because what matters for our purposes is the a priori features of the world of the discursive knower in general, not the human knower in particular.

[35]Just to be clear, the idea isn't that these cognitive structures have evolved under pressure from natural selection, therefore they can be counted on to deliver truths. (This is a well-known fallacy: even if our cognitive structures have evolved to be fitness-enhancing, what's fitness-enhancing to believe may not be true.) Here empirical Realism comes first and then figures in our accounts of cognitive development and evolution.

it does. But the mysteries of the quantum world don't change the fact that much, indeed most, of the macro-world *is* structured in accord with our ordinary notions: there are individual apples; the presence of this apple tree is a consequence of the planting of that seed. And the fact that much of the macro-world is this way, combined with the fact that we evolved in direct contact with that macro-world, explains why our basic modes of cognition are tuned to these particular structures. So our updated and naturalized Kantian position is that much of the world has these features, that logic applies to a situation insofar as it does have those features, and that our cognitive machinery has evolved to detect those features.

Thus we arrive at a non-Platonistic Realism: logical laws are true in any situation with the right physical structuring; their truth is contingent on the presence of that structuring. Moreover, on the epistemological side, we tend to believe the laws of logic independently of any experience because of our hard-wiring, so we know them in a sense a priori, and we tend to think of them as necessary, that is, we tend to built them into our very idea of a 'possible world'[36]—and all this happens despite the fact that they wouldn't be true if the world were different and in fact don't seem to hold in the actual micro-world. Naturally our language will reflect these fundamental modes of thought, but their grounding goes deeper than that, into the structure of the world we live in: it isn't simply a matter of linguistic convention; we adopt the meanings we do for a reason. Also, though Kant would say that no discursive cognizer can think any other way, it seems best to be more circumspect: because of our hard-wiring, it's very difficult for us to think any other way—as demonstrated by our inability to date to come up with a viable interpretation of quantum mechanics—but there's no clear reason to suppose that this is impossible in principle.

Finally, let me note that the physical structuring of the world doesn't seem to me to validate the full range of classical logic: for example, there are tadpoles and there are frogs, but there are also borderline cases; $2 + 2$ being 4 isn't grounded on the fact that nothing red all over is also green all over. This means that LEM and the material conditional aren't part of this more rudimentary empirical logic; rather, they appear as idealizations introduced into that logic for good reasons. From this perspective, it isn't enough for defenders of various deviant logics to point out that a given aspect of classical logic is an idealization; what's needed is an argument that this idealization is ineffective or misleading, either in particular circumstances or in general. Unless this case is made, and a more effective, less misleading alternative

[36]Descartes espoused an historical precursor to this view: the laws of logic are contingent—God could have made them other than they are—but, having made them as they are, He made us so that we believe them to be not only true but necessarily true. See his [1991], pp. 23–26 (letters to Mersenne in 1630), p. 235 (letter to Mesland in 1644), pp. 358–359 (letter to Arnauld in 1648), p. 363 (letter to More in 1649).

offered, we do best to continue to use classical logic, but to do so with care, just as we use idealized scientific theories with care, recognizing that idealizations can lead the us astray: if we ignore friction too broadly, we can't explain how we manage to walk; if we apply classical reasoning too stubbornly, a vague predicate can lead us into a sorites puzzle.

Thus ends my survey. We've touched on a broad range of historical and contemporary positions on the ground of logical truth, classified very roughly by their resemblance, in the first instance, to well-known schools of thought in the philosophy of mathematics—Platonism, Intuitionism, Formalism—and in the second, to an implausible position in the philosophy of mathematics—non-Platonistic Realism. After Frege, Kant, the Psychologists, Ayer, the ***Tractatus***, and Carnap, this last group included the simple inductivism of our pseudo-Mill, Quine's holism, Putnam's quantum logic, and my own updated and naturalized version of Kant. I hope this leaves you with some sense of the richness of the subject, and perhaps also of its relevance and importance to the intellectual mission of the Association for Symbolic Logic.

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DEPARTMENT OF LOGIC AND PHILOSOPHY OF SCIENCE
UNIVERSITY OF CALIFORNIA, IRVINE
IRVINE, CA 92697, USA
E-mail: pjmaddy@uci.edu