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2010 NORTH AMERICAN ANNUAL MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

George Washington University
Washington, D.C.
March 17–20, 2010

The North American Annual Meeting of the Association was held on the campus of George Washington University, March 17–20, 2010. The program included the Twenty-first Annual Gödel Lecture, one tutorial, nine one-hour invited talks, five special sessions, and six sessions of contributed talks. The ASL hosted a welcoming reception on the first night of the conference.

The members of the Program Committee were William Gasarch, Joel Hamkins, Alexie Kolesnikov, Robert Rynasiewicz, Peter Selinger, and Reed Solomon (Chair). The members of the Local Organizing Committee were Jennifer Chubb, Ali Enayat, Ali Eskandarian, Michèle Friend, John Goodrick, Valentina Harizanov (Chair), and Alexie Kolesnikov. There were 104 registered participants at the meeting, including 43 graduate students. Generous financial support was provided by: the George Washington University Office for Research, Office for Graduate Studies and Academic Affairs, College of Arts and Sciences, and Department of Mathematics; the National Science Foundation; and the Association for Symbolic Logic.

The Twenty-first Annual Gödel Lecture, *Complexity of propositional proofs*, was delivered by Alexander Razborov (University of Chicago). The tutorial, *Quantum information processing: a new light on the quantum formalism and quantum foundations*, was presented by Bob Coecke (Oxford University) and consisted of three one-hour lectures. The hour-long invited addresses at the meeting were:

Zoé Chatzidakis (Université Paris 7), *Some consequences of the Canonical Base Property*.

Bjørn Kjos-Hanssen (University of Hawai'i at Mānoa), *Democracy is the best form of randomness extraction*.

Nicolaas P. Landsman (Radboud University Nijmegen), *Intuitionistic quantum logic*.

Lawrence S. Moss (Indiana University), *Natural logic*.

Dilip Raghavan (University of Toronto), *Cofinal types of ultrafilters*.

Tom Scanlon (University of California, Berkeley), *The logic of differentiating numbers*.

Ernest Schimmerling (Carnegie Mellon University), *Inner model theory*.

Henry Towsner (University of California, Los Angeles), *Infinite models for finite combinatorics*.

Rebecca Weber (Dartmouth College), *Degree invariance in the* Π^0_1 *classes*.

There were five special sessions (with organizers in parentheses): Categorical Logic (Pieter Hofstra), Computational Complexity (Richard Lipton), Logic and the Foundations of Physics (Andreas Doering), Model Theory (Michael C. Laskowski), and Set Theory (Su Gao). These included a total of thirty speakers.

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Abstracts of the invited talks and the contributed talks (given in person or by title) by members of the Association for Symbolic Logic follow.

For the Program Committee
REED SOLOMON

Abstract of the invited 21st Annual Gödel Lecture

► ALEXANDER RAZBOROV, *Complexity of propositional proofs*.
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The underlying question of propositional proof complexity is amazingly simple: when interesting propositional tautologies possess efficient proofs in a given propositional proof system? This theory makes an integral part of more general theory of feasible provability, the latter being widely considered as the proof theory for the world of efficiently computable objects. Other motivations for studying complexity of propositional proofs come from algebra, automated theorem proving and, of course, computational (especially circuit) complexity.

Given its mixed origin, the methods currently employed in this area are also very diverse. We will try to illustrate some of them and give the audience at least some feeling of the current state of the art in the area. Time permitting, the list of topics we hope to discuss includes:

- Connections to the classical Proof Theory, notably Bounded Arithmetic.
- Do major open problems in Complexity Theory like $NP \not\subseteq P/poly$ possess feasible proofs?
- Outreach applications: integrality gaps for LP/SDP relaxation procedures and Learning Theory.

Abstract of the invited tutorial on Quantum Computing

► BOB COECKE, *Quantum information processing: a new light on the quantum formalism and quantum foundations*.
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In these lectures we survey the key examples of quantum information processing, which include protocols such as quantum teleportation and dense coding, the main quantum algorithms such as Grover's and Shor's, quantum key distribution protocols such as BB84 and Ekert91, as well as non-standard quantum computational models such as measurement-based quantum computing. We discuss how these pose a new challenge for an operational account on quantum foundations. These foundational considerations point at a new avenue in the quest to find a more appropriate language to describe quantum phenomena, a challenge posed by John von Neumann merely 3 years after his publication of the Hilbert space quantum mechanical formalism in 1932, which has led to the the (failed) program of Birkhoff–von Neumann quantum logic, his continuous geometries, as well as his important contributions in operator algebras. We show how ideas from type theory, categorical logic, linear logic and graph theory contribute to such a high-level quantum formalism.

Abstracts of invited plenary talks

► ZOÉ CHATZIDAKIS, *Some consequences of the Canonical Base Property*.
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This property is a property of types of finite rank in a supersimple theory. It was first discovered by Pillay and Ziegler [2], and further investigated by myself, and by Moosa and Pillay [1] who coined the name. I will define this property, and discuss some of its consequences and possible applications.

[1] R. Moosa and A. Pillay, *On canonical bases and internality criteria*, ***Illinois Journal of Mathematics***, vol. 52 (2008), no. 3, pp. 901–917.

[2] A. Pillay and M. Ziegler, *Jet spaces of varieties over differential and difference fields*, ***Selecta Mathematica, New Series***, vol. 9 (2003), no. 4, pp. 579–599.

► BJØRN KJOS-HANSSEN, *Democracy is the best form of randomness extraction*.
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We consider the problem of deterministically extracting a Martin-Löf random infinite binary sequence from a similarly random sequence that has been corrupted by adaptive bit-fixing. A similar study for finite strings, yielding analogous results, was made by Buhrman et al. [1].

On the positive side, each sequence Y that differs from a random sequence X in a little less than $\sqrt{n}$ many of the first n bits computes a random sequence by a majority vote procedure.

On the negative side, for each Turing reduction procedure Φ and each sufficiently random X, there is a sequence Y differing from X in only slightly more than $\sqrt{n}$ of the first n bits such that $\Phi(Y)$ is not random.

[1] Harry Buhrman, Lance Fortnow, Ilan Newman, and Nikolai Vereshchagin, *Increasing Kolmogorov complexity*, ***Symposium on Theoretical Aspects of Computer Science 2005***, Lecture Notes in Computer Science, vol. 3404 (2005), pp. 412–421.

► NICOLAAS P. LANDSMAN, *Intuitionistic quantum logic*.
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Following Birkhoff and von Neumann [2], quantum logic has traditionally been based on the lattice of closed linear subspaces of some Hilbert space, or, more generally, on the lattice of projections in a von Neumann algebra A (for example, just think of A as the $n \times n$ complex matrices). Unfortunately, the logical interpretation of these lattices is impaired by their nondistributivity and by various other problems. Reviewing joint work with Chris Heunen and Bas Spitters [3, 4, 5], I explain a possible resolution of these difficulties, suggested by a combination of the philosophical ideas of Niels Bohr and the mathematical setting of topos theory and categorical logic [7]. Our general idea has been called 'Bohrification': a noncommutative C*-algebra (with enough projections) is turned into a commutative one by reinterpreting it in an appropriate topos (namely the sheaves on the partially ordered set $C(A)$ of commutative subalgebras of A), in which also its associated lattice of projections, though initially nondistributive, becomes (internally) Boolean. Thus the duality theorems of Gelfand and Stone for commutative C*-algebras [1] and Boolean lattices [6], respectively, apply, and *internally* (i.e., in the topos at hand) one has the usual Boolean logic of classical physics. *Externally* (i.e., in ordinary set theory), though, 'Bohrified' quantum logic turns out to be intuitionistic! Conceptually, this is the case because instead of single projections, elementary propositions now turn out to be families of projections indexed by $C(A)$. Such families form a Heyting algebra in a natural way, so that the associated propositional logic is intuitionistic: distributivity is recovered at the expense of the law of the excluded middle.

[1] B. BANASCHEWSKI AND C. J. MULVEY, *A globalisation of the Gelfand duality theorem*, ***Annals of Pure and Applied Logic***, vol. 137 (2006), pp. 62–103.

[2] G. BIRKHOFF AND J. VON NEUMANN, *The logic of quantum mechanics*, ***Annals of Mathematics***, vol. 37 (1936), pp. 823–843.

[3] C. HEUNEN, N. P. LANDSMAN, AND B. SPITTERS, *A topos for algebraic quantum theory*, ***Communications in Mathematical Physics***, vol. 291 (2009), pp. 63–110, arXiv:0709.4364.

[4] ———, *Bohrification of operator algebras and quantum logic*, ***Synthese***, to appear, arXiv:0905.2275.

[5] ———, *Bohrification*, ***Deep beauty***, (H. Halvorson, editor), Cambridge University Press, to appear, arXiv:0909.3468.

[6] P. T. JOHNSTONE, ***Stone spaces***, Cambridge University Press, 1982.

[7] S. MAC LANE AND I. MOERDIJK, ***Sheaves in geometry and logic: A first introduction to topos theory***, Springer, 1992.

► LAWRENCE S. MOSS, *Natural logic*.
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A central motivation for modern logic is that it provides a tool to represent natural language meaning and inference. One of the motivations of first-order logic is precisely that one can translate significant aspects of language into it, and in particular one can use it in connection with the foundations of mathematics. This talk returns to the topic of language and logic, and proposes new logical systems for the area.

Quine points out that "If we were to devise a logic of ordinary language for direct use on sentences as they come, we would have to complicate our rules of inference in sundry unilluminating ways." But few people have actually tried to provide logics for sentences close to "the way they come," and so I will be able to review much of what has been done. One leading idea is that the target logics for translations should have a decidable validity problem, ruling out full first-order logic. Indeed, we are interested in finding decidable fragments of language, just as others have asked for decidable fragments of first-order logic. We also axiomatize the logics, and then we can see whether the resulting systems are illuminating or not. The overall topic of this research would be interesting to those pursuing natural language semantics. It also could be of interest to historians of logic, since in effect, we are asking what traditional logic would have evolved into if it had the mathematical tools available to people like Frege. There are many technical results in the talk, and they are closest to algebraic logic, model theory and descriptive complexity theory.

► DILIP RAGHAVAN, *Cofinal types of ultrafilters*.
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A directed set D is said to be Tukey reducible to another directed set E, written $D \leq_T E$, if there is a function $f\colon D \to E$ which maps unbounded subsets of D to unbounded subsets of E. We say D and E are Tukey equivalent, written $D \equiv_T E$, if $D \leq_T E$ and $E \leq_T D$. The notion of Tukey equivalence tries to capture the idea that two directed posets "look cofinally the same", or have the same "cofinal type". As such, it provides a device for a "rough classification" of directed sets based upon their "cofinal type", as opposed to an exact classification based on their isomorphism type. This notion has recently received a lot of attention in various contexts in set theory. In joint work with Todorčević, I have investigated the Tukey theory of ultrafilters on the natural numbers, which can naturally be viewed as directed sets under reverse containment. In the case of ultrafilters, Tukey reducibility is coarser than the well studied Rudin–Keisler reducibility (RK reducibility). I will present some recent progress on the Tukey theory of ultrafilters, focusing on the question "under what conditions is Tukey reducibility actually equivalent to RK reducibility?".

► TOM SCANLON, *The logic of differentiating numbers*.
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Through the suggestive analogy between rational functions and rational numbers, some authors have suggested that we treat the Fermat quotient operator ($\delta_p \colon \mathbb{Z} \to \mathbb{Z}$) given by $x \mapsto \frac{x-x^p}{p}$ (for a prime number p) as a discrete version of "$\frac{d}{dp}$,"Â† or differentiation with respect to the prime p.Â† For the most part, the language of differentiation with respect to primes is just another fanciful part of the intentionally mystifying yoga of the field of one element, but when developed as a rigorous theory on its, especially with Buium's theory of p-jets, has allowed the importation of function theoretic arguments to number theory.Â† In this talk, I will explain how the theory of p-jets may be expressed as the first-order theory of certain valued fields enriched with automorphisms and analytic structure.Â† Quantifier elimination and completeness theorems then permit us to understand the definable sets which arise and to consider the theory uniformly in the prime.

► ERNEST SCHIMMERLING, *Inner model theory*.
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The talk will be on some results in inner model theory from recent years. It should be accessible to a general audience of logicians.

► HENRY TOWSNER, *Infinite models for finite combinatorics*.
Department of Mathematics, University of California at Los Angeles, Los Angeles, CA 90095-1555, USA.
E-mail: `hpt@math.ucla.edu`.

Infinitary methods based on ultraproducts have been useful for addressing a number of questions in Ramsey theory and additive combinatorics, including Hindman's Theorem, Szemeredi's Theorem, and (thanks to a recent paper by Hrushovski) the non-commutative Freiman problem. I will discuss the new developments which have made more problems susceptible to these methods, as well as techniques for extracting explict bounds from the resulting proofs.

► REBECCA WEBER, *Degree invariance in the* Π^0_1 *classes*.
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A collection of Turing degrees $\mathcal{C}$ is invariant over a structure $\mathcal{X}$ if there is some subset $\mathcal{S}$ of $\mathcal{X}$, invariant under automorphisms of $\mathcal{X}$, such that the collection of degrees of members of $\mathcal{S}$ is exactly $\mathcal{C}$. That is, there is a collection of representatives of the degrees in $\mathcal{C}$ that is invariant under automorphisms of $\mathcal{X}$. A great many invariant classes have been established for $\mathcal{X} = \mathcal{E}$, the computably enumerable sets. In particular, there is a result by Cholak and Harrington that gives the invariance over $\mathcal{E}$ of the high$_n$ and non-low$_n$ degrees for $n \geq 2$. I have translated this invariance result to $\mathcal{X} = \mathcal{E}_\Pi$, the Π^0_1 classes, over which previously only the array noncomputable degrees were known to be invariant. This talk will survey invariance and discuss the method of translation; the construction is in a substructure of $\mathcal{E}_\Pi$ and in fact takes place in an isomorphic setting of c.e. ideals. As should go without saying, effort will be made to make the talk accessible to anyone familiar with basic computability theory.

Abstracts of invited talks in the Special Session on Categorical Logic

► STEVE AWODEY, *Homotopy type theory*.
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Recent work by several researchers [1, 2, 3, 4, 5, 6] has exposed a new and surprising connection between the constructive type theory of Martin-Löf and homotopy theory. The key idea is that the type theory's intensional equality relation between terms, which has heretofore resisted a semantic interpretation, can be modeled by the homotopy relation between continuous maps of spaces. The new semantics are developed using the topologist's tools of Quillen model categories and weak ω-groupoids. The latter infinite-dimentional structures provide an algebraic bridge between type theory and homotopy theory, allowing not only a homotopical interpretation of constructive mathematics, but also the possibility of new type-theoretic applications in topology.

This talk surveys some of these recent developments.

[1] S. AWODEY, P. HOFSTRA AND M. A. WARREN, *Martin-Löf complexes*, submitted, on the arXiv as arXiv:0906.4521, 2009.

[2] S. AWODEY AND M. WARREN, *Homotopy theoretic models of identity types*, ***Mathematical Proceedings of the Cambridge Philosophical Society***, vol. 146 (2009) pp. 45–55.

[3] N. GAMBINO AND R. GARNER, *The identity type weak factorisation system*, submitted, on the arXiv as arXiv:0803.4349v1, 2008.

[4] R. GARNER, *Two-dimensional models of type theory*, ***Mathematic Structures in Computer Science***, to appear.

[5] R. GARNER AND B. VAN DEN BERG, *Types are weak ω-groupoids*, submitted, on the arXiv as arXiv:0812.0298, 2008.

[6] P. L. LUMSDAINE, *Weak ω-categories from intensional type theory*, ***Typed Lambda-calculus and its applications*** (***Brasilia, Brazil, 2009***), (Pierre-Louis Curien, editor), Lecture Notes in Computer Science, vol. 5608, 2009, pp. 172–187.

► MICHAEL MAKKAI, *Model theory for essentially algebraic 2-dimensional categories*.
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I present 2-dimensional generalizations of results in the paper M. Makkai and A. M. Pitts, *Some results on locally finitely presentable categories*, ***Transactions of the American Mathematical Society***, vol. 299 (1987), pp. 473–496. Let *Cat* resp. *Cat**, denote the 2-category of all small categories, functors and natural transformations, resp. isomorphism natural transformations. By an essentially algebraic 2-category, resp. 2^*-category, I mean the 2-category of all *Cat*-models, resp. *Cat**-models, of a fixed finite weighted (indexed) bilimit sketch. Many commonly known 2-dimensional categories of structured categories are essentially algebraic. Examples are the 2-category of small finite-limit categories, lex functors and all natural transformations, and the 2^*-category of all small elementary toposes, logical functors and isomorphism natural transformations. The main goal is to obtain recognition results analogous to 2.3 Proposition in *loc. cit.* in the 2-dimensional setting.

► ROBERT PARÉ, *Double categories of models*.
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First order theories are built on a language containing operation symbols and predicate symbols. Although the operation symbols can be reduced to predicate symbols (and vice-versa in the multi sort version we are considering), they play quite different roles and are best kept separate. Operations are interpreted as functions and predicates as relations so models are most naturally interpreted in the double category of sets with functions as horizontal arrows and relations as vertical. This then leads to horizontal and vertical morphisms of models. It is this extra structure on the category of models we investigate.

► BRIAN REDMOND, *Categorical structures for lower complexity*.
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I shall discuss the use of polarized categories to capture lower complexity computations showing how these are related to fibrational models. In particular, the notion of initial algebra in polarized settings will be discussed and the manner in which it corresponds to "comprehended recursion" in the fibrational setting. These settings give rise to a type theory on which one can base a reasonably expressive programming language which has an implementation called Pola.

Joint work with Mike Burrell and Robin Cockett.

► MICHAEL SHULMAN, *Unbounded quantifiers via 2-categorical logic*.
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The internal logic of an elementary topos is a higher-order type theory, which is equiconsistent with a weak form of set theory (bounded Zermelo). One obstacle to extending this to stronger set theories is that the internal logic admits no "unbounded quantifiers," i.e., quantifiers over all sets rather than over elements of some fixed set. It is well-known that this can be overcome by embedding the topos S in a "category of classes," such as the category of sheaves on S. In such a category one can construct a "class of all sets," so that the internal logic of the category of classes allows "unbounded quantification" over "sets" (objects of S).

However, this is not an elementary construction, and the logic obtained thereby is not expressible in the first-order theory of a topos, nor is it unique or canonical. A more natural choice (though still not elementary) is to embed S in the 2-category $\mathrm{St}(S)$ of *stacks* on S. Now the self-indexing of S provides a canonical representative of the "class of all sets" in $\mathrm{St}(S)$. We can then generalize the internal logic of a category to the internal logic of a 2-category, and reinterpret the internal logic of $\mathrm{St}(S)$ as a "stack semantics" on S that generalizes the usual "sheaf semantics." Moreover, the stack semantics of the self-indexing can be expressed in the first-order theory of S, and contains an embedded copy of the usual internal logic of S. Thus, we can extend the correspondence between topos theory and set theory to axioms with unbounded quantifiers, by interpreting these axioms in the stack semantics to obtain "intrinsic" strong axioms for toposes.

Abstracts of invited talks in the Special Session on Computational Complexity

► JIN-YI CAI, *Computational complexity theory and holographic algorithms*.
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Valiant pioneered the theory of Holographic Algorithms. Information is represented by a superposition of linear vectors in a holographic mix. This mixture creates the possibility for exponential sized cancellations of fragments of local computations. The underlying computation is done by invoking the Fisher–Kasteleyn–Temperley method for counting perfect matchings for planar graphs, which uses Pfaffians and runs in polynomial time. In this way some seemingly exponential time computations can be done in polynomial time, and some minor variations of the problems are known to be NP-hard or #P-hard. Holographic algorithms challenge our conception of what polynomial time computations can do, in view of the P vs. NP question. Holographic Reductions can also be used to prove #P-hardness. A number of complexity dichotomy theorems have been proved using this method. A dichotomy theorem for a class of counting problems states, for every problem in the class, either it is solvable in polynomial time, or it is #P-hard.

► KENNETH W. REGAN, *High-degree polynomials and arithmetical lower bounds*.
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When Boolean complexity problems are "arithmetized", they usually translate into arithmetical polynomials of low degree, i.e., degree polynomial in the number n of variables. Arithmetical complexity problems centering on the permanent have low degree to begin with. However, poly-size arithmetical circuits can compute polynomials of exponential degree, and there are implicit ways to represent polynomials of doubly-exponential degree or even higher. The issue is, can such high-degree polynomials carry information beyond what low-degree polynomials can capture, information that is relevant to these low-degree complexity questions? We interpret the "Algebrization" barriers as statements about the limitations of low-degree information, and examine possible cases for surmounting them.

► NISHEETH VISHNOI, *Computational models optimal assuming the Unique Games Conjecture*.
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In this talk, I will present computational models for a large class of NP-hard problems, such as vertex cover and maximum cut, which turn out to give the best approximation ratios for them if one believes in the Unique Games Conjecture. The key step in these is to write a linear/semi-definite programming relaxation for these problems, solutions to which have enough information to construct the reduction from Unique Games to the respective problem while the soundness analysis of these reductions rely on rounding algorithms for these relaxations. Thus, reductions can be constructed in an automatic manner and are optimal assuming the Unique Games Conjecture.

► RYAN WILLIAMS, *Time-space lower bounds for NP-hard problems*.
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A fertile area of recent research has found concrete polynomial time lower bounds for solving hard computational problems on restricted computational models. Among these problems are Satisfiability, Vertex Cover, Hamilton Path, MOD6–SAT, and Majority-of-Majority–SAT, to name a few. The proofs of such lower bounds all follow a certain proof-by-contradiction strategy.

I will survey some of the results in this area, giving an overview of the techniques involved.

Abstracts of invited talks in the Special Session on Logic and Foundations of Physics

► SAMSON ABRAMSKY, *Coalgebras, Chu spaces, and representations of physical systems*.
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We study the use of coalgebras to provide mathematical representations of physcial theories, in particular of quantum systems. Coalgebra has been developed as a rich mathematical paradigm for systems modelling within Computer Science over the past 15 years or so, and provides mathematical tools such as final coalgebras, bisimulation and coalgebraic logic. We see it as a promisng medium for modelling quantum systems, particularly from the point of view of quantum information, and also of providing some new perspectives on foundational issues.

In previous work [1], we used Chu spaces to represent quantum systems. Chu spaces arise very naturally in Mackey-style representations of physical systems in terms of their states, the questions which can be asked of a system, and the probability of obtaining a 'yes' answer to a question in a given state. We showed in [1] that the notion of morphism of Chu spaces provides a full and faithful representation of the groupoid of physical symmetries on Hilbert spaces, *i.e.*, the unitaries and antiunitaries, into Chu spaces. We also showed that this result still holds if the unit interval of probabilities is collapsed to three values, but *not* to two, as in the usual 'possibilistic' semantics.

We revisit this earlier work from [1], and investigate the use of coalgebra as an alternative setting. On the one hand, coalgebras allow the dynamics of repeated measurement to be captured, and provide a rich mathematical framework. However, the standard coalgebraic framework does not accommodate contravariance, which is an important element of the Chu space framework, and is too rigid to allow physical symmetries to be represented.

We introduce a fibrational structure on coalgebras in which contravariance is represented by indexing. We use this structure to give a universal semantics for quantum systems based on a final coalgebra construction. We characterize equality in this semantics as projective equivalence. We also define an analogous indexed structure for Chu spaces, and use this to obtain a novel categorical description of the category of Chu spaces, as the Grothendieck category over an indexed category in which each fibre is a comma category. We use the indexed structures of Chu spaces and coalgebras over a common base to define a truncation functor from coalgebras to Chu spaces. This truncation functor exhibits Chu spaces as a full subcategory of a fibred category of coalgebras. We use this truncation functor to lift the full and faithful representation of the groupoid of physical symmetries on Hilbert spaces into Chu spaces, obtained in our previous work, to the coalgebraic semantics.

[1] S. Abramsky, *Big toy models: Representing physical systems as Chu spaces*, to appear in ***Synthese***.

► BOB COECKE, *Quantum structuralism and picturalism*.
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We report on progress in an approach, initiated jointly with Abramsky, which aims to provide quantum theory with a purely diagrammatic calculus, a logical foundation, and increased degrees of axiomatic freedom, while still retaining full expressiveness. Key to this approach is the fact that monoidal categories provide a natural framework to reason about systems, processes and their interactions of any kind. An axiomatization of the notion of complementary observables exposes the flows of information in sophisticated quantum informatic protocols involving complex entanglements [6], and the correspondingly emerging concept of relative phases provides new insights in the origin and peculiarity of quantum non-locality [8]. We also obtain a purely diagrammatic distinction of the classical and the quantum, and of their interaction [7].

Diagrammatic QM introductions:

[1] Kindergarten quantum mechanics arXiv:quant-ph/0510032

[2] Quantum picturalism arXiv:0908.1787

Categories for physicists:

[3] Introducing categories to the practicing physicist arXiv:0808.1032

[4] Categories for the practicing physicist arXiv:0905.3010

Latest developments:

[5] A new description of orthogonal bases arXiv:0810.0812 (with Dusko Pavlovic and Jamie Vicary)

[6] Interacting quantum observables arXiv:0906.4725 (with Ross Duncan)

[7] Classical and quantum structuralism arXiv:0904.1997 (with Eric Paquette and Dusko Pavlovic)

[8] The group theoretic origin of non-locality for qubits http://web.comlab.ox.ac.uk/publications/publication3026-abstract.html (with Bill Edwards and Rob Spekkens)

► JOHN HARDING, *Daggers, kernels, Baer *-semigroups, and orthomodularity*.
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There has been considerable recent interest in using categorical methods to address foundational issues in quantum mechanics. Here the objects of a suitable category are quantum systems and morphisms are processes. It is natural to connect this work to the older subject of quantum logic by constructing from each object in a category an orthomodular structure to represent the propositions of the quantum system represented by the object. It is our purpose here to look more closely at the axiomatics related to this task. In particular, we consider axiomatics related to Baer *-semigroups, partial semigroups, and various constructions involving dagger categories, kernels, and biproducts. Much of this is a survey of results from the 60's and 70's, adapted to the categorical setting, with a few small, but hopefully useful observations thrown in.

► PETER SELINGER, *Planar traced categories*.
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It is well-known that graphical languages ("string diagrams") are a useful tool for reasoning about monoidal categories. There has recently been a surge in interest in such languages, due to the work of Abramsky and Coecke on dagger compact closed categories as a logical foundation for quantum mechanics. In general, these graphical languages come in many different flavors, for example, symmetric, braided, planar, traced, autonomous, etc. In this talk, I will focus on a case that has not apparently been studied before, namely, that of planar traced categories.

► ALLEN STAIRS, *Could logic be empirical*?
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In 1968, Hilary Putnam published a paper called "Is Logic Empirical?" in which he argued that quantum mechanics provides the basis answering "yes". In particular, Putnam argued, quantum mechanics has shown us that the distributive law breaks down for propositions about quantum systems. Several years later, in a still-unpublished talk, Saul Kripke offered a trenchant critique of Putnam's argument. I revisit this dispute and consider whether there is any possibility of finding a middle ground between the two.

► MAARTEN VAN DEN NEST, *Quantum computations that can be simulated classically*.
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The study of quantum computations that can be simulated efficiently classically is of interest for numerous reasons. From a fundamental point of view, such an investigation sheds light on the intrinsic computational power harnessed in quantum mechanics as compared to classical physics. More practically, understanding which quantum computations do not offer any speed-ups over classical computation provides insights in where (not) to look for novel quantum algorithmic primitives. In this talk we discuss classical simulation of quantum computation from several perspectives. First we review some well-known examples of classically simulatable quantum computations. We further discuss novel simulation methods that are centred on classical sampling methods ('weak simulation'), and show how these techniques outperform existing methods that rely on the exact computation of measurement probabilities ('strong simulation'). Using weak simulation methods, several new classes of simulatable quantum circuits are generated. Finally, we focus on famous quantum algorithms (Deutsch-Jozsa, Simon, Shor) and investigate the origin of their computational power, or their lack thereof.

► NOSON S. YANOFSKY, *An operadic view of computation*.
Computer Department, Brooklyn College, CUNY, 2900 Bedford Avenue, Brooklyn, NY 11210, USA.
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We look at the relationship between programs, algorithms and functions. To us, an algorithm is an equivalence of programs, and a function is an equivalence class of algorithms. Two programs are in the same equivalence class if they are "essentially" the same. Two algorithms are in the same equivalence class if they perform the same function. We will examine the types of structures that one can find on the set of programs, algorithms and functions. Questions of complexity will also be considered.

All this is done within the language of PROPs which are generalizations of operads. An operad is a topological /combinatorial / homotopical way of describing algebraic and higher-dimensional algebraic structures. Operads and PROPs are adept at describing the structures and invariances needed in some branches of modern mathematics and theoretical physics. The theory of operads has become popular in homological algebra, (higher-dimensional) category theory, algebraic geometry, string field theory and Topological Quantum Field Theory.

We shall conclude by describing a correspondence between our work and some work in low-dimensional topology and TQFT.

Abstracts of invited talks in the Special Session on Model Theory

► SALIH AZGIN, *Reverse quantifier elimination for valued fields*.
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The question of quantifier elimination for algebraic structures in appropriate languages can be reversed. In this direction van den Dries, Macintyre and McKenna proved: If a field has QE in the language of rings then it is algebraically closed. If a field has QE in the language ordered rings then it is real closed. Similar questions for valued fields are considered by the same authors and there are recent developments by Yimu Yin. In both cases the positive answers are limited to rank 1 value groups. It turns out that by assuming special forms of relative quantifier elimination the results can be carried to finite rank value groups.

► ALFRED DOLICH, *On notions of "dependence minimality"*.
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I will discuss various competing notions intended to capture the idea of being minimal with respect to not having the independence property (i.e., of being "dependence minimal"). Among these are VC-minimality (introduced by Adler in [1]), dp-minimality (introduced by Shelah in [2]), and having minimal VC-density (studied by Aschenbrenner, Dolich, Haskell, Macpherson, and Starchenko). In particular we will assess the interactions between these various notions and consider their import and examples in various situations including that of divisible ordered Abelian groups, valued fields, and uncountably categorical or, more generally, stable theories.

[1] HANS ADLER, *Theories controlled by formulas of Vapnik–Chervonenkis codminsion 1*, preprint available at www.amsta.leeds.ac.uk/~adler

[2] SAHARON SHELAH, *Strongly dependent theories* preprint available at www.shelah.logic.at

► JOHN GOODRICK AND MICHAEL C. LASKOWSKI, *The Schröder–Bernstein property for weakly minimal theories*.
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For a countable, weakly minimal theory T, we show that the Schröder–Bernstein property (any two elementarily bi-embeddable models are isomorphic) is equivalent to each of the following:

1. For any U-rank-1 type $q \in S(\mathrm{acl}^{eq}(\emptyset))$ and any automorphism f of the monster model $\mathfrak{C}$, there is some $n < \omega$ such that $f^n(q)$ is not almost orthogonal to $q \otimes f(q) \otimes \cdots \otimes f^{n-1}(q)$;
2. T has no infinite collection of models which are pairwise elementarily bi-embeddable but pairwise nonisomorphic.

We conclude that for countable, weakly minimal theories, the Schröder–Bernstein property is absolute between transitve models of ZFC. Time permitting, we will discuss the possibility of generalizing this result beyond weakly minimal theories.

► JANA MAŘÍKOVÁ, *Valuations on o-minimal fields*.
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Let R be an o-minimal expansion of a real closed field and V a proper convex subring. We give a first-order axiomatization of the class of all (R, V) such that the corresponding residue field with structure induced from R via the residue map is o-minimal.

► ALICE MEDVEDEV, *Groups in T_A*.
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On good days [1], the class of models of a nice theory T with a distinguished automorphism admits a model-companion T_A , which is then also nice [4]. For example, when T is the theory of algebraically closed fields, T_A is ACFA. This talk is about groups definable in T_A. We observe that very recent results in [2] apply to this setting showing that such groups are always subgroups of T-definable groups. We then replace Zariski closures by forking calculus in the proofs of the characterization of groups definable in ACFA in [3].

[1] JOHN BALDWIN AND SAHARON SHELAH, *Model companions of T_{Aut} for stable T*, ***Notre Dame Journal of Formal Logic***, vol. 42 (2001), no. 3, pp. 129–142.

[2] T. BLOSSIER, F. O. WAGNER AND A. MARTIN-PIZARRO, *Géométries relatives* ***MODNET preprint server***, vol. 178 (2009), no. 178, pp. 1–20.

[3] Z. CHATZIDAKIS AND E. HRUSHOVSKI, *Model theory of difference fields*, ***Transactions of the American Mathematical Society***, vol. 351 (1999), no. 8, pp. 2997–3071.

[4] Z. CHATZIDAKIS AND A. PILLAY, *Generic structures and simple theories*, ***Annals of Pure and Applied Logic***, vol. 95 (1998), no. 1–3, pp. 71–92.

► SERGEI STARCHENKO, *On definability of Riemann theta functions*.
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In this talk we show that all Riemann theta functions, restricted to suitable domains, are definable in the o-minimal structure $\mathbb{R}_{an,exp}$.

As a consequence we obtain a definable in $\mathbb{R}_{\mathrm{an,exp}}$ embedding of a universal family of polarized complex tori $\{\mathbb{C}^g/\Lambda_\tau : \tau \in \mathbb{H}_g\}$ into a projective space.

► ALEX WILKIE, *Semi-germs of definable holomorphic functions*.
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We continue our investigation into those properties of complex holomorphic functions that are special to the polynomial bounded, o-minimal situation. One result is as follows. Consider a polynomially bounded o-minimal expansion R of the real field and let M be a sufficiently saturated elementary extension of R. Let $C(M)$ $(= M \times M)$ denote the complex plane of M, and let $\mu(M)$ be the set of infinitesimals of $C(M)$. Let $n > 0$. One could consider the ring of germs of M-definable holomorphic functions about the origin in $C(M)^n$, but more interesting (in view of Zilber's conjecture on the complex exponential field—and I shall explain this remark in the talk) is the ring of semi-germs, meaning the ring of M-definable (WITH parameters) holomorphic functions defined on neighbourhoods of $\mu(M)^n$. I shall briefly sketch a proof that this is a Noetherian ring. The polynomial boundedness of M seems to be crucial here and I do not know whether o-minimality alone is sufficient.

Abstracts of invited talks in the Special Session on Set Theory

► ANDREAS BLASS, *F_σ-generic ultrafilters.*
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This will be a survey talk about the ultrafilters produced by forcing with F_σ filters. Already in the 1970's, Daguenet used this forcing (though with topological terminology) to show that the continuum hypothesis gives "many" P-points with no selective ultrafilters below them in the Rudin–Keisler ordering. In the 1980's, Laflamme gave a combinatorial description of F_σ filters and proved additional combinatorial properties of the resulting generic ultrafilters. Recently, reals Mathias-generic with respect to these ultrafilters have played an important role in work of Dorais in reverse mathematics, and Krautzberger has studied the analogs in the world of idempotent ultrafilters.

► NATASHA DOBRINEN, *Tukey types of ultrafilters.*
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We give an overview of recent results in the theory of Tukey types of ultrafilters on countable sets. If $\mathcal{U}$ and $\mathcal{V}$ are ultrafilters, we say that $\mathcal{U}$ is Tukey above $\mathcal{V}$, $\mathcal{U} \geq_T \mathcal{V}$, if there is a function $f: \mathcal{U} \to \mathcal{V}$ which maps cofinal subsets of $\mathcal{U}$ to cofinal subsets of $\mathcal{V}$. The study of cofinal types of partially ordered sets grew out of the study of Moore–Smith convergence in topology. When restricted to ultrafilters, the Tukey ordering on ultrafilters is actually a generalization of the well-studied Rudin–Keisler ordering with some analogies and some surprising twists.

We present a canonization theorem for cofinal maps from a p-point into another ultrafilter in terms of a continuous map from $\mathcal{P}(\omega)$ into $\mathcal{P}(\omega)$, embeddings of different structures into the structure of Tukey classes of ultrafilters, a property which is preserved downwards by cofinal maps, and some progress towards solving a problem of Isbell of whether there is a model of ZFC in which there is only one Tukey type of nonprincipal ultrafilters.

Much of the work explicated in this talk is joint with Stevo Todorcevic.

► JOEL DAVID HAMKINS, *Set-theoretic geology.*
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Set-theoretic geology is the study of the structure of the ground models of the set-theoretic universe. Although forcing is customarily viewed as a method for constructing outer as opposed to inner models of set theory, many natural questions arise when one inverts this perspective and considers forcing as a method of describing inner models, namely, the inner models over which the universe was obtained by forcing. A class W is a *ground* for V, if V was obtained by set forcing over W. The model V satisfies the *Ground Axiom* if there are no such W properly contained in V. Although apparently second-order, this axiom is nevertheless first-order expressible in ZFC. A model W is a *bedrock* of V if W is a ground of V and satisfies the Ground Axiom. The *mantle* of V is the intersection of all grounds of V. The *generic mantle* of V is the intersection of all grounds of all set forcing extensions of V. One of the main initial results of geology is that every model of ZFC is the mantle and generic mantle of another model of ZFC, and this can be proved while also controlling the HOD of the final model, as well as the generic HOD, the intersection of the HODs of all forcing extensions. The principle open question of set-theoretic geology is: Is the collection of ground models downward directed? Equivalently, if two models of set theory have a common forcing extension, must they have a common ground? The Downward Directed Grounds hypothesis, asserting a positive answer, is true in every model for which we are able to determine the answer and leads to a robust theory.

This is joint work with Gunter Fuchs and Jonas Reitz, both at CUNY.

► BART KASTERMANS, *Formalizing set theory*.
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We'll discuss how to formalize set theory using a proof assistant.Â† The examples are taken from results on cofinitary groups.

► JOHN KRUEGER, *On the weak reflection principle*.
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The *Weak Reflection Principle* for ω_2, or WRP(ω_2), is the statement that every stationary subset of $P_{\omega_1}(\omega_2)$ reflects to an uncountable ordinal in ω_2. The *Reflection Principle* for ω_2, or RP(ω_2), is the statement that every stationary subset of $P_{\omega_1}(\omega_2)$ reflects to an ordinal in ω_2 with cofinality ω_1. Let κ be a κ^+-supercompact cardinal and assume $2^\kappa = \kappa^+$. Then there exists a forcing poset $\mathbb{P}$ which collapses κ to become ω_2, and $\Vdash_{\mathbb{P}} \mathrm{WRP}(\omega_2) \wedge \neg\mathrm{RP}(\omega_2)$.

► JINDRICH ZAPLETAL, *Homogeneous sets of outer measure one*.
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Let A be a coanalytic set of countable subsets of a Polish probability measure space. If there is an outer measure one set all of whose countable subsets belong to A, then there is a perfect set with this property. This is motivated by a previous result of Matrai about Baire category.

► MARTIN ZEMAN, *Inner model reflection*.
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It has been known for some time that if Martin's maximum, MM, holds and M is any inner model (that is, not necessarily fine-structural) that computes ω_2 correctly then M contains all reals. There are variations on this result concerning other forcing axioms. This gives rise to a natural question: What can be said about reals of inner models M that correctly compute ω_2? We show that under anti-large cardinal hypotheses, such models still contain reals that code sufficiently large canonical fine-structural inner models. As an immediate consequence we obtain that, depending on the respective anti-large cardinal hypotheses, such models satisfy the corresponding degree of correctness.

This is a joint work with Caicedo.

Abstracts of contributed talks

► JEAN-MARTIN ALBERT, *Hilbert spaces over non-archimedean fields*.
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We study quantifier elimination and notions of independence in elementary classes of non-archimedean Hilbert spaces over fixed non-archimedean valued fields in the context of continuous logic.

[1] ITAÏ BEN YAACOV, ALEXANDER J. BERENSTEIN, C. WARD HENSON AND ALEXANDER USVYATSOV, *Model theory for metric structures*, ***Model theory with applications to algebra and analysis***, vol. II, (Z. Chatzidakis, D. Macpherson, A. Pillay, and A. Wilkie, editors), Cambridge University Press, New York, 2008, pp. 315–427.

[2] A. C. M. VAN ROOIJ, ***Non-archimedean functional analysis***, Monographs and textbooks in pure and applied mathematics, 51, Marcel Dekker, Inc., 1978.

► CAN BAŞKENT, *Completeness of public announcement logic in topological spaces*.
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Public Announcement Logic (PAL, henceforth), first suggested in late 80s and has gained popularity with the recent developments in dynamic epistemology, is a dynamic epistemic logic where the epistemic status of the knowers is updated by an external truthful announcement [5, 2]. It is very well known that PAL is complete for traditional Kripke semantics of epistemic logic. In this work, we show that PAL is complete with respect to topological semantics of modal logic [1]. Notice that topological semantics of modal logic is historically the first semantics for modal logic [3]. In topological semantics, recall that modalities are defined in terms of interior or closure operators. Thus, our result establishes a connection between external epistemic updates and such topological reasoning. Moreover, we also provide a construction to achieve the same result in non-topological epistemic logics such as subset space logic [4, 1]. In such logics, the epistemic status of the knowers may not necessarily form a topological space. Yet, PAL is well-behaved in such systems as well.

[1] CAN BAŞKENT, *Topics in subset space logic*, ***Institute for Logic, Language and Computation***, ILLC Publication Series, MoL-2007-05 (2007), Universiteit van Amsterdam.

[2] HANS VAN DITMARSCH, WIEBE VAN DER HOEK AND BARTELD KOOI, ***Dynamic epistemic logic***, Springer, 2007.

[3] J. C. C. MCKINSEY AND ALFRED TARSKI, *The algebra of topology*, ***The Annals of Mathematics***, vol. 45 (1944), no. 1, pp. 141–191.

[4] LAWRENCE S. MOSS AND ROHIT PARIKH, *Topological reasoning and the logic of knowledge*, ***Proceedings of TARK IV***, (Yoram Moses, editor), pp. 95–105, 1992.

[5] JAN PLAZA, *Logic of public communication*, ***4th International Symposium on Methodologies for Intelligent Systems***, (M. L. Emrich, M. S. Pfeifer, M. Hadzikadic and Z. W. Ras, editors), pp. 201–216, 1989.

► KATALIN BIMBÓ, *Relational semantics for an extension of* R_+.
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Relevance logics were originally defined to include a De Morgan negation. However, other negations have been introduced into positive relevance logics, for instance, in [3] and [2]. I define an adequate relational semantics for the logic that is obtained by extending R_+ (the positive fragment of the logic of relevant implication) by the De Morgan laws and by the contraposition axioms $(A \to B) \to (\neg B \to \neg A)$, $(A \to \neg B) \to (B \to \neg A)$ as well as the so-called "non-constructive reductio" axiom.

[1] K. Bimbó and J. M. Dunn, ***Generalized Galois logics. Relational semantics of nonclassical logical calculi***, CSLI Lecture Notes, vol. 188, CSLI Publications, Stanford, 2008.

[2] G. Robles, *A note on the non-involutive Routley star*, ***Bulletin of the Section of Logic***, vol. 37 (2008), pp. 19–27.

[3] A. Urquhart, *Duality for algebras of relevant logics*, ***Studia Logica***, vol. 56 (1996), pp. 263–276.

► JUSTIN BRODY, *On rational limits of Shelah–Spencer graphs*.
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It was noticed by Baldwin and Shelah that for α irrational in $(0, 1)$, a model of the almost-sure theory of the random graphs $G(n, n^{-\alpha})$ could be obtained via Hrushovski's amalgamation construction. The resulting models are stable and AE-axiomatizable. We will examine various ways of generalizing the construction for r rational in $(0, 1)$. The main result will be that the resulting model either has a theory which is ω-stable and nearly model complete or else has a theory which is essentially undecidable.

► MINGZHONG CAI, *Array nonrecursiveness and relative recursive enumerability*.
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Array nonrecursive (**ANR**) degrees were introduced in [3]. They share a lot of nice properties with $\overline{\mathbf{GL_2}}$ degrees. Examples include the 1-generic bounding property, the cupping property ([3]) and relative recursive enumerability ([2]).

We study the problem on the definability of the **ANR** degrees, primarily motivated by the fact that the Turing jump is definable from **ANR** degrees (see [5]). Our main result is that **ANR** degrees are definable from the notion of relatively recursively enumerable (**RRE**) degrees, i.e., these degrees that are strictly r.e. above another degree. More precisely, we have:

$$\mathbf{a} \in \mathbf{ANR} \iff \forall \mathbf{b} > \mathbf{a}(\mathbf{b} \in \mathbf{RRE}).$$

This answers some questions in [1] and the proof also gives a solution to the strong minimal cover problem for n-**REA** degrees, extending a previous result for r.e. degrees ([4]).

[1] K. Ambos-Spies, D. Ding, W. Wang and L. Yu, *Bounding non-GL_2 and R.E.A.*, ***The Journal of Symbolic Logic***, vol. 74 (2009), no. 3, pp. 989–1000.

[2] M. Cai and R. A. Shore, *Domination, forcing, array nonrecursiveness and relative recursive enumerablity*, to appear.

[3] R. Downey, C. Jockusch and M. Stob, *Array nonrecursive degrees and genericity*, ***London Mathematical Society Lecture Notes Series, 224*** (S. B. Cooper, T. A. Slaman and S. S. Wainer, editors), University Press, 1996, pp. 93–105.

[4] S. Ishmukhametov, *Weak recursive degrees and a problem of Spector*, ***Recursion Theory and Complexity***, (M. Arslanov and S. Lempp, editors), de Gruyter, 1999, pp. 81–89.

[5] R. A. Shore, *Direct and local defintions of the Turing jump*, ***Journal of Mathematical Logic***, vol. 7 (2007), pp. 229–262.

► WILLIAM C. CALHOUN, *Turing degrees of K_m-trivial reals*.
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Monotone complexity, K_m, is a variant of Kolmogorov complexity that was introduced independently by Levin and Schnorr. A real α is K_m-trivial if there is a constant c such that $K_m(\alpha \upharpoonright n) \leq K_m(n) + c$ for all natural numbers n. This definition is the same as the definition of the K-trivial reals, except that prefix-free complexity has been replaced by monotone complexity. We will use a strengthening of an argument by Stephan to show that each Turing degree $\mathbf{d} \geq \mathbf{0}'$ contains a K_m-trivial real. This is in striking contrast with the situation for the K-trivial reals. All K-trivial reals are Δ^0_2 (Chaitin) and low, since they are low for random (Hirschfeldt and Nies). Since every K-trivial real is also K_m-trivial, the set of Turing degrees of K_m-trivial degrees contains all Turing degrees of K-trivial reals as well as all Turing degrees greater than or equal to $\mathbf{0}'$. We will consider the question of whether there are any other Turing degrees that contain K_m-trivial reals.

► WESLEY CALVERT AND RUSSELL MILLER, *BSS-reducibility and algebraic real numbers*.
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Blum, Cucker, Shub, and Smale introduced BSS computation, a generalization of Turing computation in which elements of an arbitrary ring (usually $\mathbb{R}$) are each considered to constitute a single finite piece of information. Meer and Ziegler considered the sets $\mathbb{A}_d$ of algebraic real numbers of degree $\leq d$, and the sets $\mathbb{A}_{=d}$ of algebraic real numbers of degree exactly d, asking whether $\mathbb{A}_{d-1}$ can compute $\mathbb{A}_d$ (in the sense of oracle BSS computation on $\mathbb{R}$), and what reducibility relations exist on the sets $\mathbb{A}_{=d}$. We give answers to these questions, and to some more general questions arising from them. The main result is that $\mathbb{A}_{=d}$ can be computed below an oracle $\cup_{n \in S} \mathbb{A}_n$ if and only if S contains a nonzero multiple of d. It follows that the partial order of the power set of ω, under inclusion, embeds into the partial order of BSS-semidecidable degrees under BSS reducibility.

[1] L. Blum, F. Cucker, M. Shub, and S. Smale, ***Complexity and real computation***, Springer, 1998.

[2] K. Meer and M. Ziegler, *An explicit solution to Post's Problem over the reals*, ***Journal of Complexity***, vol. 24 (2008), pp. 3–15.

► FRANÇOIS G. DORAIS AND CARL MUMMERT, *Stationary and convergent strategies in Choquet games*.
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We study a family of games that generalize the strong Choquet game from descriptive set theory. Two types of winning strategies for the second player are of interest: stationary strategies, in which the strategy only depends on the most recent move of the first player, and convergent strategies, in which the second player tries to make the intersection of the sets played as small as possible. We prove a sufficient topological condition for the existence of a stationary winning strategy in the strong Choquet game, and obtain a characterization of the spaces for which the second player has a convergent winning strategy.

► RODNEY G. DOWNEY, ASHER M. KACH, STEFFEN LEMPP AND DANIEL C. D. TURETSKY, *Comparing effective notions of categoricity, Part I.*
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A computable model $\mathcal{M}$ is called *computably categorical* if any computable model $\mathcal{M}_0 \cong \mathcal{M}$ is computably isomorphic to $\mathcal{M}$; *relatively computably categorical* if *any* model $\mathcal{M}_0 \cong \mathcal{M}$ is isomorphic to $\mathcal{M}$ by an isomorphism computable in $\mathcal{M}_0$; and *n-decidable* if the Σ_n-fragment of the elementary diagram of $\mathcal{M}$ is computable. We will also need relativized versions of the second notion, i.e., a computable model is *relatively Δ^0_n-computably categorical* if any model $\mathcal{M}_0 \cong \mathcal{M}$ is isomorphic by an isomorphism computable in $\mathcal{M}_0^{(n)}$; and *relatively arithmetically categorical* if any model $\mathcal{M}_0 \cong \mathcal{M}$ is isomorphic by an isomorphism arithmetical in $\mathcal{M}_0$.

It is easy to see that any relatively computably categorical model is computably categorical. Goncharov [1], using effective versions of Scott families, showed that the converse holds for 2-decidable models. On the other hand, Kudinov [2] showed that the converse fails even for 1-decidable models.

We extend these results as follows:

THEOREM.
(1) *Any 1-decidable computably categorical model is relatively Δ^0_2-computably categorical.*
(2) *There is a computably categorical model which is not relatively arithmetically categorical.*

It is not hard to see, using Goncharov's [1] characterization of relative computable categoricity and a simple construction, that this notion is Σ^0_3-complete. The characterization of the complexity of computable categoricity remains open, and appears very hard in light of the second part of the above theorem; in fact, while it is trivially Π^1_1, and Π^0_4-hard by White [3], it is not even known whether computable categoricity is arithmetical or not.

Part I of this talk will present definitions, background and the theorem.

[1] SERGEY S. GONCHAROV, *Autostability and computable families of constructivizations*, ***Algebra and Logic***, vol. 14 (1975), no. 6, pp. 392–409.

[2] OLEG V. KUDINOV, *An autostable 1-decidable model without a computable Scott family of ∃-formulas*, ***Algebra and Logic***, vol. 35 (1996), no. 4, pp. 255–260.

[3] WALKER M. WHITE, *On the complexity of categoricity in computable structures*, ***Mathematical Logic Quarterly***, vol. 49 (2003), no. 6, 603–614.

► RODNEY G. DOWNEY, ASHER M. KACH, AND DANIEL C. D. TURETSKY, *Comparing effective notions of categoricity, Part II.*
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This talk will continue Part I (by Steffen Lempp) by sketching the proofs of the following results:

THEOREM.
(1) *Any* 1-*decidable computably categorical model is relatively* Δ^0_2-*computably categorical.*
(2) *There is a computably categorical model which is not relatively arithmetically categorical.*

The proof of (1) proceeds by showing that the types of tuples of elements can be isolated by computable infinitary Π_1-formulas. The proof of (2) involves building intricate nested graphs.

We will also briefly discuss the difficulties in classifying the precise complexity of the index set of computably categorical models, a problem which remains open.

► MARTÍN ESCARDÓ AND PAULO OLIVA, *Instances of bar recursion as products of selection functions.*
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Abstracting from the booleans $\mathbb{B}$ to an arbitrary set R, the usual logical quantifiers $\forall, \exists\colon \mathbb{B}^{\mathbb{B}^X}$ can be viewed as particular instances of what we call *generalised quantifiers* $\phi\colon R^{R^X}$. A generalised quantifier ϕ is said to be *attainable* if for some functional $\varepsilon\colon X^{R^X}$ we have $\phi p \stackrel{R}{=} p(\varepsilon p)$, for all $p\colon R^X$. In this case, ε is called a *selection functions* for ϕ. Hilbert's epsilon terms, for instance, play precisely the role of selection functions for the existential quantifier. We define in [1] a notion of sequential game based on generalised quantifiers, and show that optimal strategies in such games can be effectively computed via a product of selection functions $\varepsilon \otimes \delta$. Such calculation of optimal strategies is at the heart of several well-known constructions, e.g., backtracking (Algorithms), backward induction (Game Theory), Bekič's lemma (Fixed Point Theory), and bar recursion (Proof Theory). Exploiting the connection with bar recursion, we have recently shown that modified bar recursion is primitive recursively equivalent to the infinite iteration of the binary product of selection functions, i.e.,

$$\mathsf{MBR}_s(\varepsilon) = \varepsilon_s \otimes \lambda x^{X_n}.\mathsf{MBR}_{s*x}(\varepsilon),$$

whereas Spector's (dialectica) interpretation of full classical analysis corresponds to a "controlled" iteration of the same product, namely

$$\mathsf{SBR}^{\omega}_s(\varepsilon) = \begin{cases} \mathbf{0} & \text{if } \omega(s * \mathbf{0}) \le |s|, \\ \varepsilon_s \otimes \lambda x^{X_n}.\mathsf{SBR}^{\omega}_{s*x}(\varepsilon) & \text{otherwise,} \end{cases}$$

where $s\colon X_0 \times \cdots \times X_{n-1}$ and $\varepsilon_s\colon X_n^{R^{X_n}}$ and $\omega\colon \Pi_{i=0}^{\infty} X_i \to \mathbb{N}$.

[1] MARTÍN ESCARDÓ AND PAULO OLIVA, *Selection functions, bar recursion and backward induction*, ***Mathematical Structures in Computer Science***, accepted for publication, Nov 2009 (preprint available `http://www.cs.bham.ac.uk/~mhe/papers/selection-escardo-oliva.pdf`).

► VINCENT GUINGONA, *Dependence, isolated extensions, and definability of types.*
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In this talk, I discuss a new characterization of dependence for formulas in terms of isolated extensions of types. I prove that a formula φ is dependent if and only if all φ-types can be extended to a φ-isolated *elementary φ-extension* (to be defined in the talk). Even in the stable setting, this is a new result. I discuss some corollaries to this result and explore parallels to definability of types in the stable setting. I conclude the talk with a discussion of a related notion I call *uniform definability of types over finite sets* (UDTFS). It is known that UDTFS implies dependence, but it is not yet known if UDTFS characterizes dependence. I exhibit some partial results toward showing characterization.

► JAAKKO HINTIKKA, *Definitions require quantifier independence*.
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Trivially, in an explicit definition the definiens must not refer to the definiendum. Less trivially, its quantifiers must be independent of the definendum. This requirement is not expressible in the usual first-order logic, only in IF logic. It is what Poincaré originally meant by the Vicious Circle Principle. Russell misinterpreted the nature of the dependence in question and interpreted it in terms of membership, the dependence of a set on its members. This is how he was led to his theory of types.

► KAREL HRBACEK, *Nonstandard analysis based on the concept of level*.
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We propose a framework for presentation of nonstandard methods that is loosely modeled on the physicists' concept of scales of magnitude. We take the point of view that every mathematical entity appears at some *level* $\mathbf{V}$. In addition to **ZFC** we postulate:

- Given $x_1, \dots, x_k$, there is a coarsest level $\mathbf{V}$ such that $x_1, \dots, x_k$ appear at $\mathbf{V}$.
- Given a level $\mathbf{V}$, there exist real numbers ε *ultrasmall relative to* $\mathbf{V}$ (i.e., such that $0 \neq |\varepsilon| < r$ for every $r > 0$ that appears at $\mathbf{V}$).
- A number, function or set that is uniquely defined (without any reference to levels) from parameters at some level $\mathbf{V}$, appears itself at the level $\mathbf{V}$.

These postulates suffice to define and calculate derivatives and integrals in the style of Leibniz. A few additional axioms make possible a fully rigorous development of the theory of "infinitesimal calculus" [2]. The approach has been classroom-tested in two high schools in Geneva [3].

Our framework is (equivalent to) a fragment of **GRIST**, an extension of *Relative Internal Set Theory* (**RIST**) proposed by Y. Péraire [4]. **GRIST** is developed in [1]; it is a conservative extension of **ZFC** that is complete and ω-categorical "over **ZFC**," and universal among theories of its kind.

This is joint work with R. O'Donovan and O. Lessmann.

[1] K. Hrbacek, *Relative set theory: Internal view*, ***Journal of Logic and Analysis***, vol. 1 (2009), no. 8, pp. 1–108.

[2] K. Hrbacek, O. Lessmann, and R. O'Donovan, *Analysis with ultrasmall numbers*, 2009, 16 pp., submitted.

[3] R. O'Donovan, *Teaching analysis with ultrasmall numbers*, ***Mathematics Teaching–Research Journal Online***, vol. 3 (2009), no. 3, pp. 1–22.

[4] Y. Péraire, *Théorie relative des ensembles internes*, ***Osaka Journal of Mathematics***, vol. 29 (1992), pp. 267–297.

► TAMARA LAKINS, *Ramsey's theorem for trees*.
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In joint work with Damir Dzhafarov (University of Chicago) and Jeff Hirst (Appalachian State University), we formulate a polarized version of Ramsey's theorem for trees. For exponents greater than two, the reverse mathematics and computability theory associated with this theorem parallels that of its linear analogue. For pairs, the situation is more complex. In particular, there are many reasonable notions of stability in the tree setting, complicating the analysis of the related results.

► BENEDIKT LÖWE, *Patterns of cofinality and measurability on small uncountable cardinals*.
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In a recent paper, Apter, Jackson and the present author determined exactly the ZF-consistent patterns of cofinality and measurability for the first three uncountable cardinals: combinatorially, there are 60 possible patterns, of which 13 are inconsistent and 47 are consistent assuming large cardinals. In this talk, we shall give an overview of these results and discuss some further results (also joint with Apter and Jackson), in particular extensions to larger patterns. We also discuss obstacles to getting an exhaustive analysis for the larger patterns.

► PETER LEFANU LUMSDAINE, *Fixed-point theorems, constructively*.
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The Bourbaki–Witt and Tarski fixed-point theorems for chain-complete posets are two minor gems of early set theory, easily and elegantly proven with transfinite induction.

In constructive logic, the situation is less simple; categorical models provide various independence results. Both theorems fail in the effective topos (a model of computability) [1], but hold in any sheaf topos (generalised forcing models). A syntactically-constructed topos shows that they are strictly weaker than typical ordinal-existence principles.

This is joint work with Andrej Bauer (Ljubljana).

[1] ANDREJ BAUER, *On the failure of fixed-point theorems for chain-complete lattices in the effective topos*, ***Electronic Notes in Theoretical Computer Science***, vol. 249 (2009), pp. 157–167.

► TYLER MARKKANEN, *Separating the degree spectra of structures*.
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In computable model theory, the (Turing) degree spectrum of a countable structure $\mathfrak{A}$ is the set $\mathrm{DgSp}(\mathfrak{A}) = \{\deg_T(\mathfrak{B}) \mid \mathfrak{A} \cong \mathfrak{B}\}$ and is one way to measure the computability-theoretic properties of $\mathfrak{A}$. Given various *classes* of structures, such as linear orders, groups, and graphs, two classes $\mathcal{K}_1$ and $\mathcal{K}_2$ can be separated in the following way: we say that $\mathcal{K}_1$ is *distinguished over* $\mathcal{K}_2$ *with respect to degree spectrum* if there is a structure $\mathfrak{A} \in \mathcal{K}_1$ such that $\mathrm{DgSp}(\mathfrak{A}) \neq \mathrm{DgSp}(\mathfrak{B})$ for all structures $\mathfrak{B} \in \mathcal{K}_2$. We will investigate this separation idea and look at specific choices for $\mathcal{K}_1$ and $\mathcal{K}_2$—for example, we can show that with respect to degree spectrum, linear orders are distinguished over finite-component graphs, equivalence structures, rank-1 torsion-free abelian groups, and daisy graphs. From these proofs, we will see a pattern for some of the structures over which linear orders are distinguished with respect to degree spectrum.

► ANDREA PEDEFERRI, *Second order categoricity and invariance*.
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Second order logic has been always considered as problematic by modern logicians: the lack of completeness and the subsequent lack of a sound deductive system seem to rule out second order from the realm of "pure" logic. However, second order logic provides, with its expressive power, the possibility to give categorical characterizations of infinite structures. Accordingly, on the one hand we do not call second order a proper logic due to its being "uncontrollable". On the other hand, we consider the Löwenheim–Skolem Theorem as a corner stone of the "controllable" first order, although it states the incapability of a theory to "control" its models.

I think limitative theorems of first order are only *desiderata*, and it is time to move on to a more general level. According to Tarski's definition of logical notions as invariants, I believe second order logic can naturally belong to logic *tout court*. Since second order logic is categorical, its deductive system is invariant for all the models of a theory formalized in second order logic. By following Tarski's definition this deductive system could then be called logical. It seems to me that this grasps the real sense of what "logical" means and thus it allows the problems related to the completeness to be overcome.

► LYNN SCOW, *The relationship of Ramsey classes to the modeling property for certain generalized indiscernible sequences*.
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For any infinite set C in a model, we can find an indiscernible sequence such that the n-types of the sequence come from n-types realized in C. This is an instance of the "modeling property" for indiscernible sequences, and it results from an application of compactness and the finite Ramsey theorem for sets.

More general indiscernible sequences were introduced in [3] and have found recent interest in [1] in the form of ordered graph indiscernibles. These generalized indiscernible sequences also have their version of the "modeling property", resulting from a Ramsey theorem specific for ordered graphs. Nešetřil's notion of a *Ramsey Class* is particularly helpful in understanding this relationship. We will sketch the case of ordered graphs and provide a more general result that relates Ramsey classes to the modeling property for certain generalized indiscernible sequences.

[1] M. C. Laskowski and S. Shelah, *Karp complexity and classes with the independence property*, ***Annals of Pure and Applied Logic***, vol. 120 (2003), pp. 263–283.

[2] J. Nešetřil, *Homogeneous structures and Ramsey classes*, ***Combinatorics, Probability and Computing***, vol. 14 (2005), pp. 171–189.

[3] Saharon Shelah, ***Classification theory and the number of non-isomorphic models***, North-Holland, 1978.

► PAUL SHAFER, *Coding second-order arithmetic into the closed Medvedev degrees*.
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For sets $\mathcal{A}, \mathcal{B} \subseteq \omega^\omega$, we say $\mathcal{A}$ *Medvedev reduces* to $\mathcal{B}$ ($\mathcal{A} \leq_M \mathcal{B}$) if there is a Turing functional Φ such that Φ^f is total and in $\mathcal{A}$ for all f in $\mathcal{B}$. We say $\mathcal{A}, \mathcal{B} \subseteq \omega^\omega$ are *Medvedev equivalent* ($\mathcal{A} \equiv_M \mathcal{B}$) if $\mathcal{A} \leq_M \mathcal{B}$ and $\mathcal{B} \leq_M \mathcal{A}$. The *Medvedev lattice* is the degree structure $(\mathcal{P}(\omega^\omega)/\equiv_M, \leq_M)$. Muchnik reducibility is the non-uniform version of Medvedev reducibility: $\mathcal{A}$ *Muchnik reduces* to $\mathcal{B}$ ($\mathcal{A} \leq_w \mathcal{B}$) if for all $f \in \mathcal{B}$ there is a $g \in \mathcal{A}$ such that $g \leq_T f$. The *Muchnik lattice* is the degree structure $(\mathcal{P}(\omega^\omega)/\equiv_w, \leq_w)$.

Lewis, Nies, and Sorbi [1] and Shafer independently proved that the first-order theories of the Medvedev degrees and the Muchnik degrees are recursively isomorphic to the third-order theory of arithmetic. We continue in this vein by restricting our attention to *closed* degrees (those degrees with representatives closed in ω^ω) and *compact* degrees (those degrees with representatives compact in ω^ω). We prove that the first-order theories of the closed Medvedev degrees, the compact Medvedev degrees, the closed Muchnik degrees, and the compact Muchnik degrees are recursively isomorphic to the second-order theory of arithmetic. This talk will focus on the closed Medvedev degrees case.

[1] ANDREW E. M. LEWIS, ANDRÉ NIES, AND ANDREA SORBI, *The first order theories of the Medvedev and Muchnik lattices*, ***CiE 2009: Mathematical Theory and Computational Practice,*** (***Heidelberg, Germany***), (Klaus Ambos-Spies, Benedikt Löwe and Wolfgang Merkle, editors), vol. 5635, Springer, 2009, pp. 324–331.

► AHUVA C. SHKOP, *Schanuel's conjecture and algebraic roots of exponential polynomials*.
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In this paper, I will prove that assuming Schanuel's conjecture, an exponential polynomial with algebraic coefficients can have only finitely many algebraic roots. Furthermore, this proof demonstrates that there are no unexpected algebraic roots of any exponential polynomial. This implies a special case of Shapiro's conjecture: if $p(x)$ and $q(x)$ are two such exponential polynomials with algebraic coefficients which have common factors only of the form $\exp(g)$ for some exponential polynomial g, p and q have only finitely many common zeros.

[1] ANGUS MACINTYRE, *Schanuel's conjecture and free exponential rings*, ***Annals of Pure and Applied Logic***, vol. 51 (1991), no. 3, pp. 241–246.

[2] H. S. SHAPIRO, *The expansion of mean-periodic functions in series of exponentials*, ***Communications on Pure and Applied Mathematics***, vol. 11 (1958), pp. 1–21.

[3] AHUVA C. SHKOP, *Henson and Rubel's theorem for Zilber's pseudoexponentiation*.

[4] LOU VAN DEN DRIES, *Exponential rings, exponential polynomials and exponential functions*, ***Pacific Journal of Mathematics***, vol. 113 (1984), no. 1, pp. 51–66.

[5] BORIS ZILBER, *Pseudo-exponentiation on algebraically closed fields of characteristic zero*, ***Annals of Pure and Applied Logic***, vol. 132 (2005), no. 1, pp. 67–95.

► MATTHEW P. SZUDZIK, *The computable universe hypothesis*.
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When can a model of a physical system be regarded as computable? We provide the definition of a *computable physical model* to answer this question, comparing and contrasting our definition to Kreisel's notion of a mechanistic theory [1]. Several examples of computable physical models are given, including models which feature discrete motion, models which feature non-discrete continuous motion, and probabilistic models such as radioactive decay. Various common operations on computable physical models are discussed, such as the operation of coarse-graining, the formation of statistical ensembles, and the use of quantum mechanical operators.

The definition of a computable physical model also allows us to precisely formulate the conjecture that all the laws of physics are computable. We call this conjecture the *computable universe hypothesis*.

[1] G. KREISEL, *A notion of mechanistic theory*, ***Synthese***, vol. 29 (1974), pp. 11–26.

► JAMES WORTHINGTON, *Determinizing, forgetting, and automata in monoidal categories*.
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We encode automata as representations of monoids in monoidal categories, which explains the strong connections between automata and bialgebras. Using this framework, we provide adjunctions between categories of weighted or nondeterministic automata and categories of deterministic automata. One component of this adjunction is essentially a forgetful functor from a category of semimodules to **Set**. Using these adjunctions, and final objects in categories of deterministic automata, we provide a complete proof system for the equivalence of certain automata. In this proof system, the only "rules of inference" are morphisms of actions. We also discuss the complexity of constructing these proofs in certain cases, as well as determinizing alternating automata.

Abstracts of talks presented by title

► JOHN CORCORAN, *Boole's expansion principle*.
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Boole's *Expansion Principle BEP* is one of his revolutionary innovations now accepted in mainstream logic (*Principia Mathematica* *4.42–4.45). BEP implies that, for an arbitrary proposition, no matter how few its non-logical concepts, there is a logically equivalent proposition containing all non-logical concepts, no matter how many, of any given pertinent proposition. Take "One is even" as the arbitrary proposition and let the given proposition be "Two precedes three plus four". "One is even" is logically equivalent to "*Either* one is even and two precedes three plus four *or* one is even but two does not precede three plus four". Analogs hold in many modern logics. Exceptions include some "relevance logics" designed to restrict "consequences" of a given proposition to propositions containing only non-logical concepts in the given proposition, thus banning classical non-exclusive disjunction. BEP also implies that the set of logical equivalents of an arbitrary proposition contains occurrences of all non-logical concepts pertinent to the domain of investigation. The logical-equivalence class of "Zero is zero" contains occurrences of "odd", "even", "prime", "perfect", "divisor", "the successor of", etc. BEP is a conceptual or *intensional* complement to Boole's extensional *Principle of Wholistic Reference*: each proposition refers to the whole universe of discourse as such, regardless how limited the referents of its non-logical terms (This BULLETIN vol. 12 (2006), pp. 515–516). For example, Aristotle's "Every square is a rectangle", considered as a proposition of a general theory of geometric figures, would be treated by Boole as "Being a figure that is square is being a figure that is rectangular that is square" where 'figure' names the universe of geometrical discourse. With 1 as the universe of geometrical figures, a Boolean equation for this is:

$$(1 \cdot s) = ((1 \cdot r) \cdot s).$$

► JOHN CORCORAN AND GEORGE BOGER, *Protasis in Prior Analytics*: *Proposition or premise*?
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In some senses of *argument*, an argument is a system containing a constituent set of propositions called its *premises* and a constituent single proposition called its *conclusion*. Typically, texts presenting arguments begin with premise representations and end with conclusion representations—reflecting etymologies of *premise* and *conclusion*.

In artifically narrow senses of *proposition*, a proposition is a system containing a constituent term called its *subject* and a constituent term called its *predicate*. The following is an extended analogy.

As *argument* is to *premise* and to *conclusion*,
so *proposition* is to *subject* and to *predicate*, respectively.

Argument and *proposition* are sortal nouns like *term*: the other words are relational nouns like *constituent*. They do not indicate sorts but rather are used to express relations from one sort to another. Calling something a premise (or conclusion) is incomplete—without including *of which argument or arguments*. In a broader sense, *proposition* is coextensive but not synonymous with *premise* (or *conclusion*) *of some argument*. Calling something a subject (or predicate) is incomplete—without including *of which proposition or propositions*. In certain contexts, *term* is coextensive but not synonymous with *subject* (or *predicate*) *of some proposition*.

The word *pro-tasis* is etymologically a near equivalent of *pre-mise*, *pro-position*, and *antecedent*—all having positional, relational connotations now totally absent in contemporary use of *proposition*. Taking *protasis* for *premise*, Aristotle's statement (24a16)

A protasis is a sentence affirming or denying something of something ...

is not a definition of *premise*—intensionally: the relational feature is absent. Likewise, it is not a general definition of *proposition*—extensionally: it is too narrow. This paper explores recent literature on these issues.

► JOHN CORCORAN AND RICHARD MAIN, *Numerically-indexed Alternative Constituent Format*.

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This BULLETIN vol. 15 (2009), p. 133, discussed the *Alternative Constituent Format* or *ACF* for presenting linguistic and logical data. A string in the ACF is called an *Alternative Constituent String* or *ACS*.

ACS1. (Zero * One* Two) is closer to four than (six * to six * six is).

Above, nine interrelated sentences are compactly presented for easy comparison in one ACS. The three with 'six' are structurally ambiguous; the three with 'to six' are true; and the three with 'six is' are false.

In this lecture we numerically index the sentences presented in an ACS, thereby converting it into a *Numerically-indexed ACS*, or *NACS*.

NACS1. (Zero 123*One 456*Two 789)
is closer to four than (six 147*to six 258*six is 369).

The first occurrence of a given numeral indicates one "choice" among the first set of alternative constituents and the second occurrence indicates one "choice" among the second set. The sentence chosen using '5' is:

NACS1.5. One is closer to four than to six.

Numerical indexing facilitates discussion of the data and issues presented. Moreover, it allows reducing the number of "choices" in an ACS. If the NACF is used for testing in a logic course, grading is simplified. NACF tests can be machine graded. For example, the correct answer to the following question is: 1, 4.

Q1. The Law of (Excluded Middle 12* Non-Contradiction 34) is:
"(every 13* no 24) proposition is (either13 * both 24) true (or 13* and 24) false".

To answer, write the numbers (or fill-in the bubbles) of the sentences accepted. This lecture presents many additional and more challenging questions on classical logic, formalized arithmetic, set theory, and string-concatenation theory.

► CYRUS F. NOURANI, *Fragment Kleene models on product languages*.
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Fragment consistent model techniques (Author 1996–2005) are applied to generate Kleene models.

Definition. A preorder $\ll$ on a Σ-algebra A is said to be morphic iff for every $\sigma \varepsilon \Sigma$. and ai, bi εA_{si}, and B_{si}, respectively, If $a_i \ll b_i$ for i ε [n] then, $\sigma_A(a1, \ldots, an) \ll \sigma_B(b1, \ldots, bn)$.

Kleene structures have been computing language Models interest, for example, regular expression languages. The obvious structural properties are monoidal with commutative idempotent operations that are Kleene-$*$ closed.

Proposition. Kleene structures can be granted with an initial model characterization with morphic preorders.

A String ISL algebra (Author 2005) is a Σ-algebra with an additional property that the signature Σ's has a subsignature Λ that is only on 1–1 functions. A tree game degree is the game state a tree is at with respect a model truth assignment, e.g., to the parameters on the Boolean functions on a game tree.

Theorem. Let T be a ISL language theory. T is
(a) a sound logical theory iff every axiom or proof rule in T preserves the tree game degree;
(b) a complete logical theory iff there is a function–set pair $\langle$F,S$\rangle$ defining a canonical structure M such that M has a generic diagram definable with the functions F.

Say that a Kleene ISL algebra is an algebra A (A,+,0,.,1,$*$) such that (A,+,0) and (A,;,1) are monoids, with + commutative and idempotent, and Kleene $*$-closed.

Lemma 2. String ISL algebra homomorphically extending a algebra A (A,+,0,.,1,$*$) such that (A,+,0) and (A,;,1) are monoids, with + commutative and idempotent, is Kleene.

► DOLPH ULRICH, *New results concerning single axioms for four subsystems of **BCI***.
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Where $\mathbf{B} = CCpqCCrpCrq$, $\mathbf{C}^{011} = CCpCCqtCruCCqtCpCru$, and $\mathbf{I} = Cpp$, the author reported in [1] the 15-symbol single axiom *CCpqCCCrrCqsCps* for the substitution–detachment system with **B**, $\mathbf{C}^{011}$, and **I** as axioms, noting that no shorter single axiom for $\mathbf{BC}^{011}\mathbf{I}$ exists. He can now add that this formula is in fact unique: it is the *only* single axiom for that system of length 15.

With $\mathbf{C}^{001} = CCpCqCruCqCpCru$ in place of $\mathbf{C}^{011}$, there are two more single axioms of length 19 for $\mathbf{BC}^{001}\mathbf{I}$ in addition to the axiom *CpCCqrCCpCCssCrtCqt* provided in [1], namely *CpCCpCqrCCCssCrtCqt* and *CCCppCqrCsCCsCrtCqt*. No shorter theorems of $\mathbf{BC}^{001}\mathbf{I}$ are single axioms for it, so there are exactly three shortest possible single axioms for this system.

With $\mathbf{C}^{\mathbf{010}} = CCpCCqtrCCqtCpr$, to the single axiom $CCpqCCCCCrrssCqtCpt$ reported in [1] for $\mathbf{BC}^{\mathbf{010}}\mathbf{I}$ can be added $CCCppCqrCCrsCCCqstt$, $CCCCpqCCCrrCqsCpstt$, $CCpqCCCrrCqsCCCpstt$, and $CCpqCCCCCrrCqsCpstt$. Again, these are shortest possible, and the list is exhaustive: no other formulas of the same length will do as single axioms for $\mathbf{BC}^{\mathbf{010}}\mathbf{I}$.

Finally, for $\mathbf{BC}^{\mathbf{001}}\mathbf{C}^{\mathbf{010}}\mathbf{I}$, the 27-symbol single axiom given in [1] can be improved. Each of the following seven formulas is a single axiom for this system of length 23: $CCCCpqCCrCqs-CpsCCttuCru$, $CCCpCCpCqrCsrCCttuCCsqu$, $CCCCpCqrCCsqCsrCCttuCpu$, $CCCCp-CqrCsrCCttuCCsqCpu$, $CCCppCqrCsCCCCsCrtCqtuu$, $CCpqCCCCrCqsCpsCCttuCru$, and $CpCCCCpCqrCsrCCttuCCsqu$. No shorter single axioms for $\mathbf{BC}^{\mathbf{001}}\mathbf{C}^{\mathbf{010}}\mathbf{I}$ can be found, but the author conjectures that there exist additional hitherto undiscovered single axioms of this same length.

[1] D. ULRICH, *Single axioms for a class of substructural logics contained in* ***BCI***, ***The Bulletin of Symbolic Logic***, vol. 12 (2006), p. 166.

► XUNWEI ZHOU, *Mutually-inversistic granular computing*.
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Granular computing is based on fuzzy set, rough set, interval analysis, etc. The author constructs mutually-inversistic granular computing, including mutually-inversistic fuzzy logic, mutually-inversistic interval logic, mutually-inversistic rough set, mutually-inversistic fuzzy rough set, mutually-inversistic rough fuzzy logic. They are all based on mutually-inversistic logic and set theory the author constructs. Mutually-inversistic fuzzy logic is a fuzzy extension of mutually-inversistic logic, it can mine fuzzy association rules. Mutually-inversistic interval logic is the extension of mutually-inversistic fuzzy logic to intervals, it can mine interval association rules. Mutually-inversistic rough set is formed by constructing rough set on mutually-inversistic set theory, defining lower approximation by $\subseteq^{-1}$ ($\subseteq^{-1}$ is mutually inverse being contained connective, $P \subseteq^{-1} Q$ means all Ps are Qs), upper approximation by $|\cap^{-1}$ ($|\cap^{-1}$ is mutually inverse intersection connective, $P| \cap^{-1} Q$ means some Ps are Qs). Mutually-inversistic rough set can mine not only decision rules but also mutually inverse intersection connections. Fuzzy rough set and rough fuzzy set are the integrations of fuzzy set and rough set. Their cores are the same, that is, the lower and upper approximations of a fuzzy set. Mutually-inversistic fuzzy rough set is the integration of mutually-inversistic rough set and fuzzy rough set, it can mine not only decision rules with membership grade but also mutually inverse intersection connections with membership grade. Mutually-inversistic rough fuzzy logic is the integration of mutually-inversistic fuzzy logic and rough fuzzy set, it can mine the lower and upper approximations of the strength of support of the partition fuzzy association rules.