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2009–2010 WINTER MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

Moscone Center West, San Francisco, CA
January 15–16, 2010

The 2009–2010 Winter Meeting of Association of Symbolic Logic was held on January 15–16, 2010 at the Moscone Center West, in conjunction with the annual Joint Mathematical Meetings. The members of the Program Committee were Michael Beeson, Bjørn Kjos-Hanssen, and John Steel (Chair). The program consisted of seven invited 50-minute talks and 9 contributed talks. The ASL hosted a welcoming reception on January 15 at the Marriott Hotel. In addition, there was a joint AMS-ASL Special Session, **Surreal numbers**, organized by Philip Ehrlich and Lou van den Dries, on January 13, and an ASL–AMS–MAA co-sponsored panel discussion on Hilbert's 10th problem on January 14.

The 50-minute invited addresses were:

Fernando J. Ferreira (Universidade de Lisboa), *Bar-recursive interpretations of classical analysis*.

John Harrison (Intel Corporation), *Decidability and undecidability in theories of real vector spaces*.

François Loeser (Ecole Normale Superieure), *Model theory and non-Archimedean tame topology, with a view towards Berkovich spaces*.

Chris Miller (Ohio State University), *Expansions of o-minimal structures by trajectories of definable vector fields*.

Joseph S. Miller (University of Wisconsin-Madison), *Randomness and computational strength*.

Slawomir J. Solecki (University of Illinois at Urbana-Champaign), *Ramsey theory for finite structures*.

Stevo Todorcevic (University of Paris 7 and University of Toronto), *Tukey reducibility as a setup for classifying mathematical structures*.

Abstracts of the invited talks and contributed talks given (in person or by title) by members of the Association for Symbolic Logic follow.

For the Program Committee
JOHN STEEL

Abstracts of invited talks

► FERNANDO FERREIRA, *Bar-recursive interpretations of classical analysis*.
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Following the work of K. Gödel published in 1958, C. Spector gave in 1962 a remarkable characterization of the provably recursive functionals of full second-order arithmetic (a.k.a. analysis). Spector's interpretation relies on a form of well-founded recursion known as bar-recursion. Spector's interpretation is rather indirect, via the negative translation. We re-prove Spector's result avoiding the passage through intuitionistic logic, relying instead on the very simple direct interpretation of Peano arithmetic given by J. Shoenfield in 1967. More specifically, finite-type Peano arithmetic, extended with the bar-recursive functionals and the quantifier-free axiom of choice, proves the principle of dependent choices and, therefore, full numerical comprehension.

The bounded functional interpretation was introduced in 2005 by P. Oliva and the present author. In its version for classical arithmetic, it is an interpretation which "injects" some non set-theoretical uniformities into Peano arithmetic. The interpretation refutes extensionality as well as simple forms of comprehension for number theoretical functions. Nevertheless, due to its Soundness Theorem, the interpretation is able to extract correct uniform bounds from certain theorems of mathematics (and this opens the way to possible contributions to Proof Mining).

We recently showed that the bounded functional interpretation is compatible (via bar-recursive functionals) with full numerical comprehension.

The above reported work is in collaboration with P. Engrácia.

► JOHN HARRISON, *Decidability and undecidability in theories of real vector spaces*.
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It's natural to formulate theories of real vector spaces using a 2-sorted first-order language with a sort for the scalars and a sort for the vectors. Introduction of coordinates reduces the theory of a vector space of a specific finite dimension to the first-order theory of the real numbers, known to be decidable since Tarski. Experience in the actual formalization of mathematics motivates an investigation into decision problems for various more general 2-sorted first-order theories of vector spaces, with or without an inner product, norm, assumption of completeness, or restriction on dimension.

Solovay, Arthan and the speaker have carried out a systematic study of decidability for such theories. The theories of real vector spaces, inner product spaces, and Hilbert spaces turn out to be decidable and to admit quantifier elimination in a slightly expanded language. However, similar theories of normed spaces, Banach spaces and metric spaces are not even arithmetical: the theory of 2-dimensional Banach spaces, for example, has the same many-one degree as the set of truths of second-order arithmetic. However, by restricting quantifier alternations, we can arrive at some decidable fragments of these theories, a fact that has proved useful in mechanizing proofs.

Joint work with Robert M. Solovay and Rob Arthan.

► FRANÇOIS LOESER, *Model theory and non-Archimedean tame topology, with a view towards Berkovich spaces*.
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In the model theory of algebraically closed valued fields, certain types—called stably dominated—started to play an important role after recent work of Haskell, Hrushovski and Macpherson. These types are, in a sense, controlled by their stable part. In this talk I will present joint work with E. Hrushovski devoted to the study of the topology of spaces of stably dominated types. It follows form our results that these spaces can be considered as tame in Grothendieck's sense. When working over a field with a real valued norm, they are closely related to Berkovich spaces. This allows us to deduce tameness results for the topology of Berkovich spaces that are analytifications of algebraic varieties. In particular we show that such spaces retract onto finite CW-complexes.

► CHRIS MILLER, *Expansions of o-minimal structures by trajectories of definable vector fields*.
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An expansion of the real field is said to be o-minimal if every definable set has finitely many connected components. Such structures are a natural setting for studying "tame" objects of real analytic geometry such as nonoscillatory trajectories of real analytic planar vector fields. More generally, o-minimality is preserved under expanding by nonoscillatory trajectories of definable planar vector fields. But what happens when o-minimality is not preserved? We see in some cases the best behavior that we could reasonably hope for, while in some others the worst possible, and we do not know at present of any other outcomes. I will make all this precise in a survey of the current state of the art.

► JOSEPH S. MILLER, *Randomness and computational strength*.
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One of the themes that has emerged in the study of effective randomness is that there is a negative correlation between randomness and computability-theoretic strength. Not only are more random reals less useful as oracles, but assuming that a random real is computationally weak implies a greater degree of randomness. We will explore the evidence for these assertions and look at some of the reasons why this inverse relationship should not be entirely surprising. As part of understanding this pattern, we will consider what it means for one real to be "more random" than another.

► SLAWOMIR SOLECKI, *Ramsey theory for finite structures*.
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I will present a self-dual Ramsey theory for structures equipped not only with relations, but also with functions. These results expand and build on the work of Ramsey, Nesetril, Rodl, Abramson and Harrington on the direct relational theory and of Graham, Rothschild and Promel on the dual relational theory.

► STEVO TODORCEVIC, *Tukey reducibility as a setup for classifying mathematical structures*.
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This will be an overview of recent advances on the Tukey classification program of mathematical structures such as graphs, partial orders, ultrafilters, etc. We will also expose some possible lines of further research and some open problems.

Abstracts of contributed talks

► NATE ACKERMAN, *Trees, sheaves and definition by recursion*.
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We will show there is a topological space for which presheaves are the same thing as trees. We will further show that there is a sheaf on this topological space which has an important relationship with Baire space.

We will then use these connections to show how a definition by transfinite recursion can be thought of as an operation on sheaves, and how the well-definedness of such a definition can be thought of as a property of the sheaf we are working on.

This will then allow us to define a second order tree as a sheaf on a tree and to expand our notion of definition by transfinite recursion to all well-founded second order trees (even those which are ill-founded as normal trees).

We will then mention how these techniques can be used to prove a variant of the Suslin-Kleene Separation theorem.

► NATHANAEL L. ACKERMAN, CAMERON E. FREER, AND DANIEL M. ROY, *The computability of conditional probability distributions*.
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Computing conditional probabilities with respect to measure zero events is a fundamental operation in Bayesian analysis. Constructive definitions (e.g., [2] and [1]) often involve limits that are not necessarily computable, or are limited to special cases.

We examine regular conditional probability distributions of computable random variables. A coding argument shows that for every Δ^0_2 set X, there is a computable random variable whose regular conditional distribution computes X. On the other hand, standard results on martingales can be used to show that such regular conditional distributions are uniformly $0'$-computable.

We also discuss circumstances in which conditioning is computable, and describe the relationship between conditioning and computability of the probability density.

[1] M. M. RAO, ***Conditional measures and applications***, 2nd ed., Pure and Applied Mathematics, CRC Press, Boca Raton, FL, 2005.

[2] TUE TJUR, *A constructive definition of conditional distributions*, Institute of Mathematical Statistics, University of Copenhagen, preprint no. 13, 1975.

► WESLEY CALVERT, *Probabilistic computation and stochastic processes*.
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We present a foundation for the effective theory of random variables and stochastic processes. To accomplish this, we show that certain elements of the Itô stochastic calculus have a natural expression in an extension of continuous first-order (CFO) logic and are effectively true in the sense of randomized computation. In particular, the Itô stochastic integral of a computable stochastic process is also computable, and the existence of solutions to a stochastic differential equation holds in the effective category.

► BILLY JOE LUCAS, *A bimodal deontic evidential logic for the right to prove truths*.
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Let L be a bimodal language for a normal concept of deontic obligation (with O as box, and P as $\sim O \sim$) and a normal concept of proof (sufficient evidence), with uE as (an agent indexed) box. Is it a *conceptual* truth that one has a right (absence of moral prohibition) to prove any truth? The bimodal logic RPT provides a yes answer to the question(s).

DEFINITION. RPT: $p \vdash PuEp$.

THEOREM 1. *A universal frame* $\langle W, W^{\circ}, W^{uE}, R^{\circ}, R^{uE} \rangle$ *is canonical for RPT iff* $\forall x \, \exists y [xR^{\circ}y \cdot \forall z(yR^{uE}z \supset z = x)]$.

PROOF. Let x be an arbitrary canonical set of wffs $\in W$, and α be an arbitrary $\in x$. By $PuE\alpha \in x$, $\exists y xR^{\circ}y$ (thus $y \in W^{\circ}$) and $uE\alpha \in y$. Now let z be an arbitrary canonical set of wffs such that $yR^{uE}z$, and we get $\alpha \in z$. ⊣

THEOREM 2. $\vdash_{RPT} O(uEp \supset \langle uE \rangle uEp) \supset (p \supset uEp)$.

In words, it being impermissible that you have a proof you cannot prove you have implies you have a proof of every truth. (Moreover, not $\Vdash_{RPT} p \supset uEp$.) Of note is that the proof imposes no special conditions on uE or O.

THEOREM 3. *For* $uE \geq KD \Vdash_{RPT} \sim P(uEp \cdot uE \sim PuEp)$.

In words, one has no right to prove both p yet that one is deontically forbidden to prove p. In better words: If there is an exception to your RPT, you have no right to prove that fact. Thus, *modulo* choice of logic for a concept of proof, RPT proves one has no right to refute RPT.

► GRIGORI MINTS, *A simple epsilon substitution method for ID1*.
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Epsilon substitution method was introduced by Hilbert as a basic tool of his consistency program. Its extension to the impredicative case of ID1 was outlined in my paper [2] and carried out by T. Arai [1]. Unfortunately this formulation and termination proof are very complicated. In this talk much simpler formulation and termination proof are presented. Simplification is based on simple recursion-theoretic considerations.

[1] T. ARAI, *Epsilon substitution method for* $ID_1(\Pi^0_1 \vee \Sigma^0_1)$, ***Annals of Pure and Applied Logic***, vol. 121, no. 2–3, pp. 162–208.

[2] G. MINTS, *An approach to an epsilon-substitution method for ID1*, preprint, Institute Mittag-Leffler, no. 45, MLI, Stockholm 2001.

► NICHOLAS RUPPRECHT, *"Universal" Schnorr tests*.
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A Schnorr test relative to some oracle A may informally be called "universal" if it covers all Schnorr tests. Since no true universal Schnorr test exists, such an A cannot be computable. We prove that the sets with this property are exactly those with high Turing degree. Our method is closely related to the proof of Terwijn and Zambella's characterization of the oracles which are low for Schnorr tests [1]. We also consider the oracles which compute relativized Schnorr tests with the weaker property of covering all computable reals. The degrees of these oracles strictly include the hyperimmune degrees and are strictly included in the degrees not low for Schnorr tests; the latter separation is proved using a set theoretic forcing argument.

[1] SEBASTIAAN A. TERWIJN AND DOMENICO ZAMBELLA, *Computational randomness and lowness*, ***The Journal of Symbolic Logic***, vol. 66 (2001), no. 3, pp. 1199–1205.

► LYNN SCOW, *Characterizations of stable and NIP theories by generalized indiscernible sequences*.
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Shelah's characterization of stable theories is well known: a theory is stable if and only if in a large enough model of the theory, any indiscernible sequence is an indiscernible set. [1] In this talk I will provide characterizations of both stable and NIP theories using generalizations of the above characterization.

[1] SAHARON SHELAH, ***Classification theory and the number of non-isomorphic models***, North-Holland, 1978.

► DEMIRHAN TUNC, *Expansion of Presburger Arithmetic with a predicate for* $\lfloor \sqrt{2}x \rfloor$.
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I will discuss quantifier elimination and decidability results for the structure

$$(\mathbb{Z}, +, <, \lfloor \sqrt{2}x \rfloor)$$

where $\lfloor \sqrt{2}x \rfloor$ denotes the predicate for the greatest integer less than $\sqrt{2}$ multiplied by x for every integer x. Similar structures appear naturally in expansions of the field of reals by multiplicative subgroups.

► AMY TURLINGTON, *Computable dimension of Heyting algebras*.
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The computable dimension of a computable structure is the number of (classically isomorphic) computable copies of the structure up to computable isomorphism. The possible values for the computable dimension of computable (general) lattices and computable Boolean algebras are known. In this talk we look at the computable dimension of intermediate lattices which are distributive but not Boolean algebras.

In particular, we consider Heyting algebras, which arise as models of intuitionistic logic. Heyting algebras may be thought of as "weaker" Boolean algebras in the sense that each element does not necessarily have a complement.

Abstracts of papers submitted by title

► JOHN CORCORAN, *The proposition: a historical survey*.
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The proposition has been taken as the object of belief, disbelief, understanding, and doubt: generally as *object* of *propositional attitudes*, that which can be said to be believed, disbelieved, understood, etc. It has also been taken as the object of grasping, judging, assuming, affirming, denying, and inquiring: generally as *object* of *propositional actions*, that which can be said to be grasped, judged true or false, assumed for reasoning purposes, etc. Each such attitude or action is mental and subjective: each is a certain person's attitude or action. A person begins, performs, and ends an action in a limited time interval; a person's attitude is established at a time but then persists. Attitudes are sometimes established through actions: judging a proposition establishes an attitude of belief or disbelief. The proposition has also been taken as the subject of truth and falsity: generally as the *subject* of *propositional properties*, that which can be said to be true, false, tautological, informative, inconsistent, etc. It has also been taken as the *subject* and *object* of *logical relations*, e.g., that which can be said to imply, be implied, contradict, be contradicted, etc. *Prima facie*, such properties and relations are non-mental and objective. Certain forms of these approaches complement one another. The proposition has also been taken as certain declarative *sentences* and as certain *sentence meanings*. Each approach admits of variations: e.g., in some cases a propositional attitude is thought to *presuppose* prior existence of its proposition—its object. In other cases propositions are thought to be *produced* by attitudes or in some other way to depend for their existence on attitudes. This paper treats the above and other views of the proposition throughout the history of logic.

► JOHN CORCORAN AND GEORGE BOGER, *Stating and misstating implications and consequences*.
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The proposition "$(0 + 0) = 0$" logically implies "$(0 + 0) = (0 + (0 + 0))$": the latter is a logical consequence of the former. The previous sentence uses modern conventions to state one *implication* and one *consequence*. The following six sentences might be used in misstating the above implication.

The truth of "$(0 + 0) = 0$" implies the truth of "$(0 + 0) = (0 + (0 + 0))$".
If "$(0 + 0) = 0$" is true, then "$(0 + 0) = (0 + (0 + 0))$" will also be true.
If "$(0 + 0) = 0$" were true, then "$(0 + 0) = (0 + (0 + 0))$" would be true.
"$(0 + 0) = 0$" guarantees "$(0 + 0) = (0 + (0 + 0))$".
$(0 + 0) = 0$; therefore, $(0 + 0) = (0 + (0 + 0))$.
If $(0 + 0) = 0$, then $(0 + 0) = (0 + (0 + 0))$.

We survey attempts to state implications and consequences from ancient to modern times and critically evaluate their philosophical presuppositions. To initiate logic as a discipline, it was necessary first to notice implications and consequences and then to devise locutions for stating them and methods for establishing them.

It was also necessary to notice non-implications and non-consequences and then to devise appropriate locutions and methods. "$(0+0) = 0$" is not a logical consequence of "$(0+0) = (0 + (0 + 0))$": the latter does not imply the former. The previous sentence states one *non-consequence* and one *non-implication*. We treat Aristotle, the Stoics, Ockham, Boole, Russell, Tarski, and others.

► JOHN CORCORAN AND LEONARD JACUZZO, *Allegations of the impossibility of counter-model independence proofs*.
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Two related problem types are central to logic: consequence problems and independence problems. Both concern a "given" set of propositions, the *premises*, and a "given" single proposition, the *conclusion*. No assumptions are made concerning the truth-values of the "given" propositions. *Consequence problems* have the form: to show that the conclusion is a consequence of the premise set—if it is. *Independence problems* have the form: to show that the conclusion is not a consequence of the premise set—if it is not. Traditionally, a proposition not a consequence of a set of propositions is said to be *independent* of the latter.

By a lengthy *deduction* that Andrew Wiles discovered, the Fermat conjecture was shown to be a consequence of arithmetic axioms. Consequence problems were solved by *deduction*: deducing the conclusion from the premises using a series of steps each conforming to accepted "rules of deduction". In sharp asymmetry, by means of *reinterpreting* 'number', 'zero', and 'successor' so as to produce true propositions from the Zero and Successor Axioms and a false proposition from the Mathematical Induction Axiom, the latter was shown to be independent of the other two axioms of Gödel's 1931 axiomatization of arithmetic. Independence problems were solved by *reinterpretation*: reinterpreting the non-logical constants so as to produce true premises and false conclusion.

However, some accomplished logicians allege that all such "counter-model" independence proofs are fallacious. More radically, other recognized scholars argue that no independence proofs whatever are possible unless the given premises are known to be true and the conclusion false. This lecture identifies mistakes underlying these and similar allegations by Frege, Łukasiewicz, Oliver, Massey, and others.