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2011 EUROPEAN SUMMER MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC LOGIC COLLOQUIUM '11

Barcelona, Catalonia, Spain
July 11–16, 2011

Logic Colloquium '11, the 2011 European Summer Meeting of the Association for Symbolic Logic, was hosted by the University of Barcelona from July 11 to July 16, 2011. The lectures were held at the Faculty of Philosophy of the University of Barcelona located in the heart of the city. Major funding for the conference was provided by the Association for Symbolic Logic (ASL), the U.S. National Science Foundation (NSF), the Generalitat de Catalunya, MICINN (Spanish Government), I-MATH Network (Spain), the University of Barcelona, and IMUB (Mathematics Institute of the University of Barcelona).

The success of the meeting was due largely to the hard work of the Local Organizing Committee under the leadership of its Chair, Joan Bagaria (Barcelona). The other members were David Asperó (Barcelona), Albert Atserias (Universitat Politècnica de Catalunya), Fèlix Bou (Barcelona), Enrique Casanovas (Barcelona), Víctor Dalmau (Universitat Pompeu Fabra), Pilar Dellunde (Universitat Autònoma de Barcelona), María Esteban (Barcelona), Ramon Jansana (Barcelona), Joost Joosten (Barcelona), Daniel Palacín (Barcelona), Joris Potier (Barcelona), and David Virgili (Barcelona). The Program Committee consisted of Klaus Ambos-Spies (Heidelberg, Chair), Jeremy Avigad (Pittsburgh), Henk Barendregt (Nijmegen), Enrique Casanovas (Barcelona), Stephan Kreutzer (Oxford), Heike Mildenberger (Freiburg), Andrea Sorbi (Siena), Tinko Tinchev (Sofia), Albert Visser (Utrecht), and Boris Zilber (Oxford).

The main conference topics were: *Computability Theory and Logic for Computer Science, Foundations of Mathematics and Philosophy of Logic, Model Theory, Proof Theory and Constructive Mathematics, and Set Theory*. The program included the Gödel lecture, two tutorial courses, thirteen invited plenary lectures, and twenty invited lectures in five special sessions. There were eighty-nine contributed papers and 214 participants from thirty-two countries. Twenty students and recent Ph.D.'s were awarded ASL travel grants, and four obtained awards from the ASL's NSF grant.

The Gödel lecture was delivered by Anand Pillay (University of Leeds), with the title: *First order theories*.

The following tutorial courses were given:

Georges Gonthier (Microsoft Research), *Formal mathematics in practice*.
Martin Ziegler (Universität Freiburg), *Hrushovski's amalgamation method*.

The following invited plenary lectures were presented:

Steve Awodey (Carnegie Mellon University), *Homotopy type theory*.
George Barmpalias (Chinese Academy of Sciences Beijing), *Chaitin's halting probability and the compression of strings using oracles*.
Lev D. Beklemishev (Steklov Mathematical Institute), *Topological semantics of provability logic*.

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Raf Cluckers (Université Lille), *New transfer principles for motivic integrals*.
Todd Eisworth (Ohio University), *Colorings at successors of singulars*.
Sergey Goncharov (Russian Academy of Sciences), *Problems of autostability and spectrum of autostability*.
Henri Lombardi (Université de Franche-Comté), *Constructive semantics for classical formal proofs*.
Jordi López-Abad (ICMAT Madrid), *Large cardinals and basic sequences*.
Kenneth Manders (University of Pittsburgh), *Strategies in the philosophy of mathematics*.
Martin Otto (Technische Universität Darmstadt), *Tractable finite models*.
Jan Reimann (Pennsylvania State University), *Dualities and dichotomies in algorithmic information theory*.
Andrzej Rosłanowski (University of Nebraska), *News of the creature world—not only forcing*.
Yde Venema (University of Amsterdam), *A modal distributive law*.

The proceedings of Logic Colloquium '11 will be published in a special issue of the Annals of Pure and Applied Logic.

More information about the meeting can be found at the conference webpage:

`http://logic2011.org/`.

Abstracts of invited and contributed talks given in person or by title by members of the Association follow.

For the Program Committee
KLAUS AMBOS-SPIES

Abstract of the invited Twenty-second Gödel Lecture

► ANAND PILLAY, *First order theories*.
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I will discuss logic and model theory, both in terms of history and of current trends, and try to defend the thesis that stable theories are the "perfect" first order theories.

Abstracts of invited plenary talks

► STEVE AWODEY, *Homotopy type theory*.
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Homotopy type theory refers to a new interpretation of Martin-Löf's system of intensional, constructive type theory into abstract homotopy theory. Propositional equality is interpreted as homotopy and type isomorphism as homotopy equivalence. Logical constructions in type theory then correspond to homotopy-invariant constructions on spaces, while theorems and even proofs in the logical system inherit a homotopical meaning. A survey of early results is in [1].

In parallel, Vladimir Voevodsky (IAS) has recently proposed a comprehensive, computational foundation for mathematics based on this homotopical interpretation of type theory. The Univalent Foundations Program posits a new "univalence axiom" relating propositional equality on the universe with homotopy equivalence of small types. The program is currently being implemented with the help of the automated proof assistant Coq. For more on the Univalent Foundations Program, see [2].

This program is currently experiencing a lively development by several collaborators. Much up to date information is collected at [3]. This talk will survey some of these recent developments.

[1] STEVE AWODEY, *Type theory and homotopy*, forthcoming, available at [3].

[2] VLADIMIR VOEVODSKY, *Univalent foundations program*, unpublished manuscript, available at [3].

[3] *Homotopy type theory*, on the web at `www.homotopytypetheory.org`.

► GEORGE BARMPALIAS AND ANDREW E. M. LEWIS, *Chaitin's halting probability and the compression of strings using oracles*.

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If a computer is given access to an oracle—the characteristic function of a set whose membership relation may or may not be algorithmically calculable—this may dramatically affect its ability to compress information and to determine structure in strings which might otherwise appear random. This leads to the basic question, "given an oracle A, how many oracles can compress information at most as well as A?"

This question can be formalized using Kolmogorov complexity. We say that $B \leq_{LK} A$ if there exists a constant c such that $K^A(\sigma) < K^B(\sigma) + c$ for all strings σ, where K^X denotes the prefix-free Kolmogorov complexity relative to oracle X. The formal counterpart to the previous question now is, "what is the cardinality of the set of $\leq_{LK}$-predecessors of A?"

We completely determine the number of oracles that compress at most as well as any given oracle A, by answering a question of Miller [1, Section 5] which also appears in Nies [2, Problem 8.1.13]; the class of $\leq_{LK}$-predecessors of a set A is countable if and only if Chaitin's halting probability Ω is Martin-Löf random relative to A.

[1] JOSEPH S. MILLER, *The K-degrees, low for K degrees, and weakly low for K sets*, ***Notre Dame Journal of Formal Logic***, vol. 50 (2010), no. 4, pp. 381–391.

[2] ANDRÉ NIES, ***Computability and randomness***, Oxford Logic Guides, Oxford University Press, 2009.

► LEV D. BEKLEMISHEV, *Topological semantics of provability logic*.

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Topological semantics of provability logic is well-known since the work of Harold Simmons and Leo Esakia in the 1970s. The diamond modality can be interpreted as a derived set operator acting on a scattered topological space. Although quite natural and complete, this semantics has not been much used in the study of provability logics because of the more convenient Kripke semantics. The situation turns out to be different for the polymodal provability logic GLP that has been applied to proof-theoretic analysis of Peano arithmetic. GLP is known to be incomplete w.r.t. any class of Kripke frames.

We study natural topological models of GLP where modalities correspond to derived set operators on a poly-topological space $(X, \tau_0, \tau_1, \dots)$. We call such a space *GLP-space* if, for all n, topologies τ_n are scattered, $\tau_n \subseteq \tau_{n+1}$, and $d_n(A)$ is open in τ_{n+1}, for any $A \subseteq X$. Here $d_n(A)$ denotes the set of limit points of A w.r.t. topology τ_n. GLP-spaces exactly the spaces validating all the axioms of GLP. We show that GLP is complete w.r.t. the class of GLP-spaces (joint work with David Gabelaia).

On the other hand, completeness w.r.t. natural ordinal GLP-spaces turns out to be dependent on large cardinal axioms of set theory and various facts on reflecting stationary sets. In particular, it is consistent (relative to ZFC) that GLP is incomplete. However, under the assumption of large cardinal axioms one can establish at least some partial completeness results. Under the assumption V=L we show that the bimodal fragment of GLP is complete w.r.t. the cardinal $\aleph_\omega$ (taken with the interval topology and the club filter topology).

[1] LEV BEKLEMISHEV, *Ordinal completeness of bimodal provability logic GLB*, ***Logic Group Preprint Series***, no. 282 (March 2010), Department of Philosophy, Utrecht University.

[2] LEV BEKLEMISHEV, GURAM BEZHANISHVILI, AND THOMAS ICARD, *On topological models of GLP*, ***Ways of proof theory*** (Ralf Schindler, editor), Ontos Verlag, Heusendamm bei Frankfurt, Germany, 2010, pp. 133–152.

► RAF CLUCKERS, *New transfer principles for motivic integrals*.
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Motivic integrals are an abstract version of p-adic integrals for big p. The latter in itself is a flexible tool to describe (families of) finite sums. In this talk we will only need the more concrete view of "uniform in p, p-adic integrals" instead of the abstract view on motivic integrals. With F. Loeser, [3], we obtained a first transfer principle for these integrals, which allows one to change the characteristic of the local field when one studies equalities of integrals. This transfer principle in particular applies to the Fundamental Lemma of the Langlands program as is shown in [2]; see also the overview papers [4] and [5]. In work in progress with I. Halupczok and J. Gordon, [1], we obtain a second transfer principle which allows one to change the characteristic of the local field when one studies integrability conditions of motivic exponential functions. This in particular solves an open problem of 1973 about the local integrability of Harish–Chandra characters in (large enough) positive characteristic; see [6].

Let us explain some concepts in more detail. There are two classes of non-archimedean locally compact fields, namely the characteristic zero ones (the p-adic fields $\mathbb{Q}_p$ and their finite field extensions), and the positive characteristic ones (namely the Laurent series fields $\mathbb{F}_q((t))$, where $\mathbb{F}_q$ is a finite field). These fields are named *local fields*. Since they are locally compact, they carry a Haar measure. To compare statements about integrals over $\mathbb{Q}_p$ with statement about integrals over $\mathbb{F}_p((t))$, one works with first order languages which are able to describe the data that go in the integrals in a field independent way. For example, the set one integrates over should be a definable set. Now if one has a uniform description of some integrals, one can consider properties like whether equalities between integrals hold or not, or, the integrand being integrable or not. The transfer principles show that these questions, in a general setting, only depend on the residue field of the local field, and thus, the particular question holds over $\mathbb{Q}_p$ if and only if it holds over $\mathbb{F}_p((t))$, for each individual but large p. We do this for the rich class of motivic exponential integrals, which includes oscillating behavior coming from additive characters.

[1] R. CLUCKERS, J. GORDON, AND I. HALUPCZOK, *Transfer principle for integrability over local fields and motivic constructible exponential functions*, in progress, (2011).

[2] R. CLUCKERS, AND T. HALES, AND F. LOESER, *Transfer principle for the fundamental lemma*, ***Stabilisation de la formule des traces, variétés de Shimura, et applications arithmétiques, I***, (M. Harris, editor), International Press, 2011, to appear.

[3] R. Cluckers and F. Loeser, *Constructible exponential functions, motivic Fourier transform and transfer principle*, ***Annals of Mathematics***, vol. 171 (2010), pp.1011–1065.

[4] T. Hales, *The work of Ngô Bao Châu*, ***Notices of the AMS***, vol. 58 (2011), no. 3, pp. 453–458.

[5] ———, *The fundamental lemma and the Hitchin fibration* [*after Ngô Bao Châu*], ***Séminaire Bourbaki***, (2011), arXiv:1103.4066.

[6] Harish-Chandra, ***Admissible invariant distributions on reductive p-adic groups***, Preface and notes by Stephen DeBacker and Paul J. Sally, Jr. University Lecture Series, 16. American Mathematical Society, 1999.

► TODD EISWORTH, *Colorings at successors of singulars*.
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We will survey recent results concerning the question of whether the negative square-brackets partition relation $\kappa \nrightarrow [\kappa]^2_\kappa$ necessarily holds if κ is the successor of a singular cardinal. For those unfamiliar with the partition calculus, the square-brackets partition relation

$$\kappa \rightarrow [\kappa]^2_\kappa \tag{1}$$

asserts that for any $f\colon [\kappa]^2 \rightarrow \kappa$, there is set $H \subseteq \kappa$ such that the range of the restriction of f to $[H]^2$ is a proper subset of κ. Said another way, (1) asserts that whenever we color the pairs from κ with κ colors, there is a set of size κ on which the coloring omits at least one value. The negation of (1),

$$\kappa \nrightarrow [\kappa]^2_\kappa, \tag{2}$$

therefore asserts the existence of a function $f\colon [\kappa]^2 \rightarrow \kappa$ with the property that for any $A \subseteq \kappa$ of size κ and any $\theta < \kappa$, there are $\alpha < \beta$ in A for which $f(\alpha, \beta) = \theta$.

An old result of Todorcevic tells us that (2) will hold whenever κ has a non-reflecting stationary subset. In particular, we have (2) whenever κ is the successor of a regular cardinal. Must this also be true for κ successor of singular? This question remains open, but we have discovered that many "approximations" to (2) hold (in ZFC) at successors of singulars, and that these "approximate colorings" can often be used to gain information. In this talk, we will discuss these approximate colorings, give several applications, and then draw some conclusions concerning the structure of any counterexample to (2).

► SERGEY GONCHAROV, *Problems of autostability and spectrum of autostability*.
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This talk will connected with basic problem of the theory of computable models about non-equivalent effective representations of models. We will discuss problem of autostability relative to decidable representations of models for decidable theories [2], [3]. Another problems are connected with the levels of autostability relative to some hierarhies [6], [4] and degrees [1], [5].

[1] E. B. Fokina, I. Kalimullin, and R. Miller, *Degrees of categoricity of computable structures*, ***Archive Mathematical Logic***, vol. 49 (2010), pp. 51–67.

[2] S. S. Goncharov, *Autostability of prime models with respect to strong constructivizations*, (Russian) ***Algebra Logika***, vol. 48 (2009), no. 6, pp. 729–740.

[3] ———, *On the autostability of almost prime models with respect to strong constructivizations* (Russian), ***Uspekhi Matematicheskikh Nauk***, vol. 65 (2010), no. 5(395), pp. 107–142.

[4] ———, *Computability and computable models*, ***Mathematical Problems from Applied Logic II*** (D. Gabbay, S. S. Goncharov, and M. Zakharyashev, editors),

[5] ———, *Degrees of autostability relative to strong constructivizations*, ***Proceedings of the Steklov Institute of mathematics***, vol. 274 (2011), pp. 1–11.

[6] S. Goncharov, V. Harizanov, J. Knight, C. McCoy, R. Miller, and R. Solomon, *Enumerations in computable structure theory*, ***Annals of Pure and Applied Logic***, vol. 136 (2005), no. 3, 219–246. International Mathematical Series, Springer, New York, vol. 5 (2007), pp. 99–214.

► HENRI LOMBARDI, *Constructive semantics for classical formal proofs*.
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Classical mathematics are expected to work within set theory à la ZFC.

Nevertheless, the intuition behind ZFC is not at all correctly translated in a theory admitting countable models. And the presence of oddities as Banach-Tarski's Theorem is counterintuitive.

There is a lack of clear semantics for this (very abstract) theory, and the Hilbert's programme, which was settled in order to secure set theory, has failed in its original form, asking finitary proofs of consistance.

This is in strong constrast with the facts that many concrete results obtained by suspicious arguments inside ZFC become completely secured after further work (see [1, 9, 10]), and that no contradiction has appeared in this theory after a century of practical use.

Mathematicians and logicians who do not think that ZFC has a clear content aim to solve the mystery of its fairly good concrete behaviour. A possible issue is to develop a systematic way of finding constructive semantics at least for classical proofs.

A first idea is that classical abstract objects, e.g. models of consistant first order theories, are not needed to exist truely in order to be used constructively when reasoning by contradiction. Proving that a model cannot exist is in general a direct proof that the corresponding formal theory proves False, a concrete positive result (see [5, 6, 7]).

More generally we think that classical arguments leading to concrete results within geometric theories can always be constructivized, as suggested by Barr's Theorem, which cannot have a fully constructive proof (see [2, 3, 4, 8, 11, 12]).

This new kind of Hilbert's programme has obtained some striking successes, and possibly offers a way of a more systematic and intuitive approach to constructivization of classical mathematics. This could give a concrete solution of the above mystery.

[1] E. Bishop, ***Foundations of constructive analysis***, McGraw Hill, 1967.

[2] T. Coquand, *On seminormality*, ***Journal of Algebra***, vol. 305 (2006), pp. 585–602.

[3] ———, *Space of valuations*, ***Annals of Pure and Applied Logic***, vol. 157 (2009), pp. 97–109.

[4] T. Coquand and H. Lombardi, *A logical approach to abstract algebra*, ***Mathematical Structures in Compututer Science***, vol. 16 (2006), pp. 885–900.

[5] M. Coste, H. Lombardi, and M.-F. Roy, *Dynamical method in algebra: Effective Nullstellensätze*, ***Annals of Pure and Applied Logic***, vol. 111 (2001) pp. 203–256.

[6] H. Lombardi, *Dimension de Krull, Nullstellensätze et Évaluation dynamique*, ***Mathematical Zeitschrift***, vol. 242 (2002), pp. 23–46.

[7] ———, *Algèbre dynamique, espaces topologiques sans points et programme de Hilbert*, ***Annals of Pure and Applied Logic***, vol. 137 (2006), pp. 256–290.

[8] H. Lombardi and C. Quitté, ***Algèbre commutative. Méthodes constructives***, To appear. See the page web of the first author.

[9] P. Martin-Löf, *The Hilbert-Brouwer controversy resolved?*, ***One hundred years of intuitionism*** (***1907–2007***), (Mark Van Atten et al., editors) Publications des Archives Henri Poincaré, Birkhaüser Basel, 2008, pp. 243–256.

[10] R. MINES, F. RICHMAN, AND W. RUITENBURG, ***A course in constructive algebra***, Universitext, Springer-Verlag, (1988).

[11] H. SCHWICHTENBERG AND C. SENJAK, *Minimal from classical proofs*, to appear in ***CALCO-Tools 2011***.

[12] I. YENGUI, *Making the use of maximal ideals constructive*, ***Theoretical Computer Science***, vol. 392 (2008), pp. 174–178.

▶ J. LOPEZ-ABAD, *Large cardinals and basic sequences*.
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The aim of this talk is to present recent progresses in applications of infinite combinatorics to the geometry of Banach spaces. It will be a survey of how the existence of certain large cardinals affect to the structure of the separable subspaces of a given Banach space.

[1] S. A. ARGYROS, J. LOPEZ-ABAD, AND S. TODORCEVIC, *A class of Banach spaces with few non-strictly singular operators*, ***Journal of Functional Analysis***, vol. 222 (2005), no. 2, pp. 306–384.

[2] P. DODOS, J. LOPEZ-ABAD, AND S. TODORCEVIC, *Unconditional basic sequences in spaces of large density*, ***Advances in Mathematics***, vol. 226 (2011), pp. 3297–3308.

[3] W. T. GOWERS AND B. MAUREY, *The unconditional basic sequence problem*, ***Journal of the American Mathematical Society***, vol. 6 (1993), pp. 851–874.

[4] J. L. KRIVINE, *Sous-espaces de dimension finie des espaces de Banach réticulés*, ***Annals of Mathematics. Series 2***, vol. 104 (1976), no. 1, p. 129.

[5] S. TODORCEVIC, ***Walks on ordinals and their characteristics***, Progress in Mathematics 263, Birkhäuser Verlag, Basel, 2007.

[6] B. S. TSIRELSON, *Not every Banach space contains an imbedding of ℓ_p or c_0*, ***Functional Analysis and its Applications***, vol. 8 (1974), pp. 138–141.

▶ KENNETH MANDERS, *Strategies in the philosophy of mathematics*.
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What intellectual strategies can enhance our philosophical conception of the power of mathematical understanding?

Comparative case studies (contrasting different, and differently effective, mathematical approaches to a body of mathematical questions) can make philosophically explanatory differences between approaches visible that we may not have been noticing. Contrasting Descartes' method for the analysis of geometrical construction problems with the style of reasoning in the early books of Euclid, for example, shows that informationally equivalent expressive means may differ crucially in their ability to separately express, and hence treat separately, different aspects of a situation. Similarly, the history of Knot classification shows that the principal, and as yet not fully met, challenge is to implement strong enough expressive means for knot types that can be deployed without specifiying type-irrelevant features.

These ideas promise novel possibilities for formal-logical analyses of mathematical subject areas, but they will require formalizations sensitive to differences of expressive means that make an intelligibility difference.

Approaches that benefit foundational studies may hinder understanding other philosophical issues. For example: classification of proof strength benefits by focus on only the strongest ingredients needed to obtain a result, by disregarding stages at which lesser means are required; but Descartes's geometrical method is powerful precisely by segmenting into stages using only partial information on a problem even though information omitted in some stages is required in others. Modern-algebra technique exploits the same strategy, e.g., treating algebraic extensions as vector spaces, or as multiplicative groups, in some stages while ultimately needing the full algebra structure over the base field. Again, classification of proof strength benefits from reductions to a uniform linguistic basis (set theory, second-order arithmetic) that suppress proof-strength neutral differences peculiar to specific mathematical contexts, but intelligibility within those contexts may be explanable only by attending to those differences.

▶ MARTIN OTTO, *Tractable finite models*.

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Tractability of logics or structures is a key issue in algorithmic model theory. In computational problems, tractability usually refers to problem instances that guarantee decidability or manageable complexity. In more purely model-theoretic problems, we may similarly look for criteria that render structures amenable to certain techniques for the analysis of expressiveness and definability. For instance, a (finite) structure may be considered *tractable* if the types of its tuples are governed by some comparatively simple invariants. These invariants may be combinatorial or themselves logical, like certain partial types. Classically, models with suitable saturation or homogeneity properties satisfy strong variants of such criteria, which make them ideal for the analysis of first-order definability via back—and—forth techniques. Classically or otherwise, the isolation of suitable tractability criteria, which are achievable through admissible model constructions or transformations, is often crucial for model theoretic analysis.

In finite and algorithmic model theory, however, the smooth classical notions of tractability are not available or can at best be approximated (and must at least be finitely parameterised, e.g., by quantifier ranks or locality radii). For instance, even if the problem at hand is compatible with bisimulation equivalence, which would allow for tree-unfoldings in a classical setting, such acyclic unfoldings may necessarily lead to infinite structures. Specific variants of tractability of the flavour indicated above play a fundamental role in *expressive completeness* arguments, especially also for non-classical preservation theorems, of which finite model theory meanwhile provides an impressive range. In this talk I want to illustrate the importance of the identification of structural criteria for tractability in finite models, and the combinatorial challenges involved in the construction of tractable finite models. For instance, besides local variants of tree-like behaviour, certain forms of qualified hypergraph acyclicity [2, 5, 6] can play a major role in the tractability of finite structures, both w.r.t. algorithmic issues and w.r.t. the analysis of first-order definability, and for the finite and algorithmic model theory of guarded logics [1, 3]. Guarded logics themselves are examples of highly tractable logics in the computational sense; their finite model theory moreover has intriguing connections with constructions studied in classical model theory [4] (cf. [3, 5]).

[1] H. Andéka and J. van Benthem and I. Németi, *Modal languages and bounded fragments of predicate logic*, ***Journal of Philosophical Logic***, vol. 27 (1998), pp. 217–274.

[2] V. Barany and G. Gottlob, and M. Otto, *Querying the guarded fragment*, ***Proceedings of the 25th IEEE Symposium on Logic in Computer Science***, 2010, pp. 1–11, preprint of journal version (33 pages) available online.

[3] E. Grädel, *On the restraining power of guards*, ***The Journal of Symbolic Logic***, vol. 64 (1999), pp. 1719–1742.

[4] B. Herwig and D. Lascar, *Extending partial isomorphisms and the profinite topology on free groups*, ***Transactions of the AMS***, vol. 352 (2000), pp. 1985–2021.

[5] I. Hodkinson and M. Otto, *Finite conformal hypergraph covers and Gaifman cliques in finite structures*, ***The Bulletin of Symbolic Logic***, vol. 9 (2003), pp. 387–405.

[6] M. Otto, *Highly acyclic groups, hypergraph covers and the guarded fragment*, ***Proceedings of the 25th IEEE Symposium on Logic in Computer Science***, 2010, pp. 12–21, preprint of journal version (42 pages) available online.

► JAN REIMANN, *Dualities and dichotomies in algorithmic information theory*.
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The last decade has seen tremendous progress towards the calibration of information content of reals and their logical complexity. Many of the most important results can be seen as an instance of two fundamental dichotomies: If a real does not exhibit randomness or non-trivial information content, then it has rather low definability strength. The same conclusion holds, on the other hand, if a real exhibits relatively strong randomness.

We suggest that these phenomena have a dual interpretation in the space of probability measures on Cantor space. We will illustrate this by means of a number of examples: lowness properties, fractal dimensions, randomness and cardinality, probabilistic and algorithmic independence.

We show how the dual approach can be used to produce new insights on computational properties of reals, even in cases seemingly unrelated to randomness.

► ANDRZEJ ROSŁANOWSKI, *News of the creature world—not only forcing*.
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In the talk I will review recent developments and progress concerning *creatures* which are at the heart of the method of building forcing notions using *norms on possibilities*. This method has a long history already. A systematic study of the technique and a number of its applications were presented in Rosłanowski and Shelah [4, 5, 7, 6] and several other papers.

Let $\mathbf{H}$ be a function defined on the set ω of natural numbers such that $\mathbf{H}(i)$ is a finite non-empty set for all $i \in \omega$. *A creature* for $\mathbf{H}$ is a tuple

$$t = (\mathbf{nor}[t], \mathbf{val}[t], \mathbf{dis}[t], m^t_{\rm dn}, m^t_{\rm up})$$

such that

- $\mathbf{nor}[t]$ is a non-negative real number, $\mathbf{dis}[t]$ is an arbitrary object and $m^t_{\rm dn} < m^t_{\rm up} < \omega$ and
- $\mathbf{val}[t]$ is a non-empty finite family of functions f with domains $\mathrm{dom}(f)$ included in the interval $[m^t_{\rm dn}, m^t_{\rm up})$ and values $f(i) \in \mathbf{H}(i)$ (for $i \in \mathrm{dom}(f)$).

When we deal with a fixed family K of creatures, $\mathbf{nor}[t]$ can be thought of as the measurement of how big is the family $\mathbf{val}[t]$ (but one of the important points is that often this measurement depends on the side parameters given by $\mathbf{dis}[t]$).

A condition in a forcing notion built with the use of creatures may be represented as an infinite sequence of creatures $\bar{t} = \langle t_n \colon n < \omega\rangle$, where we demand that the norms $\mathbf{nor}[t_n]$ go to ∞ (with various possible interpretations of this statement). The partial order on the set $\mathbb{Q}$ of conditions is defined by introducing one more component: an operation Σ that determines how the creatures can be put together and/or shrinked. (Typically, passing from $\bar{t} = \langle t_n \colon n < \omega\rangle \in \mathbb{Q}$ to a stronger condition $\bar{s} = \langle s_m \colon m < \omega\rangle \in \mathbb{Q}$ implies that sets $\mathbf{val}[s_m]$ are included in $\{f_0 \cup \cdots \cup f_k \colon f_i \in \mathbf{val}[t_{n_i}] \text{ for } i \leq k\}$.)

Under some assumptions on the pair (K, Σ), the corresponding forcing notion $\mathbb{Q} = \mathbb{Q}(K, \Sigma)$ can be guaranteed to be proper, sometimes even ccc.

The first new application of creatures discussed in the talk concerns partition theorems. For a condition $\bar{t} \in \mathbb{Q}(K, \Sigma)$ we consider the set

$$\mathrm{pos}(\bar{t}) = \{\mathbf{val}[s_m] \colon \bar{s} \in \mathbb{Q}(K, \Sigma) \text{ stronger than } \bar{t} \text{ and } m < \omega\}.$$

These are subsets of the family $\mathcal{F}_{\mathbf{H}}$ of all finite partial functions f on ω with values $f(i) \in \mathbf{H}(i)$ for $i \in \mathrm{dom}(f)$. In some sense, sets of the form $\mathrm{pos}(\bar{t})$ are "large" and we are naturally interested in results of the form "if the (countable) set $\mathcal{F}_{\mathbf{H}}$ is divided into finitely many pieces, then one of the parts includes $\mathrm{pos}(\bar{t})$ for some $\bar{t} \in \mathbb{Q}(K, \Sigma)$". First steps in this direction are done in Rosłanowski and Shelah [3].

The second direction discussed in the talk concerns a property (of pairs (K, Σ)) called *halving*. It plays a significant role in the paper Rosłanowski, Shelah and Spinas [8] where it is used to argue that some special forcing notions of the type $\mathbb{Q}(K, \Sigma)$ are proper. However, these arguments are quite delicate and this fact in turn is used to construct two very simple proper forcing notions whose product is not proper (and collapses the continuum).

The halving together with even stranger property called *decisiveness* allows constructructions of CS products which in some ways behave like CS iterations. The first construction of this type was given in Kellner and Shelah [1]. Subsequently, this method was used in Rosłanowski and Shelah [2] to show the consistency of "there are no monotone Borel hull operation for neither the meager nor the null ideals".

[1] Jakob Kellner and Saharon Shelah, *Decisive creatures and large continuum*, ***The Journal of Symbolic Logic***, vol. 74 (2009), pp. 73–104, `math.LO/0601083`.

[2] Andrzej Roslanowski and Saharon Shelah, *Monotone hulls for $\mathcal{N} \cap \mathcal{M}$*, ***Periodica Mathematica Hungarica***, `1007.5368`.

[3] ———, *Partition theorems from creatures and idempotent ultrafilters*, preprint, `1005.2803`.

[4] ———, *Norms on possibilities I: forcing with trees and creatures*, ***Memoirs of the American Mathematical Society***, vol. 141 (1999), no. 671, pp. xii–167, `math.LO/9807172`.

[5] ———, *Sweet & Sour and other flavours of ccc forcing notions*, ***Archive for Mathematical Logic***, vol. 43 (2004), pp. 583–663, `math.LO/9909115`.

[6] ———, *How much sweetness is there in the universe?*, ***Mathematical Logic Quarterly***, vol. 52 (2006), pp. 71–86, `math.LO/0406612`.

[7] ———, *Measured creatures*, ***Israel Journal of Mathematics***, vol. 151 (2006), pp. 61–110, `math.LO/0010070`.

[8] Andrzej Roslanowski, Saharon Shelah, and Otmar Spinas, *Nonproper products*, ***Bulletin of the London Mathematical Society***, submitted, `0905.0526`.

► YDE VENEMA, *A modal distributive law*.
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In this talk we will discuss a coalgebraic reformulation of the syntax and semantics of modal logic, based on the so-called cover modality which was introduced, independently, by Moss and by Janin & Walukiewicz. As our main topic, we will underline the fundamental importance of an axiomatic law in this language which shows how conjunctions distribute over the cover modality. As an important application of this law, we will see how every modal formula can be rewritten into a distributive normal form, in which the use of conjunctions is severely restricted. We will show some immediate consequences of this observation, such as a direct, inductive definition of the uniform interpolant of a modal formula. We will discuss manifestations and repercussions of this law in the theories of algebra, automata, and topology, and finish the discussion by putting the cover modality and its distributivity axiom into a wider coalgebraic perspective.

Abstracts of invited tutorials

► GEORGES GONTHIER, *Formal mathematics in practice*.
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The last decade has seen great progress in formally verified proofs. In pure mathematics we have moved from high school and undergraduate material on to graduate and even research-level results, such as the four-color theorem [5], the Kepler conjecture or the odd order theorem. Application of formal proof to computer systems has progressed beyond academic examples to useful compilers and system kernels, as well as commercial chip design and payment systems.

To ramp up to such complex theories one has to abandon the naive early 20^{th} century view that a mathematical theory is just a collection of first-order proposition over a limited set of symbols, that a proof is just a series of assertions, and that using a theory merely involves resolving a few variables by unification. Modern formal theorem proving relies on an array of techniques drawn from logic, type and language theory, and software engineering to define, verify and compose mathematical theories. In this series of tutorials we will demonstrate many of the techniques we have developed and used for these three tasks.

How to write formal mathematics: In practice, mathematical expressions convey an *intended use* as well as a strictly logical meaning. It is essential to capture this in order to make sense of "abusive" notation, and to ensure that a computer proof system behaves sensibly. We will show that the combination of an expressive type system—the one supported by the Coq system [1, 3]—with automatic type inference makes it possible to write formal math whose "look and feel" is close to textbooks.

How to give formal proofs: Formal proofs are hard to write because they must cover every detail. Computer proof systems help by filling in most of these, so in practice a formal proof is given by a *script* guiding the system. While assertions are an important part of scripts, they are by no means only possible proof steps, and not always the best choice—even in textbooks. The variety of steps supported by our ssreflect extension [6, 8] to Coq can capture concisely and clearly even "loose" textbook proofs.

How to build formal theories: Mathematics often achieves concision through modularity, breaking down complex constructions into a combination of generic operations, and devising general theories that apply in many contexts. Using examples from the ssreflect library [7, 2, 4] we will show how to combine the techniques we have seen to implement such modular theories.

[1] Y. Bertot and P. Castéan, ***Interactive theorem proving and program development Coq'Art: The calculus of inductive constructions***, Texts in Theoretical Computer Science, EATCS, 2004.

[2] Y. Bertot, G. Gonthier, S. Ould Biha, and I. Pasca, *Canonical big operators*, ***Theorem proving in higher-order logics***, Lecture Notes in Computer Science, vol. 5170, Springer, 2008, pp. 86–101.

[3] The Coq development team, editors, *The Coq proof assistant reference manual, version 8.3*, INRIA Technical Report, 2010, `http://coq.inria.fr/refman`.

[4] F. Garillot, G. Gonthier, A. Mahboubi, and L. Rideau, *Packaging mathematical structures*, ***Theorem proving in higher-order logics***, Lecture Notes in Computer Science, vol. 5674, Springer, 2009, pp. 327–342.

[5] G. Gonthier, *A modular formalisation of finite group theory*, ***Notices of the American Mathematical Society***, vol. 55 (2008), no. 11, pp. 1382–11393.

[6] G. Gonthier and A. Mahboubi, *An introduction to small scale reflection in Coq*. ***Journal of Formalized Reasoning***, vol. 3 (2010), pp. 95–152.

[7] G. Gonthier, A. Mahboubi, L. Rideau, E. Tassi, and L. Théry, *A modular formalisation of finite group theory*, ***Theorem proving in higher-order logics***, Lecture Notes in Computer Science, vol. 4732, Springer, 2007, pp. 86–101.

[8] G. Gonthier, A. Mahboubi, and E. Tassi, *A small scale reflection extension for the Coq system*, INRIA Technical Report, 2011, `http://hal.inria.fr/inria-00258384`.

► MARTIN ZIEGLER, *Hrushovski's Amalgamation Method*.
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In [1] Hrushovski constructed a strongly minimal theory which was not locally modular but did not interpret an infinite field, refuting Zilber's Conjecture. He used the method also to show that two almost arbitrary strongly minimal theories can be fused together to form a new strongly minimal theory ([2]). This led to the construction of many other exotic theories, as for example the bad field recently found by Baudisch, Hils, Pizarro and Wagner. This is an algebraically closed field K of characteristic 0 with a distinguished non-trivial proper multiplicative subgroup G such that (K, G) has finite Morley rank.

In the tutorial I will give an exposition of the basic theorems of [1].

[1] EHUD HRUSHOVSKI, *A new strongly minimal set*, ***Annals of Pure and Applied Logic***, vol. 62 (1993), pp. 147–166.

[2] ———, *Strongly minimal expansions of algebraically closed fields*, ***Israel Journal of Mathematics***, vol. 79 (1992), pp. 129–151.

Abstracts of invited talks in the Special Session on Computability Theory and Logic for Computer Science

► LAURENT BIENVENU, *Randomness extraction: a computability perspective*.
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Imagine you want to generate a random sequence of bits, but all you have at your disposal is an imperfect source of randomness (for example a biased coin or the bits representing your computer's memory which contains some randomness but also some regularities). Is it possible to algorithmically turn this weak source into a perfect (or close-to-perfect) one? This is the problem of randomness extraction, which is a very active research topic in mathematics and computer science. There are of course many ways to interpret the above question, and in this talk we will discuss this question from the perspective of computability theory. The main tool computability theory offers to tackle questions related to randomness is Kolmogorov complexity, which measures the 'amount of randomness' contained in finite objects (such as binary strings). Objects are defined to be 'random' if they have maximal Kolmogorov complexity. We can therefore focus on a particular interpretation of the above question: given an object of non-maximal Kolmogorov complexity, is it possible to effectively transform this object into one of maximal (or close-to-maximal) Kolmogorov complexity? We will survey the impressive progress which has been made over the last five years regarding this last question.

► JÖRG FLUM, *A halting problem and its consequences for logic and proof theory*.
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We consider a parameterized halting problem for nondeterministic Turing machines and show that statements on the complexity of it are equivalent to statements on

- the length of proofs for sentences of first-order logic;
- the property of a certain logic to capture polynomial time;
- the existence of optimal propositional proof systems;
- the existence of hard sequences for all algorithms deciding the set of propositional tautologies.

► GEORG GOTTLOB, *Tree-likeness, structural decompositions, and logical master theorems*.
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E-mail: Georg.Gottlob@comlab.ox.ac.uk.

This talk reviews structural problem decomposition methods, such as tree decompositions. It is argued that these notions can be applied in two distinct ways: Either to show that a problem is efficiently solvable when a width parameter is fixed, or to prove that the unrestricted (or some width-parameter free) version of a problem is tractable by using a width-notion as a mathematical tool for directly solving the problem at hand. In both cases, logical characterizations of tractable languages (so called master theorems) ensuring tractability play an important role. Application examples involving such logical conditions will be given.

► TONIANN PITASSI, *Propositional proof complexity: The state of the art and some new results*.
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In this talk, we will first survey the state of the art in proving lower bounds for propositional proof systems. We will then discuss the following recent results.

First, we related the automatizability question in proof complexity to the complexity of simple stochastic games. The complexity of simple stochastic games (SSGs) has been open since they were defined by Condon in 1992. Despite intensive effort, the complexity of this problem is still unresolved. Building on work of Atserias and Mineva, we prove that if depth-3 Frege systems are weakly automatizable, then SSGs are solvable in polynomial-time.

Secondly, we give a general transformation which turns polynomial-size Frege proofs to subexponential-size bounded-depth Frege proofs, thus implying that exponential lower bounds for bounded-depth systems implies Frege lower bounds. As a consequence of our transformation, we are able to shed some light on the question of weak automatizability for bounded-depth Frege systems.

This is joint work with Lei Huang, Yuval Filmus, and Rahul Santhanam

Abstracts of invited talks in the Special Session on Foundations of Mathematics and Philosophy of Logic: Logic, Reasoning and Probability

► SERGEI ARTEMOV AND TUDOR PROTOPOPESCU, *A bimodal analysis of knowability*.
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Knowability is analyzed in a logical framework with the alethic modalities $\Box$ (necessary), $\Diamond$ (possible), and the epistemic modality $\mathbf{K}$. Modalities $\Box/\Diamond$ represent, in an abstract way, the process of discovery. The Church–Fitch knowability paradox [2, 4] purports to show that the *verificationist knowability* principle, *all truths are knowable*, $F \to \Diamond\mathbf{K}F$, implies the *omniscience* principle, *all truths are known*, $F \to \mathbf{K}F$, which does not appear acceptable.

First we note that the formulation of verificationist knowability in classical logic, as $F \to \Diamond\mathbf{K}F$ is not intuitively valid once F is allowed to change from true to false during the verification process. For example, *even if it was raining when Holmes asked Watson to check for the rain, but the rain had stopped by the time Watson looked outside, Watson's answer will be "no rain"*; though true, it is not possible for Watson to verify "it is raining".

We then provide a new semantical proof that, classically, verificationist knowability is indeed equivalent to the omniscience principle. This analysis indicates that the classical understanding of verificationist knowability is not intrinsically valid and should be augmented by some new features. One solution naturally stems from our semantical analysis: the verificationist knowability principle is valid for *stable truths*, those that remain true in the process of discovery, yielding the *stable knowability principle* $\Box F \to \Diamond \mathbf{K} F$. We show that with this principle, the Church–Fitch paradox disappears.

Yet another confirmation of the correctness of the stable knowability principle comes from the intuitionistic analysis of knowability. In intuitionistic logic, the Church-Fitch argument yields only $p \to \neg\neg \mathbf{K} p$, which some, e.g., [3] argue is acceptable as an intuitionistic expression of knowability. We invoke Gödel's translation of intuitionsitic to modal logic [5] to find out what sort of classical bi-modal principles correspond to intuitionistic knowability, and show that it is equivalent to the classical principle of *monotonic knowability* $\Box F \to \Diamond\Box \mathbf{K} F$ which implies stable knowability but differs from it by asserting that there is a state after which F stays known. This additional feature appears because of the limited expressive power of the intuitionistic language and semantics, where it is implicit in the understanding of the intuitionistic truth relation $\Vdash$. Monotonic knowability can be regarded as another resolution of the knowability paradox.

Furthermore, we offer a logical analysis of the *total knowability* principle: *it is possible to know whether a proposition holds or not*, $\Diamond \mathbf{K} F \vee \Diamond \mathbf{K} \neg F$ (cf. [6]) which provides a more general view of knowability. We show that total knowability is strictly stronger than stable knowability and yet does not succumb to the knowability paradox. This indicates that total knowability is another option to consider for expressing the idea that all truths are knowable.

These observations open the door to a bimodal framework for the systematic study of knowability (cf. [1, 7, 8, 9] for other bimodal approaches to the knowability paradox).

[1] J. C. Beall, *Knowability and possible epistemic oddities*, in [10], pp. 105–127.

[2] A. Church, *Referee reports on Fitch's "a definition of value"*, in [10], pp. 13–20.

[3] *M. Dummett*, *Fitch's paradox of knowability*, in [10], pp. 51–52.

[4] F. Fitch, *A logical analysis of some value concepts*, ***The Journal of Symbolic Logic***, vol. 28 (1963), no. 2, pp. 135–142.

[5] K. Gödel, *Eine Interpretation des intuitionistischen Aussagenkalkuls*, ***Ergebnisse eines mathematischen Kolloquiums***, vol. 4 (1933), pp. 39–40. English translation in (S. Feferman et al., editors), ***Kurt Gödel Collected Works***, vol. 1, pp. 301–303, Oxford University Press, Oxford, Clarendon Press, New York, 1986.

[6] J. Melia, *Anti-realism untouched*, ***Mind***, vol. 100 (1991), no. 3, pp. 341–342.

[7] G. Priest, *Beyond the limits of knowledge*, in [10], pp. 93–104.

[8] W. Rabinowicz and C. Segerberg, *Actual truth, possible knowledge*, ***Topoi***, vol. 13 (1994), no. 2, pp. 101–116.

[9] G. Restall, *Not every truth is knowable: At least not all at once*, in [10], pp. 339–354.

[10] J. Salerno, ***New essays on the knowability paradox***, Oxford University Press, Oxford; New York, 2009.

► MICHIEL VAN LAMBALGEN, *Causality, probability and logic*.
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The purpose of this talk is to compare the explanatory potential of logical models (in particular logic programming) and Bayesian probability in explaining causal inference and learning in infants, young children and adults.

We first note some structural similarities between logic programs and Bayesian causal networks; e.g., the closed world assumption and negation as failure can be seen as causal principles. We then review a number of tasks involving causal inference and goal attribution in infants, belief attribution in young infants, and non-monotonic reasoning in adults. The conclusion will be that despite the similarities noted above, Bayesian causal inference is not sufficiently non-monotonic to account for the data, which are better explained by logic programming. We close with some remarks on so called 'executive function' which provides the best motivation for assigning logic programming a role in higher cognition.

► HANNES LEITGEB, *The logic of belief: qualitative vs. quantitative*.
Chair of Logic and Philosophy of Language, Alexander von Humboldt Professor, Fakultät für Philosophie, Wissenschaftstheorie, und Religionswissenschaft, Ludwig-Maximilians-Universität München, Ludwigstr. 31, D-80539 München, Deutschland.
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In many areas in philosophy and science, one can find concepts of belief that occupy different scales of measurement. Most importantly, there is qualitative belief—believing that A is the case—and quantitative belief—the degree of belief in A is x. And the formal structure of the corresponding logics of belief is pretty clear as well: doxastic logic and the theory of belief revision on the qualitative side, the axioms of absolute and conditional probability theory on the quantitative side. But how do these logics of belief relate to each other? Is there a unified theory in which claims about qualitative belief can be derived from claims on quantitative belief and vice versa? We will present a theory that does achieve this goal, and given some quite common bridge principles concerning the two types of belief we will prove that it is the only theory that does the job.

► JEFF PARIS, *Pure inductive logic*.
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Inductive Logic á la Carnap & Johnson was founded on the idea that the probabilities we assign in our everyday lives may be *logical* in the sense that they are determined by our knowledge via general rationality considerations. [So this is the 'Inductive Logic' of Philosophy and is not the same as the 'Inductive Logic' of Computer Science.] As a potentially applicable topic Inductive Logic is nowadays generally considered within Philosophy to have failed, largely on account of Nelson Goodman's GRUE Paradox.

However as a 'pure' mathematical investigation into rationality, with its relevance to uncertain reasoning and the aspirations of AI, it survives and has recently seen some interesting developments. In particular the serious investigation into Polyadic Inductive Logic (Carnap et al. only ever considered unary properties) has uncovered some new rational principles which appear to be remarkably powerful, even when reflected down to the unary level.

In my talk I shall try to give a fairly informal introduction to one such principle.

Abstracts of invited talks in the Special Session on Model Theory

► MATTHIAS ASCHENBRENNER, *Vapnik-Chervonenkis density in model theory*.
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VC dimension, introduced in probability theory by Vapnik and Chervonenkis, is a numerical parameter which can be used to measure the combinatorial complexity of a family of sets. Its relation to Shelah's notion of theories without the independence property (also known as "NIP" or "dependent" theories) is familiar to model-theorists, due to work of Laskowski, who showed that the complete theory of a first-order structure is NIP if and only if each uniformly definable family of sets has finite VC dimension. Another parameter associated to a family of sets, of relevance in combinatorics and probability theory, is its VC density. I will describe how one can compute uniform bounds (often optimal) on the VC density of definable families of sets in various NIP theories.

Joint work with A. Dolich, D. Haskell, D. Macpherson, and S. Starchenko.

► AYHAN GÜNAYDIN, *Exponential polynomial equations*.
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We discuss some questions in the the setting of complex exponentiation arising from Zilber's pseudo-exponentiation (see [2]). The main problem is to prove "Exponential Closure Property (EC)" in $(\mathbb{C}, \exp)$. In [1] Marker proves it for curves over $\bar{\mathbb{Q}}$. We present a new approach to solve the same problem for curves defined over $\mathbb{C}$.

[1] DAVID MARKER, *A remark on Zilber's pseudoexponentiation*, ***The Journal of Symbolic Logic***, vol. 71 (2006), no. 3, pp. 791–798.

[2] BORIS ZILBER, *Exponential sums equations and the Schanuel conjecture*, ***Journal of London Mathematical Society***, vol. 65 (2002), no. 1, pp. 27–44.

► ITAY KAPLAN, *Directionality in NIP*.
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In this talk I'll discuss the notion of directionality: the possible spectra of co-heirs of a type over a model in an NIP theory. There are 3 categories of dependent theories: small, medium and large directionality. I'll discuss this notion and give some examples and consequences. If time permits I will also discuss non-forking spectra, a work in progress with Artem Chernikov.

This is joint work with Saharon Shelah.

► ALEX USVYATSOV, *Various notion of smallness in dependent theories*.
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I will discuss several ideals on the algebra of definable sets (e.g. forking, thorn-forking, strict dependence) and relations between them in dependent theories (theories with NIP).

Abstracts of invited talks in the Special Session on Proof Theory and Constructive Mathematics

► JOSEF BERGER, *On Dickson's lemma*.
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Dickson's lemma says that for any two functions $f, g\colon N \to N$ there exist indices $i < j$ such that $f(i) \leq f(j)$ and $g(i) \leq g(j)$. Using classical logic, the proof is very simple. It is not so easy to find a constructive proof. We present a new constructive proof and discuss its implementation in the interactive proof system MINLOG.

► L. GORDEEV, *Proof-theoretic conservations of weak weak intuitionistic constructive set theories*.
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J. Myhill [4] and H. Friedman [1] introduced several constructively meaningful principles and formal systems of *weak* extensional intuitionistic set theory of proof-theoretic strengths shown [1] to range between that of standard first- and second-order Arithmetic, **PA** (or **HA**) and $\mathbf{PA}_2$ (or $\mathbf{HA}_2$), respectively, thus being essentially weaker than standard classical set theory **ZF**. Furthermore [1] posed a deeper problem conjecturing that intuitionistic set theories under consideration are conservative extensions of the underlying arithmetical intuitionistic formalisms. These conjectures (et al.) have been confirmed [3] for the set theories of proof-theoretic strengths up to Howard Ordinal $|\mathbf{ID}_1|$; the proofs were based on the author's constructive semantics [2] of analogous weak set theories. Moreover [3] strengthened Friedman's conjectures by also proving conservations in the presence of consistent combinations of other constructive principles like an anti-foundation axiom **Cpl** and/or finite-types axiom of choice $\mathbf{AC}_{\mathrm{FT}}$. Actually for every weak set theory **T** in question [3] expressed the solution in the most conservative form $\mathbf{T} \vdash A \Leftrightarrow \mathbf{HA} + TI\ (< |\mathbf{T}|) \vdash A$, for any arithmetical statement A, where $TI\ (< |\mathbf{T}|)$ denotes the arithmetical transfinite induction scheme below proof theoretic ordinal of **T**.

Working in classical logic and using analogous set theoretic semantics K. Sato [5, 6] introduced several weak refinements of basic weak set theory—both intensional and extensional, finite and infinite—and determined their proof-theoretic ordinals. Notably Sato's *weak weak* classical set theories are less expressive, than Myhill-Friedman's *weak* intuitionistic formalisms which can (arguably) simulate the full expressive power of **ZF**. In particular, Zermelo's classical power set axiom

Pow: "*for every x there exists the set of all subsets of x*"

has natural constructive interpretation in the form

Exp: "*for every x and y there exists the set of all functions from x to y*"

occurring in Myhill-Friedman's formalisms. Since constructive functions are thought to simulate only algorithms, **Exp** is weaker than **Pow** in the intuitionistic environment. This might on one hand illuminate proof-theoretic weakness of Myhill–Friedman's intuitionistic constructive formalisms, and on the other hand explain the lack of **Exp** in Sato's classical ones. However, adding **Exp** to the intuitionistic versions of the *weak weak* set theories would hardly affect their proof-theoretic strengths. It is thus natural to investigate proof-theoretic strengths of the resulting extended *weak weak* intuitionistic constructive set theories and ask whether they are conservative extensions of the underlying arithmetical intuitionistic formalisms. In this talk we investigate the corresponding intuitionistic conservations of the *weak weak* extensional set theories with **Exp** and infinity.

[1] H. Friedman, *Set-theoretic foundation for constructive analysis*, ***Annals of Mathematics***, vol. 105 (1977), pp. 1–28

[2] L. Gordeev, Constructive models for set theory with extensionality, ***The L.E.J. Brouwer Centenary Symposium*** (***Noordwijkerhout 1981***), North-Holland, 1982, pp. 123–147

[3] ———, *Proof-theoretical analysis: weak systems of functions and classes*, ***Annals of Pure and Applied Logic***, vol. 38 (1988), no. 1, pp. 1–122.

[4] J. Myhill, *Constructive set theory*, ***The Journal of Symbolic Logic***, vol. 40 (1975), pp. 374–382.

[5] K. Sato, *The strength of extensionality I—weak weak set theories with infinity*, ***Annals of Pure and Applied Logic***, vol. 157 (2009), pp. 234–268.

[6] ———, The strength of extensionality II—weak weak set theories without infinity, ***Annals of Pure and Applied Logic***, vol. 162 (2011), pp. 579–646.

► BAS SPITTERS, *Efficient real computation in constructive type theory*.

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In 1967 Bishop proposed to use constructive analysis as a language for exact real computations. Martin-Löf proposed constructive type theory as an actual programming language. This language now forms the base of both proof assistants and programming languages. I will report on our effort to actually carry out this research program: we provide an efficient computer verified implementation of exact real numbers in the Coq proof assistant.

► GUNNAR WILKEN, *Derivation lengths classification of Goedel's T as a prototype of higher order rewrite systems*.
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Inspired by recent suggestions from William A. Howard on a manuscript providing a complete derivation lengths classification of the unrestricted lambda calculus version of Goedel's T (joint work with Andreas Weiermann), I will explain the technique of non-unique ordinal assignment and the collapsing procedure in order to obtain sharp upper bounds for the heights of reduction trees in T. Compared to the earlier manuscript the approach has become considerably more accessible. It combines Howard's 1970 non-unique assignment of ordinals below the proof-theoretic ordinal of Peano arithmetic to primitive recursive functionals of finite type with Weiermann's technique of collapsing into the finite ordinals. We expect that the methods developed here can be applied to other higher order rewrite systems resulting in new powerful termination orderings since T is a paradigm for such systems.

Abstracts of invited talks in the Special Session on Set Theory

► ANDREW BROOKE-TAYLOR AND SY-DAVID FRIEDMAN, *The limits of large cardinal compatibility for* $\square$.
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A well known result of Solovay is that $\square_\alpha$ must fail for all α greater than or equal to a supercompact cardinal. Jensen refined this result, showing that if κ is subcompact then $\square_\kappa$ fails. Jensen's result is more or less optimal for $\square_\kappa$ with κ a large cardinal, but is not the end of the story for other cardinals: indeed, Solovay's result shows that forcing $\square_\alpha$ at all cardinals which are not subcompact necessarily entails the destruction of stronger large cardinal properties.

By generalising the notion of subcompactness, we obtain an optimal result regarding the consistency of square with large cardinals. Specifically, we show that $\square_\alpha$ must fail whenever there is a $\kappa \leq \alpha$ that is α^+-subcompact (appropriately defined). Further, under the GCH, $\square_\alpha$ may be forced to hold at all other cardinals, preserving all β-subcompact cardinals of the ground model for all β, along with other large cardinals. Similar results are also obtained for stationary reflection with a small strengthening of the large cardinal considered, and we also illuminate the situtation for weak squares.

► LUCA MOTTO ROS, *The descriptive set theoretical complexity of the embeddability relation on uncountable models*.
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Given an uncountable cardinal κ, we investigate the complexity of the embeddability relation on models of size κ of a given $\mathcal{L}_{\kappa^+\kappa}$-sentence. For R, S quasi-orders on topological spaces X, Y, we say that R is κ^+-*Borel reducible* to S ($R \leq_B^\kappa S$ in symbols) if there is a κ^+-Borel measurable map $f \colon X \to Y$ such that $x \mathrel{R} y \iff x \mathrel{S} y$ for every $x, y \in X$, and we say that R and S are κ^+-*Borel bireducible* if $R \leq_B^\kappa S \leq_B^\kappa R$. We call *tree* any partial order in which the set of predecessors of any element is linearly ordered.

Generalizing some results which are well-known for the countable case, we show that under very mild conditions on κ the following results hold:

THEOREM 1. *The embeddability relation on trees of size κ is κ^+-Borel complete for analytic quasi-orders on 2^κ: that is, every analytic quasi-order R on 2^κ is κ^+-Borel reducible to the embeddability relation on trees of size κ.*

THEOREM 2. *For every analytic quasi-order R on 2^κ, there is an $\mathcal{L}_{\kappa^+\kappa}$-sentence φ all of whose models are trees such that R is κ^+-Borel bireducible with the embeddability relation on the models of φ.*

Theorem 1 extends some results of [5, 2], while Theorem 2 extends the results of [4, 1, 3]. Although the statements of Theorems 1 and 2 are direct generalizations of those already obtained for $\kappa = \omega$, the techniques used to prove them are radically different from the ones used in the countable case.

[1] A. ANDRETTA AND L. MOTTO ROS, *κ-Souslin quasi-orders and definable cardinality*, in preparation.

[2] J. E. BAUMGARTNER, *A new class of order types*, ***Annals of Mathematical Logic***, vol. 9 (1976), pp. 197–222.

[3] R. CAMERLO, A. MARCONE, AND L. MOTTO ROS, *Invariantly universal analytic quasi-orders*, ***Transactions of the American Mathematical Society***, (to appear).

[4] S. D. FRIEDMAN AND L. MOTTO ROS, *Analytic equivalence relations and bi-embeddability*, ***The Journal of Symbolic Logic***, vol. 76 (2011), no. 1, pp. 243–266.

[5] A. LOUVEAU AND C. ROSENDAL, *Complete analytic equivalence relations*, ***Transactions of the American Mathematical Society***, vol. 357 (2005), no. 12, pp. 4839–4866.

► MARCIN SABOK, *Ramsey theory on Polish spaces*.
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A typical problem in the Ramsey theory on a Polish space X looks as follows: given an analytic subset $E \subseteq X^2$ and a σ-ideal I on X, find a Borel set $B \notin I$ such that $[B]^2 \subseteq E$ or $[B]^2 \cap E = \phi$. If E is an equivalence relation, this means that on the set B the relation E is one of the two trivial ones: the identity or everything. An easier problem: given an analytic equivalence relation E on X, find a Borel set $B \notin I$ such that $E \restriction B$ is strictly simpler in the Borel-reducibility order than E (e.g., smooth when E is not smooth itself). There are many classical result of this kind that deal with particular cases of σ-ideals or analytic relations. A more uniform approach can be realized in the language of definable proper forcing. Given a Polish space X and a σ-ideal I on it, we study the quotient forcing P_I of all Borel sets B in X which do not belong to I. The forcing properties of P_I often depend on the analytical and descriptive properties of the σ-ideal I, but the common feature of most of the interesting cases is that the forcing P_I is proper. It turns out that definable proper forcing can be successfully used in the study of Borel equivalence relations and Ramsey theory on Polish spaces. I will discuss recent result in this area. Since we have the two factors that come into the play: the σ-ideal I and the analytic relation E, the appropriate Ramsey theorem will be true only under specific assumptions on I and E.

This is joint work with Jindřich Zapletal and Vladimir Kanovei. [1]

[1] V. Kanovei, M. Sabok, and J. Zapletal, ***Canonical Ramsey theory on Polish spaces***, Cambridge Tracts in Mathematics, to appear.

► LYUBOMYR ZDOMSKYY, *On lifting elementary embeddings for strong cardinals*.
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An important technique in large cardinal set theory is that of lifting an elementary embedding $j: V \to M$ to an elementary embedding $j^*: V[G] \to M[G^*]$ of corresponding generic extensions, where G (resp. G^*) is a $\mathbb{P}$-generic (resp. $j(\mathbb{P})$-generic) filter over V (resp. M). If $\mathbb{P}$ is κ^+-distributive, where κ is the critical point of j, then it is often possible to lift j.

Friedman and Thompson suggested in [2] to use *fusion* as a substitute for distributivity. Another example of the use of fusion for lifting elementary embeddings is given in [3].

These results suggest to look for some general fusion-like properties of a poset which would guarantee that iterations with supports of size $\leq \kappa$ do not collapse κ^+, preserve the strongness of κ, and are enjoyed by the posets considered in [2, 3]. This is going to be the main theme of my talk.

Joint work with S. D. Friedman and R. Honzik, see [1].

[1] S. D. Friedman, R. Honzik, and L. Zdomskyy, *Reasonably bounded forcing notions, fusion, and extending elementary embeddings*, in preparation.

[2] S. D. Friedman and K. Thompson, *Perfect trees and elementary embeddings*, ***The Journal of Symbolic Logic***, vol. 73 (2008), no. 3, pp. 906–918.

[3] S. D. Friedman and L. Zdomskyy, *Measurable cardinals and the cofinality of the symmetric group*, ***Fundamenta Mathematicae***, vol. 207 (2010), no. 2, pp. 101–122.

Abstracts of contributed talks

► ASHOT ABAJYAN, *Polynomial lower bounds to the lengths of proofs in some weak systems*.
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It is well known that the study of proof complexity in propositional systems is important due to tight relation of proof complexity to the main problem of complexity theory: $P = NP$. In particular, Cook and Reckhow proved that $NP = co\ NP$ iff there is a polynomially bounded proof system for classical tautologies. Although the first superÂ¸polynomial lower bounds to the lengths of proofs were shown by Tseitin for a Resolution, but Resolution and some other weak systems are vital to applications as the design of efficient SAT solvers, and hence proving lower and upper bounds of proof complexity in such systems is a very important field of logic.

The notion of quasi-hard determinative formulas is introduced and it is shown that such quasi-hardness is enough for the receiving "good" bounds to the lenghts of proofs in weak systems.

Let φ be a propositional formula, $X = \{x_1, x_2, \ldots, x_n\}$ be the set of distinct variables of φ and $X' = \{x_{i_1}, x_{i_2}, \ldots, x_{i_m}\}$ $(1 \leq m \leq n)$ be some subset of X.

Definition 1. Given $\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_m) \in E^m$, the conjunct $K^\sigma = \{x_{i_1}^{\sigma_1}, x_{i_2}^{\sigma_2}, \ldots, x_{i_m}^{\sigma_m}\}$

is called *φ-determinative* for φ if assigning σ_j $(1 \leq j \leq m)$ to each x_{i_j}, we obtain the value of φ (0 or 1) independently of the values of the remaining variables.

DEFINITION 2. We call the minimal number of variables in a φ-determinative conjunct the *determinative size* of φ and denote it by $d(\varphi)$.

DEFINITION 3. Let φ_n $(n \geq 1)$ be a sequence of minimal tautologies. If for some n_0, $\forall n \geq n_0$, $d(\varphi_n) < d(\varphi_{n+1})$ then the formulas $\varphi_{n_0}, \varphi_{n_0+1}, \varphi_{n_0+2}, \ldots$ are called *quasi-hard determinative*.

Resolution over linear equation $(R(lin))$ and *Resolution* (R) systems are considered. Let us denote by $Tsgf_n$ $(n \geq 2)$ the Tseitin graph formulas which are constructed on the base of complete n-vertices graphs, only one vertex of which is marked with 1 and the rest with 0. Every formula $Tsgf_n$ is unsatisfiable, therefore has some refutation in those systems, so:

THEOREM. *$Tsgf_n$ quasi-hard formulas have the same polynomial-size lower and upper bound refutations in R and $R(lin)$ systems.*

► RYOTA AKIYOSHI AND GRIGORI MINTS, *Complete cut-elimination theorem for Ω-rule*.
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Ω-rule was introduced by W. Buchholz to give an ordinal analysis for a subsystem of analysis with iterated Π^1_1-comprehension. Using collapsing of proofs, he showed that any proof of *arithmetical* sequent can be transformed into a cut-free proof. In this talk, we propose an extension of Ω-rule and prove the complete cut-elimination theorem: any derivation of *arbitrary sequent* can be transformed into its cut-free derivation by the standard rules (with induction replaced by ω-rule).

We treat a subsystem of Π^1_1-CA (of the same strength as ID_1). If time is permitting, extension to full Π^1_1-CA is outlined.

[1] RYOTA AKIYOSHI AND GRIGORI MINTS, *Analysis and Extension of OmegaÂ¸rule*, preprint.

[2] WILFRIED BUCHHOLZ, *Explaining the Gentzen–Takeuti reduction steps*, ***Archive for Mathematical Logic***, vol. 40, pp. 255–272.

► VLADISLAV AMSTISLAVSKIY, *Elementary theories of functional spaces*.
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In the paper [1] the decidability of theory of structure $(C(\mathbb{R}), \leq)$ was proven, where $C(\mathbb{R})$ is the space of continuous functions from $\mathbb{R}$ to $\mathbb{R}$, $\leq$ is pointwise order. This required a generalization of the method of interpretations offered by Kudinov O.V. The decidability was proven by m-reducibility of the theory of structure $(C(\mathbb{R}), \leq)$ to the theory of structure $(O(\mathbb{R}), \subseteq, \emptyset)$, where $O(\mathbb{R})$ is the space of open subsets of $\mathbb{R}$, $\subseteq$ is the relation of inclusion. And the theory of this structure is decidable [2]. In this paper the assertion that generalizes this result is proven.

Let X be a perfectly normal space; $\mathfrak{A} \rightleftharpoons (C(X), \leq, <)$, where $C(X)$ is the space of continuous functions from X to $\mathbb{R}$, $\leq$ is pointwise order and relation $<$ is defined as follows $f < g$ if and only if $f(x) < g(x)$ for any $x \in X$; $\mathfrak{B} \rightleftharpoons (O(X), \subseteq, \emptyset)$, where $O(X)$ is the space of open subsets of X, $\subseteq$ is the relation of inclusion. Then the following statement is valid.

THEOREM. *The theories of these structures are m-equivalent.*

The space $\mathbb{R}$ was extended to a perfectly normal space X for the following arguments.

First it was assumed to generalize the statement as much as possible, and second, in the proof it was used the fact that each closed set of perfectly normal space is functionally closed, which ultimately enabled us to prove this theorem. The proof of the theorem in one direction was carried out using the classical method of interpretation described in [2] and to prove the converse again took its generalization.

[1] VLADISLAV AMSTISLAVSKIY, *Elementary theories of continuous functions spaces*, ***Vestnik NGU***, 2011 (to appear).

[2] MICHAEL O. RABIN, *Decidable theories*, ***Handbook of mathematical logic***, Studies in Logic and the Foundations of Mathematics, vol. 90 (Jon Barwise, editor), Elsevier science B.V., Amsterdam, 1999, pp. 595–627.

► MOHAMMAD ARDESHIR AND SEYED MOJTABA MOJTAHEDI, *Ultimately directed frames and the weak principle of excluded middle*.
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We introduce ultimately directed-frame Kripke models for the first order intuitionistic logic, *IQC*. We prove that *IQC* plus the axiom schema $\neg\neg\forall x(\neg\varphi \vee \neg\neg\varphi)$ is complete for ultimately directed Kripke models. This result is a refinement of the earlier result in [1], which proves soundness and completeness theorems of *IQC* plus the axiom schema $\forall x(\neg\varphi \vee \neg\neg\varphi)$ for directed Kripke models. We then conclude that HA (in fact any extension of HA which does not imply the axiom schema $\neg\neg\forall x(\neg\varphi \vee \neg\neg\varphi)$) is not complete for any class of Kripke models which does not contain a full binary subtree.

[1] G. CORSI and S. GHILARDI, *Directed frames*, ***Archive for Mathematical Logic***, vol. 29 (1989), no. 1, pp. 53–67.

► SERIKZHAN BADAEV, *Principal numberings of finite families*.
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We use the symbol Σ_n^{-1} to denote the class of level n of the Ershov hierarchy of sets, [2], whereas as usual Σ_n^0 denotes the class of level n of the arithmetical hierarchy. A numbering α of a family $\mathcal{A} \subseteq \Sigma_n^i$, with $i \in \{-1, 0\}$, is Σ_n^i*-computable* if $\{\langle x, m\rangle : x \in \alpha(m)\} \in \Sigma_n^i$, i.e., the sequence $\alpha(0), \alpha(1), \dots$ of the members of $\mathcal{A}$ is uniformly Σ_n^i. A numbering α of a family $\mathcal{A}$ is *principal* if it is Σ_n^i-computable and every Σ_n^i-computable numbering of $\mathcal{A}$ is reducible to α.

It is well-known that every class Σ_{n+1}^i, $i \in \{-1, 0\}$, has a principal numbering. We have complete description of finite families in the arithmetical hierarchy which have principal numberings:

THEOREM 1 (Lachlan, [3]). *Every finite family of Σ_1^0-sets has a principal numbering.*

THEOREM 2 (Badaev, Goncharov, Sorbi, [1]). *Let $\mathcal{A}$ be any finite family of Σ_{n+2}^0 sets. Then $\mathcal{A}$ has a principal numbering if and only if $\mathcal{A}$ contains a least set under inclusion.*

In contrary with these results, we have

THEOREM 3. *For every $n \geq 2$, there exists a family which consists of two Σ_n^{-1}-sets and has no principal numbering.*

[1] S. A. BADAEV, S. S. GONCHAROV, AND A. SORBI, *Arithmetical numberings: completeness and universality*, ***Computability and Models***, (S. B. Cooper and S. S. Goncharov, editors), Kluwer/Plenum Publishers, New York, 2003, pp. 11–44.

[2] YU. L. ERSHOV, ***Theory of numberings***, Nauka, Moscow, 1977.

[3] A. H. LACHLAN, *Standard classes of recursively enumerable sets*, ***Zeitschrift für Matematische Logik und Grundlagen der Mathematik***, vol. 10 (1964), no. 1, pp. 23–42.

► JOHN BALDWIN, WILLIAM HOWARD, AND ANDREAS MUELLER, *Purity in geometric proofs*.
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There has been considerable discussion recently (e.g., Arana, Detlefsen, Hallett, Mancosu) of 'purity of method'. In particular, the question of the role of three dimensional methods in the proof of Desargues theorem in the plane has been studied. We address another aspect of this situation; the purity of Hilbert's 'converse': a Desarguesian plane can be embedded in 3-space. We note a corollary of Hilbert's result which relies on the completeness theorem. Hilbert's argument has manifestly algebraic elements. We provide a 'geometric proof' (developed with William Howard) and analyze the issue of why this is a 'more pure' proof than Hilbert's. A somewhat different geometric proof was given by Levi in 1939.

► NIKOLAY BAZHENOV, *On categoricity spectra of Boolean algebras*.
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We study degrees of computable categoricity of Boolean algebras. We use a terminology from [1, 3].

Let's recall some definitions from [2]. Let $\mathbf{d}$ be a Turing degree. A computable structure $\mathfrak{A}$ is called $\mathbf{d}$-*computably categorical* if, for every computable structure $\mathfrak{B}$ isomorphic to $\mathfrak{A}$, there exists a $\mathbf{d}$-computable isomorphism from $\mathfrak{A}$ onto $\mathfrak{B}$.

Let $\mathfrak{A}$ be a computable structure. The *categoricity spectrum* of $\mathfrak{A}$ is a set

$$\mathrm{CatSpec}(\mathfrak{A}) = \{\mathbf{d} \mid \mathfrak{A} \text{ is } \mathbf{d}\text{-computably categorical}\}.$$

A Turing degree $\mathbf{d}_0$ is the *least degree of categoricity* of $\mathfrak{A}$ if $\mathbf{d}_0$ is the least degree in $\mathrm{CatSpec}(\mathfrak{A})$.

We establish the following generalization of McCoy's [4] result:

THEOREM 1. *Let $\mathfrak{B}$ be a computable Boolean algebra. Then $\mathfrak{B}$ is Δ^0_2-categorical if and only if it is relatively Δ^0_2-categorical.*

COROLLARY 2. *Let $\mathbf{d}$ be a Turing degree such that $\mathbf{0} < \mathbf{d} < \mathbf{0}'$. Then $\mathbf{d}$ can't be the least degree of categoricity of a computable Boolean algebra.*

We also prove

THEOREM 3. *Let n be a natural number. There exists a computable Boolean algebra $\mathfrak{B}_n$ such that $\mathbf{0}^{(n)}$ is the least degree of categoricity of $\mathfrak{B}_n$.*

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[1] C. J. ASH AND J. KNIGHT, ***Computable structures and the hyperarithmetical hierarchy***, Studies in Logic and the Foundations of Mathematics, Elsevier Science, Amsterdam, 2000.

[2] E. B. FOKINA, I. KALIMULLIN, AND R. MILLER, *Degrees of categoricity of computable structures*, ***Archive for Mathematical Logic***, vol. 49 (2010), no. 1, pp. 51–67.

[3] S. S. GONCHAROV, ***Countable Boolean Algebras and Decidability***, Siberian School of Algebra and Logic, Consultants Bureau, New York, 1997.

[4] C. F. D. McCoy, Δ_2^0-*categoricity in Boolean algebras and linear orderings*, ***Annals of Pure and Applied Logic***, vol. 119 (2003), no. 1–3, pp. 85–120.

► MATTEO BIANCHETTI, *Foundations of mathematics and philosophy of logic: Logical Consequence* (*LC*).

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Etchemendy [1, 2] claims: Tarski's definition of LC (TLC) [3], based on the notion of truth-preservation and formality, is both (1) conceptually and (2) extensionally inadequate. About (1), Etchemendy reasons: TLC does not explain why we are justified to infer a conclusion from certain premises before knowing the truth-value of the sentences in an argument. About (2), Etchemendy reasons: TLC relies on non-logical assumptions (e.g., in ZF we need the axiom of infinity to show that $\sigma =$ "There are at most n elements", for some n, is not logically valid) and, so, TLC can overgenerate.

(1*) Reply to (1): there are both intuitive (the metatheory allows to build any possible situation) and historical (Bolzano, Pieri, ...) considerations that support the idea that LC is essentially relative to the class of cases considered possible and relevant to evaluate truth-preservation.

(2*) Reply to (2): (a) for every n, σ is not a logical truth in ZF minus the axiom of infinity, (b) Etchemendy's examples with σ-sentences shows that he misunderstands the role of a meta-theory, that does not need to determine a complete ontology, (c) the fact that TLC can overgenerate only shows that LC is, here, a relational concept and, by (1*), we can consider LC as such.

[1] J. Etchemendy, ***The concept of logical consequence***, Harvard University Press, Cambridge, Massachussetts, 1990.

[2] ———, *Reflections on consequence*, in Patterson [2008], ***New essays on Tarski and philosophy***, Oxford University Press, Oxford, New York, pp. 263–299, 2008.

[3] A. Tarski, *O pojciu wynikania logicznego* (*On the concept of logical consequence*), ***Prezglad Filozoficzny***, vol. 39 (1936), pp. 56–68, translated in Tarski [1983], ***Logic, semantics, metamathematics***, II ed., translated by J. H. Woodger, introduction by J. Corcoran, Hackett Publishing Company, Indianapolis, Indiana, pp. 409–420.

► HOVHANNES BOLIBEKYAN AND TIGRAN MURADYAN, *On some loop detection strategies for minimal propositional logic*.

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Several proof systems of I. Johansson's minimal logic of predicates were introduced in [1]. In general backward proof search in propositional fragments is non-terminating in a sense that the rules can generate the same sequent again. To ensure termination of backward proof search one needs to check the same sequent is not generated again in the branch. Some loop detection technics were investigated in [2].

We consider loop detection strategies for propositional fragment of minimal logic of I. Johansson based on adding context to the sequents. Introduced systems with loop checking are proved to be equivalent to the systems from [1].

[1] H. R. Bolibekyan and A. A. Chubaryan, *On some proof systems for I.Johansson's minimal logic of predicates*, ***Proceedings of the Logic Colloquium 2003***, p. 56.

[2] D. Gabbay, ***Algorithmic proof with diminishing resources, Part 1***, Springer Lecture Notes in Computer Science, 533, pp. 156–173, 1991.

► JÖRG BRENDLE AND JANA FLAŠKOVÁ, *The generic existence of certain $\mathcal{I}$-ultrafilters*.
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Ultrafilters from a given class $\mathcal{C}$ *exist generically* if each filter base of size less than $\mathfrak{c}$, the cardinality of continuum, can be extended to an ultrafilter belonging to $\mathcal{C}$. The generic existence of ultrafilters can be characterized in terms of cardinal equalities. Given a class of ultrafilters $\mathcal{C}$ one can define a cardinal $\mathfrak{ge}(\mathcal{C})$, which denotes the minimal cardinality of a filter base which cannot be extended to an ultrafilter from $\mathcal{C}$. It is easy to see that ultrafilters from $\mathcal{C}$ exist generically if and only if $\mathfrak{ge}(\mathcal{C}) = \mathfrak{c}$.

The concept of $\mathcal{I}$-ultrafilters was introduced by Baumgartner: Let $\mathcal{I}$ be a family of subsets of a non-empty set X such that $\mathcal{I}$ contains all singletons and is closed under subsets. Given an ultrafilter $\mathcal{U}$ on ω, we say that $\mathcal{U}$ is an *$\mathcal{I}$-ultrafilter* if for any $F\colon \omega \to X$ there is $U \in \mathcal{U}$ such that $F[U] \in \mathcal{I}$.

In this talk, we focus on $\mathcal{I}$-ultrafilters defined for $X = \omega$ and $\mathcal{I}$ an ideal on ω. We present some general properties of the cardinal invariant $\mathfrak{ge}(\mathcal{I}$-ultrafilters) for an arbitrary ideal $\mathcal{I}$ on ω and we give several different estimates of the cardinal invariant for some particular ideals such as the density ideal $\mathcal{Z}_0$ or the summable ideal $\mathcal{I}_{1/n}$.

► PETR CINTULA AND CARLES NOGUERA, *Generalized disjunctions in (infinitary) structural consequence relations*.
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This paper is an abstract investigation on disjunction connectives in an Abstract Algebraic Logic approach. The most comprehensive treatment so far, summarizing and generalizing previous works, was presented by Czelakowski in his monograph "Protoalgebraic Logics" (Kluwer, 2000) where he considered generalized disjunction connectives given by a (possibly parameterized and infinite) set of formulae satisfying the Proof by Cases Property (PCP). This approach allowed to characterize disjunctions in terms of other properties of logics and their semantics, namely: distributivity of the lattice of theories (or filters over any algebra) and the behavior of prime filters and substitutions. However, all these results were restricted to *finitary* logics. This papers develops the theory mainly by generalizing it to *infinitary* logics. In particular we:

- identify a strong form of the proof by cases property (sPCP), which coincides with the PCP in finitary logics.
- present a hierarchy of logics based on the form of the disjunction they posses (a single formula or a (parameterized) set of formulae) and the strength of the proof by cases property it satisfies.
- generalize the known results in two ways: (1) we prove their *modified* versions (e.g., replacing PCP by sPCP and distributivity by infinite distributivity) which hold in complete generality and coincide with known ones in the finitary case; (2) we extend the validity of known results from finitary logics to the strictly larger class of logics where the finitely meet-irreducible theories form a basis of the closure system of all theories (this class includes not only all finitary logics, but also all semilinear logics, e.g., infinitary Łukasiewicz logic).

- show several consequences of the presence of a suitable disjunction in a given logic, e.g., a method of axiomatizing: (1) its extensions semantically given by a positive universal class of its models and (2) the intersection of any finite set of its axiomatic extensions.

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► ANDRÉS CORDÓN–FRANCO, ALEJANDRO FERNÁNDEZ–MARGARIT, AND F. FÉLIX LARA–MARTÍN, *On the optimality of a conservation result for local reflection in arithmetic*.
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Let T be a recursively enumerable (r.e.) first-order arithmetic theory extending *Kalmár Elementary Arithmetic EA* and let $Rfn_\Gamma(T)$ denote the *local reflection principle* for T restricted to sentences in the set of formulas Γ. In [1] L. D. Beklemishev proved that $T+Rfn_{\Sigma_2}(T)$ is conservative over $T+Rfn_{\Sigma_1}(T)$ with respect to $\mathcal{B}(\Sigma_1)$ (= boolean combinations of Σ_1) sentences; and raised the problem of determining whether this conservation result is best possible.

Question 1 (Beklemishev). Is $T + Rfn_{\Sigma_2}(T)$ Π_2-conservative over $T + Rfn_{\Sigma_1}(T)$?

For $T \subseteq \mathcal{B}(\Sigma_1)$, Question 1 can be seen as an instance of the following general question on arithmetic theories (let $Th_\Gamma(S)$ denote the set of all Γ-consequences of S).

Question 2. Is $Th_{\Pi_2}(Th_{\Sigma_2}(S))$ deductively equivalent to $Th_{\mathcal{B}(\Sigma_1)}(S)$?

Interestingly, a basic model-theoretic argument gives us that the dual version of Question 2 has a positive answer. Namely, if S implies Δ_0-induction then $Th_{\Sigma_2}(Th_{\Pi_2}(S))$ and $Th_{\mathcal{B}(\Sigma_1)}(S)$ are deductively equivalent. In this work we prove that, however, Questions 1 and 2 have, in general, a negative answer. We prove that: (a) if T is a finite Π_2-extension of EA closed under Σ_1-collection rule, then $T + Rfn_{\Sigma_2}(T)$ is not Π_2-conservative over $T + Rfn_{\Sigma_1}(T)$ provided that $T + Rfn_{\Sigma_1}(T)$ is consistent; and (b) if $T \subseteq \mathcal{B}(\Sigma_1)$ and $S = T + Rfn_{\Sigma_2}(T)$ is consistent, then $Th_{\Pi_2}(Th_{\Sigma_2}(S))$ is strictly stronger than $Th_{\mathcal{B}(\Sigma_1)}(S)$. We derive both results from the following *unboundedness theorem* for the Π_2-consequences of $T + Rfn_{\Sigma_2}(T)$.

THEOREM 1. *Suppose T is a finite Π_2–extension of EA closed under Σ_1-collection rule. Then $Th_{\Pi_2}(T + Rfn_{\Sigma_2}(T))$ is not contained in any consistent, r.e. extension of T by $\mathcal{B}(\Sigma_1)$-sentences.*

Our methods are model-theoretic. We use Beklemishev's results *à la* Kreisel–Lévy relating local reflection principles to parameter-free induction, and then work with certain *local* variants of the Σ_1-induction scheme.

[1] L. D. BEKLEMISHEV, *Reflection principles and provability algebras in formal arithmetic*, ***Russian Mathematical Surveys***, vol. 60 (2005), no. 2, pp. 197–268.

► MARCOS CRAMER, *Parsing and disambiguation of symbolic mathematics in Naproche*.
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The Naproche system is a system for linguistically analysing and proof-checking mathematical texts written in a controlled natural language. The aim is to have an input language that is as close as possible to the language that mathematicians actually use when writing textbooks or papers.

Mathematical texts consist of a combination of natural language and symbolic mathematics, with symbolic mathematics obeying its own syntactic rules. For example, its syntax includes prefix, suffix, infix and circumfix operators; new operators may be introduced by definitions and used in the subsequent text. We discuss the difficulties that a program for parsing and disambiguating symbolic mathematics must face and present how these difficulties have been tackled in the Naproche system. One of these difficulties is the fact that information provided in the preceding context—including information provided in natural language—can influence the way a symbolic expression has to be disambiguated, as in the following example:

The expression $a(x+y)$ can be read in two different ways, depending on the reading of a: If a is a function symbol, then $a(x+y)$ is read as the result of applying the function a to $x+y$. If on the other hand a, x and y are all real numbers, then $a(x+y)$ is read as the product of a and $x+y$. Whether a is a function or a real number should have been specified (whether explicitly or implicitly) in the preceding text.

We describe how such difficulties are solved in the Naproche system, and compare this solution to Ganesalingam's solution [1].

[1] MOHAN GANESALINGAM, ***The language of mathematics***, Ph.D. thesis, University of Cambridge, 2009.

► LAURA CROSILLA AND ANDREA CANTINI, *Constructive set theory with operations: weak systems*.

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Constructive operational set theory [2] is a variant of Myhill and Aczel's constructive set theory in which a notion of operation or rule is introduced aside the notion of constructive set. It also represents a constructive version of Feferman's recent operational set theory and of Beeson's intuitionistic set theory with rules ([1], [4], [5], [6]). In constructive operational set theory, operations apply both to sets and to operations themselves. Consequently, while sets are fully extensional, operations are non-extensional and partial.

We shall present the salient elements of a fully explicit, finitely axiomatized system, ESTE, of constructive sets and operations, which was shown to be as strong as PA in [3]. We shall then hint at some recent work on extensions of such a system by means of operations which internally represent transitive closure and unique decoration of graphs.

[1] M. BEESON, *Towards a computation system based on set theory*, ***Theoretical Computer Science***, vol. 60 (1988), pp. 247–340.

[2] A. CANTINI AND L. CROSILLA, *Constructive set theory with operations*, (A. Andretta, K. Kearnes, and D. Zambella editors), ***Logic Colloquium 2004***, Association for Symbolic Logic, Lecture Notes in Logic, 29, 2008.

[3] ———, *Explicit constructive set theory*, ***Ways of proof theory***, (R. Schindler, editor), Ontos Series in Mathematical Logic, Frankfurt, 2010.

[4] S. FEFERMAN, *Operational Set Theory and small large cardinals*, ***Information and Computation***, vol. 207 (2009), no. 10, pp. 971–979.

[5] G. JÄGER, *On Feferman's operational set theory OST*, ***Annals of Pure and Applied Logic***, vol. 150 (2007), pp. 19–39.

[6] ———, *Full operational set theory with unbounded existential quantification and powerset*, ***Annals of Pure and Applied Logic***, vol. 160 (2009), no. 1, pp. 33–52.

► SERGEY DROBYSHEVICH, *A double negation operator in logic N^**.

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We study a double negation operator in logic N^*. The logic N^* in the language $\mathcal{L} = \langle \vee, \wedge, \to, \neg \rangle$ [3] is an extension of logic N suggested by Dosen [2], in which the negation is interpreted as an impossibility operator. N^* is complete with respect to the class of frames, in which the validity of negation is defined by an anti-monotonic function on the set of possible worlds.

The aim of this work is to obtain axiomatization of a double negation operator in N^* as a modal operator of necessity.

We introduce logic $N^{\Box}$ in the language $\mathcal{L}_{\Box} = \langle \vee, \wedge, \to, \Box, - \rangle$ as an extension of logic $HK\Box$ [1] with the axioms: $\Box p \wedge \Box q \to \Box(p \wedge q)$, $\Box(p \to p)$, $-\Box - (p \to p)$ and $\Box(p \vee q) \to \Box p \vee \Box q$. We define semantics for $N^{\Box}$ the following way:

Definition. $\mathcal{W} = \langle W, \leq, \sharp \rangle$ is an $N^{\Box}$-frame if: (i) W is a non-empty set, (ii) $\leq$ is a partial ordering on W, (iii) $\sharp \colon W \to W$ is a monotonic function on W. An $N^{\Box}$-model is an $N^{\Box}$-frame together with valuation satisfying intuitionistic heredity.

The validity of formulas is defined the usual way (the minus sign stands for inuitionistic negation) except for the case of necessity: $\mu, x \vDash \Box\phi$ iff $\mu, x^{\sharp} \vDash \phi$. We then prove

Theorem. *$N^{\Box}$ is complete with respect to the class of $N^{\Box}$-frames.*

Proposition. *$N^{\Box}$ has a finite model property and it is decidable.*

We define natural translation τ from $N^{\Box}$ into N^*: (i) $\tau(p) = p$, $p \in Prop$; (ii) $\tau(\phi * \psi) = \tau(\phi) * \tau(\psi)$, *for* $* \in \{\vee, \wedge, \to\}$; (iii) $\tau(-\phi) = \tau(\phi) \to \neg(p \to p)$; (iv) $\tau(\Box\phi) = \neg\neg\tau(\phi)$ and prove

Proposition. *For any ϕ in the language $\mathcal{L}_{\Box}$, we have $\phi \in N^{\Box}$ iff $\tau(\phi) \in N^*$.*

This work was supported by Russian Foundation for Basic Research, project RFBR-09-01-00090-a.

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[3] S. P. Odintsov, *Combining intuitionistic connectives and Routley negation*, ***Siberian Electronic Mathematical Reports***, vol. 7 (2010), pp. 21–41.

► EKATERINA FOKINA, ANDREY FROLOV, AND ISKANDER KALIMULLIN, *Structures with no degree of categoricity*.
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The degree of categoricity of a computable structure is the least Turing degree **x** such that the structure is **x**-computably categorical. For example, it is proved in [1] that every c.e. degree is the degree of categoricity of some rigid computable structure. R. Miller [2] have showed that some computable structures can have no degree of categoricity. However, these examples are not rigid structures. We present new series of computable structures with no degree of categoricity which are rigid. Moreover, for every nonzero c.e. degree **x** there is a rigid **x**-computably categorical structure with no degree of categoricity.

[1] E. Fokina, I. Kalimullin, and R. Miller, *Degrees of categoricity of computable structures*, ***Archive for Mathematical Logic***, vol. 49 (2010), no. 1, pp. 51–67.

[2] R. Miller, *d-Computable categoricity for algebraic fields*, ***The Journal of Symbolic Logic***, vol. 74 (2009), no. 4, pp. 1325–1351.

► MARCO FORTI, *Asymptotic equinumerosities via quasi-selective ultrafilters*.
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Equinumerosities are equivalence relations that preserve the basic properties of equipotency for *finite* sets, including the Euclidean principle "the whole is greater than the part".

An *asymptotic equinumerosity* is an equivalence relation $\approx$ on $\mathbb{W} = \bigcup_k \mathcal{P}(\mathbb{N}^k)$ s.t.

E0: If $X_n = \{x \in X \mid \forall i\,(x_i \leq n)\}$, then

$$\forall n \in \mathbb{N}\, |X_n| \leq |Y_n| \Longrightarrow \exists Z \subseteq Y \text{ s.t. } X \approx Z;$$

E1: $X \approx Y \Longleftrightarrow X \setminus Y \approx Y \setminus X$;

E2: Exactly one of the following conditions holds:

$$X \approx Y;\ X' \approx Y \text{ for some } X' \subset X;\ X \approx Y' \text{ for some } Y' \subset Y.$$

E3: $X \approx X' \Longrightarrow X \times Y \approx X' \times Y$.

The corresponding *numerosities* are the non-negative part of a *discretely ordered ring*, where sums, products, and ordering correspond to disjoint unions, Cartesian products, and proper subsets. In general equinumerosities require special ultrafilters. Asymptotic equinumerosities require a new class of ultrafilters on $\mathbb{N}$, called "*quasi-selective*" because they are intermediate between selective ultrafilters and P-points.

A nonprincipal ultrafilter $\mathcal{U}$ on $\mathbb{N}$ is *quasi-selective* if

$$\forall n\,(g(n) \leq n) \Longrightarrow \exists U \in \mathcal{U}.\, \exists h \text{ nondecreasing s.t. } \forall n \in U\,(g(n) = h(n))$$

Remark that the family $F_{\mathcal{U}}$ of functions that are nondecreasing modulo $\mathcal{U}$ is much larger, including, *e.g.*, all primitive recursive functions.

THEOREM 1 ([1]). *There exists a biunique correspondence between asymptotic equinumerosities $\approx$ on $\mathbb{W}$ and quasi-selective ultrafilters $\mathcal{U}$ on $\mathbb{N}$, given by*

$$X \approx Y \Longleftrightarrow \{n \in \mathbb{N} \mid |X_n| = |Y_n|\} \in \mathcal{U}. \quad (\sharp)$$

The map $\mathfrak{n}\colon X \mapsto [\langle |X_n| \rangle_{n\in\mathbb{N}}]_{\mathcal{U}}$ induces an isomorphism of ordered semirings between $\mathbb{W}/\approx$ and the initial segment of the ultrapower $\mathbb{N}^{\mathbb{N}}_{\mathcal{U}}$ containing all polynomially bounded sequences. So asymptotic numerosities can be taken to be special nonstandard integers.

[1] A. BLASS, M. DI NASSO AND M. FORTI, *Quasi-selective ultrafilters and asymptotic numerosities*, ArXiv:1011.2089.

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In this talk, I discuss U. Kohlenbach's work *Applied Proof Theory: Proof Interpretations and their Use in Mathematics* from a foundational and historical perspective. I put the emphasis on the continuity of applied proof theory with the Hilbert programme and I show that Kronecker's idea of finitist foundations survives within the Hilbert programme (with infinite descent) in the foundations of mathematics. The idea of proof mining is meant to underline the project of extracting effective bounds from ineffective classical proofs in analysis and the logical tools used in that perspective are reviewed. Finally, I stress the foundational significance of the work of Kohlenbach and others in applied proof theory.

[1] YVON GAUTHIER, ***Internal logic. Foundations of mathematics from Kronecker to Hilbert***, Kluwer, Dordrecht/Boston/London, 2002.

[2] ———, *Hilbert programme and applied proof theory*, ***Logique et Analyse***, forthcoming, 2011.

[3] Ulrich Kohlenbach, ***Applied proof theory. Proof interpretations and their use in mathematics***, Springer-Verlag, Berlin/Heidelberg, 2008.

▶ ALEXANDER GAVRYUSHKIN, *Decidable Fraïssé limits*.
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In [1], B. Csima, V. Harizanov, R. Miller, and A. Montalbán found a criterion for Fraïssé limit to be computable. In present paper we study the following question. What properties a class of finitely generated structures must satisfy in order to have a decidable Fraïssé limit? We find several necessary and several sufficient conditions. These conditions enable us to prove the following statements.

We say that a model is *p-almost prime* if it is prime over some realization of the finite type-p, and is *p-limit* if it is a union of isomorphic p-almost prime models forming an ω-chain under elementary embeddings, but it is not almost prime. A model is *almost prime* (*limit*) if it is p-almost prime (p-limit) for some p.

Theorem. *Let a complete theory T has a decidable saturated model. Let $\{\mathfrak{A}_0, \mathfrak{A}_1, \dots\}$ be a maximal set of pairwise nonisomorphic almost prime models of the T such that $\mathfrak{A}_i$ is elementary embeddable into $\mathfrak{A}_j$, for all i and j. Then there is a decidable p-limit model, where $\mathfrak{A}_i$ is p-almost prime for some i.*

This theorem provides a positive answer to the Morley Problem in the following weakened form:

Corollary. Let T be an Ehrenfeucht theory all whose types are decidable types. Suppose that for any finite type p consistent with T there exists at most one p-limit model. Then all countable models of T are decidable.

Note that for all n there is an example of complete theory T_{mn} having exactly m countable models (that is Ehrenfeucht theory) such that (1) T_{mn} satisfies the Corollary; (2) T_{mn} has at lest n limit models.

[1] B. Csima, V. Harizanov, R. Miller, and A. Montalbán, *Computability of Fraïssé limits*, ***The Journal of Symbolic Logic***, vol. 76 (2011), no. 1, pp. 66–93.

▶ DENIS HIRSCHFELDT, KEN KRAMER, RUSSELL MILLER, AND ALEXANDRA SHLAPENTOKH, *The complexity of computable categoricity for algebraic fields*.
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A computable structure $\mathcal{A}$ is *computably categorical* if every computable structure $\mathcal{B}$ classically isomorphic to $\mathcal{A}$ is in fact computably isomorphic to $\mathcal{A}$. We present recent results on this topic, focusing on our proof that the property of computable categoricity for computable algebraic fields is Π^0_4-complete. (A field is *algebraic* if it is an algebraic extension of its prime subfield.) It follows that there exists a computably categorical algebraic field F for which the more general property of relative computable categoricity fails. This F is in some sense pathological. We describe a structural Σ_3^0 condition which is equivalent to relative computable categoricity. On the other hand, we will also explain why the separate pathological situation of finite computable dimension > 1 is impossible for these fields: every computable algebraic field either is computably categorical or else has infinitely many computable isomorphism classes.

► SIMON IOSTI, *Binding groups and the Tannakian duality*.
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This talk aims at explaining the notion of a binding group in a first order structure, and to describe an algebraic context where those groups can be used. Binding groups are closely related to the model-theoretical notion of internality; they can be seen as abstract Galois groups, in the sense that they are groups of automorphisms of the structure (or of a definable part of the structure) fixing some definable set. Under some particular assumptions, Hrushovski proved in [1] that those groups and their actions on the structure are type-definable (with parameters). Kamensky used this result in [2] to prove a version of the Tannakian duality, which states among other things that an algebraic group G defined over a field k can be completely recovered from the category of its finite-dimensional representations over k, together with a forgetful functor to the category of the finite-dimensional k-vector spaces. In his proof, he uses a kind of Galois correspondence for binding groups. I will explain how the Tannakian duality can be generalized using the ideas of Kamensky, and how to make this Galois correspondence explicit.

[1] Ehud Hrushovski, ***Computing the Galois group of a linear differential equation***, Banach Center Publications, vol. 58 (2002), pp. 97–138.

[2] Moshe Kamensky, *Model theory and the Tannakian formalism*, http://www.nd.edu/mkamensk/papers/tannakian.pdf, 2009.

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We present a quantifier-free Hilbert-style axiomatization, based on classical propositional logic, of a system of relational syllogistic formalizing the following binary relations between classes (of objects):

$$a \leq b \overset{\text{def}}{\Leftrightarrow} \forall x (x \in a \Rightarrow x \in b)$$

$$\langle Q_1 Q_2 \rangle (a, b)[\alpha] \overset{\text{def}}{\Leftrightarrow} (Q_1 x \in a)(Q_2 y \in b)\left((x, y) \in \alpha\right),$$

where a, b denote arbitrary classes, $Q_1, Q_2 \in \{\forall, \exists\}$, and α denotes an arbitrary binary relation between objects. The language of the logic contains only variables denoting classes, determining the set of class terms, and variables denoting binary relations between objects, determining the set of relational terms. Both classes of terms are closed under the standard Boolean operations. The set of relational terms is also closed under taking the converse of a relation α^{-1}. The results of the paper are the completeness theorem with respect to the intended semantics and the computational complexity of the satisfiability problem. The completeness proof uses techniques from the modal logics of region-based theories of space [1, 2] and Boolean Modal Logic [4, 3]. The complexity of the satisfiability problem is the same as that of BML [5].

[1] Ph. Balbiani, T. Tinchev, and D. Vakarelov, *Modal Logics for region-based theory of space*, ***Fundamenta Informaticae***, vol. 81 (2007), pp. 29–82.

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[4] G. Gargov, S. Passy, and T. Tinchev, *Modal Environment for Boolean speculations*, ***Mathematical Logic and its Applications*** (D. Skordev, editor), Plenum Press, New York, 1987, pp. 253–263.

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► JOOST J. JOOSTEN, *On the closed fragment of* GLP_Λ.

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The polymodal provability logic GLP_Λ has modalities $[\alpha]$ for each ordinal $\alpha \in \Lambda$. A natural interpretation of $[\alpha]$ is "provable in T together with all true hyperarithmetical sentences of level α". The closed fragment of these logics is decidable and can be used to calculate proofÂ¸theoretical ordinals. Although full GLP_Λ lacks natural Kripke semantics, the closed fragment does allow these structures whose depths are intimately related to the corresponding proofÂ¸theoretic ordinals. In this presentation we will expose these structures and some of their properties.

► ANNIKA KANCKOS AND JAN VON PLATO, *Normal derivability in classical predicate logic*.

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In a step of indirect inference in natural deduction, a negative temporary assumption ¬A is closed and A concluded. When assumptions are closed otherwise, they are subformulas of the conclusion, as in implication introduction, or of the major premiss, as in those elimination rules that close assumptions, and the subformula property of normal derivations follows. The situation is different with indirect proof, because the conclusion A can be a major premiss in an elimination, and it can be lost trace of even in the absence of non-normal instances of the rest of the rules. Solutions to the problem have included the leaving out of disjunction and existence from the language and the restriction of conclusions of indirect proof to atomic formulas, by which they cannot be major premisses of elimination rules. Other solutions involve restrictions in the way elimination rules can be instantiated, and yet others contain global proof transformations.

In the present work, it is shown that derivations in the full language of predicate logic can be so transformed by standard methods of local permutations that no conclusion of an indirect step of proof is the major premiss of an elimination rule. For the rest, normal form is defined as for intuitionistic derivations, in particular, no *a priori* restrictions are imposed on rule instances: Normal derivations and whatever rule instances they may contain come out purely as results of a normalization procedure. It follows in particular that normal derivations have the subformula property. The situation is particularly clear if natural deduction is formulated in terms of general elimination rules (cf. von Plato 2001) with the definition: A derivation is *normal* if all major premisses of elimination rules are assumptions. This definition can be applied directly to classical natural deduction for predicate logic.

[1] J. von Plato, *Natural deduction with general elimination rules*, ***Archive for Mathematical Logic***, vol. 40 (2001), pp. 541–567.

► VLADIMIR KANOVEI AND VASSILY LYUBETSKY, *An infinity which needs the axiom of choice*.
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In the early years of set theory, Du Bois Reymond introduced a vague notion of *infinitary pantachie* meant to symbolize an infinity bigger than the infinity of real numbers. Hausdorff reformulated this concept rigorously as a maximal *chain* (a linearly ordered subset) in a partially ordered set of certain type, for instance, the set $\mathbb{N}^{\mathbb{N}}$ under eventual domination. Hausdorff demonstrated the existence of a pantachy in any partially ordered set, using the axiom of choice **AC**. We prove, using the Solovay model and advances in modern descriptive set theory, that the pantachy existence theorem fails in the absense of **AC**, and moreover, even if **AC** is assumed, hence pantachies do exist, one may not be able to come up with an individual, effectively defined example of a pantachy.

► CHRIS KAPULKIN, *Homotopy theoretic models of type theory*.
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Connections between Intensional Martin-Löf Type Theory and homotopy theory have been recently intensively studied, see for example: [1, 2, 3]. By interpreting type theory in Quillen model categories—or more generally, in categories equipped with a (natural) weak factorization system—we obtain a wide class of models for a type theory being an extension of a theory with only one type constructor Id. In such an interpretation we allow only fibrant objects in a category to be interpretations of types.

However, this interpretation does not (yet) take into account one of the key features of type theory that is the type constructors Π and Σ which under Curry-Howard Isomorphism correspond to universal and existential quantifiers. In this talk, I will propose a set of conditions on a model category that make it into a model of a type theory that is any extension of the theory with Id-, Π-, and Σ-types. These conditions will be further examine in a number of examples known from algebraic topology and algebraic geometry: the category **Gpd** of groupoids, the category **SSets** of simplicial sets, and the category of simplicial presheaves.

In the last part of the talk, I will try to show how the homotopy theoretic models can be used as a tool to prove independence of certain type theoretic rules such as η.

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► MAKOTO KIKUCHI AND TAISHI KURAHASHI, *Notes on the incompleteness theorems*.
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We present three topics about the incompleteness theorems. Firstly, we prove the second incompleteness theorem by using Priest's idea [2] of arithmetical formalization of Yablo's paradox [4], which is a liar-type paradox without self-referential nature. Then, by using some results on modal logic of provability by Solovay [3] and Visser (cf. [1]), we prove the existence of an independent Π_1 sentence whose independence is not formally provable. At last, we show that the truth of any model of arithmetic can be defined in some model and formula.

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► ARCHIL KIPIANI, *On isomorphism types of some infinite graphs*.
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In the papers [4] and [5], for any ordinal number α, we have constructed a root tree with verteces set ω_α, which has the trivial automorphisms group. Now we consider some graphs that are built with the aid of this tree, and some of their applications to combinatorial problems considered in [1]–[7].

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► NURLAN KOGABAEV, *Computable presentations of desarguesian projective planes*.
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Following [2] we treat a projective plane as partial algebraic system $\langle A, (A^0, {}^0A), \cdot \rangle$ with a disjunction of A into two subsets $A^0 \cup {}^0A = A$, $A^0 \cap {}^0A = \emptyset$ and commutative partial operation "$\cdot$" which satisfy certain axioms.

In [1] it was shown that any countable free projective plane has computable dimension 1 or ω. Furthermore, such a plane is computably categorical if and only if it has finite rank.

In the present paper we examine desarguesian projective planes from the perspective of computable dimension. Let $\mathcal{P}(F)$ be a countable desarguesian (pappian) projective plane over a skewfield F (field F). We obtain the following results:

(1) *$\mathcal{P}(F)$ has a computable presentation iff F has a computable presentation.*
(2) *The computable dimension of $\mathcal{P}(F)$ coincides with the computable dimension of F.*

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► KANAT KUDAIBERGENOV, *Some generalizations of o-minimality*.
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The notion of o-minimality was introduced more than twenty five years ago and proved to be quite useful and important. Since that time many generalizations appeared—weak o-minimality, quasi-o-minimality, etc. In this talk I will generalize o-minimality a little further and discuss some properties of the corresponding notions.

► JUI-LIN LEE, *Classical model existence and paraconsistency*.
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In [2] we studied the classical model existence property *CME* (every consistent set has a classical model) and constructed several proof systems satisfying *CME*. While constructing such proof systems, the choice of unsatisfiable set (which serves as a contradiction) does make differences on shaping proof systems. In order to get a better understanding on this, in [3] a general study of consistency was developed.

However, paraconsistent logic is defined as a logic with non-explosive contradictory theory. Note that a theory Γ is *contradictory* in logic L iff from Γ one can derive α and $\neg\alpha$ in L for some α. This definition somehow depends on $\neg$.

In this talk we will investigate the concept of paraconsistency to see whether this concept can be generalized according to the idea in [3].

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► MARGARITA LEONTYEVA, *Strongly computable copies of Boolean algebras with elementary characteristic* $(\infty, 0, 0)$.
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The important notion in theory of Boolean algebras is elementary characteristic. It is a triple of elements $(ch_1(\mathfrak{B}), ch_2(\mathfrak{B}), ch_3(\mathfrak{B}))$ from $\omega \cup \{\infty\}$ which fully describes elementary theory of Boolean algebra.

By strongly computable structure we understand a computable structure with computable first order diagram.

Let T be the elementary theory of a Boolean algebra $\mathfrak{B}$. For every such theory, except one which corresponds to $ch_1(\mathfrak{B}) = \infty$, Yu. L. Ershov in [1] found a finite sequence of unary predicates $P_0, \dots, P_n$ defined by first order formulas such that an arbitrary Boolean algebra of this theory is strongly computable iff it is computable and all predicates $P_0, \dots, P_n$ are also computable.

The following problem was been considered: if $S \subseteq \{P_0, \dots, P_n\}$ is a subset, $\mathfrak{B}$ is computable, and all predicates from S are computable in $\mathfrak{B}$, then can we assert that $\mathfrak{B}$ has strongly computable copy?

At Logic Colloquium 2010 we presented the complete answer for this question. The following question is what do we have in the case of $ch_1(\mathfrak{B}) = \infty$? That is the topic which is going to be discussed this time.

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► EVGENY LEUTSKY, *Spectrum of constructive models of uncountably categorical theories*.
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DEFINITION 1. Let T be an uncountably but not totally categorical theory. Let us define the spectrum of constructive models of T by

$$SCM(T) = \{\kappa \leq \omega \mid M_\kappa \text{ is constructive}\}.$$

DEFINITION 2. The model is called n-constructive if its Σ_n-diagram is computable.

Spectrum of constructive models of uncountable categorical theories only consisting from prime models was founded with the aid of the example of the prime model constructed in [3] for finite relational binary language and the example of the prime model constructed in [1] for infinite language. The other spectra of computability were founded in [2] for infinite languages. The question remains opened concerning the possibility of obtention of the same results (as [2]) for the final languages.

We investigate the complexity of the prime model $\mathcal{M}_0$ that has been constructed in [3] by an finitely generated group G. The theory T of this model is uncountably categorical and $SCM(T) = \{0\}$.

First we apply the Marker's operators ($\forall$-, $\exists$-extention) n times, for any $n > 0$, to prime model $\mathcal{M}_0$ and obtain

THEOREM 3. *The model $(\mathcal{M}_0)_{(\forall\exists)^n}$ is n-computable, but not $(n+k)$-computable for any $k > 0$ and $n > 0$.*

Now we can apply theorem 3 to prove the following result of this paper:

THEOREM 4. *If the model $(\mathcal{M}_0)_{(\forall\exists)^{n+1}}$ is $(n+1)$-computable, then group G has a solvable word problem.*

Also I want to remark that the question about other spectra is open, but with help of new methods [4] some interesting results were obtained.

THEOREM 5. *Let $n \geqslant 0$. There exists a strongly minimal first-order theory such that the prime model is n-computable and all other models of this theory are $(n+2)$-computable.*

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► ROBERT LUBARSKY AND HANNES DIENER, *Principles weaker than BD-N*.
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BD-N is a very weak foundational principle, first identified in [3] through its being equivalent (under Countable Choice) to sequentially continuous functions being continuous. BD-N holds in all major schools of mathematics (Brouwer's intuitionism, Russian computable or recursive constructivism, and classical mathematics), and it has been shown to be equivalent with many other properties. Thus it is of central importance in reverse (constructive) mathematics. There were other interesting theorems it was seen to imply, but the reverse implication had not been proven to be true. These are a version Riemann's well-known theorem that a conditionally not absolutely convergent sequence can be rearranged to have any limit [1], and the closure of the anti-Specker spaces, a kind of compactness, under products [2]. We show that these statements are strictly weaker than BD-N, while still not being provable in constructive set theory alone. (That the closure of the anti-Specker spaces is weaker than BD-N was shown in [4].)

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► KERKKO LUOSTO, *Dichotomy and trichotomy related to unary quantifiers and arithmetics*.
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Generalized quantifiers were introduced by Andrzej Mostowski in 1957 (in the unary simple case) and Per Lindström in 1966 (in the general case). Up to 1980's generalized quantifiers were mostly studied from the general model theoretic point of view, but from 1990's onwards, the emphasis has been on finite model theoretic aspects. Consequently, the definability theory of generalized quantifiers has gained in importance.

In this work I have studied the expressive power of logics with unary quantifiers in the ordered structure of natural numbers $\langle \mathbb{N}, \leq \rangle$. Let $\mathcal{O}_\omega$ be the class of expansions of $\langle \mathbb{N}, \leq \rangle$. Let $S \subseteq \mathbb{N}$ be fixed, and consider first the logic $\mathrm{FO}(\mathsf{C}_S)$ where the cardinality quantifier C_S expresses "there are S-many". Then we get the following dichotomy: either

S is eventually periodic,
$\mathrm{FO}(\mathsf{C}_S) < \mathrm{FO}(\mathsf{I})$ $(\mathcal{O}_\omega)$,
$\mathrm{FO}(\mathsf{C}_S)$-theory of $\langle \mathbb{N}, \leq \rangle$ is decidable,
and addition and multiplication are not definable in $\langle \mathbb{N}, \leq \rangle$,

or

> S is aperiodic,
> $\mathrm{FO}(\mathsf{C}_S) \geq \mathrm{FO}(\mathsf{D})$ $(\mathcal{O}_\omega)$,
> the corresponding theory $\mathrm{Th}_{\mathrm{FO}(\mathsf{C}_S)}(\langle \mathbb{N}, \leq \rangle)$ is undecidable,
> and addition and multiplication are definable.

Here, I is the equicardinality (or Härtig) quantifier, and D is the general divisibility quantifier.

More generally, if we consider logics $\mathrm{FO}(Q)$ with a unary quantifier Q instead of a cardinality quantifier C_S, we get a trichotomy with a new case falling in the middle of the two old cases and corresponding roughly to Presburger arithmetic. The proofs rely on quite basic combinatorics of words and the result of Krynicki and Lachlan that $\mathrm{FO}(\mathsf{I})$-theory of $\langle \mathbb{N}, \leq \rangle$ is decidable.

► RADU MARDARE, *Probabilistic logic and the metric space of logical formulas*.
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A central questions in the field of probabilistic systems is "*when do two systems behave similarly up to some observational error?*". *Probabilistic bisimulation* relates models with identical behaviours [3] and it is characterized by a multimodal logic with operators indexed with probabilities [1, 2, 5]. In spite of its elegant theory, the concept of probabilistic bisimulation is too strict for applications where the parameter of a model are always approximations. Thus the interest moves from identical probabilistic behaviours, to similar ones. Instead of probabilistic bisimulation, it has been introduced a class of metrics [4]: two processes are at distance zero iff they are bisimilar, and are close when they differ by a small amount in their probabilistic behaviours. It has been also shown that these metrics can be defined on top of the multimodal logic that characterizes the bisimulation: the satisfiability relation $P \Vdash \phi$ (for P a probabilistic process and ϕ a logical formula) can be extended to a function d such that $d(P, \phi) \in [0, 1]$ measures the degree of satisfiability between P and ϕ. d induces a distance D on processes by $D(P, P') = sup\{|d(P, \phi) - d(P', \phi)|, \phi \in L\}$, where $\mathcal{L}$ is the set of logical formulas. Because $\mathcal{L}$ is infinite, D is not computable. Moreover, if the process is infinite or extremely complex, evaluating $d(P, \phi)$ is already problematic.

We show that relying on a complete axiomatization of the probabilistic logic [2] and on the fact that this logic enjoys the finite model property [5], we can derive a pseudometric $\overline{d}$ on the set of logical formulas such that for any process P and any formulas ϕ, ψ, $d(P, \phi) \leq d(P, \psi) + \overline{d}(\phi, \psi)$. Moreover, the construction of the small model for a consistent formula provides an algorithm to compute the value of $\overline{d}(\phi, \psi)$. $\overline{d}$ characterizes the similarity between logical formulas in terms of provability: $\overline{d}(\phi, \psi) = 0$ iff $\vdash \phi \leftrightarrow \psi$ and $\overline{d}(\phi, \psi)$ measures the amount of common hypothesis that can prove one while disproving the other.

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► JUAN CARLOS MARTINEZ, *Consistency results for cardinal sequences of LCS spaces*.
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By an LCS *space*, we mean a locally compact, Hausdorff, scattered space. If X is an LCS space and α is an ordinal, we denote by $I_\alpha(X)$ the α^{th}-Cantor–Bendixson level of X, i.e., $I_\alpha(X) =$ set of isolated points of $X \setminus \bigcup\{I_\beta(X)\colon \beta < \alpha\}$. The *height of* X is defined by $\mathrm{ht}(X) =$ the least ordinal α such that $I_\alpha(X)$ is finite. Then, *the cardinal sequence of* X is defined by $\langle |I_\alpha(X)|\colon \alpha < \mathrm{ht}(X)\rangle$.

Many authors have studied the possible sequences of infinite cardinals that can arise as the cardinal sequence of an LCS space (equivalently, of a superatomic Boolean algebra). By results due to La Grange and Juhász and Weiss, it is known that if $s = \langle \kappa_\alpha\colon \alpha < \omega_1\rangle$ is a sequence of infinite cardinals, then s is the cardinal sequence of some LCS space iff $\kappa_\beta \leq \kappa_\alpha^\omega$ for $\alpha < \beta < \omega_1$. The situation becomes, however, more complicated when one considers cardinal sequences of length greater than ω_1. By a classical result of Baumgartner and Shelah, it is consistent with ZFC that there is an LCS space X of height ω_2 such that $|I_\alpha(X)| = \omega$ for each $\alpha < \omega_2$. And by a classical result of Roitman, it is consistent with ZFC that there is an LCS space X of height $\omega_1 + 1$ such that $|I_\alpha(X)| = \omega$ for each $\alpha < \omega_1$ and $|I_{\omega_1}(X)| = \omega_2$. Then, our aim in this talk is to present some consistency results for cardinal sequences proved recently, including substantial generalizations of the theorems of Roitman and Baumgartner–Shelah.

► MUSTAFA MANAT AND ANDREA SORBI, *Note on computable numberings in the Ershov hierarchy*.

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We give a sufficient condition for an infinite computable family $\mathcal{A}$ of Σ^{-1}_a sets, to have positive but undecidable numberings, where a is a notation for a nonzero computable ordinal. This extends a theorem proved by Talasbaeva for the finite levels of the Ershov hierarchy. The results of Talasbaeva's paper [5] and of this paper are partly motivated by the observation that for the arithmetical hierarchy questions about the existence of Friedberg numberings for a family may be reduced to the existence of positive numberings. Indeed, Goncharov and Sorbi [3] show that if a family of Σ^0_n sets, $n \geq 2$, has positive numberings then it has Friedberg numberings as well. In particular the family of all Σ^{-1}_a sets has positive undecidable numberings. As an immediate consequence of a theorem due to Ospichev, we point out also that for every ordinal notation a there are families of Σ^{-1}_a sets having positive numberings, but no decidable numberings: this answers for all levels (whether finite or infinite) of the Ershov hierarchy, a question originally raised, only for the finite levels, by Badaev and Goncharov.

[1] S. Badaev, *Positive enumerations*, ***Sibirskiĭ Matematicheskiĭ Zhurnal***, vol. 18 (1977), no. 3, pp. 483–496.

[2] S. Badaev and S. Goncharov, *The theory of numberings: open problems*, (P. A. Cholak, S. Lempp, M. Lerman, and R. A. Shore, editors), ***Computability theory and its applications***, vol. 257, Contemporary Mathematics, pp. 23–38, American Mathematical Society, Providence, 2000.

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► JOSÉ M. MÉNDEZ, GEMMA ROBLES, AND FRANCISCO SALTO, *A general class of logical matrices for the variable-sharing property*.
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A logic S has the *variable-sharing property* (vsp) if in all theorems of S of the form $A \to B$, A and B share at least one propositional variable. According to Anderson and Belnap, the vsp is a necessary property of any relevant logic (cf. [1]).

The aim of this paper is to define a class of logical matrices called "Relevant logical matrices" (RLM). We shall prove that any logic verified by an RLM has the vsp and related properties predicable of the logic of entailment E and of the logic of relevant implication R (cf. [1]).

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► RYSZARD MIREK, *Filtration in non-classical logics*.
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N. Tennant in [1] distinguishes between prevalid and valid sequents/sets of sentences. The inference from Γ to Δ is prevalid iff: $\Gamma \models \Delta$ and for no proper subsets of Γ and Δ, we have $\Gamma' \models \Delta'$. In this way Lewis paradoxes $\alpha, \neg\alpha\colon \beta$ and $\alpha\colon \beta, \neg\beta$ are not provable. This is because the proper subsets respectively there are $\alpha, \neg\alpha\colon \emptyset$ and $\emptyset\colon \beta, \neg\beta$. In this way one can construct the filter logics. Proof theoretically, using the Gentzen sequent calculus, there are no applications of the structural rules of thinning and cut. Another approach would be Parry's system of analytic implication which develops Kant's dictum that the "predicate is contained in the subject". In this system $\alpha\colon \beta$ is provable if all variables in β also occur in α. We have no Lewis paradoxes but collapses the notion of validity and prevalidity. It is possible to define validity in terms of model-preservation. This approach we can find in Jaskowski's *discussive/discursive* logic. Although an original interpretation is for modal logic $S5$ but there are possible non-modal axiomatizations proposed by Furmanowski/Kotas, da Costa etc. [2] Generally, that discursive logic is paraconsistent. My aim is to compare different filter logics and different strategies to avoid the paradoxes of implication as in relevance logic or endorse inconsistent views as in paraconsistent logic.

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[2] Vladimir L. Vasyukov, *A new axiomatization of Jaskowski's discussive logic*, ***Logic and Logical Philosophy***, vol. 9 (2001), pp. 35–46.

► TAKAKO NEMOTO, *A system of explicit mathematics and* Π_3 *reflection*.
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The system T_0 of explicit mathematics was introduced by Feferman [1], which was originally designed as a framework for Bishop style constructive mathematics. In [3], limit axiom and Mahlo axiom for explicit mathematics are suggested. It is also proved that the proof theoretic strengths of the systems T_0 plus these axioms do not exceed those of KPi ($\mathsf{KP}+$ "each set is contained in an admissible set") and KPm ($\mathsf{KP}+$ "Π_2 reflection on admissible sets"), respectively, by embedding systems of explicit mathematics into systems of first order inductive definition. In this talk, we consider an extension of T_0 with axiom of 2-universe introduced in [2]. The axiom is motivated by the notion of 2-admissibility introduced in [4], and is proved there to be equivalent to Π_3 reflection property of ordinals. We compare the proof theoretic strengths of this extension and of $\mathsf{KP} + \Pi_3$ reflection by modifying the technique developed in [3].

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► VLADISLAV NENCHEV, *Undecidability of logics for mereological and mereotopological relations*.
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In this paper we prove *hereditary undecidability* for several first-order theories, which languages consist of mereological and mereotopological relations. We use a method described by Ershov in [1]. Provided we already have a theory that is hereditary undecidable, if we prove that it is *relatively elementary interpretable* in a target theory of our choice, then that target theory is also hereditary undecidable. Here we talk of two such interpretations. One for theories based on mereological relations, that has the relation *overlap* in their language. The second is for theories based on *contact algebras*.

Mereology is an ontological discipline, that deals with *regions* of space and relations between them. Typical ones are *part-of*, *overlap* and its dual relation—*underlap* (denoted $\leq$, O and U respectively). They express common situations between regions, such as one region being part of another or that they share a common sub-region. Tarski showed that the mathematical equivalent of mereology are complete Boolean algebras. Thus let $\underline{B} = (B, 0, 1, ., +, *)$ be a nondegenerate Boolean algebra. Then $\leq$ is just the Boolean ordering and O is defined: $x \,\mathsf{O}\, y$ iff $x.y \neq 0$. Theories about these relations are studied in [3] and [2]. Furthermore all relations from RCC-5, a subsystem of the Region Connection Calculus (Randell, Cui and Cohn), can be defined with $\leq$ and O.

Mereotopology is an extension of mereology with some topological relations like *contact*. Tarski showed that standard point models of mereotopology are regular closed (or open) sets of some topological spaces with a topological definition of the contact. Since mereology can be identified with Boolean algebras, mereotopology can be identified with *contact algebras*, which are Boolean algebras with added contact relation C (see [4]). Besides the first-order theory of contact algebras, there are other well-known theories that can be based on them. For instance the theory of mereotopological structures from [3] or the Egenhofer–Franzosa RCC-8 system.

Finally [2] and [4] present another development of mereological and mereotopological relations—*stable* and *unstable* versions of them. They represent dynamic versions of the initial relations. The formal construction defines the dynamic versions over a collection of (static) structures using quantifiers over the structures. We use either of the two interpretations to show undecidability, depending on the choice of the static structures.

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► JOSEPH W. NORMAN, *Undecidability is not a crime: dynamical systems debunk Gödel's incompleteness myth*.
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The method of computational algebraic logic is introduced and used to demonstrate that Gödel's incompleteness conjectures are wrong. Gödel described a formula that he claimed to be correct but formally undecidable: impossible to prove or disprove within the formal system PM [2]. Therefore he judged any system like PM essentially *incomplete* and incapable of proving even its own consistency. Gödel's formula is 'undecidable' in the sense that its value is not in the set {TRUE, FALSE} of elementary truth values; by analogy $\sqrt{1}$ is 'undecidable' relative to the set $\mathcal{R}$ of real numbers. Each formula needs a more complex mathematical structure to represent its value. *Discrete dynamical systems* are the 'complex numbers' required for Gödel's self-referential formula. Using an extension of Boole's method of translating logical formulas into polynomials [1], Gödel's formula is revealed to be just like the recurrence relation $x_t = 1 - x_{t-1}$ with each $x \in \{0, 1\}$: both expressions yield the system $0 \rightleftarrows 1$ with two states $x = 0$ and $x = 1$ connected by a periodic orbit. The graph shows exactly what is exceptional about Gödel's formula: the derived system has no steady state (fixed point); furthermore, it encodes for x an alternating sequence $(\dots, 0, 1, 0, 1, \dots)$ of values. Recognizing Gödel's construction as a dynamical system dispels the myth of incompleteness: it is no more problematic for formal reasoning that Gödel's formula has no unique value in $\{0, 1\}$ than it is that Fibonacci's formula has no unique value in $\{0, 1, 2, 3, 5, 8, \dots\}$.

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► ROLAND SH. OMANADZE, *$Q_{1,N}$-reducibility and splittings of r-maximal sets*.
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Tennenbaum (see [3, p. 159]) defined the notion of Q-reducibility on sets of natural numbers as follows: a set A is Q-reducible to a set B (in symbols: $A \leq_Q B$) if there exists a computable function f such that for every $x \in \omega$ (where ω denotes the set of natural numbers),

$$x \in A \Leftrightarrow W_{f(x)} \subseteq B.$$

We say in this case that $A \leq_Q B$ via f. If $A \leq_Q B$ via a computable function f such that for all x, y,

$$x \neq y \Rightarrow W_{f(x)} \bigcap W_{f(y)} = \emptyset \text{ and } \bigcup_{x \in \omega} W_{f(x)} \text{ is computable}$$

then we say that A is $Q_{1,N}$-reducible to B, and denoted $A \leq_{Q_{1,N}} B$ (see [1]).

A set A is hemi r-maximal (see [2]) if there are an r-maximal set M and a nontrivial splitting M_0, M_1 of M such that $A = M_0$.

THEOREM 1. *There exist two c.e. sets having no c.e. least upper bound in the $Q_{1,N}$-reducibility ordering.*

THEOREM 2. *If C, D are hemi r-maximal sets then*

$$C \equiv_{Q_{1,N}} D \Leftrightarrow C \equiv_1 D.$$

THEOREM 3. *Let* **a** *be the $Q_{1,N}$-degrees of Friedberg splitting of any r-maximal set. Then for all c.e. sets $A, B \in$* **a**, *we have $A \equiv_1 B$.*

THEOREM 4. *The Q-degree of a hemi r-maximal set contains an infinite number of pairwise m-incomparable c.e. m-degrees.*

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► ALEXANDRA OMAR AZIZ, *Type-definable multiplicative subgroups of U-rank 1 in separably closed fields*.
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Groups of the form $p^\infty G(L) = \bigcap_n p^n(G(L))$, where G is a semiabelian variety over a separably closed field L of characteristic $p > 0$, played an important role in Hrushovski's proof of the Mordell–Lang conjecture for function fields in positive characteristic. These are type-definable in the structure $(L, +, \cdot)$. If G is an extension of an abelian variety by $\mathbb{G}_m$, the group $p^\infty G(L) \cap \mathbb{G}_m(L)$ can be regarded as a type-definable subgroup of the multiplicative group, and it has U-rank 1. We give a characterization of type-definable multiplicative subgroups of U-rank 1 in terms of their quotient over the connected component (which is always $(L^{p^\infty})^*$). In particular, we show that not all type-definable multiplicative subgroups of U-rank 1 arise from extensions of abelian varieties by $\mathbb{G}_m$.

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In [3] are considered open problems of numberings theory. One of the questions is about computable [5] Friedberg numberings of Σ_a^{-1}-sets [1], a is Kleene notation of constructive ordinal. In [2] was constructed example of computable single valued numbering of family of all c.e. sets. In [4] this result was generalized to all finite levels of Ershov hierarchy. In work are considered some modifications of the construction of Goncharov, Lempp and Solomon.

In work is proved, that any Σ_a^{-1}-computable family S of sets has a positive undecidable $\Sigma^{-1}_{2^{(a+_{\mathcal{O}}a)}}$-computable numbering.

Also, was proved

THEOREM 1. *If S—Σ_a^{-1}-computable family of sets and $T \subset S$ has a Σ_b^{-1}-computable Friedberg numbering, then there is $\Sigma^{-1}_{2^{(a+_{\mathcal{O}}b)}}$-computable Friedberg numbering of S.*

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Quite recently Maclagan [2] has shown that there are no infinite antichains in the set of monomial ideals in a polynomial ring in finitely many variables over a field. Aschenbrenner and Pong [1] investigated her result extensively from the viewpoint of the theory of well partial orders and they computed several related (and very interesting) ordinal invariants.

We examine a finitary version of the theorem, additionally introducing a parameter function and determining a phase transition threshold for the provability of that theorem in a weak fragment of Peano Arithmetic. Let MM_f be the following statement:

For every d, l there exists an M such that for every sequence $I_0, \ldots, I_M$ of monomial ideals in $K[X_d, \ldots, X_0, Y]$, with $|I_i| \leq l + f(i)$ for all $i \leq M$, there exist $i < j < M$ with $I_i \supseteq I_j$.

Take $f_\alpha = {}^{F_\alpha^{-1}}\sqrt[(i)]{\log i}$ where F_α is the αth function in the Wainer hierarchy. Then:

THEOREM 1.

$\mathrm{I}\Sigma_2 \nvdash \mathrm{MM}_{id}$.
$\mathrm{I}\Sigma_2 \nvdash \mathrm{MM}_{f_\alpha}$ *for* $\alpha = \omega^\omega$.
$\mathrm{I}\Sigma_2 \vdash \mathrm{MM}_{f_\alpha}$ *for* $\alpha < \omega^\omega$.

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The logic TW is, essentially, the result of dropping the contraction axiom

$$(\mathrm{C})\ [A \to (A \to B)] \to (A \to B)$$

and the *reductio* axiom

$$(\mathrm{R})(A \to \neg A) \to \neg A$$

from Ticket Entailment Logic. In [1] a semantics for logics including TW plus axiom (R) and rule (γ), disjunctive syllogism,

$$(\gamma)\ (\vdash A \ \ \& \vdash \neg A \vee B) \Rightarrow \vdash B$$

is defined.

The aim of this paper is to provide a semantics for relevant logics including TW plus the rules *reductio*

$$(\mathrm{r}) \vdash A \to \neg A \Rightarrow \neg A$$

contraction

$$(\mathrm{c}) \ \vdash A \to (A \to B) \Rightarrow A \to B$$

and (γ).

The "normalization" strategy and the ideas and results of the "calculus of intensional T-theories" firstly defined in [2] have been here adapted.

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Herbrand consistency has been used for Π_1-separting $\mathrm{I}\Delta_0 + \mathrm{Exp}$ from $\mathrm{I}\Delta_0$, where the usual, Hilbert style, consistency predicate fails, since $\mathrm{I}\Delta_0 + \mathrm{Exp} \vdash \mathrm{Con}(\mathrm{I}\Delta_0)$. But inside $\mathrm{I}\Delta_0 + \mathrm{Exp}$, the notion of Herbrand consistency has not been able to Π_1-separate the hierarchy $\mathrm{I}\Delta_0$, $\{\mathrm{I}\Delta_0+\Omega_i\}_i$, $\mathrm{I}\Delta_0+\mathrm{Exp}$, and Π_1-separation of $\mathrm{I}\Delta_0+\Omega_{i+1}$ from $\mathrm{I}\Delta_0+\Omega_i$ is an open problem. We only know that $\mathrm{I}\Delta_0 + \mathrm{Exp}$ is not a Π_1-conservative extension of $\mathrm{I}\Delta_0 + \bigwedge_i \Omega_i$ ([2]).

Here we investigate whether Herbrand consistency of a theory can or cannot be proved by itself or a super-theory of it. It was proved in [1] that $\mathrm{I}\Delta_0+\Omega_2$ cannot prove $\mathrm{HCon}(\mathrm{I}\Delta_0+\Omega_2)$, and by using some of its techniques it was shown in [3] that $\mathrm{I}\Delta_0 + \bigwedge_i \Omega_i \nvdash \mathrm{HCon}(\mathrm{S} + \Omega_1)$ for some finite fragment $\mathrm{S} \subseteq \mathrm{I}\Delta_0$. The author showed in [4] that $\mathrm{I}\Delta_0 \nvdash \mathrm{HCon}(\mathrm{I}\Delta_0)$, when the standard axiomatization of $\mathrm{I}\Delta_0$ is taken into account, and (unlike [6]), the sentence $\forall x\, \exists y\, (y = x \cdot x)$, totality of the squaring function, has not been added to its axioms. Then in [5] the result of [3] was partially extended by showing $\mathrm{I}\Delta_0 + \bigwedge_i \Omega_i \nvdash \mathrm{HCon}(\mathrm{I}\Delta_0)$.

One new result in this direction is the existence of a finite fragment $T \subseteq \mathrm{I}\Delta_0$ for which $\mathrm{I}\Delta_0 \nvdash \mathrm{HCon}(T)$, also $\mathrm{I}\Delta_0 + \Omega_m \nvdash \mathrm{HCon}(T)$ for any m; or in other words, more generally, $\mathrm{I}\Delta_0 + \bigwedge_i \Omega_i \nvdash \mathrm{HCon}(T)$. Also, it can be shown that there exists a Π_1-sentence S such that $\mathrm{I}\Delta_0 \vdash S$ and $\mathrm{I}\Delta_0 + \bigwedge_i \Omega_i \nvdash \mathrm{HCon}(\mathrm{Q} + S)$. This generalizes a result of [6] which showed the existence of a $\mathrm{I}\Delta_0$-derivable Π_1-sentence V such that $\mathrm{Q} + V \nvdash \mathrm{HCon}(\mathrm{Q} + V)$.

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► SAM SANDERS, *A tale of three Reverse Mathematics*.
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Reverse Mathematics (RM) is a program in the foundations of Mathematics initiated by Harvey Friedman ([1, 2]) and developed extensively by Stephen Simpson ([6]). The aim of RM is to determine which minimal axioms prove theorems of ordinary mathematics. Historically, Nonstandard Analysis has played an important role in RM ([4, 7]). An important open problem is whether RM can be done in weak theories, like $I\Delta_0 + \exp$. We develop Reverse Mathematics in the system ERNA, a version of Nonstandard Analysis based on $I\Delta_0 + \exp$ ([5]). We show that ERNA's RM has intimate connections to Friedman-Simpson style RM, in particular Recursion Theory, and to Constructive RM ([3]). Our results have applications in Physics and the Philosophy of Science. In particular, we argue that Reverse Mathematics is robust in the same way this term is used in the exact sciences.

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We show that any model $\mathfrak{A}$ canonically extends to a model $\boldsymbol{\beta}\mathfrak{A}$ (of the same language) consisting of ultrafilters over it with the standard compact Hausdorff topology on ultrafilters. The extension lifts various relationships between models. Continuous extensions of homomorphisms of $\mathfrak{A}$ into $\mathfrak{B}$ are homomorphisms of $\boldsymbol{\beta}\mathfrak{A}$ into $\boldsymbol{\beta}\mathfrak{B}$. Moreover, if a model $\mathfrak{C}$ carries a compact Hausdorff topology which is compatible with its structure (in the same way as the topology of $\boldsymbol{\beta}\mathfrak{A}$ is compatible with the structure of the latter), then continuous extensions of homomorphisms of $\mathfrak{A}$ into $\mathfrak{C}$ are homomorphisms of $\boldsymbol{\beta}\mathfrak{A}$ into $\mathfrak{C}$. These facts about homomorphisms remain true for embeddings and other relationships between models. Thus the construction provides a direct generalization of the Stone–Čech (or Wallman) compactification of discrete spaces to arbitrary models.

The construction, together with the mentioned basic results, appeared very recently in [6]. It has some precursors: ultrafilter extensions of semigroups were known from 70s and used as a powerful tool in number theory, algebra, topological dynamics, and ergodic theory, see [1, 4]; other sources of close constructions were semantics of modal logic and universal algebra, see [2, 3]. We present several new results from [5, 7]. Certain ultrafilters form submodels; in particular, given any infinite cardinal κ, κ-complete ultrafilters form a submodel of $\boldsymbol{\beta}\mathfrak{A}$, and under certain conditions κ-uniform ultrafilters form a closed submodel of $\boldsymbol{\beta}\mathfrak{A}$. In certain simple cases ultrafilter extensions can be elementary; in general the equational theory of $\boldsymbol{\beta}\mathfrak{A}$ is quite different from the equational theory of $\mathfrak{A}$. We investigate formulas that are preserved under ultrafilter extensions; finally we mention some applications in algebra.

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▶ STANISLAV O. SPERANSKI, *Quantification over propositional formulas in probability logic: computability issues*.

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We generalize the standard language for reasoning about probabilities proposed in [1] by allowing meta-variables in propositions under the probability sign and quantifiers bounding these variables. Since each propositional formula can be viewed as a Bernulli random variable within a *probability structure* (see [1] for the definition), our treatment may be interpreted as quantification over such entities.

Let $Prop$ and $\mathcal{X}$ be (infinite) computable collections of *propositional symbols* and *parameters* (or *variables*) respectively. Assume the logical connectives are $\wedge$ and $\neg$. Propositional formulas are introduced in the usual way. Analogously *parametric propositional formulas* are built up from $Prop \cup \mathcal{X}$ using logical connectives.

Definition 1. The notion of $\mathcal{L}$-*term* is defined by induction:

1. for a parametric propositional formula φ, the expression $\mu(\varphi)$ is an $\mathcal{L}$-term;
2. if $f(x_1, \dots, x_n)$ is a polynomial with rational coefficients and $t_1, \dots, t_n$ are $\mathcal{L}$-terms, then the expression $f(t_1, \dots, t_n)$ is an $\mathcal{L}$-term.

Definition 2. For any two $\mathcal{L}$-terms t_1 and t_2, $t_1 \leqslant t_2$ is an $\mathcal{L}$-*atom*. Next the set of $\mathcal{L}$-*formulas* is the smallest set $For_{\mathcal{L}}$ containing all $\mathcal{L}$-atoms such that: if Φ and Ψ are in $For_{\mathcal{L}}$ and α is in $\mathcal{X}$, then $\neg\Phi$, $\Phi \wedge \Psi$, $\forall\alpha\,\Phi$ and $\exists\alpha\,\Phi$ are in $For_{\mathcal{L}}$.

The semantics of quantifier-free $\mathcal{L}$-sentences is provided in [1] and is fairly standard. It can be easily extended to all $\mathcal{L}$-sentences if we treat quantifiers $\forall$ and $\exists$ (semantically) as infinite conjunctions and disjunctions ranging over all propositional formulas.

Note that the validity problem for quantifier-free $\mathcal{L}$-sentences is known to be decidable [1]. However, our first theorem establishes the undecidability of the validity problem even for the so-called *linear* $\mathcal{L}$-sentences. In item 2 of Definition 1 replace "polynomial" with "linear combination of $x_1, \ldots, x_n$" to get *linear $\mathcal{L}$-terms*. Then the other definitions are relativized to obtain *linear $\mathcal{L}$-atoms*, *-formulas*, etc.

THEOREM 3. *The validity problem for linear $\mathcal{L}$-sentences is undecidable.*

Nevertheless the decidability with respect to all $\forall\exists$-sentences in $\mathcal{L}$ holds.

THEOREM 4. *The validity problem for $\forall\exists$-sentences in $\mathcal{L}$ is decidable.*

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We consider computability on abstract structures via the formalism based on the notion of Σ-definability in admissible sets and, in particular, in HF-superstructures (see [1]). Providing a natural generalization of the classical computable model theory, this approach can be used for measuring the complexity of abstract structures, no matter countable or not.

Concerning the conjecture stated by Yu. L. Ershov in [2], we study some properties of Σ-degrees of uncountable models of c-simple theories (i.e., first-order theories which are ω-categorical, model complete, decidable and with a decidable set of atoms). In particular, we prove that there exist c-simple theories with a finite signature which do not have uncountable models Σ-definable in $\mathbb{HF}(\mathfrak{L})$, where $\mathfrak{L}$ is a dense linear order. Recall that in the class of c-simple theories with infinite computable signatures such examples were found in [3]. We also discuss some corrections in the proofs of positive results in this direction [4], and present one more positive result which says that erery model of a c-simple theory is generalized Σ-low.

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► SERGEY SUDOPLATOV, *On classification of countable models of complete first-order theories*.
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URL Address: http://math.nsc.ru/~sudoplatov/. In fact I represent two Organizations:

A classification of Ehrenfeucht theories with respect to Rudin—Keisler preorders and distribution functions of limit models is described in [1, 2, 3]. It is generalized to the class of small theories in [3, 4, 5]. We propose a further generalization to the class of all complete first-order theories taking Rudin—Keisler preorders and finite diagrams of models as basic characteristics. We get possible hierarchies of countable models of theories with continuum many types using upward directed sets $\langle X; \leq\rangle$ (being systems of types) with at most countably many elements under each element, having only countable $(\leq\ \cap \geq)$-equivalence classes and countable or continual sets of common elements over finite sets.

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▶ CARLO TOFFALORI, *Generalized lattices over local Dedekind-like rings*.
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This is part of a joint work with Gena Puninski (Moscow) on *Dedekind-like rings*. These rings include several familiar examples, such as integral group rings $\mathbf{Z}[G]$ where G is a cyclic group of finite square-free order, and were intensively studied by Klingler and Levy. In particular [2] and [3] deal with *local* Dedekind-like rings.

A notion of *generalized lattice* over orders over Dedekind domains was considered in [1], [4], [5]. These papers also examine when a generalized lattice over an order is a direct sum of ordinary—that is, finitely generated—lattices.

Our work introduces a notion of *generalized lattice* over a local Dedekind-like ring Λ. They are the direct summands of a direct sum of arbitrarily many copies of $\Lambda \oplus \Gamma$ where Γ is the integral closure of Λ in its total ring of quotients and both Λ and Γ are viewed as modules over the ring Λ. In this setting lattices can be seen as direct summands of direct sums of finitely many copies of $\Lambda \oplus \Gamma$.

Our main question is to classify the rings Λ for which every generalized lattice is a direct sum of lattices. We also compare the first order theories of lattices and generalized lattices over Λ.

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▶ HSING-CHIEN TSAI, *Decidability and undecidability of the elementary mereological theories*.
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A mereological theory is formed with axioms pertaining to a binary relation "being a part of". Hence its formal language has only one non-logical symbol, a binary predicate P, which stands for the aforementioned relation. It is believed that P must at least define a partial ordering, thus stand the following three basic axioms. (P1) $\forall x Pxx$; (P2) $\forall x \forall y((Pxy \wedge Pyx) \to x = y)$; (P3) $\forall x \forall y \forall z((Pxy \wedge Pyz) \to Pxz)$. It is also convenient to define three additional predicates as follows. Proper Part: $PPxy =_{df} Pxy \wedge \neg Pyx$; Overlap: $Oxy =_{df} \exists z(Pzx \wedge Pzy)$; Underlap: $Uxy =_{df} \exists z(Pxz \wedge Pyz)$.

In the literature, there are some other mereological axioms which are arguably still philosophically motivated, such as extensionality principle (EP): $\forall x \forall y(\exists z PPzx \to (\forall z(PPzx \leftrightarrow PPzy) \to x = y))$, weak supplementation principle (WSP): $\forall x \forall y(PPxy \to \exists z(PPzy \wedge \neg Ozx))$, strong supplementation principle (SSP): $\forall x \forall y(\neg Pyx \to \exists z(Pzy \wedge \neg Ozx))$, finite sum (FS): $\forall x \forall y(Uxy \to \exists z \forall w(Owz \leftrightarrow (Owx \vee Owy)))$, finite product (FP): $\forall x \forall y(Oxy \to \exists z \forall w(Pwz \leftrightarrow (Pwx \wedge Pwy)))$, complementation (C): $\forall x(\neg \forall z Pzx \to \exists z \forall w(Pwz \leftrightarrow \neg Owx))$ and the existence of the greatest member (G): $\exists x \forall y Pyx$.

It turns out that any theory which can be formed by using the foregoing axioms is in a sense a subtheory of the elementary theory of Boolean algebras. Now it is known that the theory of partial orderings is undecidable while the elementary theory of Boolean algebras is decidable. It is then interesting to look into the behaviors in terms of decidability of those mereological theories located in between. I will give a comprehensive picture of the said issue, and especially offer solutions to the open problems which I have raised in a paper published previously [1].

[1] HSING-CHIEN TSAI, *Decidability of mereological theories*, ***Logic and Logical Philosophy***, vol. 18 (2009), no. 1, pp.45–63.

► REGINA TUKHBATULLINA AND NIKOLAY BAZHENOV, *On computable categoricity of Boolean algebra $\mathfrak{B}(\omega)$ distinguished by automorphism.*

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This paper deals with degrees of computable categoricity of computable Boolean algebra $\mathfrak{B}(\omega)$ distinguished by automorphism. Research of this question began in [3].

Let $\mathfrak{A}$ be a computable model. Categoricity spectrum is the following set

$$\mathrm{CatSpec}(\mathfrak{A}) = \{\mathbf{d} \mid \mathfrak{A} \text{ is } \mathbf{d}\text{-computably categorical}\}.$$

A Turing degree $\mathbf{d}_0$ is the least degree of categoricity of $\mathfrak{A}$ if $\mathbf{d}_0$ is the least degree in $\mathrm{CatSpec}(\mathfrak{A})$.

The following results are obtained in this paper:

THEOREM 1. *Let* $\mathbf{d}$ *be a computable enumerable Turing degree. Then there is a computable Boolean algebra distinguished by automorphism* $\mathfrak{A}^* = \langle \mathfrak{A}, \phi \rangle$ *such that* $\mathfrak{A} \simeq \mathfrak{B}(\omega)$ *and* $\mathbf{d}$ *is the least degree of categoricity of* $\mathfrak{A}^*$.

THEOREM 2. *Let* $\mathbf{d}$ *be a computable enumerable Turing degree. Then there is a computably categorical Boolean algebra distinguished by automorphism* $\mathfrak{A}^* = \langle \mathfrak{A}, \phi \rangle$ *such that* $\mathfrak{A} \simeq \mathfrak{B}(\omega)$ *and a set of atoms of* $\mathfrak{A}$ *has degree* $\mathbf{d}$.

THEOREM 3. *There is a computable Boolean algebra distinguished by automorphism* $\mathfrak{A}^* = \langle \mathfrak{A}, \phi \rangle$ *such that*

1. $\mathfrak{A}^*$ *isn't computably categorical*
2. *If* $\mathfrak{B}^* = \langle \mathfrak{B}, \psi \rangle$ *is a computable representation of* $\mathfrak{A}^*$ *then a set of atoms of* $\mathfrak{B}$ *has degree* $\mathbf{0}'$.

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[3] R. R. TUKHBATULLINA, *Autostability of Boolean algebra $\mathfrak{B}_\omega$ distinguished by automorphism*, ***6th conference on Computability in Europe 2010,*** (***Ponta-Delgada, Portugal***), p. 369–377.

► J. A. TUSSUPOV, *Isomorphisms and algorithmic properties structures with two equivalences*.
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We will consider the problems on algorithmic complexity of isomorphic and definable properties on models and connections with Scott families. Necessary definitions can be viewed in a book [1].

The structure $\mathcal{A}$ with binary predicate $P(x, y)$ is called the structure with bipartition binary predicates $P(x, y)$ if for the sets $K_1 = \{x : \mathcal{A} \models \exists y P(x, y)\}$ and $K_2 = \{x : \mathcal{A} \models \exists y P(y, x)\}$ satisfy the conditions: $K_1 \cap K_2 = \emptyset$ and $K_1 \cup K_2 \neq \emptyset$.

Let $\sigma_0 = \langle P^2(x, y)\rangle$ is signature with the bipartition binary predicates $P(x, y)$ and $\sigma_1 = \langle E_0^2(x, y), E_1^2(x, y)\rangle$ is signature with two equivalences $E_0^2(x, y)$, $E_1^2(x, y)$.

Let $\mathcal{A}$ structure of signature σ_i, where $i = 0, 1$.

THEOREM 1.I. *For each computable successor ordinal α there is computable structure $\mathcal{A}$ of signature σ_i that is Δ^0_α categorical but not relatively Δ^0_α (and without formally Σ^0_α Scott family).* ⊣

THEOREM 2.I. *For each computable successor ordinal α there is computable structure $\mathcal{A}$ of signature σ_i that is Δ^0_α with a relation that is intrinsically Σ^0_α but not relatively intrinsically Σ^0_α.* ⊣

THEOREM 3.I. *For each computable successor ordinal α for each finite n there is computable structure $\mathcal{A}$ of signature σ_i with Δ^0_α dimension n.* ⊣

THEOREM 4.I. *For each computable successor ordinal α there is structure $\mathcal{A}$ of signature σ_i with presentations in just the degrees of sets X such that $\Delta^0_\alpha(X) \neq \Delta^0_\alpha$. In particular, for each finite n there is structure $\mathcal{A}$ of signature σ_i with presentations in just the non-low$_n$ degrees.* ⊣

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► DIMITER VAKARELOV, *Dynamic modalities*.
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In this note we introduce a new modal logic with four dynamic modalities:

$\Box^\forall$—**always necessary**, $\Box^\exists$—**sometimes necessary**, and their duals—$\Diamond^\forall$—**always possibly**, and $\Diamond^\exists$—**sometimes possibly**. The formal semantics consists of the class of all frames in the form $(W, \{R_i : i \in I\})$ where I is a nonempty set of indices (intuitively interpreted as moments of time or situations) and for each $i \in I$, R_i is a binary relation in W. The interpretation is as follows:

$x \models \Box^\forall\alpha$ iff $(\forall i \in I)(\forall y \in W)(xR_i y \to y \models \alpha)$,
$x \models \Box^\exists\alpha$ iff $(\exists i \in I)(\forall y \in W)(xR_i y \to y \models \alpha)$, and dually for $\Diamond^\forall\alpha$ and $\Diamond^\exists\alpha$:
$x \models \Diamond^\forall\alpha$ iff $(\forall i \in I)(\exists y \in W)(xR_i y$ and $y \models \alpha)$,
$x \models \Diamond^\exists\alpha$ iff $(\exists i \in I)(\exists y \in W)(xR_i y$ and $y \models \alpha)$.

Axiomatization. $\Diamond^\forall\alpha =_{def} \neg\Box^\exists\neg\alpha$, $\Diamond^\exists\alpha =_{def} \neg\Box^\forall\neg\alpha$):

(I) axioms for the classical propositional logic,

(II) $\Box^\forall(\alpha \Rightarrow \beta) \Rightarrow (\Box^\forall\alpha \Rightarrow \Box^\forall\beta), \Box^\forall(\alpha \Rightarrow \beta) \Rightarrow (\Box^\exists\alpha \Rightarrow \Box^\exists\beta), \Box^\forall\alpha \Rightarrow \Box^\exists\alpha$.
Modus ponens $\frac{\alpha.\alpha\Rightarrow\beta}{\beta}$, Necessitation $\frac{\alpha}{\Box^\forall\alpha}$.
We prove the completeness theorem and decidability via fmp.

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Realizability semantics for intuitinistic set theories were first proposed by S. Kleene in 1945 [5]. Inspired by the definition of realizability for higher-order Heyting arithmetic due to Kreisel and Troelstra [6], the realizability was first applied to set theories by Myhill [8], and Friedman [3]. More recently, realizability models of set theories were investigated by Beeson [1, 2] (for non-extensional set theory) and Khakhanyan [4] (directly for extensional set theory). In [2] Beeson has proved some effectivity properties for non-extensional set theory ZFI (see also [10]). More lately realizability for extensional set theory was investigated by MacCarty [7] (by another approach based on so-called realizabity structures $V(A)$ over any applicative structure A) and Rathjen [9].

The most interesting applications of the realizability method to set theories concern principles, germane to Russian constructivism and Brower's intuitionism, that are classically refutable. For example, the Church Thesis CT, the Uniformization Principle UP, the Unzerlegbarkeit Principle UZ, and assertion that every function of $\mathbb{N}^\mathbb{N}$ into $\mathbb{N}$ is continuous, are realizable.

Let T be ZFI2 or ZFI2+DCS where DCS is the Double Complement of Sets Principle. We define a realizability $eR\varphi$ and prove that if T $\vdash \varphi$ then T $\vdash eR\varphi$ for some natural number e. The same holds for T+ECT+UP+UZ+AC$^{\omega,\omega}$+each of Dependent Choices Principles RDC, BDC, DC. Also we prove that is T+M $\vdash \varphi$ then T+M$^-$ $\vdash eR\varphi$ for some e, where M and M$^-$ are the Markov Principle and the weak Markov Principle. Then we use the realizability to establish several equiconsistency results and the following conservativity result: For every formula φ, if T $\vdash eR\varphi$ for some e, then T+φ and T have the same negative arithmetical theorems.

Analogous results about Constructive Set Theory CZF have been got by Rathjen in [9].

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[8] J. MYHILL, *Some properties of intuitionistic Zermelo–Fraenkel set theory*, (A. Mathias and H. Rogers, editors), Cambridge School in Mathematical Logic, vol. 337, Lecture Notes in Mathematics, Springer, Berlin, 1973, pp. 206–231.

[9] M. RATHJEN, *Realizability for constructive Zermelo–Fraenkel set theory*, ***Proceedings. Logic Colloquium '03***, to appear.

[10] A. V. VLADIMIROV, ***Effective properties for intuitionistic set theory***, Mathematical Notes, to appear, 2011.

► MITKO YANCHEV, *Complexity of the satisfiability problem: modal language with quantitative and qualitative modalities*.
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K. Fine, [1], introduced the graded modal language—an extension of the classical propositional modal language with counting modal operators. Formally, for any natural number n, $\Diamond_n$ is an unary modal operator. In Kripke models, $\mathfrak{M}, x \Vdash \Diamond_n \phi$ means "more than n successors of x satisfy ϕ". S. Tobies, [3], proves that the satisfiability problem for the formulae of the graded modal language is PSPACE-complete.

An augmentation of that language with qualitative capabilities is considered by E. Pacuit and S. Salame, [2]. They add the unary majority operator M (and its dual $W = \neg M \neg$). The meaning of $M\phi$ is "the most of the successors of the current world have the property ϕ". The appropriate semantics for $M\phi$ has frames where every world can see finitely many worlds. Namely in the class of such frames it is natural to interpret the generalized qualitative modalities M^r, where r is a rational number in $(0, 1)$. So, if $|R(x)|$ is the number of successors of x, and $|R_\phi(x)|$ is the number of those of them satisfying ϕ, then $\mathfrak{M}, x \Vdash M^r \phi$ iff $|R_\phi(x)| > r|R(x)|$. It is proven that the set of all valid formulae GGML form a decidable non-normal modal logic.

The main result in the present talk is the PSPACE-completeness of the satisfiability problem for that extended propositional modal language with both quantitative and qualitative modalities. For, a complete tableaux system is found.

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► XUNWEI ZHOU, *Mutually-inversistic set theory*.
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Mutually-inversistic set theory is isomorphic with mutually-inversistic logic. Mutually-inversistic logic reflects the real world from both intension and extension perspectives, while mutually-inversistic set theory reflects the real world from extension perspective only. There are 14 set operators in mutually-inversistic set theory, including $\sim$, $\cap$, $\cup$, $\subseteq^{-1}$ (mutually inverse being contained), $|\cap^{-1}$ (mutually inverse intersection). P $\subseteq^{-1}$ Q means that all Ps are Qs. P$|\cap^{-1}$ Q means that some Ps are Qs. In mutually-inversistic set theory, elements are divided into zeroth level, first level, second level elements, sets are divided into zeroth level, first level, second level sets. Elements are divided into element constant and element variable, sets are divided into set constant and set variable. Individuals are elements, and n-tuples are also elements. Mutually-inversistic set theory restricts the use of empty set and universal set, thus it is free of Russell's paradox, the largest cardinal paradox, the largest ordinal paradox. Intersection is divided into classical intersection, extended intersection, first intersection, second intersection horizontally, fact intersection and empirical-mathematical intersection vertically. The difference between a function and a binary relation is not that function is single valued relation, but that a binary function connecting two terms is still a term, e.g., $3 + 5$ is still a term, while a binary relation connecting two terms is a proposition, e.g., $3 = 5$ is a false proposition. Set operators are divided into composition operators and connection operators.

$\sim$, $\cap$, $\cup$ are composition operators. $\subseteq^{-1}$ and $|\cap^{-1}$ are connection operators. A composition operator connecting two propositions is still a proposition, e.g., $P \cap Q$ is still a proposition. A connection operator connecting two propositions is the connection between the two propositions, e.g., $P \subseteq^{-1} Q$ is the connection between P and Q. Horizontally, function and binary relation are second level constants, composition operator and connection operator are third level constants. Vertically, function and composition operator are the auxiliary, binary relation and connection operator are the main. The main has the properties of reflectivity, anti-reflectivity, symmetry, anti-symmetry, transitivity. The auxiliary has the properties of associativity, commutativity, binary bijection, idempotency, complement idempotency.

Abstracts of talks presented by title

► LUCA BELLOTTI, *On some early consistency proofs for fragments of arithmetic*.
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I consider some early consistency proofs in the Hilbert School (by Ackermann, von Neumann and Herbrand) for fragments of arithmetic, before Gödel's results. The basic questions I shall try to answer are: What did they prove? Where (in what theory) can one prove what they proved? Which presentÂ¸day formal theories can 'mimic' those contentual proofs, without violating Gödel's theorem on consistency proofs?

► JOHN CORCORAN, *Quantifier-range-variable: the logical form of quantifications*.
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Variable-enhanced mathematical English sentences—as E below—expressing propositions about numbers translate into first-order sentences—as F below.

E: Every number x is either zero or the successor of some number y.
F: $\forall x\,(x = 0 \vee \exists y\, x = sy)$.

English *quantifications*—phrases such as *every number x* and *some number y*—correspond to symbolic quantifications $\forall x$ and $\exists y$. However, all fully-expressed quantifications have the form QRV (Tarski [2], pp. 6–9). Q expresses the *quantifier type*—often universal or existential. R is the *range-indicator*—usually a common noun. V is the variable. $\forall x$ and $\exists y$ above are fully expressed by $\forall \mathrm{N}x$ and $\exists \mathrm{N}y$, where N ("number") indicates the range of x and y. This formal observation, due to Anil Gupta [1], had been suggested informally by others (e.g., Tarski [2], pp. 6–9).

A sentence lacking one or more of Q, R, or V might still express a quantified proposition—but only elliptically.

(1) for every x, $x < sx$
(2) $x < sx$

The above two sentences express the proposition that every number precedes its successor.

(3) $x + y = y + x$

Sentence 3 expresses the proposition that for every number x, for every number y, x plus y is y plus x. Sentence (1) omits the range-indicator; (2) and (3) omit the whole quantification. This literature-research paper, expanding this BULLETIN, vol. 5 (1999) pp. 418–419, documents implicit acceptance or rejection of the QRV theory of quantification form by recognized logicians such as Andrews, Boolos, Carnap, Dedekind, Enderton, Frege, Goldfarb, Hilbert, Jeffrey, Kneebone, Lemmon, Moore, Nagel, Whitehead, and others.

[1] A. GUPTA, ***Logic of common nouns***, Yale University Press, New Haven, 1980.
[2] A. TARSKI, ***Introduction to logic***, Dover, New York, 1995.

► JOHN CORCORAN AND GEORGE BOGER, *Implicational-future sentences*.
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The proposition "No dog is a cat" logically implies "No male cat is a dog".

The previous sentence—whose metalinguistic verb *implies* is in the timeless-present tense—uses modern conventions to express an *implication*. The antecedent and consequent propositions are both normally expressed—as above—also using present-tense verbs. Presupposing further suitable conventions, a present-tense conditional sentence expresses the same implication.

If no dog is a cat, then no male cat is a dog.

Surprisingly, logic writings from Aristotle on reveal many implications idiomatically expressed using future-tense verbs. The following form of expression is common in Aristotle's original writings and in translations.

If no dog is a cat, then no male cat *will be* a dog.

The present-tense antecedent proposition "No dog is a cat" does not logically imply the future-tense "No male cat will be a dog"—which is not the intended implication's consequent.

Other *implicational-future* sentence forms occur.

If no dog is a cat, then it *will* follow that no male cat is a dog.

The intended implication's antecedent and consequent are given with present-tense verbs; the future-tense verb is metalinguistic. This idiom lacks a literal meaning: (1) the logical consequences of a proposition—be it "No dog is a cat" or any other—are not contingent on its truth and (2) all such consequences are fixed without reference to time.

We survey future-tense implication statements. We also analyze and critically evaluate discussions of such idioms. Could logicians, or their translators—who understand the timeless nature of logical implication—permit themselves to use such misleading idioms without alerting their readers?

We treat Aristotle, the Stoics, Galen, Ockham, Locke, Boole, Russell, Tarski, and others.

► JOHN CORCORAN AND JUSTIN LEGAULT, *Discussing incommensurability in Aristotle*.
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Given two arbitrary things, the first *is incommensurable with* the second if it is meaningless (incoherent, a *category* mistake) to say that the first is larger than, smaller than, or equal to the second—e.g., a time and a volume. No two *homogeneous* magnitudes—two times, two volumes, two lines, two numbers, and so on—are incommensurable [*sc*. with each other] in *this* sense.

In another sense, given two homogeneous magnitudes, the first *is incommensurable with* the second if the first *isn't* a multiple of any magnitude that divides the second without remainder—i.e., if no "unit" magnitude "measures" both evenly. In this sense, incommensurability is the complement of *commensurability*: given two homogeneous magnitudes, the first is *commensurable* (co-measurable) with the second if the first *is* a multiple of some magnitude that divides the second without remainder—i.e., if some "unit" magnitude "measures" both evenly.

Aristotle's ***Analytics*** apparently mentions several times the theorem that in every square the diagonal is incommensurable with the side: in every square each of the two diagonals is incommensurable with each of the four sides (41a27+29, 46b29, 50a37, 65b18).

However, in each case, Aristotle wrote "the diagonal is incommensurable"—which appears more categorical (or predicative) than relational. Taken literally this locution *miscategorizes* the *relation* of incommensurability as a *predicate*. Taken elliptically it suggests insensitivity to miscategorization.

Such miscategorizing parallels the absurdity of taking inequality to be a property. "(Every12* No34) diagonal is (in-13*24)commensurable" parallels "(Every12* No34) side is (un-13* 24)equal".

We ask whether and if so how and to what extent Aristotle's imprecision is noticed, discussed, and assessed. Scholars treated include Barnes, Bochenski, Crivelli, Heath, Jenkinson, Łukasiewicz, Owen, Smith, Striker, Tredennick, and Ross.

► JOHN CORCORAN AND HASSAN MASOUD, *Refutation and proof*.
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We survey usage of two words, *refutation* and *proof*—and their cognates and synonyms—in the sciences and humanities with special attention to history and philosophy of logic, history and philosophy of mathematics, and mathematical logic. The treatment is descriptive, analytic, and lexicographic. Authors considered include Aristotle, Galen, Ockham, Boole, De Morgan, Peirce, Frege, Russell, Hilbert, Lewis, Łukasiewicz, Tarski, Church, and Quine.

In one ordinary usage, a *refutation* of a proposition (sentence, statement) is a cogent demonstration that it is not *true*; a *refutation* of an argument (inference, implication) is a cogent demonstration that it is not *valid*—that its conclusion does not follow from its premise set; and a *refutation* of a proof (demonstration, deduction) is a demonstration that it is not *cogent*, not *conclusive*—that its step-by-step chain of reasoning does not establish its conclusion for its intended audience, e.g., contemporary mathematicians.

Rebuttals are attempted refutations; conversely, refutations are successful rebuttals. *Rebuttal* and *refutation* are attempt/success correlatives.

Proof has somewhat parallel usages, which however are controversial among logicians. In one usage, a *proof* of a proposition is a cogent demonstration that it is *true*; a *proof* of an argument is a cogent demonstration that it is *valid*; a *proof* of a proof is a cogent demonstration that it is conclusive—perhaps despite initial appearances.

The word *proof* has no exact attempt/success correlative. In fact, it suffers from attempt/success ambiguity. Above, both senses were used. The attempt sense—as in "many proofs of the Fermat conjecture were refuted"—is more common. *Argument* often serves as an attempt correlative to *proof* in the success sense. Refutations of propositions are sometimes called *disproofs*.

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Premises rhymes with nemesis.

The words 'premises' and 'nemesis' are used autonymously above. A word that denotes itself—that is *used* to *mention* itself—in a given statement is used *autonymously*. *Autonymy* is the phenomenon of autonymous usage.

The word premises rhymes with the word nemesis.

In the preceding statement, 'premises' and 'nemesis' are used parautonymously—as they are in this statement. A word denoted in a given statement by an expression containing that word is used *parautonymously*. *Parautonymy* is parautonymous usage. Autonymy cannot "iterate"; parautonymy can—at least in "semi-ideographic" contexts.

'Plato' begins with pee.
' 'Plato' ' begins with a left quotation mark.

Ancient Greek and Medieval Latin writings were usually phonographic—readable aloud. Both languages permitted autonymy and both had phonographic devices for parautonymy. The two-letter Ancient Greek word *to*—also used for the definite article—marks parautonymy: *aristotelês* denotes Aristotle, *to aristotelês* denotes Aristotle's Greek name, the word *aristotelês*. In Medieval Latin, which lacks articles, the otherwise meaningless two-letter word *li* marks parautonymy: *aristoteles* denotes Aristotle, *li aristoteles* denotes Aristotle's Latin name, the word *aristoteles*.

Autonymy permits ambiguity not present in parautonymy.

Socrates is Greek.
The name Socrates is Greek.

This study—sharpening Corcoran's discussions in this BULLETIN, vol. 12 (2006), pp. 219–240—surveys logic-related autonymy and parautonymy (and use-mention distinctions). We discuss previously ignored questions: whether translations should respect the two distinctions of autonymy versus parautonymy and phonography versus ideography. For example, should autonymy and parautonymy in Greek be rendered respectively as autonymy and parautonymy in English? Should Greek intended phonographically be rendered in English that admits being read phonographically?

► SEBASTIAN EBERHARD AND THOMAS STRAHM, *Towards the unfolding of feasible arithmetic*.

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The notion of *unfolding a schematic formal system* was introduced in Feferman [1] in order to answer the following question:

Given a schematic system S*, which operations and predicates, and which principles concerning them, ought to be accepted if one has accepted* S*?*

A paradigmatic example of a schematic system S is the basic system NFA of non-finitist arithmetic. In Feferman and Strahm [2], three unfolding systems for NFA of increasing strength have been analyzed and characterized in more familiar proof-theoretic terms; in particular, it was shown that the full unfolding of NFA, $\mathcal{U}(\mathsf{NFA})$, is proof-theoretically equivalent to predicative analysis.

More recently, the unfolding notions for a basic schematic system of finitist arithmetic, FA, and for an extension of that by a form BR of the so-called bar rule have been worked out in Feferman and Strahm [3]. It is shown that $\mathcal{U}(\mathsf{FA})$ and $\mathcal{U}(\mathsf{FA} + \mathsf{BR})$ are proof-theoretically equivalent, respectively, to primitive recursive arithmetic, PRA, and to Peano arithmetic, PA.

The aim of the present contribution is to elucidate the concept of unfolding in the context of a natural schematic system FEA for *feasible arithmetic*. We will sketch various unfoldings of FEA and indicate their relationship to weak systems of explicit mathematics and partial truth.

[1] SOLOMON FEFERMAN, *Gödel's program for new axioms: Why, where, how and what?*, ***Gödel '96*** (Brno, Czech Republic), (Petr Hájek, editor), Lecture Notes in Logic vol. 6, Springer, 1996, pp. 3–22.

[2] Solomon Feferman and Thomas Strahm, *The unfolding of non-finitist arithmetic*, ***Annals of Pure and Applied Logic***, vol. 104 (2000), no. 1–3, pp. 75–96.

[3] ———, *Unfolding finitist arithmetic*, ***The Review of Symbolic Logic***, vol. 3 (2010), no. 4, pp. 665–689.

▶ TATYANA IVANOVA, *Line-based reasoning in Euclidean plane*.
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In [1] the relational structure $\langle LE^2; P, C\rangle$, where LE^2 is the set of the lines in Euclidean plane, P is the predicate "strong parallel" and C is the predicate "convergent", is investigated both as a first-order structure and as a Kripke structure for the adequate propositional language.

The aim of my talk is to present recently obtained results concerning the structure $\langle LE^2; P, C, \perp\rangle$, where $\perp$ is the predicate "perpendicular". The first-order theory of $\langle LE^2; P, C, \perp\rangle$ is axiomatized and its basic model theoretic properties are shown. The complexity of the membership problem for this theory is found—it is PSPACE-complete. The modal logic of $\langle LE^2; P, C, \perp\rangle$ in the propositional modal language with three unary modalities is also axiomatized. The complexity of the membership problem for this modal logic is coNP-complete.

[1] Ph. Balbiani and T. Tinchev, *Line-based affine reasoning in Euclidean plane*, ***Journal of Applied Logic***, vol. 5 (2007), pp. 421–434.

▶ VLADIMIR STEPANOV, *Many-valued logics for dynamical semantics of the atomic self-reference statements*.
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Let $P(x)$ be a predicate formula of a fragment of the type-free language, in which predicates can take other predicate as arguments. By quantor $\mathsf{S}x$ we designate the fact, that the formula $\mathsf{S}xP(x)$ refers to itself and satisfies the self-reference axiom:

$$\mathsf{S}xP(x) \leftrightarrow P(\mathsf{S}xP(x)).$$

Let $P(x)$ be constructed by $\vee \;\; \& \; \rightarrow \leftarrow \; \neg$ from atomic predicate $Tr(x)$, which satisfies Tarsky axiom:

$$Tr(x) \leftrightarrow x.$$

Let I be interpretation of our language, described in [1], where, in particular, $\mathsf{I}(\mathsf{S}xP(x)) = \{\langle p^n(x_0)\rangle, n \in \mathbb{Z} \mid x_0 \in \{0,1\}\}$ is the set of motions of the dynamical system $(\{0,1\}, p)$ ***associated*** *with* $\mathsf{S}xP(x)$. In this case $p^n = p(p^{n-1})$. The dynamical intrepretation I generate a 16-valued logic, which is the fourth cartesian direct power of classical propositional logic $\mathbf{C}^2$ with the matrix $M_2^c = \langle\{1,0\}, \neg, \vee, \wedge, \{1\}\rangle$, i.e.:

$$\begin{aligned}\mathbf{M}^{\mathrm{c}}_{16} = (\mathbf{M}^{\mathrm{c}}_2)^4 &= \langle\{11/11, 10/01, 11/00, \ldots, 00/11, 01/10, 00/00\}, \underline{\neg}, \underline{\vee}, \underline{\wedge}, \{11/11\}\rangle \\ &= \langle\{\mathbf{T},\ \mathbf{A},\ \mathbf{V}, \ldots, \sim\!\mathbf{V},\ \sim\!\mathbf{A},\ \sim\!\mathbf{T}, \}, \sim, \underline{\vee}, \underline{\wedge}, \{\mathbf{T}\}\rangle.\end{aligned}$$

The $\mathbf{T}(\mathit{rue})$, $\mathbf{A}(\mathit{ntinomy})$, $\mathbf{V}(\mathit{oid})$ and its negations are the set of truth-values for atomic self-reference statements. Let L be the lattice diagram for the 16-valued logic in the coherent case of the interpretation I. For incoherent case the nodes of the lattice diagram L situated at one horizontal line are merged into one node named an aggregate by the following scheme:

$$\begin{aligned}\{\mathbf{A}\underline{\vee}\mathbf{V},\ \sim\!\mathbf{A}\underline{\vee}\mathbf{V},\ \mathbf{A}\underline{\vee} \sim \mathbf{V},\ \sim\!\mathbf{A}\underline{\vee} \sim \mathbf{V}\} &\rightarrow \mathbf{A}\underline{\underline{\vee}}\mathbf{V}, \\ \{\mathbf{A}\underline{\wedge}\mathbf{V},\ \sim\!\mathbf{A}\underline{\wedge}\mathbf{V},\ \mathbf{A}\underline{\wedge} \sim \mathbf{V},\ \sim\!\mathbf{A}\underline{\wedge} \sim \mathbf{V}\} &\rightarrow \mathbf{A}\underline{\underline{\wedge}}\mathbf{V}, \\ \{\quad \mathbf{A}\underline{\leftrightarrow}\mathbf{V},\quad \sim\!(\mathbf{A}\underline{\leftrightarrow}\mathbf{V})\quad\} &\rightarrow \mathbf{A}\underline{\underline{\leftrightarrow}}\mathbf{V}.\end{aligned}$$

The aggregates generated 9-valued logic with the following factor-matrix:

$$\mathbf{M}^{\mathbf{c}}_{\mathbf{16/agg}} = \mathbf{M}^{\mathbf{c}}_{\mathbf{9}} = \langle \{\mathbf{T}, \mathbf{A}, \mathbf{V}, \mathbf{A}\underline{\underline{\vee}}\mathbf{V}, \mathbf{A}\underline{\underline{\leftrightarrow}}\mathbf{V}, \mathbf{A}\underline{\underline{\triangle}}\mathbf{V}, \sim\mathbf{V}, \sim\mathbf{A}, \sim\mathbf{T}\},\ \sim,\ \underline{\underline{\vee}},\ \underline{\underline{\triangle}},\ \{\mathbf{T}\}\rangle$$

THEOREM. *The sets of tautologies for* $\mathbf{M}^{\mathbf{c}}_{\mathbf{2}}$ *and* $\mathbf{M}^{\mathbf{c}}_{\mathbf{9}}$ *coincide.*

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[1] V. STEPANOV, *Semantics of self-reference: a dynamical systems approach*, ***The Bulletin of Symbolic Logic***, vol. 10, no. 2, pp. 272.