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2011 NORTH AMERICAN ANNUAL MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

University of California at Berkeley
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March 24–27, 2011

The North American Annual Meeting of the Association was held on the campus of the University of California, Berkeley, March 24–27, 2011. The program included the Retiring Presidential Address, nine one-hour invited plenary talks, five special sessions, and nine sessions of contributed talks. The Shoenfield Prizes were presented during a welcoming reception on the first night of the conference. A conference banquet was held on the third night of the conference.

The members of the Program Committee were Samuel Buss, Jeremy Heis, Itay Neeman (Chair), Christos Papadimitriou, Theodore Slaman, and Sergei Starchenko. The members of the Local Organizing Committee were Paolo Mancosu, Thomas Scanlon (Chair), Theodore Slaman, and W. Hugh Woodin.

There were 145 registered participants at the meeting, including 58 graduate students. Generous financial support was provided by: the Group in Logic and the Methodology of Science supported by the Department of Mathematics, Department of Philosophy and the Division of Computer Science at the University of California, Berkeley; the National Science Foundation; and the Association for Symbolic Logic.

The Shoenfield Prizes for outsanding expository writing in logic were presented to John T. Baldwin for his book *Categoricity*, and to Rod Downey, Denis Hirschfeldt, Andrew Nies, and Sebastiaan Terwijn for their article *Calibrating randomness*. Penelope Maddy (UC-Irvine) delivered the Retiring Presidential Address on *The philosophy of logic*. The plenary addresses at the meeting were:

G. Aldo Antonelli (UC Davis), *Arithmetic, abstraction, and the Frege quantifier*.
Laurent Bienvenu (CNRS and Paris 7), *Algorithmic randomness over general spaces*.
Lou van den Dries (UIUC), *On the model theory of transseries*.
Deirdre Haskell (McMaster), *Model theory of analytic functions*.
Justin Moore (Cornell), *Forcing axioms and the Continuum Hypothesis*.
Christian Rosendal (UIC), *On the structure of isometry groups of separable Banach spaces*.
Neil Thapen (ASCR), *Bounded arithmetic and NP search problems*.
Simon Thomas (Rutgers), *The descriptive set theory of unitary group representations*.
Ryan Williams (IBM), *Lower bounds in complexity theory via diagonalization*.

There were five special sessions (with organizers in parentheses): Definability throughout Mathematical Logic—in honor of Leo Harrington (Theodore Slaman, John Steel), Hilbert (Wilfried Sieg), Model Theory (David Marker), Philosophy of Mathematics (John Burgess), and Set Theory (Ernest Schimmerling). These included a total of thirty-one speakers.

Abstracts of the invited talks and the contributed talks (given in person or by title) by members of the Association for Symbolic Logic follow.

For the Program Committee
ITAY NEEMAN

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Abstract of Retiring Presidential Address

► PENELOPE MADDY, *The philosophy of logic*.
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Philosophers of mathematics ask: what makes mathematical claims true and how do we come to know them? Familiar answers are offered by the schools of Formalism, Intuitionism/Constructivism, and Platonism/Realism. Perhaps less familiar are answers to the corresponding questions in the philosophy of logic: what makes logical claims true and how do we come to know them? This talk will outline an array of such positions, drawing rough parallels to views in the philosophy of mathematics.

Abstracts of invited plenary talks

► G. ALDO ANTONELLI, *Arithmetic, abstraction, and the Frege quantifier*.
Department of Philosophy, University of California, One Shields Avenue, Davis, CA 95616, USA.
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URL Address: http://philosophy.ucdavis.edu/antonelli.

This talk presents a formalization of Peano arithmetic in a first-order language comprising (*i*) a cardinality quantifier F (for "Frege") as the only logical constant other than truth-functional connectives, and (*ii*) an abstraction operator Num mapping formulas into terms in such a way that equinumerous formulas are assigned co-referential terms. While the Frege quantifier embodies a logical notion of cardinality, the abstraction operator reflects instead an *extra-logical* notion of number. The interaction between these two devices is governed by a number of axioms, one of which is a form of Hume's Principle, for which no logically privileged status is claimed. The project draws its philosophical motivation from a non-reductionist conception of logicism, a deflationary view of abstraction, and an approach to formal arithmetic that emphasizes the cardinal properties of the natural numbers over the structural ones.

[1] G. Aldo Antonelli, *The nature and purpose of numbers*, ***The Journal of Philosophy***, vol. 107 (2010), no. 4, pp. 191–212.

[2] ———, *Notion of invariance for abstraction principles*, ***Philosophia Mathematica***, vol. 18 (2010), no. 3, pp. 276–292.

[3] ———, *Numerical abstraction via the Frege quantifier*, ***Notre Dame Journal of Formal Logic***, vol. 51 (2010), no. 2, pp. 161–179.

► LAURENT BIENVENU, *Algorithmic randomness over general spaces*.
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Since Martin-Löf's definition of the notion of a random real via computable measure theory in 1966, the field of algorithmic randomness has seen tremendous progress, as witnessed by the two recent monographs of Downey and Hirschfeldt [1], and of Nies [5]. Most of the research conducted so far has focused on the space of infinite binary sequences endowed with the uniform measure, but it is possible to consider randomness for more general spaces and measures. This talk will be a survey of the research done in this direction, from the seminal work of Levin [3, 4] (perhaps better explained, and also extended in Gács [2]) to the latest developments. Topics will include: extension of Martin-Löf randomness to computable probability spaces, existence of a universal Martin-Löf test, randomness for non-computable measures, Levin's neutral measure, randomness preservation under computable transformations, interactions with computable analysis.

[1] Rod Downey and Denis Hirschfeldt, ***Algorithmic randomness and complexity***, Theory and Applications of Computability, Springer, 2010.

[2] Peter Gács, *Uniform test of algorithmic randomness over a general space*, ***Theoretical Computer Science***, vol. 341 (2005), pp. 91–137.

[3] Leonid A. Levin, *Uniform tests of randomness*, ***Soviet Mathematics Doklady***, vol. 17 (1976), no. 2, pp. 337–340.

[4] ———, *Randomness conservation inequalities: information and independence in mathematical theories*, ***Information and Control***, vol. 61(1984), no. 1, pp. 15–37.

[5] André Nies, ***Computability and randomness***, Oxford Logic Guides, Oxford University Press, 2009.

► LOU VAN DEN DRIES, *On the model theory of transseries*.
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E-mail: vddries@math.uiuc.edu.

The differential field of transseries has many features that suggest it might have a beautiful model theory. This talk is about the results that have been obtained in this direction, in collaboration with Matthias Aschenbrenner and Joris van der Hoeven.

► DEIRDRE HASKELL, *Model theory of analytic functions*.
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Model theorists have been studying analytic functions since the late 1970s. Highlights include the seminal work of Denef and van den Dries on the theory of the p-adics with restricted analytic functions, Wilkie's proof of o-minimality of the theory of the reals with the exponential function, and the formulation of Zilber's conjecture for the complex exponential. My goal in this talk is to survey these main developments and to reflect on today's open problems, in particular for theories of valued fields.

► JUSTIN MOORE, *Forcing axioms and the Continuum Hypothesis*.
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It is known that there is a strongest consistent forcing axiom, namely the forcing axiom for partial orders which preserve stationary subsets of ω_1 (also known as Martin's Maximum or MM). The consistency of this axiom was established by Foreman, Magidor, and Shelah relative to the existence of a supercompact cardinal. Since MM has proved very effective at settling statements left undecided by ZFC, it is natural to ask whether there is an analogous optimal forcing axiom which is relatively consistent with the Continuum Hypothesis. One precise way to cast this question is whether there are two Π_2 sentences in the language of $(H(\omega_2), \in, \mathrm{NS}_{\omega_1})$ which are each Ω-consistent with CH but which jointly negate CH. This problem is due to Woodin, who showed that the answer is negative if one replaces CH with ZFC.

We show that the answer to Woodin's problem is positive and in the process establish that there are two preservation theorems for not adding reals which can not be subsumed into a single iteration theorem.

This is joint work with David Aspero and Paul Larson.

► CHRISTIAN ROSENDAL, *On the structure of isometry groups of separable Banach spaces*.
Department of Mathematics, Statistics, and Computer Science, (M/C 249), University of Illinois at Chicago, 851 S. Morgan Street, Chicago, IL 60607-7045, USA.
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We shall present joint work with V. Ferenczi on the relation between the isomorphic structure of a Banach space and the possible structure of its isometry group under various renormings. It follows from the theory of Lipschitz free Banach spaces that any Polish group is topologically isomorphic with an open subgroup of index 2 of the isometry group of some separable Banach space, but unfortunately this provides little information about the relation between the Polish group and the isomorphic structure of the Banach space. We shall provide some new results on these questions, including a recent solution to a longstanding problem of G. Wood on whether $GL(X)$ always contains a maximal bounded subgroup when X is a separable Banach space.

► NEIL THAPEN, *Bounded arithmetic and NP search problems*.
Academy of Sciences of the Czech Republic.
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An NP search problem is given by a polynomial time relation $R(x, y)$, with the properties that (a) $R(x, y)$ implicitly bounds the length of y by some polynomial in the length of x, and (b) R is total, that is, for all x there is some y such that $R(x, y)$ holds. The search problem is to find such a y, given x.

Call a first-order theory bounded if it cannot prove total any function that increases the length of its input by more than a polynomial amount. For such a theory, the provably total NP search problems are a natural analogue of the provably recursive functions. Bounded arithmetic contains rich families of such theories, and I will talk about these and some corresponding classes of search problems, focusing on recent results that have extended this to relatively strong theories. These classes are interesting for their own sake and also give a natural measure of the theories' strength, which we hope can be used to give new separations between theories.

The main technical method we use is a translation of first-order proofs into uniform families of constant-depth propositional proofs, structured in such a way that checking their soundness becomes a natural combinatorial search problem.

► SIMON THOMAS, *The descriptive set theory of unitary group representations*.
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In this tribute to Greg Hjorth, I will discuss some of his many fundamental contributions to the theory of Borel equivalence relations. The talk will mainly focus on Hjorth's work on the Borel complexity of the unitary equivalence relation for the irreducible unitary representations of countable discrete groups.

► RYAN WILLIAMS, *Lower bounds in complexity theory via diagonalization*.
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The general feeling in complexity theory has been that the method of diagonalization would not be powerful enough to prove strong *lower bounds*, i.e., separations of complexity classes. Recent work has shown that diagonalization can be fruitfully combined with other ideas to prove interesting lower bounds, such as time-space lower bounds for the satisfiability problem and circuit size lower bounds for nondeterministic exponential time. I will survey the general ideas and thought processes that are used to prove these lower bounds. Ultimately the ideas are algorithmic in nature: one assumes that too-good-to-be-true algorithms exist, and through algorithmic thinking, one eventually derives a contradiction to the assumption via diagonalization.

Abstracts of invited talks in the Special Session on Definability throughout Mathematical Logic—in honor of Leo Harrington

► GREGORY CHERLIN, *Structure/nonstructure in finite model theory*.
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One of the tasks of mathematics is to separate the tame from the wild. Shelah's classification theory is based on the premise that a good definition is one whose negation is also a good definition (cf. Plato, Statesman, 262b); this works well for classes of infinite structures. In dealing with classes of finite structures, one may look for notions of tame/wild which are well separated in the sense of being *decidable* over a wide range of natural cases. This leads to the consideration of decision problems which are generally open. In other words, it is unclear whether natural measures of structure vs. nonstructure allow for a clear separation of cases in this context.

Research supported by NSF Grant DMS-0600940

[1] GREGORY CHERLIN, *Forbidden substructures and combinatorial dichotomies: WQO and universality*, ***Discrete Mathematics***, vol. 311 (2011), no. 15, pp. 1543–1584.

► PETER CHOLAK, *Definability in the computably enumerable sets*.
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In 1997–98, I spent a year on sabbatical at UC–Berkeley. At that time, Leo Harrington and I started working together on some projects in the computably enumerable sets under inclusion, $\mathcal{E}$. A number of papers, including [1] and [2], resulted. For the first half of my talk, I will discuss some of the results that appear in these papers. For the remaining half, I will discuss the collaborative progress that has been made in answering the following open questions:

QUESTION 1 (Completeness). *Which c.e. sets are automorphic* (*in* $\mathcal{E}$) *to a complete set*?

QUESTION 2 (Lowness). *Which c.e. sets are automorphic* (*in* $\mathcal{E}$) *to a low set*?

QUESTION 3 (Cone Avoidance). *Given an incomplete c.e. set* C, *for which c.e. sets* A *can we find an c.e. set* $\widehat{A}$ *automorphic* (*in* $\mathcal{E}$) *to* A *which is not Turing above* C?

[1] PETER CHOLAK, RODNEY DOWNEY, AND LEO A. HARRINGTON, *On the orbits of computably enumerable sets*, ***Journal of the American Mathematical Society***, vol. 21 (2008), no. 4, pp. 1105–1135,

[2] PETER CHOLAK AND LEO A. HARRINGTON, *On the definability of the double jump in the computably enumerable sets*, ***Journal of Mathematical Logic***, vol. 2 (2002), no. 2, pp. 219–613.

► ALEXANDER S. KECHRIS, *Borel combinatorics and ergodic theory*.
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In this talk I will discuss descriptive aspects of graph combinatorics, particularly coloring problems, and their connections with the ergodic theory of countable group actions.

► MICHAEL C. LASKOWSKI, *An update on Leo's conjecture*.
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Following Shelah's announcement of the Main Gap, where he proved that every model of a classifiable theory is prime and minimal over an independent tree of countable models, Leo Harrington asked whether every model of such a theory is actually prime and minimal over an independent tree of finite sets. While the answer is no in general, this question has led to a deeper understanding of classifiable theories. We survey a number of results, both positive and negative. On one hand, Bouscaren and Hrushovski found examples of $\aleph_0$-stable, classifiable theories that refute Leo's conjecture. On the other hand, a series of papers explain the role of 'potential isomorphism' i.e., an isomorphism lies in a generic extension of the set-theoretic universe. Additionally, Shelah has proved that Leo's conjecture actually holds if one restricts to the class of $\aleph_\epsilon$-saturated models and if one allows independent trees of ϵ-finite sets. Using technology from this result, Shelah and the speaker characterize those $\aleph_0$-stable theories that have 'few' $L_{\infty,\omega}$-inequivalent models.

► MENACHIM MAGIDOR, *Some simple observation on reflection cardinals*.
Institute of Mathematics, Hebrew University of Jerusalem, Edmond J. Safra Campus, Givat Ram, Jerusalem 91904, Israel.
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Given a property of the class of structures in a given signature, a cardinal κ is a reflection cardinal for this property if every structure having the property has a substructure of cardinality less than κ having the property. In the talk we shall discuss the existence and the size of reflecton cardinals for several properties of structures, typically properties that can be expressed in second order logic.

► ANTONIO MONTALBAN, *The boundary of determinacy within second order arithmetic*.
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We find the exact level at which determinacy stops being provable in second order arithmetic, or equivalently for this type of sentences, in ZFC without the power set axiom.

This is joint work with Richard A. Shore.

► GERALD SACKS, *Models of size ω_2 in infinitary logic*.
Mathematics Department, Harvard University, One Oxford Street, Cambridge, MA 02138, USA.
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Construction of size ω_2 models in the setting of infinitary logic via fine structure of L.

► THOMAS SCANLON, *(Non)definability in finitely generated rings*.
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It is an easy consequence of Gödel coding of finite sequences that each finitely generated commutative ring is interpretable in the natural numbers. It follows from work of J. Robinson and R. Robinson that the natural numbers are interpretable (possibly using parameters) in each infinite finitely generated commutative ring. As a key step towards proving that the theory of a finitely generated commutative ring is quasi-finitely axiomatizable (in the sense of Nies) within this class, Khelif claimed that these natural interpretations form a bi-interpretation. In the accepted sense of "bi-interpretation" (after Ahlbrandt–Ziegler) Khelif's assertion is false (though one sees from his proof that he really meant something weaker and the weaker claim suffices for his theorem). For example, it would follow from bi-interpretability that $\mathbb{Z}$ would be *definable* as a subring of $\mathbb{Z} \times \mathbb{Z}$ but the Feferman–Vaught theorem implies that no such definition exists. The dual numbers, $\mathbb{Z}[\epsilon]/(\varepsilon^2)$, present a subtler case, but we show by constructing derivations on nonstandard models of arithmetic that $\mathbb{Z}$ is not definable as a subring of $\mathbb{Z}[\epsilon]/(\varepsilon^2)$. Ultimately, we give a ring theoretic characterization of those finitely generated commutative rings which are bi-interpretable with $(\mathbb{Z}, +, \times, 0, 1)$.

This is joint work with Matthias Aschenbrenner.

► RICHARD A. SHORE, *The Hyperdegrees: rigidity, definability and (un)decidability*.
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We will discuss the rigidity of, and definability in, $\mathcal{D}_h$ the full structure of the hyperdegrees, arbitrary hyperjump ideals $\mathcal{I}$ and $\mathcal{D}_h(\mathbf{0}')$, the hyperdegrees below that of Kleene's $\mathcal{O}$. We will also consider the complexity of the theories of these structures and the dividing line, in terms of quantifier complexity, between decidability and undecidability. The techniques involved include coding and forcing arguments in the setting of second order arithmetic following Feferman or Sacks. The notions of forcing include those of Cohen, Slaman–Woodin and some unusual variants of perfect set forcing to prove results on initial segments of these structures.

► ROBERT SOARE, *Definability, automorphisms, and low sets*.
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Let $\mathcal{E}$ denote the structure of the computably enumerable (c.e.) sets under inclusion. Results on definability and automorphisms of $\mathcal{E}$ may be viewed as a game between two players. The definability player (RED) tries to produce an $\mathcal{E}$-definable property $P(X)$ for a given class such as maximal sets, creative sets, or incomplete sets (for an $\mathcal{E}$-definable solution to Post's Program). The automorphism player (BLUE) tries to produce for some property $P(X)$ some set $A \in \mathcal{E}$ with $P(A)$ and an automorphism Ψ of $\mathcal{E}$ such that $P(\Psi(A))$ fails. This shows that P is not $\mathcal{E}$-definable, such as having low degree or being hypersimple (D. A. Martin). We review Harrington's fundamental results in both areas.

Next we consider some recent results on automorphisms of low sets. A c.e. set A is *low* if $A' \equiv_T 0'$ where A' is the jump of A. It often suffices that the complement $\overline{A}$ possess the weaker property of being *semilow*, *i.e.*, $\{x\colon W_x \cap \overline{A} \neq \emptyset\} \leq_T \mathbf{0}'$.

Theorem 1 (Soare). *If A c.e. and $\overline{A}$ semilow, A is effectively automorphic to a low set.*

Theorem 2 (Soare). *In every c.e. degree there is a c.e. set effectively automorphic to a low set.*

A weaker property is that $\overline{A}$ be *semilow*$_{1.5}$, *i.e.*, $\{x\colon W_x \cap \overline{A} =^* \emptyset\} \leq_1 \emptyset''$. Maass proved that if A and B are c.e. and have semilow$_{1.5}$ complements then $\mathcal{L}^*(A) \cong \mathcal{L}^*(B)$, where $\mathcal{L}(X) = \{Y\colon X \subseteq Y\}$. P. Cholak and R. Weber asked whether Theorem 1 can be improved to semilow$_{1.5}$ in place of semilow, but Harrington and Soare refute it.

Abstracts of invited talks in the Special Session on Hilbert

► WILLIAM EWALD, *Hilbert, Bernays, and the sources of proof-theory*.
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I shall speak about what is now known, in the light of research for the Hilbert Edition, about the philosophical and mathematical interactions between Hilbert and Bernays in the early years of their collaboration, from 1917 until the early 1920s.

► ROBIN HARTSHORNE, *Hilbert's axioms for geometry*.
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I will review Hilbert's axioms for Euclidean geometry as presented in his Grundlagen der Geometrie of 1899, and discuss how they differ from earlier axiomatic treatments of geometry, in particular Euclid and his successors, and how they fit into the late 19th century program of axiomatizing different branches of mathematics.

► GRIGORI MINTS, *Development and present state of the epsilon substitution method*.
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The epsilon substitution method was introduced by D. Hilbert as the main technical tool of his foundational program. After an unsuccessful consistency proof in W. Ackermann's thesis [1] a version working for first order predicate logic and fragments of arithmetic was presented in [3] by D. Hilbert and P. Bernays, then a consistency proof for arithmetic by epsilon substitution method (based on ideas of Gentzen) was published by W. Ackermann in [2]. More recently formulations working for predicative and some impredicative systems were proposed. We explain what went wrong in W. Ackermann's thesis, what are obstacles for extension of the version from [3] to the second order case and outline the treatment of the theory of inductive definitions.

[1] W. Ackermann, *Begründung des "tertium non datur" mittels der Hilbertischen Theorie der Widerspruchsfreiheit*, ***Mathematische Annalen***, vol. 93, pp. 1–36, 1925.

[2] W. Ackermann, *Zur Widerspruchsfreiheit der Zahlentheorie*, ***Mathematische Annalen***, vol. 117, pp. 162–194, 1940.

[3] D. Hilbert and P. Bernays, ***Grundlagen der Mathematik***, Bd. 2, Springer, 1939.

Abstracts of invited talks in the Special Session on Model Theory

► URI ANDREWS, *Computable stability theory*.
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I will discuss how tools from stability theory can be used to shed light on some problems of computable model theory.

► ISAAC GOLDBRING, *Definable linear operators on Hilbert spaces*.
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Let H be an infinite-dimensional real (or complex) Hilbert space, viewed as a metric structure in the natural continuous signature for Hilbert spaces. We characterize the definable linear operators on H as exactly the "scalar plus compact" operators on H. We will also see how this shows that many "intuitively definable" linear operators on Hilbert spaces are not definable in the sense of continuous logic.

► VINCENT GUINGONA, *On uniform definability of types over finite sets*.
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In this talk, we explore uniform definability of types over finite sets (UDTFS). We examine which theories have this property and what this property implies. For example, we show that dp-minimal theories have UDTFS and explore relationships between UDTFS and VC-density.

► CAMERON DONNAY HILL, *On the geometry of L_k-canonization.*
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I will consider the class $K = \{M \models T : M \text{ is finite}\}$ of finite models of an (incomplete) $\forall\exists$-theory T such that the class of induced substructures $K_\forall = \{M/A \mid M \in K, A \subseteq M\}$ has HP, JEP and AP (equivalently, K has the amalgamation property over sets). Around K, I consider the following computational problem: **Given** a diagram $A = \text{diag}_{qf}(A; M_0)$, where $M_0 \in K$ and $A \subseteq M_0$, **Return** a model $M \in K$ such that $A \subseteq M$ and $\text{diag}_{qf}(A; M) = A$. I will explain briefly how this problem—the CONSTRUCTIBILITY problem—can be understood as a generalization of the L_k-CANONIZATION problem, wherein one wishes to devise an efficiently computable operator F over finite structures such that (i) $F(M) = F(N)$ if and only if $Th_{L^k}(M) = Th_{L^k}(N)$, and (ii) $F(M)$ is a structure and $F(M) \models Th_{L^k}(M)$. (For $2 \leq k < \omega$, L_k denotes the k-variable fragment of first-order logic.) Under a model of computation in which the primitives amount to manipulations with definable sets in the sense of model theory, I will show how to recover a model-theoretic independence relation on K given the assumption that the CONSTRUCTIBILITY problem over K is efficiently computable.

In brief, the technique begins by rendering the behavior of the proposed algorithm on a structure as a directed acyclic graph (dag). In any dag, there is a notion of *d-separation* (developed originally in statistical learning theory to model conditional independence in joint probability distributions), which has properties similar to the well-known orthogonality relation from model theory. Thereafter, one manipulates the d-separation relation to recover the independence relation.

► MARYANTHE MALLIARIS, *Saturation of ultrapowers and Keisler's order.*
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A longstanding open problem is to determine the structure of Keisler's order on countable theories. This order, introduced by Keisler in 1967, is defined in terms of saturation of ultrapowers, which gives, roughly speaking, a measure of the combinatorial complexity of a theory. The structure of Keisler's order on stable theories is known. This talk will discuss recent work in the unstable case.

► LYNN SCOW, *Tree-indexed indiscernibles and the* SOP$_n$*, for $n \leq 3$.*
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Dzamonja and Shelah introduced a generalized sort of indiscernible sequence indexed by a full binary tree, in order to better understand the properties SOP$_1$ and SOP$_2$.

In this talk, we explore the uses of a variant of this kind of tree-indexed indiscernible sequence in studying the properties SOP$_2$ and NSOP$_3$.

► PIERRE SIMON, *Distal NIP theories.*
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The class of NIP theories contains stable theories and some theories with orderings (o-minimal, valued fields). The stable case is of course well understood and we are interested in the other extreme: NIP theories which are in a sense purely unstable (e.g., o-minimal or $\mathbb{Q}_p$). I will define such a notion (called "distality"), and give equivalent characterizations in terms of indiscernible sequences, invariant types and measures. I will then briefly present some ideas towards an analysis of general NIP theories as being distal+stable.

Abstracts of invited talks in the Special Session on Philosophy of Mathematics

► CHARLES CHIHARA, *The Frege–Hilbert dispute and its relevance for a contemporary dilemma argument for realism*.
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My talk is about an important and far-reaching dispute about the nature of mathematics and logic that took place via letters between Frege and Hilbert in the years 1899 to 1903. I will focus in particular on how these titans of logic and foundational studies had radically different views about the nature of mathematical axioms and definitions. It will be my contention that these two widely differing views underlie disputes between realists and nominalist that are taking place today. To illustrate my contention, I shall analyze one contemporary dispute, in particular, in terms of the Frege–Hilberts differences.

► RICHARD PETTIGREW, *Against naturalism*.
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Naturalism in the philosophy of mathematics is a broad church. Its unifying creed is this: It is the pronouncements of science or mathematics that ought to determine our philosophy of mathematics, not the arguments of philosophy. There are more and less extreme versions of the doctrine. Extreme naturalists allow no room for philosophical arguments. Moderate naturalists come in two denominations: the first allows some room for philosophical argument, but demand a special status for the pronouncements of science as premises in these arguments and as defeaters of their conclusions; the second allows philosophy to address certain issues, such as the interpretation of mathematical discourse, but insists that it can have no authority on other issues, such as the correct rules of mathematical reasoning. The single argument for naturalism in any of its forms is the Failure Argument, which begins from the alleged failures of philosophy and the alleged successes of mathematics and science. The Failure Argument itself fails for a number of reasons. I focus on the assumptions that it makes about the boundaries between subject areas, and its misrepresentation of the nature of rational inquiry. The picture of rational inquiry that emerges leads to a complete rejection of naturalism in its extreme and moderate forms. And it suggests a picture of how the philosophy of mathematics ought to proceed that differs both from the armchair philosophy against which naturalism is a reaction, and from all forms of naturalism. I will end by considering a problem that this view faces. If we are naturalists, there can be no conflict between the conclusions of the philosophy of mathematics and mathematical practice. But if we adopt my alternative suggestion, there can be. How ought we to behave in the light of such a conflict? Should philosophers of mathematics ever presume, as David Lewis said they never should, to advise mathematicians on how to proceed?

► STEWART SHAPIRO, *Varieties of relativism concerning logic*.
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My purpose is to articulate a number of different senses in which one can be a pluralist and/or a relativist concerning logical consequence. I propose, first, that logical consequence is either polysemous or it denotes a cluster concept. In other words, there are a number of different notions that go by that names, often run together, or else there are several aspects of the notions, with varying weights. The different notions, or aspects, of consequence, turn on matters of modality, semantics, effectiveness, justification, rationality, and form. Second, most of the articulations of the pre-theoretic notions(s) of logical consequence make essential use of a boundary between logical and non-logical terminology. This suggests a sort of relativism/pluralism explicitly noted by Bernard Bolzano and Alfred Tarski: logical consequence is relative to the logical/non-logical boundary. An argument may be valid on one collection of logical terms, invalid on another. Third, it is possible that at least some aspects of the notion of logical consequence are vague: there may be borderline cases of valid arguments. If so, we have to turn to what the correct account of vagueness is. On some theories of vagueness, consequence ends up as relative to something, such as a sharpening or a conversational context, and on others, we end up with a kind of pluralism. Finally, there are a number of interesting and important mathematical theories that employ a non-classical logic, and are rendered inconsistent if classical logic is imposed. This suggests a fourth kind of relativism/pluralism: relativity to structure.

Abstracts of invited talks in the Special Session on Set Theory

► ALESSANDRO ANDRETTA AND LUCA MOTTO ROS, *κ-Souslin quasi-orders and definable cardinality*.
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A combinatorial tree on X is an acyclic connected graph whose set of vertices is X, and CT_κ is the collection of combinatorial trees on κ. The relations of embeddability and bi-embeddability on CT_κ are denoted by $\sqsubseteq_\kappa$ and $\equiv_\kappa$, while $\sqsubseteq_{<\kappa}$ and $\equiv_{<\kappa}$ denote the analogous relations on $\mathrm{CT}_{<\kappa}$, the collection of combinatorial trees of size $< \kappa$. We show that $\sqsubseteq_\kappa$ is complete for κ-Souslin quasi-orders on ${}^{\omega}2$ via a reduction which is continuous with respect to the product topology on ${}^{\kappa}2$. Our result is based on [1] (where this result was proved for $\kappa = \omega$) which in turns extends [2].

As a corollary we obtain that

THEOREM 1. *Assume $\mathsf{AD}_{\mathbb{R}}$. Then $\sqsubseteq_{<\Theta}$ is complete for arbitrary quasi-orders on ${}^{\kappa}2$, i.e., for every quasi-order R on ${}^{\kappa}2$ there is a definable reduction of R into $\sqsubseteq_{<\Theta}$. Similarly, $\equiv_{<\Theta}$ is complete for arbitrary equivalence relations on ${}^{\kappa}2$.*

[1] S.-D. FRIEDMAN AND L. MOTTO ROS, *Analytic equivalence relations and bi-embeddability*, ***The Journal of Symbolic Logic***, to appear.

[2] A. LOUVEAU AND C. ROSENDAL, *Complete analytic equivalence relations*, ***Transaction of the American Mathematical Society***, vol. 357 (2005), no. 12, pp. 4839–4866.

► ANDRÉS E. CAICEDO AND SY-D. FRIEDMAN, *Projective well-orderings of the reals and forcing axioms*.
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If the bounded proper forcing axiom BPFA holds and $\omega_1 = \omega_1^L$, then there is a lightface

Σ^1_3 well-ordering of the reals. The argument combines a well-ordering due to Caicedo–Veličković [2] with an absoluteness result for models of Martin's axiom MA, and generalizes appropriately as long as there are no inner models with ω many strong cardinals.

[1] ANDRÉS E. CAICEDO AND SY-D. FRIEDMAN, BPFA *and projective well-orderings of the reals*, submitted.

[2] ANDRÉS E. CAICEDO AND BOBAN VELIČKOVIĆ, *The bounded proper forcing axiom and well-orderings of the reals*, ***Mathematical Research Letters***, vol. 13 (2006), no. 3, pp. 393–408.

► JAMES CUMMINGS, *Forcing axioms and weak square principles*.
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We discuss the extent to which strong forcong axioms such as PFA and MM are compatible with weak versions of Jensen's square principle.

This is joint work with Menachem Magidor.

► SU GAO, *Resolvable maps and complete metrizability*.
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A classical theorem of Sierpiński and Hausdorff states that if a metrizable space is the continuous open image of a Polish space, then it is also Polish. There have been efforts to generalize this theorem by weakening the openness requirement of the map, but results of Michael and Stone showed that it cannot be weakened to quotient maps. We define a notion of resolvable map and show that complete metrizability is preserved under resolvable maps. This provides a generalization of the classical theorem. In addition, we will indicate why this generalization seems to be optimal in this direction.

This is joint work with V. Kieftenbeld, and independently by Holický and Pol.

► JOHN KRUEGER, *An equiconsistency result on partial squares*.
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A famous theorem of Jensen and Solovay states that the failure of the square principle at a regular uncountable cardinal is equiconsistent with a Mahlo cardinal. In joint work with Schimmerling, we extend this result as follows. For a regular uncountable cardinal κ, the statement that there is no stationary subset of $\kappa^+ \cap \mathrm{cof}(\kappa)$ which carries a partial square sequence is equiconsistent with a greatly Mahlo cardinal.

► PAUL LARSON, *Separating club guessing principles*.
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The principle $(+)$ is an ostensibly weak guessing principle at ω_1 which was first introduced in the context of determinacy for games of length ω_1.

We present two proofs that $(+)$ does not imply the standard club guessing principle at ω_1, one using iterated forcing and the other using $\mathbb{P}_{\max}$.

This is joint work with Tetsuya Ishiu.

► DIMA SINAPOVA, *The tree property and not SCH for small cardinals*.
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The tree property at κ^+ states that every tree with height κ^+ and levels of size at most κ has an unbounded branch. There is a tension between the tree property and the failure of the Singular Cardinal Hypothesis (SCH). Woodin and others asked if the failure of SCH at a singular κ implies the failure of the tree property at κ^+. Recently Neeman answered this question in the negative. Here we show that his result can be obtained at small cardinals. In particular, we will show that given ω many supercompact cardinals, there is a generic extension in which the tree property holds at $\aleph_{\omega^2+1}$ and SCH fails at $\aleph_{\omega^2}$.

Abstracts of contributed talks

► BERNARD A. ANDERSON AND BARBARA F. CSIMA, *Degrees that are not degrees of categoricity*.
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We say a computable structure $\mathcal{M}$ is $\mathbf{x}$-computably categorical for some Turing degree $\mathbf{x}$, if for every computable structure $\mathcal{A} \cong \mathcal{M}$ there is an isomorphism $f\colon \mathcal{M} \to \mathcal{A}$ with $f \leq_T \mathbf{x}$. We say $\mathbf{x}$ is a *degree of categoricity* if for some computable $\mathcal{M}$, we have $\mathcal{M}$ is $\mathbf{x}$-computably categorical, and for all $\mathbf{y}$, if $\mathcal{M}$ is $\mathbf{y}$-computably categorical then $\mathbf{x} \leq_T \mathbf{y}$.

Fokina, Kalimullin, and Miller [2] showed that every degree that is d.c.e. in a finite jump is a *degree of categoricity*. Csima, Franklin, and Shore [1] extended this result to transfinite jumps, and showed that every degree of categoricity is hyperarithmetical.

In this talk, we will demonstrate some degrees which are not degrees of categoricity. In particular, we will construct a Σ_2 set whose degree is not a degree of categoricity.

[1] Barbara F. Csima, Johanna N. Y. Franklin, and Richard A. Shore, *Degrees of categoricity and the hyperarithmetic hierarchy*, preprint.

[2] Ekaterina B. Fokina, Iskander Kalimullin, and Russell Miller, *Degrees of categoricity of computable structures*, ***Archive for Mathematical Logic***, vol. 49 (2010), no. 1, pp. 51–67.

► ACHILLES A. BEROS, *Arithmetic complexity in learning theory*.
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EX-learning, introduced by E. M. Gold [1], has become the fundamental notion of learning. In this model, a computable machine succeeds in identifying a *c.e.* set if, in the limit, it produces a correct code when taking an enumeration for the set as an input. A family of *c.e.* sets is *EX*-learnable if there is a single learner which can identify every set in the family. Many other models of learning have been defined and explored since—including finite learning, behaviorally correct learning and vacillatory learning as well as anomalous variants of these models. These various models are discussed in Jain, *et al*. [2].

We determine the arithmetic complexity of index sets arising from the study of algorithmic learning theory. In particular, we study the index sets of codes for *u.c.e.* families which are learnable according to the different models of learning. Hardness results witness lower bounds on the complexity and syntactic descriptions witness upper bounds. In some cases a precise characterization of the complexity is achieved. Among them, we prove Σ^0_4-completeness for *EX*-learning and both Σ^0_4 and Π^0_4-hardness for anomalous *EX*-learning.

[1] E. Mark Gold, *Language identification in the limit*, ***Information and Control***, vol. 10 (1967), no. 5, pp. 447–474.

[2] Sanjay Jain, Daniel N. Osherson, James S. Royer and Arun Sharma, ***Systems that learn: An introduction to learning theory***, 2nd edition, Learning, development and conceptual change, The MIT Press, 1999.

► KATALIN BIMBÓ AND J. MICHAEL DUNN, *From relevant implication to ticket entailment*.
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We define a consecution calculus (denoted as $LT_{\rightarrow}^{\circledR}$) for the implicational fragment of the logic of relevant implication with the constant $\boldsymbol{t}$. The calculus is obtained from the consecution calculus for $T_{\rightarrow}^{\boldsymbol{t}}$ in [1], by adding two structural rules. We prove, by combined inductions, that the single cut rule in the following form is admissible.

$$\frac{\mathfrak{A} \vdash \mathcal{A} \quad \mathfrak{B}[\mathcal{A}] \vdash \mathcal{B}}{\mathfrak{B}[\mathfrak{A}] \vdash \mathcal{B}}$$

There exist sequent calculi for $R_{\rightarrow}$ (see e.g., [3]), and adding $\boldsymbol{t}$ to those calculi is not difficult. The interest in this new consecution calculus stems from the way it is obtained from $LT_{\rightarrow}^{\boldsymbol{t}}$, which is based on semantic insights. (See [3], [2] and [1] for models of relevance logics.) We are working on the completion of our solution to the long-open decidability problem of $T_{\rightarrow}$, in which $LT_{\rightarrow}^{\circledR}$ plays a central role.

[1] KATALIN BIMBÓ, *Relevance logics*, (D. Jacquette, editor), ***Philosophy of logic***, ***Handbook of the philosophy of science***, vol. 5, (D. Gabbay, P. Thagard and J. Woods, editors), pp. 723–789, Elsevier (North-Holland), Amsterdam, 2007.

[2] KATALIN BIMBÓ AND J. MICHAEL DUNN, ***Generalized Galois Logics. Relational semantics of nonclassical logical calculi***, CSLI Lecture Notes, vol. 188, CSLI Publications, Stanford, CA, 2008.

[3] J. MICHAEL DUNN, *Relevance logic and entailment*, (D. Gabbay and F. Guenthner, editors), ***Handbook of philosophical logic***, vol. 3, pp. 117–229. D. Reidel, Dordrecht, 1986.

► WILLIAM C. CALHOUN, *Degree of randomness versus Turing degree*.
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The degree of randomness of a real α can be defined in terms of the prefix-free or monotone Kolmogorov complexity of initial segments of α. The Turing degree of α is defined quite differently in terms of the computing power of α when used as an oracle. At first glance, there appears to be little connection between the Turing degree and the degree of randomness. However, there are relationships between the two notions.

An example is provided by the K-trivial reals. These are the reals with the lowest degree of randomness (when defined in terms of prefix-free complexity). Nies has shown that the Turing degrees of K-trivial reals are closed downward and that the computably enumerable K-trivial degrees form a proper ideal in the c.e. degrees.

The situation is quite different when the degree of randomness is defined in terms of monotone complexity. Every Turing degree $\mathbf{d} \geq \mathbf{0}'$ contains a K_m-trivial real. Furthermore, every K-trivial degree is also K_m-trivial. However, work of Demuth and Kautz on 1-random reals may be used to show that some Turing degrees are not K_m-trivial.

We will consider the question of what combinations of degree of randomness and Turing degree are possible.

► WESLEY CALVERT, *Computation and logic on dynamic random graphs*.
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Effective and model-theoretic properties of classical Erdős–Renyi random graphs have been extensively studied. However, efforts to model computer, social, and biochemical networks have caused a recent increase in interest in other random graph processes, including their evolution over time. An important example is the preferential attachment process, in which a new vertex periodically enters the graph and connects to an existing node with probability proportional to the degree of the existing node.

The present talk will describe structures capturing this dynamic behavior. For instance, there is a probabilistically computable structure in continuous first-order logic which captures all possible evolutions of the preferential attachment process. Issues of completeness and categoricity will also be discussed.

► J. CARSON, V. HARIZANOV, J. KNIGHT, K. LANGE, C. MAHER, C. MCCOY, A. MOROZOV, S. QUINN, AND J. WALLBAUM, *Describing free groups and determining a basis of F_∞.*
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An index set for a computable structure (or a computable class of structures) measures, from a computability perspective, how hard it is to distinguish that structure (or that class), up to isomorphism, from other computable structures within some larger class. This talk describes the complete analysis of index sets for each free group with a finite basis, both within the class of free groups and within the class of all groups; for the free group with a countable basis, within both classes; and for the class of free groups within the class of all groups.

In each case, the argument proceeds as follows: we find a computable infinitary formula to define the group or the class of groups under consideration; this provides an upper bound on the complexity of the corresponding index set. We then use an injury argument to show that the index set is actually m-complete at that level of complexity.

Similar arguments involving definability and injury allow us to analyze the complexity of bases of computable free groups. In particular, we show that every computable free group with countable basis has a Π^0_2 basis, but there are computable free groups with no Σ^0_2 basis.

The talk intends to highlight the specific algebraic properties of free groups that establish the defining formulas and allow the injury arguments to proceed. These results depend on some properties of free groups very recently discovered by Bestvina and Feighn and further studied by one of the co-authors.

► JACOB CARSON, JESSE JOHNSON, JULIA KNIGHT, KAREN LANGE, CHARLES MCCOY, AND JOHN WALLBAUM, *The arithmetical hierarchy in the setting of ω_1.*
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We continue work from [3] on computable structure theory in the setting of ω_1, where the countable ordinals play the role of natural numbers, and countable sets play the role of finite sets. In the present paper, we define the arithmetical hierarchy—through all countable levels. We do this in two different ways. We then give analogues of the well-known results from [1] and [2] on relatively intrinsically arithmetical relations. To do this, we define computable Σ_α and computable Π_α formulas, for countable ordinals α. We do this also in two different ways, corresponding to our two definitions of the arithmetical hierarchy. We prove, for both approaches, that a relation R on a computable structure $\mathcal{A}$ is relatively intrinsically Σ^0_α if and only if it is defined in $\mathcal{A}$ by a computable Σ_α formula.

[1] J. Ash, J. Knight, M. Mannasse, and T. Slaman, *Generic copies of countable structures*, ***Annals of Pure and Applied Logic***, vol. 42 (1989), pp. 195–205.

[2] J. Chishilm, *Effective model theory versus recursive model thoery*, ***The Journal of Symbolic Logic***, vol. 55 (1990), pp. 1168–1191.

[3] N. Greenberg and J. Knight, *Computable structure theory in the setting ω_1*, ***Proceedings of first EMU workshop***,

[4] M. Vanden Boom, *The effective Borel hierarchy*, ***Fundamental Mathematics***, vol. 195 (2007), pp. 269–289.

► PETER CHOLAK, PETER GERDES, AND KAREN LANGE, *Degree-theoretic properties of n-tardy sets*.
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Harrington and Soare in [1] defined a first order nontrivial property Q in $\mathcal{E}$, the set of c.e. sets under inclusion, such that if A is a c.e. set and $Q(A)$ holds, then A is not automorphic to a complete set. This result, amongst others, shows that there are ways to relate degree-theoretic notions with definability. In [2], they showed that if $Q(A)$ holds then A is *very tardy* and specifically 2-*tardy*, i.e., satisfies a property about slowness of enumerations. We examine degree-theoretic properties of the n-tardy sets in order to answer some questions of Harrington and Soare about codeability and lowness. A set X is *codeable* if for every noncomputable set A, there is some B automorphic to A such that $X \leq_T B$.

[1] Leo Harrington and Robert Soare, *Post's program and incomplete recursively enumerable sets*, ***Proceedings of the National Academy of Sciences of the United States of America***, vol. 88 (1991), no. 22, pp. 10242–10246.

[2] ———, *Codeable sets and orbits of computably enumerable sets*, ***The Journal of Symbolic Logic***, vol. 63 (1998), no. 1, pp. 1–28.

► WILLIAM CRAIG, *Functions that underlie first-order logic*.
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For any nonempty set (universe, alphabet) U and any i, $0 \leq i < \omega$, let *i-excision* be the function $\dot{q}_i$ such that

$$\dot{q}_i(\langle x_0, \dots, x_{i-1}\rangle^\frown\langle x_i\rangle^\frown\langle x_{i+1}, \dots, x_{n-1}\rangle) = \langle x_0, \dots, x_{i-1}\rangle^\frown\langle x_{i+1}, \dots, x_{n-1}\rangle,$$

for any sequence (word) of length n, $i < n < \omega$. Then

$$\dot{q}_{i+2} = (\dot{q}_i \circ \dot{q}_{i+1} \circ \dot{q}_i^{\smile}) \cap (\dot{q}_{i+1} \circ \dot{q}_{i+1} \circ \dot{q}_{i+1}^{\smile}),$$
$$p_i = (\dot{q}_i \circ \dot{q}_{i+1}^{\smile}) \cap (\dot{q}_{i+1} \circ \dot{q}_i^{\smile}),$$

where, given any sequence (word) of length n, $i+1 < n < \omega$, p_i interchanges the terms (letters) x_i and x_{i+1}. Let $p_i^{=}$ be the restriction of p_i to the set of its fixed points. Let

$$\mathcal{O} = \{\dot{q}_i^*, \dot{q}_i^{\smile *}, p_i^*, p_i^{=*}\}_{0 \leq i < \omega}$$

be the set of unary operations which, for any set X of sequences form the direct image of X under $\dot{q}_i, \dot{q}_i^{\smile}, p_i, p_i^{=}$, respectively. As is essentially well known, the union of $\mathcal{O}$ and $\{-, \cap, \cup\}$ is in many respects adequate for first-order logic (with equality).

► QUINN CULVER, *Polynomial-clone reducibility*.
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Polynomial-clone reducibilities are generalizations of the truth-table reducibilities. A polynomial clone is a set of functions over a finite set X that is closed under composition and contains all the constant and projection functions. For a fixed polynomial clone $\mathcal{C}$, a sequence $B \in X^\omega$ is $\mathcal{C}$-reducible to $A \in X^\omega$ if there is an algorithm that computes B from A using only effectively selected functions from $\mathcal{C}$. We show that if A is a Kurtz random sequence and $\mathcal{C}_1 \nsubseteq \mathcal{C}_2$ are distinct polynomial clones, then there is a sequence B that is $\mathcal{C}_1$-reducible to A but not $\mathcal{C}_2$-reducible to A. This implies a generalization of a result first proved by Lachlan for the case $|X| = 2$. We also show that the same result holds if Kurtz randomness is replaced by Kolmogorov–Loveland stochasticity.

► PHILIP EHRLICH, *Conway names, the simplicity hierarchy and the surreal number tree*.
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In his monograph ***On Numbers and Games*** [1], J. H. Conway introduced a real-closed field containing the reals and the ordinals as well as a great many less familiar numbers including ω, $\omega/2$, $1/\omega$, $\sqrt{\omega}$, and $\omega - \pi$ to name only a few, where ω is the least infinite ordinal. Indeed, this particular real-closed field, which Conway calls *No*, is so remarkably inclusive that, subject to the proviso that numbers—construed here as members of ordered "number" fields—be individually definable in terms of sets of NBG, it may be said to contain "All Numbers Great and Small." In addition to its inclusive structure as an ordered field *No* has a rich algebraico-binary tree-theoretic structure, or *simplicity hierarchy*, that emerges from the recursive clauses in terms of which it is defined.

Among the striking simplicity-hierarchical features of *No* is that every surreal number can be assigned a canonical "proper name"—called its *Conway name* (or *normal form*)—that is a reflection of its characteristic simplicity-hierarchical properties.

In [2], answers are provided for the following two questions that are motivated by *No*'s structure as an ordered binary tree:

(i) Given the Conway name of a surreal number, what are the Conway names of its two immediate successors?

(ii) Given a chain of surreal numbers of infinite limit length, what is the Conway name of the immediate successor of the chain?

The purpose of this talk is to provide an introduction to [2].

[1] J. H. Conway, ***On numbers and games***, Academic Press, 1976.

[2] P. Ehrlich, *Conway names, the simplicity hierarchy and the surreal number tree*, ***The Journal of Logic and Analysis***, forthcoming.

► SOLOMON FEFERMAN, *Foundations of category theory: What remains to be done*.
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In the first part of [1], I argued against the thesis that category theory is in some way a global autonomous foundation for mathematics because the general notions of collection and operation are both logically and psychologically prior to the basic notions of structure required to explain the basic notions of category theory. The ensuing controversy has only strengthened my view that some background theory S of collections and operations is required to provide proper foundations of category theory, and indeed of working mathematics more generally. In the second part of [1], I pointed to the defects of set-theoretical foundations of category theory as proposed by Grothendieck, Mac Lane and others, and posed three requirements that an improved foundation in the form of a theory S ought to meet: it should (i) allow us to construct the category of all structures of a given kind such as the categories **Grp** of all groups, **Top** of all topological spaces, and **Cat** of all categories, (ii) allow us to construct for any two categories A and B the category of all functors from A to B, and (iii) allow us to establish the existence of the usual basic mathematical structures and carry out the usual set-theoretical operations on them. A minimal additional requirement on any such S is that its consistency should be established relative to currently accepted systems. I shall review what has been achieved on this program thus far and what remains to be done.

[1] Solomon Feferman, *Categorical foundations and foundations of category theory*, ***Foundations of Mathematics and Computability Theory***, (R. Butts and J. Hintikka, editors), Reidel Publishing Company, Dordrecht-Holland, 1977, pp. 149–169.

► STEPHEN FLOOD, *The reverse mathematics of a theorem of Erdős and Galvin*.
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A set is semi-homogeneous for an n-ary coloring f if it is given at most 2^{n-1} colors by f. The existence of semi-homogeneous sets follows trivially from Ramsey's theorem.

In [1], Erdős and Galvin study the size of initial segments of semi-homogeneous sets. A corollary of their main theorem shows that if ϕ is the $(n-1)$-times iterated logarithm, then for any coloring of n-tuples, there is a semi-homogeneous set which infinitely often contains $\phi(m)$ elements less than m. They also show that for ϕ, there is a coloring f such that for any set A, if there are $\phi(m)$ elements of A below m infinitely often, then A must be given at least 2^{n-1} colors by f.

We prove (by an effective combinatorial proof) that, for any exponent n, Erdős and Galvin's main theorem implies standard Ramsey's theorem for that exponent. We also obtain a reversal for all exponents except $n = 2$, by giving an effective combinatorial proof from RT^1 for exponent 1 and a proof from ACA_0 for all other exponents. We are currently working to show that RT^2 implies exponent 2 of their main theorem.

[1] Paul Erdős and Fred Galvin, *Some Ramsey-type theorems*, ***Discrete Mathematics***, vol. 87 (1991), no. 3, pp. 261–269.

► CAMERON FREER, BJØRN KJOS-HANSSEN, AND ANDRÉ NIES, *Effective aspects of Lipschitz functions*.
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Recall that a function $f\colon [0,1] \to \mathbb{R}$ is *Lipschitz* if there is a constant C such that for all x and y we have $|f(x) - f(y)| < C|x - y|$. Such functions are of bounded variation. We study computable Lipschitz functions f. The total variation $V(f, [0,1])$ is a finite left-c.e. real, but it need not be computable. Our first result characterizes the possible variation functions $V(f, [0, x])$ for computable Lipschitz functions f.

Our second result concerns the possible sets of differentiability for computable Lipschitz functions. Every computable Lipschitz function f is the difference of two non-decreasing computable functions; therefore f is differentiable at every computably random real, by a result in [1]. We show that if every computable Lipschitz function is differentiable at a real z, then z is Schnorr random.

[1] Vasco Brattka, Joseph S. Miller, and André Nies, *Randomness and differentiability*, preprint, 2010.

► PETER GERDES, *A ω-REA set forming a minimal pair with* $0'$.
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It is easy to see that no n-REA set can form a (non-trivial) minimal pair with $0'$ and only slightly more difficult to observe that no ω-REA set can form a (non-trivial) minimal pair with $0''$. Shore has asked whether this can be improved to show that no ω-REA set forms a (non-trivial) minimal pair with $0'$. We show that no such improvement is possible by constructing a set C with $0 <_T C \leq_T 0''$ forming a minimal pair with $0'$.

► GEOFFREY HELLMAN, *None of the above; or (better) some of each*.
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We begin by reviewing highlights of the recent debate among Awodey, McLarty, and myself about category and topos theory (CTT) as a foundational framework for mathematics from a modern, structuralist perspective. In hope of further progress, we then set out some key *desiderata* of such a framework, against which we can measure the strengths and weaknesses of set-theoretical, categorical, and other frameworks. For example, the first accommodates pluralism of approaches, e.g., to the mathematics of continua, less readily than the second; whereas the articulation of logical and mathematical primitives of set theory is clearer and more widely agreed upon than for categorical foundations. (It is illustrative that Awodey's and McLarty's responses to my challenge take diametrically opposite stands on the very need for assertory foundational axioms!) Next we relate the challenge I raised regarding articulating structuralism to Feferman's 1977 critique [1]: essentially, I am fleshing that out and bringing related points to bear on the question of autonomy of CTT in articulating what structures are and how they are interrelated. Finally, we emphasize an under-appreciated point of Feferman [1], that set theory should not be seen as the only alternative to CTT, even if the case can be made that certain set-theoretical concepts really are indispensable to human mathematical thought.

[1] Solomon Feferman, *Categorical foundations and foundations of category theory*, ***Foundations of Mathematics and Computability Theory*** (R. Butts and J. Hintikka, editors), Reidel Publishing Company, Dordrecht-Holland, 1977, pp. 149–169.

► HUNTER JOHNSON, *Coherence and uniformly definable types over finite sets*.
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I will discuss an open problem derived from a longstanding question in Computational Learning Theory: Does every dependent formula have a uniform type definition which works over all finite type domains?

The original learning-theoretic version of this conjecture was posed by Warmuth and Littlestone in 1986, and asks whether any concept class with finite Vapnik Chervonenkis dimension admits of an extended compression scheme [1]. The resolution of this question has applications to PAC learning and to the combinatorial complexity of definable families.

In [3], we resolved this question positively for o-minimal theories. New results in [2] improve this to dp-minimal theories.

This talk will give a new criterion for UDTFS, called coherence, which generalizes dp-minimality, and establishes the conjecture in the broadest setting to date.

[1] S. Floyd and F. Warmuth, *Sample compression, learnability, and the Vapnik–Chervonenkis dimension*, ***Journal of Machine Learning***, vol. 21 (3), (1995), pp. 269–304.
[2] V. Guingona, *Uniform definability of types over finite sets*, http://arxiv.org/ abs/1005.4924.
[3] H. R. Johnson and M. C. Laskowski, *Compression schemes, stable definable families, and o-minimal structures*, ***Journal of Combinatorial and Discrete Geometry***, vol. 43 (4), (2009), pp. 914–926.

► MUSHFEQ KHAN, *Shift-complex sequences*.
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The Martin-Löf random sequences can be characterized as those which have high initial segment complexity, in the sense that all initial segments are c-incompressible for some constant c. However, substrings of a random sequence can have very low complexity (any random sequence must have arbitarily long substrings of zeroes). What if we demand that all substrings of a sequence have uniformly high complexity, in the sense that there is a $\delta \in (0, 1)$ such that if σ is any substring, then $K(\sigma) \geq \delta|\sigma| - O(1)$? Durand, Levin and Shen showed that such sequences exist and Rumyantsev showed that Martin-Löf random sequences need not compute them. We will discuss a couple of recent results on the computational properties of these sequences.

This is joint work with Joseph S. Miller.

► YURII KHOMSKII, *MAD families and the projective hierarchy*.
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A maximal almost disjoint (MAD) family is an infinite collection of infinite subsets of ω such that all its members are pairwise almost disjoint, and which is maximal with respect to this property. A typical MAD family is constructed using the axiom of choice, and does not have an explicit definition. In fact, a classical result of A. Mathias [1] shows that there cannot be an analytic MAD family.

In this work we investigate MAD families of complexity $\mathbf{\Pi}^1_1$ and $\mathbf{\Sigma}^1_2$, for instance by asking in which models of set theory such families exist.

This is joint work with Jörg Brendle (Kobe University, Japan).

[1] A. R. D. Mathias, *Happy families*, ***Annals of Mathematical Logic***, vol. 12 (1977), pp. 59–111.

► ELAINE LANDRY, *The genetic versus the axiomatic method: responding to Feferman '77*.
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Feferman [1] argues that category theory cannot stand on its own as a structuralist foundation for mathematics: he claims that, because the notions of operation and collection are both epistemically and logically prior, we require a background theory of operations and collections. Recently [3], I have argued that in rationally reconstructing Hilbert's organizational use of the axiomatic method, we can construct a pure algebraic version of mathematical structuralism. That is, in reply to Shapiro [4], we can be structuralists all the way down; we do not have to appeal to some background theory to guarantee the truth of our axioms. In this paper, I again turn to Hilbert; I borrow his [2] distinction between the genetic method and the axiomatic method to argue that even if the genetic method requires the notions of operation and collection, the axiomatic method does not. Even if the genetic method is in some sense epistemically or logically prior, the axiomatic method stands alone. Thus, if the claim that category theory can act as a structuralist foundation for mathematics arises from the organizational use of the axiomatic method, then it does not depend on the prior notions of operation or collection.

[1] Solomon Feferman, *Categorical foundations and foundations of category theory*, ***Foundations of Mathematics and Computability Theory*** (R. Butts and J. Hintikka, eds), Dordrecht-Holland, Reidel Publishing Company, 1977, pp. 149–169.

[2] David Hilbert, *On the concept of number*, ***From Kant to Hilbert***, (W. Ewald, ed.), vol. 2, Oxford University Press, 1999m pp. 1089–96

[3] Elaine Landry, *How to be a structuralist all the way down*, ***Synthèse***, to appear.

[4] Stewart Shapiro, *Categories, structures, and the Frege–Hilbert controversy: the status of metamathematics*, ***Philosophia Mathematica***, vol. 13, no. 1, 2005, pp. 61–77.

► STEFFEN LEMPP, *On Vaught's Conjecture for Boolean algebras in $L_{\omega_1\omega}$*.
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I will discuss some joint work with Asher Kach toward Vaught's Conjecture for Boolean algebras in the infinitary logic $L_{\omega_1\omega}$.

Vaught's Conjecture for Boolean algebras in first-order logic was established by Iverson [2]; indeed, he showed that each complete first-order extension of the theory of Boolean algebras has either one or continuum many countable models. Camerlo and Gao [1] extended this to show that in the second case, the set of countable models is Borel complete; in the same paper, they also asked whether Vaught's Conjecture for Boolean algebras extends to $L_{\omega_1\omega}$.

Since every countable model has a Scott sentence in $L_{\omega_1\omega}$, Vaught's Conjecture for $L_{\omega_1\omega}$ has to be phrased as each individual sentence in $L_{\omega_1\omega}$ having either at most countably many or else continuum many models.

At the time of writing, we have established Vaught's Conjecture for Boolean algebras in $L_{\omega_1\omega}$ in two of the three cases being determined by whether rank and depth are bounded or unbounded, using Ketonen's classification [3] of countable Boolean algebras (see also Pierce [4]). We can also show that Borel completeness can fail even when there are continuum many models, and we have established a number of surprising results on the possible depths in countable Boolean algebras.

[1] Riccardo Camerlo and Su Gao, *The completeness of the isomorphism relation for countable Boolean algebras*, ***Transactions of the American Mathematical Society***, vol. 353 (2001), no. 2, pp. 491–518.

[2] Paul Iverson, *The number of countable isomorphism types of complete extensions of the theory of Boolean algebras*, ***Colloquium Mathematicum***, vol. 62 (1991), no. 2, pp. 181–187.

[3] Jussi Ketonen, *The structure of countable Boolean algebras*, ***Annals of Mathematics* (2)**, vol. 108 (1978), no. 1, pp. 41–89.

[4] R. S. Pierce, *Countable Boolean algebras*, ***Handbook of Boolean algebras***, vol. 3, North-Holland, Amsterdam, 1989, pp. 775–876.

► BENEDIKT LÖWE, *Multiplication in the hierarchy of norms*.

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A *norm* is a surjective function from the reals onto an ordinal. Given two norms φ and ψ, we can define a Wadge-like game $\mathrm{G}(\varphi, \psi)$: if player I plays x and player II plays y, player II wins if and only if $\varphi(x) \leq \psi(y)$. If player II has a winning strategy in $\mathrm{G}(\varphi, \psi)$, we write $\varphi \leq \psi$. The equivalence classes of the derived equivalence relation form the *hierarchy of norms*.

Under the assumption of the axiom of determinacy (AD), the hierarchy of norms is a wellordering. In [1], Duparc investigated the Borel fragment of the hierarchy; in [2], we investigated the full hierarchy under the assumption of AD, and gave a lower bound of Θ^2 for its length (where $\Theta := \sup\{\alpha;$ there is a surjection from $\mathbb{R}$ onto $\alpha\}$).

In this talk, we investigate further closure properties of the hierarchy of norms, in particular, the existence of a analogue of the multiplication for sets in the Wadge hierarchy, i.e., an operation $\otimes$ such that (for well-chosen φ and ψ) the ordinal rank of $\varphi \otimes \psi$ is at least big as the product of the ordinal ranks of φ and ψ. This closure property allows us to improve the lower bound to Θ^ω.

[1] Jacques Duparc, *The Steel hierarchy of ordinal valued Borel mappings*, ***The Journal of Symbolic Logic***, vol. 68 (2003), pp. 187–234.

[2] Benedikt Löwe, *The length of the full hierarchy of norms*, ***Universitá e Politecnico di Torino, Seminario Matematico***, vol. 63 (2005), pp. 161–168.

► BILLY JOE LUCAS, *Agent indexed logics for reasoning about which truths one ought to know*.
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Consider the question(s): Of all the truths there are, which are the ones I ought to know? Let uOK be an agent indexed modal box (uPK its diamond) with K as minimal option for answers. What additional rules for reasoning about answers to our question(s) should we add? T: $\vdash$uOKp $\supset$ p. T holds for all agents, by the question(s), as does $\vdash$ p $\supset$ uPKp (one has an *alethic* right to know each truth). Absent certainty about one's epistemic obligations, this seems plausible: One ought to know *which* obligations to know truth one *actually* has. If so, add 4: $\vdash$uOKp $\supset$ uOKuOKp. Thus, S4 and only 12 proper positive modalities: uOK, uPK, uOKuPK, uPKuOK, uOKuPKuOK, uPKuOKuPK. Hence, another consideration in favor of 4, one *cannot* have an infinite number of distinct epistemic obligations/rights for arbitrary p, thus avoiding a paradox for any *finite* reasoner.

THEOREM. *For p such that* uPKp *is* S4-*consistent*: *if* uPKp $\vdash_{S4}$ uOKq, *then* $\vdash_{S4}$ uOKq.

No reasonable (consistent) S4 *right* to know can impose a *new* obligation to know, thus additional support for S4. Not all rights to know truth seem of equal importance in this sense:

$$\exists p[(uPKp \cdot uOK(uPKp)], \text{ yet, } \exists q[(uPKq \cdot \sim uOK(uPKq)].$$

If so, S5 is not a viable logic for uOK.

One distinguishes *alethic* concepts of 'OK' from *deontic* obligations to know: OuK, where O is a box for a deontic logic, *vide* [1], and uK is an indexed box for knowledge; common sense requires $\exists$p(OuKp $\cdot \sim$p).

[1] BILLY JOE LUCAS, *The logic of categorical moral reasoning*, ***The Bulletin of Symbolic Logic***, vol. 12 (2006), no. 3, pp. 510–511.

► JEAN-PIERRE MARQUIS, *Category theory as a theory of abstract sets*.
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Following [1], we argue that higher-dimensional categories constitute a universe for pure mathematics that can be used as a background of a foundational approach in the same way that the cumulative hierarchy is taken as a clear and distinct interpretation of the axioms of ZF. Furthermore, one can interpret the underlying objects of this universe as abstract sets, in a specific sense of "being abstract". In this way, Feferman's argument is not only acknowledged but at the same time circumvented. In Makkai's approach, the mathematical structures of this universe are then abstracted from the given data since their criterion of identity is extracted from the data and not given a priori. This in itself does not provide an axiomatic framework and, thus, a foundation in the standard meaning of the term. We will briefly explore what these foundations could be. We can already expect that such an axiomatization would stipulate the existence of some objects, thus answering another common objection against a categorical foundation for mathematics.

[1] M. MAKKAI, *Towards a categorical foundations of mathematics*, ***Logic Colloquium '95 (Haifa)***, (Johann A. Makowsky and Elena V. Ravve, editors), vol. 11, Springer, 1998, pp. 153–190.

► COLIN MCLARTY, *Foundations as truths which organize mathematics*.
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We speak of foundations of mathematics in many senses, but this talk focuses on a foundation as a body of truths which organizes the practice of mathematics. These are not merely truths which could in principle organize practice but which do in fact. Rather than make foundations prior to practice this agrees with Russell that we rightly begin by moving "backward to the logical foundations of the things that we are inclined to take for granted in mathematics" [2, p. 2]. Such a foundation may not live up to philosophical strictures. We may pursue philosophical projects such as Chihara's nominalist "new system of mathematics" but we say as Chihara does that is not a foundation for current mathematics [1, p. vii]. The study of foundations in this sense involves notions unfamiliar in the rest of practice: e.g., forcing in set theory reveals a huge amount but no one takes forcing per se as a foundation. All the varied senses of foundations of mathematics get their interest from the fact as old as Euclid that large amounts of mathematical knowledge actually are organized as deductive consequences of axioms. While nothing here forbids formalist investigations, this conception of foundations is committed to saying mathematics is true.

[1] CHARLES CHIHARA, ***A structural account of mathematics***, Oxford University Press, 2004.

[2] BERTRAND RUSSELL, ***Introduction to mathematical philosophy***, George Allen & Unwin, 1919.

► RUSSELL MILLER, *Boolean subalgebras and computable copies*.
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It is known from the work referenced below that the spectrum of a Boolean algebra cannot contain a low$_4$ degree unless it also contains the degree $\mathbf{0}$; it remains open whether the same holds for low$_5$ degrees. We address the question differently, by considering Boolean subalgebras of the computable atomless Boolean algebra $\mathcal{B}$. For such subalgebras $\mathcal{A}$, we show that it is possible for the spectrum of the unary relation $\mathcal{A}$ on $\mathcal{B}$ to contain a low$_5$ degree without containing $\mathbf{0}$. That is, $\mathcal{A}$ is a low$_5$ Boolean subalgebra of $\mathcal{B}$, but $\mathcal{B}$ has no computable Boolean subalgebra $\mathcal{C}$ with $(\mathcal{B}, \mathcal{C}) \cong (\mathcal{B}, \mathcal{A})$.

[1] R. G. DOWNEY and C. G. JOCKUSCH, JR., *Every low Boolean algebra is isomorphic to a recursive one*, ***Proceedings of the American Mathematical Society***, vol. 122 (1994), pp. 871–880.

[2] J. F. KNIGHT and M. STOB, *Computable Boolean algebras*, ***The Journal of Symbolic Logic***, vol. 65 (2000), pp. 1605-1623.

[3] J. J. THURBER, *Every low$_2$ Boolean algebra has a recursive copy*, ***Proceedings of the American Mathematical Society***, vol. 123 (1995), pp. 3859–3866.

► DON MONK, *Maximal free sequences in* $\mathcal{P}(\omega)/\mathrm{fin}$.
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A *free sequence* in a Boolean algebra A is a sequence $\langle a_\xi : \xi < \alpha\rangle$ of elements of A (α an ordinal), such that if F and G are finite subsets of α such that $\forall \xi \in F \forall \eta \in G[\xi < \eta]$, then $\prod_{\xi \in F} a_\xi \cdot \prod_{\eta \in G} -a_\eta \neq 0$. The sequence is *maximal* iff there is no $b \in A$ such that $\langle a_\xi : \xi < \alpha\rangle \frown \langle b\rangle$ is a free sequence. $f(A)$ is the minimum of $|\alpha|$ such that A has a maximal *free sequence* indexed by an infinite ordinal α.

THEOREM. *It is relatively consistent with ZFC that the continuum is large and* $f(\mathcal{P}(\omega)/\mathrm{fin}) = \omega_1$.

► RAJA NATARAJAN, *The impact of computers on the notion of proof*.
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Recent advances in the field of 'Interactive Proof Checking' with the associated development of powerful tools such as 'Proof Assistants' have given rise to an interesting consequence—viz. the practical feasibility of importing techniques developed in the computer science community and redeploying them to improve the main activity of the working mathematician, namely the process of proof development. At the core of such redeployed techniques lie the notions of formal systems, formal reasoning, and formal proofs. However the process of formalizing mathematics is a highly non-trivial task, and gives rise to a number of challenging and interesting issues which need to be addressed in order to make the discipline of machine-supported mathematics more prevalent in the future [1, 2].

[1] A. Asperti, H. Geuvers, and R. Natarajan, *Social processes, program verification and all that*, ***Mathematical Structures in Computer Science***, vol. 19 (2009), no. 5, pp. 877–896.

[2] R. Natarajan, *Special issue on interactive theorem proving and verification*, ***Sadhana***, vol. 34 (2009), no. 1, pp. 1–220.

► ERMEK NURKHAIDAROV AND EREZ SHOCHAT, *Automorphisms of short saturated models of* PA.
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A model of PA is short if the definable closure of some element from the model is cofinal in the model. Short saturated models of PA are short elementary initial segments of saturated models of PA. In this talk, we outline a proof which shows that any automorphism of a short saturated model of PA, can be extended to an automorphism of the saturated elementary end extension of the model. This result allows us to apply results concerning automorphisms of saturated models of PA to automorphisms of short saturated models of PA.

► CHRISTOPHER PORTER, *Randomness and truth-table reducibility*.
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According to Demuth's Theorem (see [1]), if A is a Martin-Löf random real and Φ is a truth-table functional, then if $\Phi(A)$ is not computable, there is a Martin-Löf random real B such that $\Phi(A) \equiv_T B$. In this talk, I will discuss my recent work on the problem of determining whether Demuth's Theorem also holds for different definitions of algorithmic randomness, namely Schnorr randomness, computable randomness, and Kurtz randomness.

[1] Osvald Demuth, *Remarks on the structure of tt-degrees based on constructive measure theory*, ***Commentationes Mathematicae Universitatis Carolinae***, vol. 29 (1988), no. 2, pp. 233–247.

► OMAR LEÓN SÁNCHEZ, *Geometric axioms for the theory* $\mathrm{DCF}_{0,m+1}$.
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In [1] a geometric characterization of DCF_0 is given in terms of ACF_0 and points on subvarieties of prolongations of affine algebraic varieties. In this talk we present a generalisation of this characterization to $\mathrm{DCF}_{0,m+1}$ in terms of $\mathrm{DCF}_{0,m}$ using a relative notion of prolongation.

Suppose $\Delta = \{\delta_1, \dots, \delta_m\}$ are commuting derivations on a field K of characteristic zero, and $D\colon K \to K$ is an additional derivation that commutes with Δ. If V is a Δ-closed set, then the map $\overline{x} \mapsto (\overline{x}, D\overline{x})$ gives a section of a certain torsor of the Δ-tangent bundle that we call the D/Δ-prolongation of V. We show that $(K, \Delta \cup \{D\}) \models \mathrm{DCF}_{0,m+1}$ if and only if for any set of derivations $\Delta' \cup \{D'\}$, which form a basis for the space of derivations spanned by $\Delta \cup \{D\}$ over the constants, we have that $(K, \Delta') \models \mathrm{DCF}_{0,m}$ and for each Δ'-closed set V and each Δ'-closed subset W of the D'/Δ'-prolongation of V that projects onto V, there exists $\overline{a} \in V(K)$ such that $(\overline{a}, D'\overline{a}) \in W$. We also prove that this condition is axiomatizable.

[1] D. Pierce and A. Pillay, *A note on the axioms for differentially closed fields of characteristic zero*, ***Journal of Algebra***, vol. 204 (1998), no. 1, pp. 108–115.

► MICHAEL A. TYCHONIEVICH, *Metric properties of sets definable in the expansion of the real field by a logarithmic spiral*.
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We discuss some metric properties of sets definable in certain expansions of the real field, including expansions of the real field by logarithmic spirals. For example, in these structures each bounded definable curve has finite length if and only if the dimension of its frontier is 0 (preliminary report).

► DAN E. WILLARD, *A new variant of a self-justifying logic pertaining to* $I\Sigma_0$.
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Let α denote an axiom system, and d a deduction method. The ordered pair (α, d) will be called **Self Justifying** when:

1. one of α's formal theorems will state that the deduction method d, applied to the axiom system α, will produce a consistent set of theorems,
2. and the axiom system α is in fact consistent.

Examples of self-justifying systems can be found in [1, 3, 4, 6]. All such paradigms contain built-in weaknesses to evade the Incompleteness Theorem. This abstract will explain how to extend [6]'s results so that self-justification is feasible under the resolution method of automated deduction for some extensions of $I\Sigma_0$.

Let $\alpha \supseteq \beta$ denote that α can prove all β's theorems, and $\alpha \geq \beta$ denote α physically contains all β's axiom sentences. We showed in [5, 6] that there existed three equivalent encodings for $I\Sigma_0$, called Ax-1, Ax-2 and Ax-3, such that no consistent $\alpha \geq$ Ax-i can verify its own semantic tableaux consistency. *But yet* [6] showed that Ax-3's *unorthodox encoding* for $I\Sigma_0$ had the property that there existed a consistent $\alpha \geq$ Ax-3 where α could prove a theorem affirming its own Herbrand consistency.

We previously thought this result was of isolated theoretical interest because [2, 5, 6] proved it did not generalize under semantic tableaux for Ax-1, Ax-2 or Ax-3. However, it turns out [6]'s main theorem applies to most implementations of resolution because they are similar to Herbrand deduction by having the following two features:

1. Implementations of resolution typically start by skolemizing the axiom system α and skolemizing the sentence $\neg\psi$ when seeking to prove ψ from α.
2. They then prove ψ by showing this skolemized schema leads to a contradiction.

Our Resolution-based self-justification methodology is of limited generality because [6] showed it does not generalize to semantic tableaux, and because it requires use of Ax-3's logically valid [6] *but highly unusual* encoding for $I\Sigma_0$. Nevertheless, the generalization of [6]'s methods to resolution-based theorem provers is of interest.

[1] G. Kreisel and G. Takeuti, *Formally self-referential propositions for cut-free classical analysis*, ***Dissertationes Mathematicae***, vol. 118 (1974), pp. 1–55

[2] D. E. Willard, *How to extend the semantic tableaux and cut-free versions of the second incompleteness theorem almost to Robinson's arithmetic*, ***The Journal of Symbolic Logic***, vol. 67 (2002), pp. 465–496.

[3] ———, *An exploration of the partial respects in which an axiom system recognizing solely addition as a total function can verify its own consistency*, ***The Journal of Symbolic Logic***, vol. 70 (2005), pp. 1171–1209.

[4] ———, *On the available partial respects in which an axiomatization for real valued arithmetic can recognize its consistency*, ***The Journal of Symbolic Logic***, vol. 71 (2006), pp. 1189–1199.

[5] ———, *Passive induction and a solution to a Paris–Wilkie open question*, ***Annals of Pure and Applied Logic***, vol. 146 (2007), pp. 124–149.

[6] ———, *Some specially formulated axiomizations for $I\Sigma_0$ manage to evade the Herbrandized version of the second incompleteness theorem*, ***Information and Computation***, vol. 207 (2009), pp. 1078–1093.

► XUNWEI ZHOU, *Automated hypothetical inference system*.
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Take the inference "all integers are rationals, all rationals are real numbers, therefore, all integers are real numbers" as an example. The dominant hypothetical inference is:

$$\frac{\begin{array}{c}(P \to Q) \wedge (Q \to R) \to (P \to R) \\ (\text{Int} \to \text{Rat}) \wedge (\text{Rat} \to \text{Real})\end{array}}{\text{Int} \to \text{Real}}$$

It is made by computers. In recessive hypothetical inference, there is an inference rule:

$$\frac{\begin{array}{c}Q \to R \\ P \to Q\end{array}}{P \to R}$$

and Int $\to$ Real is inferred from Rat $\to$ Real and Int $\to$ Rat directly. It is made by human beings.

Rule-based system is divided into inference rule system and hypothetical inference system. Inference rule system is based on recessive hypothetical inference. In inference rule system, there are many inference rules. A human being can choose them easily, but a computer cannot. So, it is suitable for human beings. Hypothetical inference system is based on dominant hypothetical inference. It is suitable for computer. In hypothetical inference systems, hypothetical inference is the only inference rule, the other rules are treated as axioms. Since there is only one inference rule, each time the system makes inference, the system employs it. Hypothetical inference system is further divided into interactive hypothetical inference system and automated hypothetical inference system. If the axioms are searched top-down, from left to right, depth-first plus backtracking, like the way Prolog does, then it is automated hypothetical inference system, otherwise, it is interactive hypothetical inference system.

Abstracts of talks presented by title

► JOHN CORCORAN, *Aristotle's syllogistic premises*.
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Prior Analytics is often not clear whether a premise—either *per se* or *of a syllogism*—is a sentence, proposition, statement, fact, judgment, belief, or thought. To discuss the question we use numerically-indexed alternative-constituent sentences (This BULLETIN, vol. 16 (2010), p. 460).

Speakers employ the following four English *sentences* to make *statements*.

(Every 1* Some 24* No 3) square is (123 *not 4) a rectangle.

Using 1 speakers "predicate" *rectangle* of *square* universally and affirmatively: they "affirm" *rectangle* of *square* universally. With 2 speakers predicate partially and affirmatively: they affirm *rectangle* of *square* partially. With 3 speakers predicate universally and negatively: they deny *rectangle* of *square* universally. With 4 speakers predicate partially and negatively: they deny *rectangle* of *square* partially.

Speakers using (1*2* 3* 4) *express* the *proposition* that (every 1* some 24* no 3) square is (123 *not 4) a rectangle, which is true iff it is a *fact* that *rectangle* ("belongs" 123 * does not "belong" 4) to (every 14* some 2* no 3) *square*. The *sentence* (1* 3) *expresses* the proposition that (every 1* no 3) square is a rectangle, which is true iff it is a fact that *square* (is included in 1 * is excluded from 3) *rectangle* as (in 1* from 3) a whole. Alternatively, (every 1* no 3) square is a rectangle is true iff it is a fact that *rectangle* (contains 1 * excludes 3) *square*.

Beside sentences, propositions, statements, and facts—for example that every square is a rectangle—we distinguish a speaker's *judgment* (decision, determination), a speaker's *belief* established by the judgment, and the speaker's *thought* (idea)—which must have preceded the judgment—and also the speaker's subsequent thoughts that every square is a rectangle.

► JOHN CORCORAN AND THOMAS GEZELLA, *Aristotle's conception of syllogism*.
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Early in *Prior Analytics*, after stating clearly that his work concerned demonstrations, or demonstrative proofs, Aristotle presented an elliptical and vague "definition" of *syllogism* (*sullogismos*). Despite obvious deficiencies, he never explained more fully what he meant by *sullogismos*. And he never revealed awareness that readers might wonder whether his understanding of the word had evolved as his investigations developed. He never allowed doubt, however, that everything deserving of being called a demonstration—in geometry, arithmetic, astronomy, or other sciences—also warranted being called a syllogism (e.g., Book A, Chapter 4). Nor was there any doubt that demonstrations are not statements.

In the "definition" (24b18), Aristotle says that a syllogism is a *logos* (discourse) *in which* one constituent follows of necessity from others. He does not say that a syllogism is a discourse *stating that* one constituent follows of necessity from others. Aristotle never attributes to syllogisms the truth or falsity he reserves for statements or propositions.

Any work that represents Aristotelian syllogisms as statements distorts Aristotle's meaning regardless of the nature of the statements. It is as wrong to represent a syllogism as saying that its conclusion follows logically from its premises—or that its conclusion is deducible or derivable from them—as it is to represent a syllogism as a conditional or a tacitly universalized conditional having the conclusion as consequent and its premises conjoined as antecedent. Accordingly, it is a category mistake to speak of *demonstrations or proofs of syllogisms*. This paper surveys the literature for passages that misconstrue "discourse *in which*" as "discourse *stating that*"—even in authors who otherwise exhibit knowledge of Aristotle's intentions. Authors treated include David Ross, Jan Łukasiewicz, Timothy Smiley, Robin Smith, George Boger, and Gisela Striker.

► JOHN CORCORAN AND JEAN-LOUIS HUDRY, *Aristotle's syllogistic terms*.
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Prior Analytics is sometimes not clear whether a term—either *per se* or *in a premise*—is a linguistic expression, abstract concept, mental thought, mental image, sort (natural kind), class, set, property, quality, or something else. To treat the issue, we use numerically-indexed alternative-constituent sentences (This BULLETIN, vol. 16 (2010), p. 460).

(Every 1* Some 24* No 3) square is (123 *not 4) a rectangle.

In these four English *sentences* the *common* nouns *square* and *rectangle* are, represent, or correspond to the terms: *square* the subject and *rectangle* the predicate—in the original Aristotelian senses. In modern grammatical senses, the "grammatical subject" has two words (as *every square*) and the "grammatical predicate" has up to four words (as *is not a rectangle*).

In contrast, the following twelve sentences use *square* and *rectangle* as *proper names*—but of what?

Rectangle is predicated of *square* (universally 57*partially 68)

and (affirmatively 56* negatively 78).

Rectangle is (affirmed $ab*$ denied cd) of *square* (universally ac^*partially bd).

Square (is contained in ef^* excludes gh) *rectangle* (as a whole eg^* in part fh).

We discuss Aristotle's *homogeneity thesis* that there are no pragmatic, semantic, epistemic, or ontic differences between subject terms *per se* and predicate terms *per se*: everything that can be a subject term in a premise serves as a predicate term in a premise and conversely. Can the first four sentences express the same premises expressed by the second four, the third four, and the fourth four?

Contrasting with the above "homogeneity", below although *square* is a common *noun*, *rectangle* is proper *name*.

Rectangle (belongs efg * does not belong h) to (every eh^* some f^* no g) *square*.

► JOHN CORCORAN AND MATT LAVINE, *Notating Tarski's truth-definition paper*.
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Using quotes, '2 > 1' is an abstract, immaterial sentence *form*—a *type* in Peirce's sense—not a concrete, material *token*, [1], pp. 155–166. The printed sentence token you now read contains one concrete token of '2 > 1'. Tarski used no contrasting notations for other important related concepts. We supply some notational omissions.

Using strokes, |2 > 1| is the sentence token between the two stroke tokens in this sentence token you now see. Tarski called tokens "concrete", "inscriptions", "material objects": [1], pp. 156–174.

'2 > 1' is the type of |2 > 1|.

|2 > 1| is one of the tokens of '2 > 1'.

$$|2 > 1| \neq |2 > 1|$$

The finite and growing class of tokens of '2 > 1'—the class of inscriptions "equiform" with the token |2 > 1|—is [|2 > 1|], using the standard bracket notation for equivalence classes.

$$[|2 > 1|] = \{x : x \text{ is equiform with } |2 > 1|\}$$

$$|2 > 1| \in [|2 > 1|]$$

Tarski also uses common nouns, "general names", that "denote" members of equivalence classes of tokens: [1], p. 157. Using slashes, '/2 > 1/' is a common noun whose extension is [|2 > 1|].

|2 > 1| is one /2 > 1/.

Every /2 > 1/ is a token of '2 > 1'.

For every /2 > 1/ x, for every /∼1 > 2/ y, x is shorter than y.

Every /2 > 1/ is a member of [|2 > 1|], and conversely.

No /2 > 1/ is a /1 > 2/.

Only tokens of '|2 > 1|' denote; thus '|2 > 1|' is an indexical like 'now' but unlike '2 > 1' and '/2 > 1/'. '|2 > 1|' has referential force only in statements.

According to Tarski's explicit words in [1], p. 157, the following holds.

Every /2 > 1/ is a true sentence if and only if 2 > 1.

[1] ALFRED TARSKI, *The concept of truth in formalized languages*, ***Logic, semantics, metamathematics, papers from 1923 to 1938***, (John Corcoran, editor), Hackett, Indianapolis, 1983, pp. 152–278.

► JOHN CORCORAN AND JOSÉ MIGUEL SAGÜILLO, *What makes "natural deduction" natural*?

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This lecture concerns distinctions between *artificial* and natural *deduction*. In one sense, *deduction* is a process by which a person determines that given conclusions follow from given premise sets. In contrast, *a deduction* is an application of this process or a discourse that describes an actual or potential application. A *deduction system* is a collection of deductions or—not equivalently—a collection of rules that "generate" deductions (as generative grammars "generate" sentences) and which thereby may be regarded as a "description" of a process.

A *deduction system* is *natural* for given persons to the extent it faithfully models the process they use to gain knowledge that conclusions follow from premises. A *deduction system* is *artificial* or *unnatural* for given persons if it omits, restricts, or exaggerates operations they use or if it includes operations alien to those they use.

Aristotle's system is natural in ways that Boole's and Jeffrey's systems are unnatural. Aristotle's system required that the conclusion be presented as a goal before premises could be used, thereby being faithful to the fact that for many persons deduction is a goal-directed activity—not merely application of rules to premises. Moreover, Aristotle included both direct and indirect deductions whereas Boole omits indirect deductions and Jeffrey omits direct deductions.

Further, Jaskowski's system is natural in ways that Aristotle's and Frege's systems are unnatural. Jaskowski's system allows both empty and non-empty premise sets. But Aristotle's system omits empty premise sets whereas Frege's omits non-empty premise sets. Jaskowski's system has simple natural rules for connectives and quantifiers. But Aristotle's system omits them altogether and Frege's rules are artificially complicated.

► JOHN GRANT, *Information structures III: disjunctive models*.

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In the early 1970s I started a project to develop model theory for information structures without finiteness constraints. I considered two issues: incompleteness in [1] and inconsistency in [2]. In consequent years I wrote papers on these issues concerning databases and knowledgebases, such as [3] and [4].

This work continues the investigation of model-theoretic ideas for structures and concepts significant for information processing. Disjunctive information is considered here. For example, the only information available about an object's color may be that it is blue or green. In logical form, a or b is true but neither a nor b is true. Incidentally, a similar situation occurs for indeterminate systems in quantum mechanics; however, this is not a proposal for quantum logic.

The first formalization is in propositional logic with conjunction, negation, and a special kind of disjunction. There is a complete axiomatization. This is extended to predicate logic where the formalization uses a weak second-order logic. Compactness does not hold. Like incomplete and inconsistent models earlier, disjunctive models generalize classical models, but in a different way.

I plan to study further properties of disjunctive structures and other connections between model theory and information structures. Constructive helpful suggestions are welcome.

[1] John Grant, *Incomplete models*, ***Notre Dame Journal of Formal Logic***, vol. 15 (1974), pp. 601–607.

[2] ———, *Classifications for inconsistent theories*, ***Notre Dame Journal of Formal Logic***, vol. 19 (1978), pp. 435–444.

[3] ———, *Incomplete information in a relational database*, ***Fundamenta Informaticae***, vol. 3 (1980), pp. 363–378.

[4] John Grant and Anthony Hunter, *Analysing inconsistent first-order knowledgebases*, ***Artificial Intelligence***, vol. 172 (2008), pp. 1064–1093.

► NOA LAVI, *Positivstellensätze in real closed valued fields*.
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The purpose of this talk is to give a generalization of Hilbert's seventeenth problem in real closed valued fields. That is, to give an algebraic characterization, for a definable set, of the set of polynomials which get only non-negative values on it. We give a general characterization of the positive semi-definite polynomials for any definable set with a Ganzstellensatz, and we also give a representation of those polynomials in the sense of Hilbert 17th problem (that is, in terms of sums of squares) for definable sets from some kinds.

[1] E. Artin, *Über die Zerlegung definiter Funktionen in Quadrate*, ***Abhandlungen aus dem Mathematischen Seminar der Universität Hamburg***, vol. 5 (1927), pp. 100–115.

[2] G. Cherlin and M. Dickmann, *Real closed rings. II. Model theory*, ***Annals of Pure and Applied Logic***, vol. 25 (1983), no. 3, pp. 213–231.

[3] M. Dickmann, *On polynomials over real closed rings*, ***Model theory of Algebra and Arithmetic, Proceeding Karpacz 1979***, (L. Pacholski, J. Wiezejewski and A. Wilkie, editors), Springer Lecture Notes in Mathematics 843, 1980, pp. 117–135.

[4] I. Efrat, *Valuations, orderings, and Milnor K-theory*, ***Mathematical Surveys and Monographs***, vol. 124, American Mathematical Society, Providence, 2006.

[5] D. Haskell and Y. Yaffe, *Ganzstellensatz in theories of valued fields*, ***Journal of Mathematical Logic***, (2008), pp. 1–23.

[6] S. Kochen, *Integer valued rational functions over the p-adic numbers: A p-adic analogue of the theory of real fields*, ***Number Theory*** (***Proceedings of the Symposium on Pure Mathematics, vol. XII, Houston, Texas, 1969***), American Mathematical Society, Providence, R.I., pp. 57–73, 1969.

[7] A. Robinson, *On ordered field and definite functions*, ***Mathematische Annalen***, vol. 130 (1956), pp. 257–271.

► PAOLO LIPPARINI, *Ordinal compactness*.
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We extend to ordinal numbers the more usual compactness notion defined in terms of cardinal numbers.

Definition 1. Suppose that X is a nonempty set and that τ is a nonempty family of subsets of X. If α and β are nonzero ordinal numbers, we say that (X, τ) is $[\beta, \alpha]$-*compact* if and only if the following holds.

Whenever $(O_\delta)_{\delta \in \alpha}$ is a sequence of members of τ such that $\bigcup_{\delta \in \alpha} O_\delta = X$, then there is $H \subseteq \alpha$ with order type $< \beta$ and such that $\bigcup_{\delta \in H} O_\delta = X$.

When α and β are both cardinals, X is a topological space, and τ is the topology on X, we get back the classical cardinal compactness notion. See [1] for references.

We show that ordinal compactness is a much more varied notion than cardinal compactness. We prove a great deal of results of the form "Every $[\beta, \alpha]$-compact space is $[\beta', \alpha']$-compact", for various ordinals β, α, β' and α'. Usually, we are able to furnish counterexamples showing that such results are the best possible ones.

[1] J. E. VAUGHAN, *Some properties related to* $[a, b]$*-compactness*, ***Fundamenta Mathematicae***, vol. 87 (1975), pp. 251-260.

▶ D. VENKATESAN, *On finding a method to measure the information content of algorithmic procedures*.

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Due to rapid developments in the field computer hardware, we see increased computational power and hence realization of complex algorithms on them. There is a need to answer several basic questions on the nature and property of the algorithms that goes into making these software (for various reasons such as testing). Some of these questions (and answers) are well known such as (i) is there an algorithm that will take as input a description of a formal language and a mathematical statement in the language and produce as output either "True" or "False" according to whether the statement is true or false [2, 3] (which is answered negatively and reinforced by results like works of Yuri Matiyasevich on non-existence of an algorithm to decide whether general Diophantine equations have a solution). (ii) is there an algorithm that will take as input two algorithms and decide whether they are equivalent? (which is answered negatively by [1]). However these results motivate a deeper study of notion of "algorithms" itself and their mathematical models. Moschovakis [5] [7] discusses reasons and ways to obtain such abstract models (of algorithms) and motivate to obtain their properties such as their inherent "information content" (or complexity a.k.a. number of operations need to be performed). These results motivate the question (iii) is there a mechanism by which we can measure the information content of (or number of inherent operations performed by) an algorithmic procedure? This may help us (or provide a mechanism) to answer questions like ordering [4] of the algorithms on a hierarchy based on their inherent information content through a measure. We hope to map algorithms based on this measure on onto "arithmetic hierarchy" (or hyperarithmetic hierarchy) so that algorithms (based on their inherent information content) can be naturally ordered. Apart from other usages like elementary ordering, we believe that this device shall help us to answer about open questions like if for a "NP" complete algorithm is there a possible mapping on "P" complete algorithm? (from a higher level hierarchy to lower level hierarchy). We argue there can be no such mapping.

[1] ANDREAS BLASS, NACHUM DERSHOWITZ, AND YURI GUREVICH, *When are two algorithms the same?*, ***The Bulletin of Symbolic Logic***, vol. 15 (2009), no. 2, pp. 145–168.

[2] ALONZO CHURCH, *An unsolvable problem of elementary number theory*, ***American Journal of Mathematics***, vol. 58 (1936), pp. 345–363.

[3] ———, *A note on the Entscheidungsproblem*, ***The Journal of Symbolic Logic***, vol. 1 (1936), pp. 40–41.

[4] JOHN E. DONER, ANDRZEJ MOSTOWSKI, AND ALFRED TARSKI, *The elementary theory of well ordering—A metamathematical study*, ***Logic colloquim '77***, (A. Mackintyre, et al., editors), North Holland, 1978, pp. 1–55.

[5] YIANNIS N. MOSCHOVAKIS, *A mathematical modeling of pure, recursive algorithms*, (A. R. Meyer and M. A. Taitslin, editors), ***Logic at Botik '89***, Lecture Notes in Computer Science, no. 363, Springer-Verlag, Berlin, 1989, pp. 208–229.

[6] ———, ***On founding the theory of algorithms***, (H. G. Dales and G. Oliveri, editors), Truth in Mathematics, Clarendon Press, Oxford, 1998, pp. 71–104.

[7] ———, *What is an algorithm?*, ***Mathematics Unlimited—2001 and beyond***, (B. Engquist and W. Schmid, editors), Springer, 2001, pp. 919–936, (Part II).