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REVIEWS

The Association for Symbolic Logic publishes analytical reviews of selected books and articles in the field of symbolic logic. The reviews were published in ***The Journal of Symbolic Logic*** from the founding of the JOURNAL in 1936 until the end of 1999. The Association moved the reviews to this BULLETIN, beginning in 2000.

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In a review, a reference "JSL XLIII 148," for example, refers either to the publication reviewed on page 148 of volume 43 of the JOURNAL, or to the review itself (which contains full bibliographical information for the reviewed publication). Analogously, a reference "BSL VII 376" refers to the review beginning on page 376 in volume 7 of this BULLETIN, or to the publication there reviewed. "JSL LV 347" refers to one of the reviews or one of the publications reviewed or listed on page 347 of volume 55 of the JOURNAL, with reliance on the context to show which one is meant. The reference "JSL LIII 318(3)" is to the third item on page 318 of volume 53 of the JOURNAL, that is, to van Heijenoort's *Frege and vagueness*, and "JSL LX 684(8)" refers to the eighth item on page 684 of volume 60 of the JOURNAL, that is, to Tarski's *Truth and proof*.

References such as 49***5*** or 280***1*** are to entries so numbered in *A bibliography of symbolic logic* (the JOURNAL, vol. 1, pp. 121–218).

SU GAO. ***Invariant descriptive set theory.*** Pure and applied mathematics. Chapman & Hall/CRC, Boca Raton, 2009, xiv + 392 pp.

Invariant descriptive set theory is the rigorous foundational study of classification problems in mathematics. The subject is also frequently called "Borel equivalence relations," a misnomer that Gao takes aim at with the title of his book. (Indeed, many of the most important equivalence relations turn out not to be Borel.) An important figure in the field once proposed "quantum set theory" as a sexy alternative. All of these names marginalize the most attractive feature of the subject: the classification problems it studies are naturally occurring (often classical), and span dozens of areas of mathematics.

Formally, the principle objects of study are definable equivalence relations on *standard Borel spaces*, that is, sets equipped with the Borel structure of a complete separable metric space. This is an extremely large collection of spaces encompassing virtually all continua studied in mathematics. Just a few examples: The unitary group of a separable Hilbert space, the space of countable groups, the space of Borel actions of a fixed countable group, and the space of complete separable metric spaces. Numerous classification problems correspond to natural equivalence relations on a standard Borel space. For the examples above, some relations of central importance would be conjugacy, isomorphism, equivariant isomorphism, and isometry, respectively.

The key notion in the subject is a measure of complexity of an equivalence relation called *Borel reducibility*, which was first made precise by Friedman and Stanley about 30 years ago.

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Here, if E and F are equivalence relations on standard Borel spaces X and Y, then we say E is Borel reducible to F if there is a Borel function $f : X \to Y$ which satisfies

$$x \; E \; x' \iff f(x) \; F \; f(x').$$

Borel reducibility measures the complexity of equivalence relations not as sets of pairs, but as *classification problems*. That is, if E is Borel reducible to F, then the classification of elements of X up to E is no harder than the classification of elements of Y up to F.

From this point of view, the simplest classification problems are those which admit a system of complete invariants from a standard Borel space I. Indeed, so long as the invariants can be computed in an explicit (*viz*., Borel) fashion, the corresponding equivalence relation will be Borel reducible to the equality relation on I, and a classical dichotomy theorem of Silver implies that the latter relation is minimum among Borel equivalence relations with uncountably many classes. Beyond this level, the Borel reducibility hierarchy becomes a wild and rich universe. In part, the subject aims to explore the structure of this universe, and to establish well-understood relations as benchmarks against which naturally occurring problems can be measured.

We now turn to Gao's exposition, which is divided into four sections. The first gives all of the set-theoretic grounding needed to make the book self-contained. While some of the exposition here is a little hurried (indeed, it would be impossible to also give a complete course in descriptive set theory), most of the background material is interesting in its own right. Gao treats a number of elementary subjects which are of critical importance, but at the same time are not easily found elsewhere: effective descriptive set theory, the universal (Urysohn) Polish space, and the general theory of Polish group actions. The last is vital to the development, since some of the most important equivalence relations can be realized as the orbit equivalence relation of a Polish group action.

In the second section, Gao collects together the deepest and most important results from the recent literature. The highlight here is his presentation of the Glimm-Effros dichotomy: if a Borel equivalence relation is not completely classifiable, then it embeds a copy of E_0 (the almost equality relation for infinite binary sequences). Gao gives this result its own chapter, beginning with Effros's version for F_σ orbit equivalence relations, moving on to the version stated above due to Harrington-Kechris-Louveau, and culminating with Becker's generalization to orbit equivalence relations of certain Polish groups.

Such general results are rare in a subject where the principal objects of study come from such diverse areas of mathematics as group theory, model theory, functional analysis, measure theory, representation theory, and so on. But there are several, and Gao has done a good job working them into his narrative: the Silver and Glimm–Effros dichotomies mentioned above, a theorem of Dougherty–Jackson–Kechris characterizing the hyperfinite Borel equivalence relations, the Friedman jump operation, the Burgess trichotomy theorem for analytic relations, Hjorth's theory of turbulence, and many more.

The final two sections are dedicated to finding the complexity of real life classification problems. Though many examples appear throughout the text, the most interesting problems are held until after Gao has treated the general theory. The third section deals with examples from countable model theory, including the classification of countable groups, graphs, and linear orders. For this, Gao first explores in detail the close connection between countable model theory and actions of the permutation group S_∞ of the naturals. The fourth section gives one of the most important examples: the classification of Polish metric spaces. These results are likely to be of interest to any mathematician, and they serve as good motivators for the first two sections.

This book has numerous strengths which make it recommendable for several uses. It is an excellent research reference, thanks largely to its thorough style and streamlined proofs of key results. Moreover, the last chapter contains indispensable diagrams summarizing the state of the Borel hierarchy of equivalence relations. The book also serves as an introduction to the subject for the working mathematician. Such readers should look to the beginning of the second section for the motivation, the first section for necessary background, and the final sections for concrete examples.

On the back cover this book advertises itself as a course textbook, and this is also a reasonable use. A course based on this book would be fairly advanced; incoming students would benefit greatly from familiarity with Kechris's text "Classical Descriptive Set Theory." That being said, we add that Gao's exercises are wonderful: they vary in difficulty from quite reasonable to somewhat challenging, and they manage to simultaneously relate strongly to the chapter text and to expand the scope of the book.

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MARY LENG. ***Mathematics and reality.*** Oxford University Press, Oxford, 2010, x + 278 pp.

This is an interesting contribution to the debate on the allegedly *empirical* nature of mathematical knowledge. As a naturalist philosopher Leng appeals to scientific practice in attacking the famous *Indispensability Argument*. This argument, mostly due to W. V. O. Quine, contends that many mathematical statements are *empirically confirmed* through science. Leng's attack draws on various aspects of scientific practice, ranging from the role of models and idealizations in science, to the nature of scientific confirmation. This makes the book of interest to general philosophy of science, where 'confirmational holism'—Leng's central point of contention in the indispensability argument—has been long debated. The crossing of the boundary between philosophy of science and philosophy of mathematics is commendable, and Leng's study interestingly also draws inspiration from the philosophy of art, as she defends a *fictionalist* position, according to which one is rationally compelled to believe only whatever science says about the concrete world.

Those familiar with the debate on the indispensability argument will find the first half of the book covering mostly familiar ground. Here Leng sets the scene by a critical discussion of Field's nominalization programme (ch. 2), and of philosophical naturalism both in general terms (ch. 2) and in relation to mathematical and scientific practice (chs. 4 and 5). The more novel second half leads to a new fictionalist position that emerges out of Leng's analysis of some central issues in the general philosophy of science. Leng characterizes her position as an alternative to scientific realism and 'constructive empiricism', a form of anti-realism. Here, in brief, is how she arrives at it.

First, Leng draws on the currently popular 'modelling view of science', according to which scientific theories are best construed in terms of *models* that are often *idealized*: models selectively misrepresent the world that is often too messy and complex to be modelled faithfully. In the modelling view the fit between theorizing and the world is not thought of in terms of theories or models being true or false. Rather, there are more accurate and less accurate *representations* of the world. The prevalence of idealizations in science, and especially their occasional *indispensability*, begs for an explanation of how the idealized posits (e.g., continuous fluids in fluid mechanics) can be indispensable for the predictive and explanatory success of science. They are, after all, seemingly non-existent (e.g., no fluid is actually continuous). Leng's account of the indispensability of idealizations appropriates Kendal Walton's famous account of literary fiction: she likens idealizations to fiction analysed as *a game of make-believe*. This account of idealized models as fiction is then extended to mathematics: the indispensability of mathematics, like that of some idealized concrete posits, derives from its power to represent hypotheses regarding the concrete world. Finally, Leng defends her fictionalism by arguing that it can explain the success of science sufficiently well.

This tallies with her overarching ontological guiding principle: the face value ontological commitments of science are to be taken seriously only to the extent required to explain the success of science.

This book-length argument for fictionalism involves much subtle thought and many novel moves. But it is open to criticism on several fronts, with respect to both the detail and the bigger picture. I'll now move on to make some quick observations to this end.

Ontology or epistemology? Leng begins: 'The central problem of this book is the problem of ontology, that is, the question of what there is.' Here, and elsewhere, there is ambiguity as to whether Leng sees herself as doing ontology or epistemology. She's (mostly) doing the latter; Leng's is (by and large) a treatise on the epistemology of science—on what one is *rationally justified* believing in. Besides equivocal use of language, the real worry is that Leng's position doesn't relate to the Carnap–Quine debate—the backdrop of Leng's project—in the intended way: 'our ontological project can be seen as an attempt to navigate a third way between [Carnap and Quine]' (p. 134). The debate between Carnap and Quine was ontological, turning on what it *means* for a theory to be true, and what the truth of a theory entails regarding existence. Leng, by contrast, is concerned with an epistemological issue that hinges on the possibility of explaining the success of science *without assuming* that our best theories are true. (Similar worries of misconstruction arise in connection with the discussion of Yablo on ontological commitment in ch. 6.)

Scientific confirmation. Leng aims to show that the standards of confirmation internal to science speak against the kind of confirmational holism required by the indispensability argument. This line of attack is prominent in the literature, which Leng critically reviews before mounting her own offensive. There is some tension in the big picture that gets painted here (chs. 5 and 6). Leng first discusses an idea due to both Sober and Musgrave, according to which something cannot be empirically confirmable unless it is also (potentially) empirically disconfirmable (should the world fail to cooperate). Intuitively, this is clearly something that might demarcate '$2 + 2 = 4$' (say) from bona fide empirical hypotheses: if we can't conceive of gathering empirical evidence conflicting with '$2 + 2 = 4$', why think that it earns confirmation through the role it plays in science? Leng faults this intuition, echoing Colyvan, in a way that just seems to boil down to reasserting Quinean all-out confirmational holism. It isn't transparent which side indulges in heavier question-begging here. Never mind; this is a strange response to Sober and Musgrave regardless, as Leng herself *opposes* confirmational holism, and advocates a *very local* notion of confirmation.

Following Maddy, Leng asserts that in actual scientific practice 'mere indispensability' of a theoretical posit often isn't enough, and more 'direct evidence' is demanded in form of experiments or manipulations of the posits in question. Although there are well-known precedents of this gambit in the scientific realism debate, the emphasis on direct evidence is badly motivated in the present context, due to the causal undertones of manipulation and experiment talk. (Leng's insists that her notion of direct evidence isn't defined in causal terms, and that it doesn't rule out non-causal evidence by fiat, but it is difficult to buy into this in light of her exemplars.) By setting such strict, local criteria for one's realist commitments the debate gets automatically stacked against the Quinean. It is no wonder that under such conception empirical confirmation doesn't extend to mathematical abstracta, given that it doesn't even seem to extend to black holes or quarks! With such a local notion of confirmation Leng in effect opts out of the debate with Quine, rather than engages in it.

Regardless of such criticism, the book is very much worth reading by everyone in the broad audience it addresses.

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Stevo Todorcevic. ***Introduction to Ramsey spaces.*** Annals of Mathematics Studies, vol. 174. Princeton University Press, New Jersey, 2010, 287 pp.

This work is a fascinating effort in formalizing and unifying infinite-dimensional Ramsey theory, bringing together deep combinatorial methods which have had tremendous impact over the last 80 years in areas as diversified as functional analysis, dynamical systems, set theory, topology and much more.

Although Ramsey theory owes its name to its grandfather, Frank Ramsey, who in 1930 published a groundbreaking paper focused on classifying certain relational structures on the natural numbers, this general subject of finding order in apparent chaos has been around in its finite dimensional origins for a much longer period. However since Ramsey's seminal work, the subject has exploded in very wide and diversified directions, from the point of view of both techniques and applications. It is then a natural progression if not a necessity of mathematics to understand and rewrite its contributions in the correct framework, and this is what I believe to be the objective of the present work in the form of an abstract infinite-dimensional Ramsey setting encompassing the techniques and from which the various applications follow. Such an enterprise requires a true mastery and grand overview of the subject, and we could not have been better served by anyone else than Todorcevic to undertake this endeavor.

The material begins with some preliminary material introducing the familiar and classic pigeon hole results and extensions of Ramsey, semigroup colorings of Hindman, Hales–Jewett, Gowers, trees and products colorings of Halpern–Läuchli (arguably one of the finest pigeon hole results). To warm up the reader, one finds self contained proofs of these results. This quickly leads us to higher dimensions in Ramsey theory starting with Erdös–Rado, and the techniques developed by Nash–Williams, Sperner, Galvin, Ellentuck, and finally Silver and Galvin–Prikry.

Now ready for action, the Abstract Ramsey Theory is developed in Chapter 4 and is certainly the crux of the contribution. This is the general framework encompassing the intended applications in the form of Ramsey space. The intended structures have the form $(\mathcal{R}, \mathcal{S}, \leq, \leq^0, r, s)$ and satisfy four axioms. Here you can think of $\mathcal{R}$ and $\mathcal{S}$ as families of infinite sequences of objects, r and s as (functions) giving us finite approximations, $\leq$ a relation on $\mathcal{S}$, and finally $\leq^0$ a relation between elements of $\mathcal{R}$ and elements of $\mathcal{S}$. Once (Ellentuck type) basic sets of the form $[a, Y]$ have been generalized, one defines a set $\mathcal{X} \subseteq \mathcal{R}$ to be $\mathcal{S}$-Ramsey if for every nonempty basic set $[a, Y]$, there is an (appropriate) set X such that $[a, X] \subseteq \mathcal{X}$ or $[a, X] \subseteq \mathcal{X}^c$. These are of course the (abstract) Ramsey subsets one wants in all applications. The Abstract Ramsey Theorem states that such structures (satisfying the four axioms) are *Ramsey spaces*: the family of $\mathcal{S}$-Ramsey subsets of $\mathcal{R}$ is large, that is closed under the Souslin operation, and in fact coincides with similarly defined $\mathcal{S}$-Baire subsets. Moreover the $\mathcal{S}$-Ramsey null subsets coincide with the $\mathcal{S}$-meager subsets (and are σ-ideals).

One gets a sense of the power of the machinery from an immediate corollary: for every finite coloring of $\mathcal{R}$ which is Souslin-measurable under the metric topology of $\mathcal{R}$, there is X in $\mathcal{S}$ such that the basic subset $[\emptyset, X]$ is monochromatic.

The author then presents two applications in detail, the infinite-dimensional Hales–Jewett Theorem, and the Bergelson–Blass–Hindman result for infinite block sequences of located words.

The proof of the Abstract Ramsey Theorem is based on *Combinatorial forcing*, itself an abstraction of several well known results, starting form Nash–Williams all the way to Gowers' celebrated results on infinite-dimensional Banach spaces.

Moving to Chapter 5, the focus is on Topological Ramsey Theory, an abstraction of Ellentuck's work on Silver's Theorem, Galvin–Prikry, Galvin, Nash–Williams, Milliken, Taylor, Rosenthal, and leading to Dual Ramsey results of Carlson–Simpson, and finally Graham–Leeb–Rothschild.

Further applications of the Abstract Ellentuck Theorem are given in Chapter 6 for spaces of trees, including Milliken's results on strong subtrees. This further leads into Devlin's partition calculus on finite powers of the rationals, to continuous colorings of finite powers of the rationals, and perfect set results of Mycielski and Blass among other variants.

Chapter 7 covers local Ramsey Theory, where Ramsey spaces are based on infinite subsets of the natural numbers. The abstraction here starts by covering the work of Mathias, Ellentuck, and Louveau but evolves into the *Ultra-Ellentuck Theorem* and applications to results of Bourgain–Fremlin–Talagrand, Odell–Rosenthal, and Balcar–Pelant–Simon.

Results in Chapter 8 focus on semicontinuous colorings and polarized Ramsey property, largely due to the author together with DiPrisco and Llopis.

Chapter 9 concludes with Parametrized Ramsey Theory, abstraction of early work of Nash–Williams and Galvin on barriers, then Galvin–Prikry and Silver on Ramsey families of subsets of the natural numbers. It culminates into likely to be the optimal parametrized form of the infinite-dimensional Ramsey Theorem.

The text was not intended to be a definitive compilation of Ramsey spaces and applications, but certainly constitutes a phenomenal resource for anyone involved in the field. By this I mean not only for active researchers in need of these combinatorial tools, but also to emerging mathematicians requiring a grasp and overview of this body of work. A natural next step for this enterprise is to further contribute Ramsey spaces and applications, and this is already well under way.

As another beautifully written text by the author, it would be a delight to use for course material as it will surely inspire graduate students and postdoctoral fellows.

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Massimo Mugnai. *Logic and mathematics in the seventeenth century*. ***History and Philosophy of Logic***, vol. 31 (2010), pp. 297–314.

The general view of the development of logic between the fifteenth respectively the sixteenth and the first half of the nineteenth century is, in accordance to the opinions of Bocheński and the Kneales, a primarily negative one: It is a time of little innovation and looks like a dead end of development (cf. pp. 297f). In contradistinction hereto, the author wants to relativize these judgments while showing "that there is a very important feature of modern logic [to be found in the seventeenth century, M.W.] which constitutes its novelty and the main reason (maybe the only one) to evaluate it positively: namely the attempt to establish a link between *logic* and *mathematics*" (p. 298). In particular he argues in support of the thesis that especially Boole and Frege completed a process which had started already in the second half of the seventeenth century (cf. p. 311).

The author's argument is organized into three steps. First, he considers the distinctness of logic and mathematics in pre-modern times (pp. 298f). From antiquity to the Middle Ages logic and mathematics are treated as fully distinct disciplines: Besides the merely logical—Aristotelian—tradition we have the independent mathematical—Euclidean—one. Early contributions explicitly discussing the question of the relationship between logic and mathematics were written for the first time in the second half of the sixteenth century (cf. p. 299). By this second step (pp. 299–304), Mugnai shed some light on the prehistory of linking both subjects. A first strand is named "From logic to mathematics" (pp. 299–302). Whereas, e.g., in an essay written by Alessandro Piccolomini (1574) we will find the thesis that mathematics derives its certainty not from the strength of logical proof but from the nature of its objects (p. 300), Conrad Dasypodius and Christian Herlinus (1566) attempt (on the contrary) to make explicit the logical structure of each proof in Euclid's *Elements* (p. 301). The second strand "From mathematics to logics" (pp. 302–304) deals with attempts of the assimilation of some logical procedures to procedures already established in mathematics as well as with approaches of identification of the process of thinking to a calculus. The author's final step is primarily considered with an analysis of Jacob Bernoulli's *Parallelismus ratiocinii logici et algebraici* (1685) and Gottfried Wilhelm Leibniz's *Dissertatio de arte combinatoria* (1666). For both works Mugnai verifies links between logic and mathematics for justifying his historiographic thesis. In the case of Bernoulli's paper he detects several relevant features, e.g., the establishment of a comparison between logical and algebraic operations, a superiority claim of algebra over traditional logic and the employment of the distinction between comprehension and extension of a concept (pp. 305f). Instead, for Leibniz's program of the analysis and synthesis of concepts, the author emphasizes the connection of this approach to Hobbes' identification of thinking and computing (pp. 306ff.). Mugnai's analysis of partially unknown publications between 1566 and 1735 carries without doubt interesting insights and widens the horizon of the history of logic and mathematics. But the reviewer's major problem is a lack of understanding of Mugnai's central thesis. What is exactly meant if the author states "in the period [between the sixteenth and the first half of the nineteenth century] one may find the seeds [sic!] of a process which was completed by Boole and Frege and opened the door to the contemporary, mathematical form of logic" (p. 297)? This metaphorically fortified and in the whole paper not further specified thesis could be applied to several subjects in the history of logic and mathematics, since it is vague enough to allow for further interpretations. And also by a harmless interpretation the thesis could not be false, since every novelty—even in the case of Boole or Frege—owns a prehistory (and in this sense seeds in former times), at least a prehistory of stagnation or of misunderstandings or shortcomings made by precursors. Thus, the hints given by the author concerning the different links between logic and mathematics should be explored in more detail and edited as a historiographic narration, making plausible why, e.g., the above mentioned works should be regarded as components of a prehistory of Frege's approach of a foundational reduction of (parts of) mathematics to axiomatized second order predicate logic.

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