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REVIEWS

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References such as 49***5*** or 280***1*** are to entries so numbered in *A bibliography of symbolic logic* (the JOURNAL, vol. 1, pp. 121–218).

W. D. HART. ***The evolution of logic.*** Cambridge University Press, Cambridge, 2010, xi + 299 pp.

This book is the fifth in a series called "The Evolution of Modern Philosophy". In the introduction, it says "the series will constitute a library of modern conceptions of philosophy and will reveal how philosophy does not in fact comprise a set of timeless questions but has rather been shaped by broader intellectual and scientific developments to produce particular fields of inquiry addressing particular issues." Unfortunately, this book does not do as much as this introduction suggests to address the relevance of mathematical logic for the development of 20th century philosophy. What it does do instead is provide an introduction to four important results of mathematical logic that go beyond what most philosophers study, and develop the tools needed to prove each of these results. As a result, the book will probably be of most interest to people like me, who already have some familiarity and interest in mathematical logic, but aren't already familiar with all four of the results that are presented here. For those (primarily mathematicians) that are familiar with the results, there are a few items of historical interest in the discussion (though not as many as the title would suggest), but for those (primarily philosophers) who don't already have an interest in mathematical logic, the book doesn't do as much to motivate the results as it could.

The book largely consists of two parts. The first half, consisting of the first five chapters, outlines the development of mathematical logic with some standard historical context, from the work of Cantor and Frege to Gödel's theorems. The next four chapters provide the

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background and proofs of the "four monuments" this book is dedicated to—Gödel's proof of the consistency of the continuum hypothesis, Cohen's proof of the consistency of its negation, Friedberg and Muchnik's proof of the existence of intermediate Turing degrees, and Morley's proof that categoricity in one uncountable cardinality entails categoricity in all of them. The last chapter provides a sort of summary, and an attempt to motivate the philosophical interest of these four results, and more importantly, the methods that lead to them.

It is hard to tell what level of familiarity with mathematical logic is assumed in the reader of this book. The proofs of the "four monuments" become quite intricate, and will thus be hard to follow for readers that are not mathematically sophisticated. But all relevant background is included, down to the level of set notation, and the definition of an ordered pair, so that a devoted student could theoretically learn all the material from this book.

However, the writing style will be quite intimidating for those that aren't already familiar with much of the material. This is especially true because there are no sections within chapters, no headings indicating new topics, and in general no easy way to flip back a page or two to find out what some letter stands for in the current or previous proof. In textbooks this is often less of a problem, because tools are introduced in order of complexity, with well-delineated sections. But in this book, techniques are introduced as they are needed—Tarski's theory of truth is introduced as part of a digression in discussing the analytic/synthetic distinction in chapter 2, and the notions of topology are introduced in the middle of some extended discussion of model theory in chapter 9. When Hart begins a proof or a discussion of some philosophical concept, the reader doesn't know how long it will take, and how many other topics and discussions will be introduced on the way.

In the first half of the book, chapters 1 and 5 are dedicated primarily to mathematical technicalities (Cantorian set theory, and Gödel's theorems, respectively), while chapters 2, 3, and 4 are devoted more to the historical and philosophical issues that accompany them. Chapter 2 follows Kant's notions of analyticity and a priority, and shows how they developed in the "low-water mark in the history of logic" (p. 35), followed by Frege's attempt to redevelop them in terms of some notion of the dependence of some truths on others. Chapter 3 discusses the debate between Russell and Poincaré about the vicious-circle principle, the development of stratified type theory, and the contrast with Quine's "New Foundations". Chapter 4 discusses "Orayen's paradox"—the fact that Tarski's results seem to show that truth and quantification only make sense over a set-sized domain, and yet we seem to be able to assert some things with full generality. I think that these chapters could have used more contact with some contemporary literature of the past two decades (for instance, recent work by metaphysicians on notions of dependence and grounding, the use of various new non-classical logics for a hierarchy-free theory of truth, and the anglophone philosophical discussion of absolute generality—there is some interesting discussion of the Spanish language literature on this topic, and it would be interesting for the two literatures to engage one another) but for mathematicians, they will still be an interesting introduction to some philosophical topics that are connected to the mathematics.

In chapters 6 through 9 on the other hand, there is very little attempt to connect the mathematics to philosophical topics. There is some suggestion that the way the mathematical techniques are used leads us to platonist intuitions, but no strong argument for this view. In the chapter on recursion theory and Turing degrees, there is a discussion of the Kantian study of reason, which we must understand in terms of deductive reasoning, which should therefore lead us to consider the complexity of various deductive theories. But it is never clear how the study of Turing degrees (which is definitely very interesting) will shed light on any of the interesting philosophical questions.

The mathematical exposition of these results is not original (the two results in set theory follow Hilary Putnam's lecture notes from 1968, while the discussions of recursion theory and model theory follow the classic textbooks by Rogers and Marker, respectively). So the main advantage of this book is to have these four results in one place, with all of the facts that they depend on included as well. This book would be much more useful as a reference if these chapters were structured more like textbooks, but even as they stand they still suffice to learn the material. Although I was already familiar with the set-theoretic results, I found the method used to be slightly less readable than either the expositions of forcing used in the books by Kunen and Jech, or the method of Boolean-valued models introduced by Dana Scott and taught in the book by Bell. But the results from recursion theory and model theory were easier to follow, despite the fact that I was less familiar with them.

Overall, I think that the mathematics would have been more helpful if it had been more clearly organized, and the philosophy would have been more useful if it had more up-to-date references rather than primarily citing the classic discussions of Quine, Putnam, Grice, and their contemporaries. But I found that the book provided a useful way for me to learn central results of areas of mathematical logic that I wasn't as familiar with, and it provided me with a few new historical insights (for instance, according to a footnote on p. 257, Ramsey initially developed Ramsey theory in an attempt to find an algorithm for deciding validity in first-order logic—it's interesting to see how such a fruitful mathematical theory came from a doomed project of this sort).

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WILLIAM J. MITCHELL. *$I[\omega_2]$ can be the nonstationary ideal on* $\mathsf{Cof}(\omega_1)$. ***Transactions of the American Mathematical Society***, vol. 361 (2009), no. 2, pp. 561–601.

This influential paper brings an important breakthrough on the long-standing open question if Shelah's approachability ideal $I[\omega_2]$ can be trivial on ω_1-cofinal ordinals, giving an affirmative answer. Starting from a model with a cardinal κ that is κ^+-Mahlo, the author introduces a forcing notion that is proper, turns κ into ω_2 and arranges that no stationary subset of $\omega_2 \cap \mathsf{cof}(\omega_1)$ is in $I[\omega_2]$. The statement that κ is κ^+-Mahlo is a large cardinal concept somewhat stronger than Mahloness, but well below weak compactness. This large cardinal assumption is necessary for the result as was shown previously by Shelah; the argument can be found in Mitchell's paper *A weak variation on Shelah's $I[\omega_2]$*, ***The Journal of Symbolic Logic***, vol. 69 (2004), pp. 94–100.

Given a regular cardinal λ and a sequence $\vec{a} = \langle a_\alpha \mid \alpha < \lambda \rangle$ the set $B(\vec{a})$ consists of those $\nu < \lambda$ for which there is a set $c \subseteq \nu$ whose order type agrees with the cofinality of ν and all proper initial segments of c are among a_α for $\alpha < \nu$. $I[\lambda]$ is the ideal of subsets of λ generated by nonstationary sets and sets of the form $B(\vec{a})$. Shelah introduced the ideal $I[\lambda]$ in the appendix on stationary sets to his *Classification theory for non-elementary classes*, ***Lecture Notes in Mathematics***, vol. 1292, pp. 483–497. In *Reflecting stationary sets and successors of singular cardinals*, ***Archive for Mathematical Logic***, vol. 31 (1991), pp. 25–53 he discusses in detail its basic properties; in particular he establishes the consistency result that for regular κ the ideal $I[\kappa^+] \cap \mathsf{cof}(\kappa)$ is generated over the nonstationary ideal by a single stationary-costationary set. Finally he makes a substantial use of the ideal in the construction of a model that satisfies full stationary reflection along with the maximum possible amount of supercompactness. There, too, Shelah asks the question if it is consistent that $I[\kappa^+] \cap \mathsf{cof}(\kappa)$ is the nonstationary ideal.

The main ingredient in Mitchell's construction is a new method of adding a closed unbounded subset of κ while simultaneously collapsing all cardinals between ω_1 and κ to ω_1 via a forcing notion consisting of finite conditions, so that κ becomes ω_2 in the generic extension. Adding closed unbounded subsets is a natural attempt to destroy all stationary sets that are in $I[\omega_2]$. The use of finite conditions is motivated by the fact that standard forcings for adding such sets based on infinite conditions do not add any reals to the ground model, and by the result of Shelah that under CH the ideal $I[\omega_2]$ contains a stationary set of ω_1-cofinal ordinals.

Mitchell's forcing is a generalization of Abraham's reformulation of Baumgartner's forcing for adding a closed unbounded subset of ω_1 with finite conditions described in U. Abraham and S. Shelah, *Forcing closed unbounded sets*, ***The Journal of Symbolic Logic***, vol. 48 (1983), no. 3, pp. 643–657. A condition in Abraham's forcing is a finite set whose members are intervals of countable ordinals of the form $(\eta, \xi]$ and countable ordinals that do not belong to any of the intervals. In this context, $\alpha \notin (\eta, \xi]$ says that α is compatible with $[\eta, \xi)$. Given a generic filter G, the set $\omega_1 \cap \bigcup G$ is a closed unbounded subset of ω_1 and the union of all intervals form $\bigcup G$ is its complement. Thus the conditions in Abraham's forcing are approximations to both the generic club and its complement. The intervals contained in the conditions are referred to as side conditions, as they become "invisible" once we obtain the object of our interest—the generic club. As the naive generalization of Abraham's forcing from ω_1 to ω_2 automatically collapses ω_1, any generalization of interest must include some obstructions preventing such collapsing. These obstructions are materialized as countable models that are included in the finite forcing conditions as additional side conditions. The idea of using countable models as side conditions was discovered and extensively used in a different context by Todorčević already in mid 80's. The simplest example of a side condition is a finite chain of elementary substructures of H_θ linearly ordered under $\in$. If we order such finite chains by reverse inclusion, we obtain a proper poset. In fact if G is a generic filter for this poset then $\{\omega_1 \cap M \mid M \in \bigcup G\}$ is a closed unbounded subset of ω_1.

Given a stationary set $B \subseteq \kappa$ of regular cardinals the conditions in Mitchell's forcing $\mathbb{P}_B$ for adding a single closed unbounded subset of ω_2 are finite sets consisting of ordinals in $B \cup \mathsf{cof}(\omega)$, intervals of the form $(\eta, \xi]$ and countable elementary substructures of H_κ that satisfy certain compatibility conditions. As in the case of Abraham's forcing, ordinals, intervals and models included in the same condition must be compatible. For ordinals and intervals the compatibility notion is the same as in Abraham's forcing. Compatibility of models with either ordinals, intervals, or else other models is significantly more delicate; the definition is naturally motivated by the need for $\mathbb{P}_B$ to be proper. An increasing finite chain of models as in the example above is a simple instance of compatibility of models, although the general case is less restrictive. In general, one can vaguely say that a model is compatible with any of the three types of objects if any incompatibility present in $\mathbf{V}$ is "reflected" inside the model.

The forcing $\mathbb{P}_B$ satisfies a strong form of properness: Given any set X and any condition $\bar{p} \in X$ there is a condition p below $\bar{p}$ which forces $\dot{G} \cap X$ to be $\mathbb{P}_B \cap X$-generic over $\mathbf{V}$. The author calls such conditions p strongly $(\mathbb{P}_B, X)$-generic. Given any forcing notion $\mathbb{P}$, the strong $(\mathbb{P}, X)$-genericity of a condition $p \in \mathbb{P}$ can be equivalently formulated as the requirement that $\mathbb{P}/p \cap X$ is a complete subposet of $\mathbb{P}/p$ where $\mathbb{P}/p$ is the portion of $\mathbb{P}$ below p. The presence of sufficiently well-behaved strongly generic conditions for X along with the presence of strongly generic conditions for stationarily many structures M of size δ guarantees the so-called δ-approximation property for $\mathbb{P} \cap X$: Any function $f \subseteq \mathbf{V}$ in $\mathbf{V}[G]$ with domain in $\mathbf{V}$ such that $k \restriction x \in \mathbf{V}[G \cap X]$ for all $x \in \mathbf{V}$ of size δ is actually in $\mathbf{V}[G \cap X]$. This crucial property was already exploited in Mitchell's work on the tree property at ω_2 in early 70's, and is distilled in an abstract form for arbitrary forcing notion $\mathbb{P}$ in Lemma 2.22 in the current paper. When applied to the forcing $\mathbb{P}_B$, the lemma yields the countable approximation property. This in turn is used in the proof that if B is a stationary set of inaccessible cardinals then in the generic extension via $\mathbb{P}_B$ the ideal $I[\omega_2]$ restricted to $\mathsf{cof}(\omega_1)$ is generated by the nonstationary ideal together with the single set S of all non-reflecting points of B (as computed in $\mathbf{V}$), and all stationary $\mathbf{V}$-subsets of $B - S$ remain stationary in the generic extension.

A natural attempt to build a model where $I[\omega_2]$ is the nonstationary ideal on $\mathrm{cof}(\omega_1)$ is to iterate the forcing $\mathbb{P}_B$ in order to eliminate all possible stationary generators of the ideal. Clearly, there is no meaningful notion of iteration for $\mathbb{P}_B$, as its relevant properties depend on κ being Mahlo, but on the other hand a single extension via $\mathbb{P}_B$ turns κ into ω_2. Instead, the author introduces a way of incorporating κ^+ many posets $\mathbb{P}_{B_\alpha}$ into a single poset $\mathbb{P}^*$ to simultaneously add a κ^+-sequence $(D_\alpha)_{\alpha<\kappa^+}$ where each D_α is a closed unbounded subsets of $B_\alpha \cup \mathsf{cof}(\omega)$. The stationary sets B_α are carefully constructed, which makes the treatment of $\mathbb{P}^*$ similar to that of $\mathbb{P}_B$, although significantly more involved.

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Lydia Patton. *Anti-psychologism about necessity: Friedrich Albert Lange on objective inference*. ***History and Philosophy of Logic***, vol. 32 (2011), pp. 139–152.

In his posthumously published ***Logische Studien*** (1877), Friedrich Albert Lange presents a foundational account for epistemology on the basis of logic as a methodology of sciences, avoiding psychologism and Aristotelianism. In her essay, Patton attempts to examine Lange's central methodological argument "that logical inference can serve as a grounding for epistemology" (p. 140). More precisely, Patton analyzes the different steps of Lange's "theory of the role of logic in knowledge that takes our access to necessity to be trough the norms governing inference" (p. 144). For Lange, as Patton explores, the basis for necessary inference is demonstration. The thus used imagined or physical diagrams do not represent the content of the involved concepts but are only signs of the extension of the concepts. The objective content of a proof can only be captured by the whole inference.

Patton starts with some reminiscences on Johann Friedrich Herbart und Adolf Trendelenburg (pp. 140–144), since Lange's approach is founded on a critical response to Herbertian psychologism. But in contrast to Trendelenburg's rejection, using a 'logic of content', Lange favors the new 'logic of extension'. In the main part of the essay (pp. 144–150), Patton develops Lange's particular method of thought by four steps. First, all apodictic truths are of the same kind (cf. pp. 145–146). This is a result of Lange's definition of apodictic reasoning "that follows from the immediate grasp of the sense of the premises, governed by the norms that shape reasoning using the extension of concepts" (p. 146). Second, since logical and mathematical truth are of the same nature, the Kantian distinction between the analytic and the synthetic fails (Patton claims this to be an anticipation of Quine's refutation of the distinction, cf. p. 151). Third, especially the law of non-contradiction is synthetic, since even this principle has to be constructed by intuition. The last step consists of the distinction between the descriptive psychological law of non-contradiction and the normative logical law (cf. pp. 147–148). Since the psychological law of non-contradiction determines the limits of possible experience, teaching us that a particular cannot have contradictory properties at the same time, the corresponding logical law gives us prescriptions concerning consistent theses about every particular. The normativity of the logical law stems from diagrammatic reasoning: We see immediately that we cannot affirm and deny the same of a certain particular. Lange's "innovative turn" (p. 147) consists of the observation that—while using the normative law—we can resolve conflicting explanations about a particular "by showing that some assumption of the explanation leads to a conflict with the naturalist law of non-contradiction" (p. 147).

There are several points of criticism. It remains almost unclear whether Lange's transfiguration of the apodictic and his abandonment of the analytic-synthetic-distinction can be placed in contradistinction to Kant, as Patton does (cf. p. 139, p. 140, most notably on p. 145). Nowhere in the essay the author has elucidated that by way of psychological means

Lange transforms transcendental philosophy in an (for Kant) inadmissible way. Lange's epistemology is universally an empiricist conception, treating e.g., the extension of concepts as abstractions from mental representations and the inner spatial intuition (as the basis for diagrammatic reasoning) as a pure psychological one. This has nothing in common with Kant's transcendental philosophy, and Lange's critique does by no means affect Kant's conception. This becomes most obvious by considering the (especially for Kant) categorical distinction between questions of validity and questions of genesis. In his whole epistemological enterprise, Lange mingles the context of discovery with the context of justification, treating questions of normative validity as questions of becoming aware of a certain structure of our actual thought. In the mode of Lange, diagrammatic logical reasoning stems from empirical intuition and will always be accompanied by those. Lange's ***Logische Studien*** are affected by unambiguous statements like: "Ganz blind, d.h. anschauungslos ist eine Operation des Denken wohl niemals, denn die Erinnerung an das bereits gelernte und eingeübte Verfahren knüpft sich doch immer an die sinnliche Vorstellung der Zeichen" (p. 26). (Translated as "An operation of thought will be never totally blind, i.e., an empty intuition, since the memory of already learned and trained operations is always tied to the sensible representation of signs".)

Already these cursory remarks make doubtful whether Lange's approach to logic can guarantee the normative character of logical laws. Patton nowhere mentions this central problem, and it is as doubtful whether Lange's epistemological reasoning should be evaluated as "innovative" as Patton does.

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