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REVIEWS

The Association for Symbolic Logic publishes analytical reviews of selected books and articles in the field of symbolic logic. The reviews were published in ***The Journal of Symbolic Logic*** from the founding of the JOURNAL in 1936 until the end of 1999. The Association moved the reviews to this BULLETIN, beginning in 2000.

The Reviews Section is edited by Steve Awodey (Managing Editor), John Baldwin, John Burgess, Mark Colyvan, Anuj Dawar, Marcelo Fiore, Noam Greenberg, Hannes Leitgeb, Ernest Schimmerling, Carsten Schürmann, Kai Wehmeier, and Matthias Wille. Authors and publishers are requested to send, for review, copies of books to *ASL, Box 742, Vassar College, 124 Raymond Avenue, Poughkeepsie, NY 12604, USA.*

In a review, a reference "JSL XLIII 148," for example, refers either to the publication reviewed on page 148 of volume 43 of the JOURNAL, or to the review itself (which contains full bibliographical information for the reviewed publication). Analogously, a reference "BSL VII 376" refers to the review beginning on page 376 in volume 7 of this BULLETIN, or to the publication there reviewed. "JSL LV 347" refers to one of the reviews or one of the publications reviewed or listed on page 347 of volume 55 of the JOURNAL, with reliance on the context to show which one is meant. The reference "JSL LIII 318(3)" is to the third item on page 318 of volume 53 of the JOURNAL, that is, to van Heijenoort's *Frege and vagueness*, and "JSL LX 684(8)" refers to the eighth item on page 684 of volume 60 of the JOURNAL, that is, to Tarski's *Truth and proof*.

References such as 49***5*** or 280***1*** are to entries so numbered in *A bibliography of symbolic logic* (the JOURNAL, vol. 1, pp. 121–218).

GREG HJORTH. ***Classification and orbit equivalence relations.*** Mathematical Surveys and Monographs, vol. 75. American Mathematical Society, Providence, RI, 2000, xviii + 195 pp.

GREG HJORTH. *A dichotomy theorem for turbulence*. ***The Journal of Symbolic Logic***, vol. 67 no. 4 (2002), pp. 1520–1540.

We shall be considering the topic of Borel reducibility of $\mathbf{\Sigma}^1_1$ equivalence relations and some closely related topics. It is difficult to describe exactly how this part of mathematics fits into the whole of mathematics. It is a subfield of descriptive set theory, which, in turn, is a subfield of mathematical logic. But it is not only that; it is also a subfield of several other branches of mathematics. Much recent research has, in fact, been done by non-logicians, or by logicians who are not generally thought of as descriptive set theorists. And the mathematics is as diverse as the people.

About fifteen years ago, Greg Hjorth began proving theorems on this topic. He apparently can't stop, and now has over thirty publications. The work under review is only a small fraction of his research.

For E and F equivalence relations on Polish spaces X and Y, respectively, we say that E is *Borel reducible* to F if there is a Borel function $\theta\colon X \to Y$ so that for all $x_1, x_2 \in X$: $x_1 E x_2$ iff $\theta(x_1) F \theta(x_2)$. One can consider this topic from two points of view—the set theoretic point of view or the classification point of view. The former point of view is that this is "Borel cardinality": if we modify the definition by allowing θ to be an arbitrary function, then the

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definition merely states that the quotient space X/E has cardinality less than or equal to that of the quotient space Y/F; Borel reducibility is simply a Borel version of this. The latter point of view is that it is common in mathematics to classify elements of X up to E-equivalence by Borel invariants; that the invariants may themselves be equivalence classes, say members of Y/F; and this classification by invariants amounts to saying that E is Borel reducible to F.

There are a number of variations on this theme. One could consider $\mathbf{\Delta}$ *reducibility*, where $\mathbf{\Delta}$ is some pointclass other than the class of Borel sets. Set theoretic issues arise if $\mathbf{\Delta}$ is $\mathbf{\Delta}^1_2$, or some larger class. So when working in ZFC, one generally considers a class $\mathbf{\Delta}$ properly between Borel and $\mathbf{\Delta}^1_2$, such as the class of absolutely $\mathbf{\Delta}^1_2$ sets. Assuming strong axioms, one can consider larger classes, and assuming AD—which contradicts the axiom of choice—one can take $\mathbf{\Delta}$ to be the full power set of the reals (in which case, $\mathbf{\Delta}$ cardinality is cardinality). Another variation involves considering a different collection of equivalence relations. One could consider equivalence relations that are not $\mathbf{\Sigma}^1_1$, but, in practice, almost all research has involved some subcollection of the $\mathbf{\Sigma}^1_1$ equivalence relations. One important subcollection is that of Borel equivalence relations. Another is the collection of orbit equivalence relations induced by a Polish group acting continuously on a Polish space. A special case of this is the equivalence relation of isomorphism for L-structures with universe $\mathbb{N}$ (L a fixed language), which is the orbit equivalence relation of a *logic action* by the group of permutations of $\mathbb{N}$.

The work under review derives from a specific question, with an embarrassing history. Countable L-structures are classified up to isomorphism by their Scott sentence. A 1996 book by Alexander S. Kechris and the reviewer (***The descriptive set theory of Polish group actions***, Cambridge University Press, Cambridge) asked a precise version of the following vague question: Is there some sort of "generalized Scott sentence", applicable not merely to isomorphism but to arbitrary orbit equivalence relations? A negative answer to the (precise) question appeared in a 1989 paper of Harvey Friedman and Lee Stanley (*A Borel reducibility theory for classes of countable structures*, ***The Journal of Symbolic Logic***, vol. 54, pp. 894–914). Fortunately, Hjorth was also ignorant of the Friedman-Stanley Theorem, and went to work on the problem. He rediscovered their result, and then went much further to produce the results contained in the two publications under review.

Let us begin by making the question precise. A Scott sentence "is" a hereditarily countable set. Let L' be the language with one binary relation symbol, let HC denote the class of hereditarily countable sets, and for any set A, let $TC(A)$ denote the transitive closure of A. Note that a member of HC can be identified with an isomorphism-type of a countable L'-structure: identify A with $\langle A \cup TC(A), \in\rangle$. Thus, as explained above, we are classifying L-structures up to isomorphism by invariants, and the invariants are themselves the L'-isomorphism classes of HC. The function which assigns to each L-structure its Scott sentence is not Borel; it is, however, absolutely $\mathbf{\Delta}^1_2$. We can now make the question precise: Is it true that for any orbit equivalence relation, it is possible to assign elements of HC as invariants in an absolutely $\mathbf{\Delta}^1_2$ way? Call this *Question* 1. A related question, *Question* 2, is: Is it true that every orbit equivalence relation is Borel reducible to isomorphism for countable L-structures, for some L? A positive answer to Question 2 implies a positive answer to Question 1, since we can compose the Borel reducing function with the function that calculates the Scott sentence.

The answer to Question 1 (and hence to Question 2) is no. But Hjorth did more than just answer that question. He isolated a particular property of the action which is an obstruction to assigning elements of HC as invariants (and hence to classification by countable structures). The property, called *turbulence*, is defined below. (The word "turbulence" has a different definition in certain parts of mathematics which also involve group actions. The two concepts with the same name are not related.) A concrete example of a turbulent action is c_0 acting on $\mathbb{R}^{\mathbb{N}}$ by coordinatewise addition.

Consider a Polish group, G, acting continuously on a Polish space, X. The action is said to be *turbulent* if every orbit is meager and the following property holds: for all $x, y \in X$, for all open $U \subset X$, $V \subset G$ with $x \in U$ and $1 \in V$, there exists $y_0 \in G \cdot y$, a sequence g_i from V, a sequence x_i from U with $x_0 = x$ and $x_{i+1} = g_i \cdot x_i$, and a subsequence $x_{n(i)}$ of x_i which converges to y_0.

If the above definition is altered to consider only $U = X$ and $V = G$, we obtain the definition of *properly generically ergodic* (the Baire category analog of properly ergodic). Thus turbulence might be thought of as a type of local ergodicity. Hjorth's book makes this point via the famous billiard ball on a frictionless table with perfectly elastic boundaries. This corresponds to an action by the additive group of the reals, thought of as time. With generic initial conditions, we would expect the ball to eventually pass through every open neighborhood on the table; but if the ball starts on the right half of the table, we expect that it would *not* pass through every open neighborhood on the right half of the table *before* it ever crossed the boundary to the left half. The first expectation is that the action is generically ergodic; the second expectation is that the action is not turbulent. This lack of turbulence is a special instance of a general theorem: actions by locally compact groups are never turbulent.

It is proved in the book that for turbulent actions, it is not possible to assign elements of HC as invariants in an absolutely $\mathbf{\Delta}^1_2$ way (nor, under AD, in any way at all). So no action which contains a copy of a turbulent action admits such invariants. It is then natural to ask whether this is the only obstacle to the existence of a "generalized Scott sentence". *Question* 1*: Is it true that for any action that does not contain a copy of a turbulent action, it is possible to assign elements of HC as invariants in an absolutely $\mathbf{\Delta}^1_2$ way? *Question* 2*: Is it true that any such action is Borel reducible to isomorphism for countable structures?

Hjorth's book contains a positive answer to Question 1*. Question 2* is still open. Hjorth's paper contains a positive answer to Question 2* under the additional assumption that the orbit equivalence relation is Borel. For any action, the space being acted upon can be written as an increasing union of ω_1 invariant Borel sets, in such a way that the orbit equivalence relation restricted to each of the ω_1 sets is Borel. (For logic actions, this decomposition is obtained by bounding the Scott rank. Something similar holds in general.) So we know that for any action unsullied by turbulence, each of the ω_1 Borel subspaces admits classification by countable structures; but it is still open whether the entire space admits (a Borel) classification by countable structures.

There is, of course, much more than this in the book. There are asides discussing related topics. Some special cases of positive answers to the four previously mentioned questions are proved. And the turbulence of some specific actions is proved. The theory of turbulence is applied to such topics as infinite dimensional group representations and the conjugacy relation on groups of homeomorphisms of compact metric spaces.

The book contains quite a bit of background material. Most readers of this journal would feel that it contains too much background material, e.g., the definition of *ordinal*. This is put in because the author wanted to make the book—except for the last chapter—suitable for a general (mathematical) audience and totally self-contained. He has probably succeeded in doing that. Part of the background material is an expository preface, devoted to the question "What does it mean to *classify* the equivalence classes of some equivalence relation?" Hjorth's survey of the literature in several branches of mathematics reveals: first, that no two of the surveyed authors agree on the meaning; second, that each author believes it to be self-evident that his own notion of classification is the only thing that could possibly be meant by classification.

The last chapter of the book assumes the reader has a serious knowledge of set theory in general, and of AD-theory in particular. No recursion theory is used anywhere in the book. The paper, however, does use it. The Gandy-Harrington topology, a recursion theoretic tool that has frequently been used in the study of Borel reducibility, is again used for that purpose.

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E. Grädel, P. G. Kolaitis, L. Libkin, M. Marx, J. Spencer, M. Y. Vardi, Y. Venema and S. Weinstein. ***Finite model theory and its applications.*** Texts in Theoretical Computer Science. Springer, Berlin, 2007, xiii + 437 pp.

Based on a series of tutorials given at a workshop at the University of Pennsylvania in 1999, the book covers the major topics in finite model theory including applications to databases and artificial intelligence.

It can broadly be distinguished into two major parts: the first covers the foundations of finite model theory whereas the second focuses on applications to databases and artificial intelligence and on modal logics. The two parts are preceded by an introduction by Scott Weinstein which highlights the fundamental questions, the main motivation and methods that define finite model theory and derives the applications the theory has had from this common set of tools and methods.

The first major part, Chapters 2–4, contains the foundations of finite model theory. Chapter 2, by Phokion Kolaitis, focuses on methods for analysing the expressive power of logics on finite models, most notably on Ehrenfeucht–Fraïssé (EF) games which, in the absence of compactness and several other methods, are among the few tools from classical model theory that survive in the finite.

One of the major motivations for studying finite model theory is its connection to complexity theory and databases. In this context, first-order logic loses its defining rôle from classical model theory and both restrictions and extensions of FO, most notably by second-order quantifiers or fixed-point operators, gain in importance. The second part of Chapter 2, therefore, introduces various forms of fixed-point logics focusing on their model-theoretical properties. The connection to computational complexity is left for Chapter 3, where these logics are reintroduced. The chapter closes by a thorough discussion of infinitary bounded variable logics and their characterisation in terms of pebble games, which also lays the foundation for parts of Chapter 6.

Chapter 3 by Erich Grädel presents another core topic of finite model theory, descriptive complexity theory, which analyses the surprisingly close connection between the complexity of a computational problem in terms of classical measures such as time and space and the complexity in terms of the logical constructs one needs to describe the problem. The chapter introduces the concept of logics capturing complexity classes and then revisits the fixed-point logics from Chapter 2 that are used to describe complexity classes. The presentation of fixed-point logics and their complexity is based on model-checking games for these logics, in particular parity games describing least fixed-point logics. Research on parity games and game characterisations of fixed-point logics has received significant attention in recent years but this is one of the very few text book treatments explaining this connection thoroughly. The chapter closes with an exposition of counting logics leading towards a major open problem in finite model theory: whether there is a logic capturing PTIME. Finally, a glimpse into complexity issues of logics on finitely representable infinite structures is given.

Chapter 2 and 3 are by far the largest chapters in the book, making up for about half of it. The foundational part of the book is completed by a short chapter by Joel Spencer on logic and randomness giving an introduction to 0-1 laws.

The second part of the book is devoted to applications of finite model theory and to modal logics. Chapter 5, by Leonid Libkin, introduces the framework of embedded finite models and constraint databases and presents the main results obtained in this area. This chapter shows very clearly that while the book appeared 2007 it is based on a workshop in 1999. At the end of the 1990s, research on constraint databases was at its absolute height and there was no text book available presenting the subject. However, towards the mid-2000s research activity on constraint databases declined significantly and at the same time two books on the subject appeared, one dedicated exclusively to constraint databases and Libkin's excellent finite model theory book which contains a scaled down version of the present chapter. In this respect, this chapter serves as a legacy of constraint databases containing most major results obtained in this field, which is not as active as it once was.

Chapter 6, by Phokion Kolaitis, on the other hand, addresses a very active research area in presenting logical approaches to constraint satisfaction problems. Constraint satisfaction problems (CSPs) are a class of computational problems in artificial intelligence that have received significant attention in recent years. Work on their complexity usually relies on logic, graph theory or universal algebra. Chapter 6 of this book beautifully describes the logical approaches to CSPs relating them to the universal algebra approaches. In this way, this chapter serves as a brief but excellent introduction to logical approaches to CSPs.

Finally, Chapter 7 by Maarten Marx and Yde Venema presents a survey of decidability and complexity results in modal logics with a focus on the satisfiability problem for various extensions of basic modal logic.

Taken together, the individual chapters provide an excellent introduction to finite model theory and selected applications. At the level at which it is written the book can serve as a reference for beginning graduate students or advanced undergraduates looking for a guide to methods in the field. The fact that this book is based on a workshop reveals itself in some repetitions so that several concepts are introduced independently in various chapters. On the positive side it means that the chapters can largely be read independently. What does show, however, is that each chapter is written by distinguished experts in the particular field who managed to present even the classical finite model theory material in a new light concentrating on the parts of the field most relevant to current research.

And this brings me to my one reservation about the book, which is that its selection of topics is based on current trends in finite model theory towards the late 1990s and early 2000s. Some topics which gained importance in the 2000s are treated only relatively briefly, for instance the expressive power of logics on trees and data trees or more generally the finite model theory of well-behaved finite structures, decidability issues of logics on infinite structures with a finite presentation such as automatic structures or pushdown systems and others. But then no book can cover everything and the topics which are covered in this book are presented very well.

Comparing the book to existing books on finite model theory, its inclusion of topics such as constraint databases and constraint satisfaction problems distinguishes it from the classical text on finite model theory by Ebbinghaus and Flum and its broad focus sets it apart from the descriptive complexity book by Immerman. There is significant overlap with the third existing book on finite model theory, by Leonind Libkin. But the inclusion of modal logics and constraint satisfaction and the very different approach in which large parts of the material are presented makes the present book an excellent addition to the finite model theory literature, recommended to any scholar looking for an accessible introduction to finite model theory.

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JAAP VAN OOSTEN. ***Realizability: an introduction to its categorical side.*** Studies in Logic and the Foundations of Mathematics, vol. 152. Elsevier Science, Amsterdam, 2008, 328 pp.

Bravely, the author of this book begins his Preface by quoting Martin Hyland's dictum that the time is not yet ripe for a book on realizability toposes. He then goes on to explain in cogent terms why he disagrees with this dictum; but this reviewer, at least, finds himself uncomfortably agreeing with both Hyland and van Oosten. The time may indeed be ripe (indeed, in some respects it is over-ripe) for a book on realizability toposes; but perhaps it isn't quite ripe for this particular book.

The problem (to quote a favourite saying of the reviewer's) is that the study of realizability toposes is still at the 'stamp-collecting' stage: we have lots of exotic and colourful examples, which we can proudly display in our albums, but what we lack is a general theory which would indicate just where the boundaries of the subject lie.

There is an instructive comparison with the study of Grothendieck toposes: almost the first major result in that subject was Jean Giraud's celebrated theorem showing that Grothendieck's intensional notion of a topos, as a category representable as the category of sheaves on a site, was exactly matched by a purely extensional definition. In contrast, thirty years after Hyland discovered the first example of a realizability topos (and, with a little help from Andrew Pitts and the reviewer, parlayed this into the first intensional definition of a family of realizability toposes), we still have nothing approaching an extensional definition of what a realizability topos is.

With these comments in mind, let us examine what the book contains. Of the 288 pages of text, almost half (139 pages) are devoted to chapter 3 which deals with a single example, Hyland's effective topos. This is preceded by 47 pages on partial combinatory algebras and their categories of assemblies (chapter 1), plus 66 pages on triposes and the toposes one constructs from them (chapter 2), and followed by 36 pages on various extensions and generalizations of the realizability notion.

The reviewer has no quarrel with the strong focus on the effective topos: it is by far the prettiest stamp in our collection, and its remarkable and beautiful features deserve to be better known. Indeed, in the present state of the subject, any general account of it has to focus on this particular example, if only to explain to the reader why the search for a general theory is worth pursuing. Moreover, the author's treatment of the effective topos is very well-judged: everything the general reader needs to know about it is here, with the right degree of emphasis.

Where the reviewer does part company with the author is with regard to the relative prominence of the first two chapters. Whilst the tripos-to-topos construction (for which the reviewer would wish to claim partial credit, though the author awards the whole of it to the third author of the Hyland–Johnstone–Pitts joint paper) was the original access route to realizability toposes, and remains an important one, it has over the years become clear that a much gentler approach is provided by regarding them as the exact completions of their subcategories of assemblies.

The fact that any realizability topos is the exact completion of a category of assemblies does appear in the book (Corollary 2.4.5), but not nearly enough use is made of it. And the reader will search in vain for important properties of categories of assemblies, such as the facts that they are quasitoposes and contain exemplary subobjects (that is, monomorphisms satisfying the 'existence' but not the 'uniqueness' part of the definition of a subobject classifier). Even the fact that every category of assemblies has a natural number object is missing; in fact, only the effective topos is shown to possess this amenity (Proposition 3.1.1).

There are one or two other instances where the author's focus on the effective topos has caused him to miss the fact that he could have proved the same results for general realizability toposes at no extra cost. For example, in section 2.4 he introduces uniform objects, and proves (2.4.7) that the power-object of a uniform object is uniform; and in 3.2.6 he proves that all power-objects in the effective topos are uniform. But in fact it is easy to prove that all injective objects (in particular, all power-objects) in any realizability topos are uniform.

This is, then, a partially flawed account of realizability toposes; to that extent, it is not the account of the subject for which the world is ready. (Another symptom of unreadiness is the rather high density of trivial misprints, though most of them will not detain the reader for long.) But if (as the reviewer does) one accepts the author's contention that it is better to have this material available in book form than not to have it, it is still a book to be warmly welcomed.

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Reasoning in simple type theory – Festschrift in honor of Peter B. Andrews on his 70th birthday, edited by Christoph Benzmüller, Chad E. Brown, Jörg Siekmann, and Richard Statman, Studies in Logic, vol. 17. College Publications, London, 2008, 454 pp. —therein:

PETER B. ANDREWS. *Some historical reflections*. Pp. 3–34.

CHRISTOPH BENZMÜLLER, CHAD E. BROWN, and MICHAEL KOHLHASE. *Cut elimination with ξ-functionality*. Pp. 84–100.

OLIVIER HERMANT and JAMES LIPTON. *Cut elimination in the intuitionistic theory of types with axioms and rewriting cuts, constructively*. Pp. 101–134.

CHAD E. BROWN. *M-set models*. Pp. 135–158.

WILLIAM M. FARMER. *Andrews' type theory with undefinedness*. Pp. 208–227.

MARK KAMINSKI and GERT SMOLKA. *A finite axiomatization of propositional type theory in pure lambda calculus*. Pp. 228–243.

GILLES DOWEK. *Skolemization in simple type theory: the logical and the theoretical points of view*. Pp. 244–255.

MURDOCH J. GABBAY and AAD MATHIJSSEN. *The lambda-calculus is nominal algebraic*. Pp. 256–287.

FRANK PFENNING. *Church and Curry: Combining intrinsic and extrinsic typing*. Pp. 288–323.

CLAUDE KIRCHNER, FLORENT KIRCHNER, and HÉLÈNE KIRCHNER. *Strategic computation and deduction*. Pp. 324–349.

CHRISTOPH BENZMÜLLER and LARRY PAULSON. *Exploring properties of normal multimodal logics in simple type theory with Leo-II*. Pp. 386–406.

DALE MILLER. *A proof-theoretic approach to the static analysis of logic programs*. Pp. 407–426.

HONGWEI XI. *ATS/LF: A type system for constructing proofs as total functional programs*. Pp. 427–450.

This volume is the Festschrift assembled for the occasion of Peter Andrews's 70th birthday. Andrews received his PhD. from Princeton under Alonzo Church in 1964 and has been a faculty member at Carnegie Mellon University since 1963. He is one of the most influential researchers in the areas of type theory, higher-order logic, and automated reasoning and the recipient of the 2003 Herbrand Award. This collection includes a historical essay by Andrews, reprints of eight historical papers, and 12 contributed original papers.

Throughout this review, HOL refers to simple type theory with classical higher-order logic in the style of Church and Andrews without extensionality axiom, description operator, choice operator, or infinity axiom.

The historical papers (one each by Church and Henkin, five by Andrews, one by Andrews et al.) represent an excellent overview of the seminal contributions that have shaped the research area in general and Andrews's research life in particular. These technical papers are nicely complemented with Andrews's rather personal historical reflections. There he first chronicles his early work on extending Herbrand's Theorem from first-order to higher-order logic. Then he describes some key aspects of his and his students' later work on automated reasoning for HOL and the implementation in the TPS system. Thus, the reader can experience how over several decades simple ideas developed into elaborate research results.

Benzmüller, Brown, and Kohlhase prove cut elimination and completeness of a sequent calculus for a variant of HOL. In JSL LXIX 1027, the authors classified variants of HOL according to the functional extensionality conditions imposed on the models: η-conversion ($f = \lambda x : \alpha.f(x)$), ξ-conversion ($\forall x : \alpha.M = N$ implies $\lambda x : \alpha.M = \lambda x : \alpha.N$.), neither, or both. The authors handle the case with only ξ, the other three cases were known before.

Hermant and Lipton prove cut elimination semantically in the intuitionistic variant of HOL. While constructive proofs for the classical system and non-constructive proofs for the intuitionistic system are known, a constructive result for the intuitionistic system had so far been missing. They extend their result to a variant with more liberal cut rules, which they use to obtain the rule "If A rewrites to B, then infer B from A." relative to some confluent rewrite system for formulas.

Brown considers M-sets (a set with a monoid acting on it) as models of HOL inspired by the interpretation of simple type theory in a cartesian closed category. M-sets satisfy the η-rule but not necessarily the ξ-rule (see above). Thus, an extension of TPS with η can be used well to reason about them.

Farmer improves upon a variant of HOL with extensionality, description, and undefined terms that he had given erroneously in JSL LV 1269. His models induce partial interpretation functions, and the domain of a function type may contain partial functions. In particular, a model M gives no interpretation for the term $\iota(P)$ if ι is the description operator and P is a predicate that is not uniquely satisfiable in M. Case distinctions are used to avoid undefined terms of type o, i.e., formulas. He proves soundness and completeness.

Kaminski and Smolka look at propositional type theory, a variant of HOL without equality and with o as the only base type; falsity and implication are primitive, and o is interpreted as the booleans. They give a complete calculus and show that, just like for propositional logic, all elements of a model can be expressed by a term of the syntax.

Dowek discusses earlier results (MSCS XI 1) that HOL (with extensionality, infinity, choice, description) can be treated as a theory of a significantly enriched variant of first-order logic. His focus is not on the results itself but on contrasting the view of HOL as a logic and as a theory. While the former dominates today, he argues for the latter appealing to historical arguments by Davis and Wang.

Gabbay and Mathijssen look at the untyped λ-calculus from an algebraic point of view. Using nominal atoms instead of bound variables as well as a primitive freshness judgment, they give a calculus for $\alpha\beta$-equality and show soundness and completeness relative to the usual formulation of the λ-calculus.

Pfenning distills known results on type theories contrasting Church-typing (where typing is a function from terms to types) and Curry-typing (where it is a relation). He combines their advantages by giving a Church-typed λ-calculus and adding a Curry-typed sort system on top. Each sort refines a type, and there are subsorting and intersection sorts. All operations are based on canonical representatives for the $\beta\eta$-classes, and typing and sorting are decidable.

Kirchner, Kirchner, and Kirchner look at strategies as they are used both in term rewriting and in proof search, and they propose abstract definitions of strategy and normalization under strategy. Then they instantiate these definitions to obtain rewrite and proof search systems and discuss within their framework the strategies employed in various systems.

Benzmüller and Paulson use HOL to encode Kripke models for multimodal logics. While the well-known encoding in first-order logic treats $\Box_R$ as syntactic sugar for a relativized $\forall$, they can define $\Box$ as a function that is applied to R, which permits quantification over R. They use the Leo-II prover for HOL to derive some simple properties of some modal logics.

Miller studies how certain invariants of logic programs can be statically verified. He approximates collection data structures like lists or trees as multisets and formulates necessary correctness conditions as inclusions and equalities between multisets. These can be embedded into a decidable fragment of linear logic, for which an implementation is available.

Xi gives the syntax and semantics of ATS/LF, a formal system based on his ATS framework. A static type theory is used for object-level data structures and a dynamic one for meta-level proofs and programs. He shows that all well-formed dynamic functions are total and can be checked for termination. Then he sketches how one might encode some simple deductive systems in ATS/LF.

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JOHN P. BURGESS. ***Philosophical logic.*** Princeton University Press, Princeton NJ. 2009, viii + 153 pp.

This is a short text intended not so much for the readers of this journal (though there are several tidbits for them here) as for their students. The chapters conveniently begin with motivating discussion and leave the more technical material to later sections, so that beginners are not overwhelmed and those just wanting the general gist are not distracted. The reviewer used the book in a later year philosophy class of students with little prior acquaintance with formal logic, to check its intelligibility for the intended audience. It passed with flying colours. As evidence of this: most of the errors noted below were in fact spotted by students in this class. It was not always obvious, however, what the appropriate corrections should be, and a second edition from which the errors were absent would be even better.

After an opening with a chapter on classical logic, Burgess devotes successive chapters to (in the words of his titles) Temporal Logic, Modal Logic, Conditional Logic, Relevantistic Logic and Intuitionistic Logic. Thus, what we have is essentially an admirably concise introduction to non-classical logics, where this phrase is understood to subsume both extensions of and alternatives to classical propositional and predicate logic. The coverage thus coincides more or less with that of Graham Priest's text ***An Introduction to Non-Classical Logic***, (Cambridge University Press, 2001) which comes in at roughly the same length—in fact 242 pages—but in a larger font and with greater line-spacing. Had Burgess's book appeared some years ago, this would have been its main competition. But Priest's book is now in a second edition (2008) with the same publishers, an extra subtitle ("From If to Is") and, more significantly, an extra 370 pages, removing it from the ('short text') field of competitors altogether. (Priest's subtitle was intended to register the fact that now predicate rather than just propositional logic is under discussion: Burgess includes—where they raise significant issues—quantifiers and identity in his treatment already.) One important respect in which Burgess's treatment is preferable to Priest's arises from the latter's reliance on tableau proof methods, which in the reviewer's opinion are not conducive to a pedagogically clear distinction between syntactic and semantic considerations. The following remarks concentrate on Chapters 4–6.

Chapter 4, on Conditional Logic, introduces the indicative/subjunctive contrast between conditionals, mentioning that this goes under several names, including factual/counter-factual. (The reviewer has never heard of indicative conditionals being called factual.) Adams-style probabilistic accounts of validity for indicative conditionals are described, as well as novel 'qualitative' model-theoretic semantics (the 'remoteness theory'—§4.4) for a language in which no such conditional occurs within the scope of another. The discussion of probability is marred by the fact that although the conditional probability of B given A, notated by Burgess as $\pi(B|A)$ is defined in the usual way (π being a probability function), the A and B here keep getting mixed up, so that on p. 77 (line 7) an indicative conditional $A \rightarrow B$ is described as having its assertibility given according to one popular theory by $\pi(A|B)$, instead of by $\pi(B|A)$. (This typo recurs nine lines further on, and in the line after that also, where the reference to $\pi(\neg A \wedge B)$ is presumably meant to be $\pi(\neg A \vee B)$.) A version of David Lewis's triviality result, establishing that subject to certain background assumptions, the conditional probability of B given A cannot be identified with the absolute probability of any statement compounded from A and B, is given on p. 77*f.*, but it will not be easily followed by first-time readers because of the typographical errors. The third occurrence of

"B" in line (iv) should be "A", the seventh occurrence of "A" in line (vi) should be "B", and the words "follows from (vi)–(vii)" in the justification given after the proof (i)–(ix) should be "follows from (vi)–(viii)".

Chapter 5, 'Relevantistic Logic' deals with what other authors variously call relevant or relevance logics depending on subtle differences in attitude; Burgess's terminology—despite the parallel with "intuitionistic"—seems distinctly pejorative (one is reminded of "scientistic") and is in any case too long for him to persist with, so we are soon reading about 'r-logic'. Before that, Parry-inspired analytic implication, a dual form Burgess calls co-analytic implication, and the relatedness logic of R. Epstein are treated. The presentation here is again somewhat compromised by misprints, as when we are told (p. 101) that "the argument from p to $p \vee q$, and the argument from $p \wedge \neg p$ to $(p \vee q) \wedge \neg p$ are valid in relatedness logic and for analytic implication, but not for co-analytic implication"; here "analytic" and "co-analytic" need to be interchanged (with a similar correction in what follows the passage just quoted). There is another potentially confusing misprint in the sequent calculus rule for inserting disjunction on the left (p. 101, line 4), which can easily be corrected by anyone teaching a course based on this book, but might well trouble a solo reader.

There are many philosophical points about the applicability of 'r-logic' in this chapter. They include the vexed issue of intensional disjunction and the disjunctive syllogism (§5.5) in which Burgess spells out an objection (originally raised by Belnap) he had aired in journal articles in the 1980s against the claim of an intensional/extensional ambiguity in the word *or*, only the latter reading suited to inference by disjunctive syllogism. The section closes with some adverse comments on the prospects for mathematics based on logics of this kind, but unfortunately Burgess does not actually discuss any of the interesting work that has been done (Meyer, Restall, Slaney, …) on relevant—and more generally substructural—arithmetic. Not all of Burgess's verdicts are similarly unfavourable: §5.6 makes an interesting point about switching circuits, and §5.8 gives an excellent presentation of motivating considerations for the purely implicational fragment of (the relevant logic) **R** concerned with the Deduction Theorem. §5.7 is rather less sympathetic toward the radical paraconsistent motivation for relevant(istic) logic championed by Graham Priest under the name *dialetheism*. Burgess is antipathetic not only to any such motivation but even to the name, explaining to us (p. 113) why it should, by parity with other words of similar etymology, be simply *dialethism*. (Priest informs the reviewer that he has frequently encountered this suggested revision of his terminology in the past.) It must be disconcerting to be told that one has misspelt a term of one's own coinage – rather like being told one has been misspelling the name of one of one's children. It would no doubt be even more disconcerting to be told that one has been misspelling one's *own* name, as in effect happens at p. 466 of I. Ravenscroft (editor) ***Minds, Ethics, and Conditionals: Themes from the Philosophy of Frank Jackson*** (OUP, Oxford 2009), in which Priest is repeatedly referred to as Graeme Priest.

Chapter 6, on intuitionistic logic, begins in exactly the way that Chapter 1 of Dummett JSL XLIV 276 does, with the example of a non-constructive proof that there are irrational numbers a, b, with a^b being rational, described by Burgess as 'folklore' (but credited by Dummett to F. C. Benenson). This is followed by a statement of what is sometimes called the BHK interpretation of the logical constants (sentence connectives, at least) of intuitionistic logic, a justification of the axioms of a handy axiomatization of intuitionistic propositional logic, some translation results, and the Kripke semantics for the logic (together with a completeness proof, conveniently located in a section of its own) as well as for two intermediate logics. We then have some remarks on the extension of these ideas to intuitionistic predicate logic, and a discussion of choice sequences, to which we shall be returning in the following paragraph, and the desirability is stressed of showing intuitionistic logic to be complete with respect to some semantic account more closely linked to Brouwer's motivation or the

BHK interpretation (as opposed to the Kripke model theory). The treatment (p. 129) of the negation/conjunction translation of intuitionistic into classical propositional logic (deriving from an observation of Gödel's) is accompanied by the suggestion (originally made by Łukasiewicz) that this allows us to think of intuitionistic logic as an extension—with new connectives $\to$ and $\vee$—of classical logic, rather than a subsystem. The reviewer would make only the standard objection to this suggestion, namely that it relies on taking logics as sets of formulas (the theorems of the logic) rather than as consequence relations, since the consequence relations of the negation–conjunction fragments of classical and intuitionistic logic by no means coincide. Burgess appears here to be trapped in the same restricted conception of what constitutes a logic as Łukasiewicz was. A full discussion would also describe the fate of connections like this between intuitionistic and classical logic (Glivenko, Gödel, ...) at the level of *predicate* logic.

On the subject of intuitionistic predicate logic (§6.8), the equivalence and non-equivalence of quantifier prefixes (consisting of an existential or universal quantifier together with one or two negations before or after it) are detailed, essentially as on p. 29 of Dummett JSL XLIV 276 (or p. 19 of the second edition of that work; the original source is Heyting JSL XII 59). Unfortunately the clauses in the definition of truth (or satisfaction) that need to be added for the quantifiers to the Kripke semantics outlined for the propositional case are not supplied. But the treatment of the propositional case will be very useful for students, along with the canonical model completeness proof given. As already mentioned, the discussion also touches on free choice sequences, in §6.9 ('Completeness in the Intuitive Sense'), but the intrinsic difficulty—to put it politely—of the subject matter makes any typographical errors hard to deal with. For example on p. 140 the following pair, in which $\alpha(n)$ denotes the nth term of ('lawless') sequence α, are described as ("even intuitionistically") equivalent:

$$\neg\exists\alpha\,\forall n\,\alpha(n) = 0 \qquad\qquad \forall\alpha\,\neg\exists n\,\alpha(n) \neq 0.$$

Since these formulas are not (even classically) equivalent, one suspects a typo: perhaps the second formula here was meant to be $\forall\alpha\,\neg\neg\exists n\,\alpha(n) \neq 0$ or simply $\forall\alpha\,\exists n\,\alpha(n) \neq 0$. But the subsequent discussion doesn't make much sense with either of these, so the reviewer had to resort to consulting the author—a desperate measure—and was told that the "$\neq$" in the second formula should be "$=$".

In summary: this is potentially an extremely efficient as well as engagingly opinionated introduction to non-classical logics of (at least, purported) philosophical interest, though to realise that potential fully it would need to have a number of typographical errors corrected.

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Martin Peterson. ***An introduction to decision theory.*** Cambridge University Press, Cambridge, 2009, x + 317 pp.

Martin Peterson's book provides a useful introduction to the key concepts of decision theory, probability, game theory and social choice theory, targeted towards philosophers, economists, psychologists, political scientists and computer scientists. The book is well written, clearly presented and easily read, with numerous examples, exercises and solutions ensuring the reader understands the material. It is not intended to be a highly technical introduction. Peterson explains in the preface that the book includes philosophical and foundational topics, and is intended to be accessible to those with limited knowledge of mathematics, although a number of key proofs are included in boxes. The omission of some formal details is thus to be expected. For readers with a technical bent, the book may seem overly informal, however, Peterson generally hits his stated target when balancing the formal and informal approaches.

The contents should fit a semester-long course, with each of the 14 chapters covering roughly a week's worth of material. By the second half of the book, each chapter is a largely self-contained introduction to a specific topic or extension of decision theory. The level of the material makes the book suitable as a textbook for undergraduate courses or a supplementary reference for graduate courses.

The first chapters of the book cover the fundamentals of decision theory, including the history of decision theory in chapter 1, decision matrices in chapter 2, and decision-making under ignorance (including decision-making rules such as maximin, dominance reasoning and minimax regret) in chapter 3. Chapter 4 introduces the expected utility principle, and the best-known problems arising from the expected utility principle (Allais' paradox, Ellsberg's paradox, the St. Petersburg paradox and the two-envelope paradox). Chapter 5 provides a more detailed discussion of utility, and in particular the von Neumann–Morgenstern preference axioms (prescribing how a rational decision-maker's preferences ought to be arranged) and the von Neumann–Morgenstern theorem relating these axioms to the expected utility principle (a proof is provided as an appendix).

Peterson then turns to some foundational and philosophical topics. Chapters 6 and 7 are an introduction to probability theory, including Kolmogorov's axioms, conditional probability and Bayes' theorem, and the main interpretations of probability. Chapter 7 includes a discussion of Savage's decision theory and its connection to subjective probabilities. Chapter 8 is devoted to a closer examination of the preference axioms, with particular attention to transitivity, completeness and independence. The chapter is concerned primarily with pragmatic arguments for or against these axioms, such as the "money-pump" argument for transitivity. Chapter 9 deals with causal decision theory, including Newcomb's problem, the differences between causal and evidential decision theory, and a number of arguments for and against causal decision theory. Chapter 10 provides a general discussion of Bayesian decision theory. While admitting that a number of definitions of Bayesianism have been used in the literature, the chapter canvases the key claims of Bayesian decision theorists, the main arguments for and against Bayesian decision theory, and some approaches to non-Bayesian decision theory.

Chapters 11 and 12 are an introduction to game theory. After an explanation of basic game-theoretic concepts such as cooperative and non-cooperative games and zero-sum and nonzero-sum games, several methods of solving games are described, including Nash equilibria and Nash's solution to the bargaining problem. Philosophical issues and the application of game theory to areas such as ethics are also discussed. Chapter 13 is an introduction to social choice theory, including an overview of group decision problems, the theorems of Arrow, Sen and Harsanyi, and the application of social choice theory to problems in ethics and welfare.

The book concludes in chapter 14 with an overview of experimental results that suggest decision-makers do not follow the normative rules of standard decision theory, in particular, transitivity and completeness of preferences and the expected utility principle. There is also a short overview of prospect theory.

A reading list is provided that serves as a loose bibliography for each chapter, although for some topics the reading list is limited. The reading list for the probability calculus contains only one reference, and the lists for Bayesianism and game theory contain only a handful of references each. For the most part, the reading list directs the reader to the key works on each topic; there is certainly enough to begin further study.

There are some minor problems with the book. Peterson covers a broad range of topics, but sometimes at the cost of depth and detail. Of course, the book is intended to be an introduction to decision theory generally rather than an in-depth analysis of particular topics, so a certain amount of brevity is understandable. Chapters 6 and 7 on probability form a useful introduction to the philosophy of probability, but do not add a great deal to the book as an introduction to decision theory. Much of the mathematics of chapter 6 seems to have been assumed knowledge in earlier parts of the book and is not needed in the later parts. The philosophical content of chapter 7 is a useful summary of the main interpretations of probability, but only the subjective interpretation is discussed further. Examples and exercises are occasionally dealt with quickly enough that a reader with a weak mathematical background may find difficulty in understanding how a result was reached. This sometimes manifests as oversimplified or one-word answers to problem questions, and at other times as derivations with intermediate steps skipped. When assistance from an instructor is available, of course, this should not be a major problem. Finally, there are a number of typographical errors. These generally occur in examples and exercises rather than in key definitions or formulae, and are likely to be understood by the attentive reader as typographical errors. A list of corrections is available on the author's website at `www.martinpeterson.org/errata.pdf`.

These shortcomings, however, are merely minor flaws in an otherwise excellent introductory text.

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G. Metcalfe, N. Olivetti and D. Gabbay. ***Proof theory for fuzzy logics.*** Applied Logic Series, vol. 36. Springer, 2009, viii + 276 pp.

Fuzzy logic is a branch of many-valued logic designed for the formalization of truth-degree based reasoning under vagueness. In recent years the investigation of fuzzy logics has become a flourishing field of logical research mainly as a result of the fruitful application of t-norm based algebraic semantics by Hájek and others. The book is a thorough expert introduction to fuzzy logics written with a proof-theoretic orientation. Its main intention is to "convince the reader that Gentzen systems admitting cut elimination—in particular, hypersequent calculi—provide an appropriate and useful framework for investigating fuzzy logics".

The book is composed of nine chapters each of which, except for the introductory and the concluding chapter, is either based on or integrates (co-authored) research by at least one of its authors that has been published during the last decade or so. Each chapter, again with the exception of the first and the last one, concludes with rich historical information on its theme.

Ch. 1 introduces the topic and gives an overview of the book. We give a detailed description of the contents of the subsequent chapters.

Ch. 2 provides an introduction to the basics of algebraic semantics for fuzzy logics. It first explains the truth-degrees a formula may take in terms of partially ordered sets (posets), defines intervals for these sets, and describes several properties and special cases of posets. It then defines the functions on posets which serve to interpret the logical connectives. The definitions are guided by "design choices" of posets and basic properties functions on them should have for the interpretations to be appropriate. Conjunction connectives are interpreted in terms of t-norms (binary functions on the real unit interval which are associative, commutative, increasing and have 1 as a unit element) and disjunction connectives by t-conorms which are like t-norms except for having 0 as unit elements. The logics determined by the fundamental t-norms take center stage: Łukasiewicz Logic (Ł), Gödel Logic (G) and Product Logic (P). The elementary theorem is discussed which says that each continuous t-norm is the ordinal sum of a family of t-norms order isomorphic to either the Łukasiewicz or Product t-norm. Next, the traditional t-norm approach is extended to an approach based on uninorms. In contrast to t-norms and t-conorms, uninorms allow the unit element to be any real in the interval. Implication is defined by residua for uninorms and negation by functions which are antitonic (i.e., non-increasing) and assign 0 to 1 and conversely.

The discussion of the connectives is followed by a definition of (bounded) (pointed) commutative residuated lattices ((b)(p)crls) to obtain a handy algebraic framework for the investigation of non-classical logics. The bpcrls which are based on residuated uninorms are identified as those which are most relevant for fuzzy logics and the fundamental properties of these lattices are listed. With an eye on Ch. 3 the notion of a Dedekind–MacNeille completion is defined which says that for special classes of (b)pcrls each countable dense chain from the class can be embedded into a standard algebra of the class. Finally, languages for fuzzy logics are defined, validity of a formula in an algebra is explained, and several logics are related to the (classes of) algebras on which they are based. The chapter is pivotal for the book in so far as it feeds the algebraic intuitions which guide the proof-theoretic investigations of the remaining chapters.

Ch. 3 presents several Hilbert systems for propositional fuzzy logics which are obtained by supplementing the basic system of additive-constant-free Multiplicative Additive Intuitionistic Linear Logic (MAILL) with additional axiom schemes and rules. (Additive constants are $\top$ and $\bot$. Their presence in an algebra indicates that its truth value set is bounded, e.g., by 0 and 1.) The presentation of the axioms draws on the intuitions stimulated by Ch. 2 in that it always suggests how they relate to the various kinds of (b)pcrls. The presentation of the axioms is followed by a proof of a local deduction theorem (designed for systems lacking weakening and contraction axioms) which uses the method of so-called confusions of theories. The correspondence between derivability in any extension of the basic system and validity in the appropriate class of (b)pcrls is made precise in terms of soundness and completeness results. These results are first shown for certain systems with respect to linearly ordered algebras and then for systems which are also equipped with a density rule with respect to algebras of the previous kind that in addition possess the property of density. In both cases completeness is established by appeal to Lindenbaum algebras. The last completeness proof concerns extensions of the basic system with respect to standard algebras (i.e., algebras in which the function for multiplicative addition is a residuated uninorm and the multiplicative falsity constant is any element in the real unit interval) by making use of the Dedekind–MacNeille completion. Of special interest (with respect to density elimination in Ch. 5) is the fact that the addition of the density rule to any extension of the basic system (supplemented with the axioms for prelinearity and distributivity) ensures completeness with respect to dense chains. The completeness results allow insights into the interconnections between extensions of the basic system (which are depicted in a helpful diagram).

Chapters 4–6 are dedicated to the structural proof theory of fuzzy logics, the main theme of the book, with Gentzen systems (i.e., sequent and hypersequent calculi) as the format for proofs.

Ch. 4 presents a hypersequent system for additive-constant-free MAILL as the core system (the classical counterpart being its multiple-conclusion version). It then shows how, holding the logical rules fixed, sequent and hypersequent calculi for many fuzzy logics can be obtained by adding extra structural hypersequent rules (e.g., communication, split, mix, various kinds of contraction, cancellation, density). The authors discuss the effects of these rules and how they and their sequent versions (if any) interact. Soundness and completeness with respect to Hilbert systems are established for many hypersequent and sequent calculi by translating hypersequents into formulae and matching structural hypersequent rules with axioms. The matching relations between these rules and axioms are displayed in a useful table which nicely assists these results in making the connections to (b)pcrls of Ch. 2 transparent.

Ch. 5 presents ingenious proofs of the admissibility of cut, cancellation and density for several hypersequent calculi and discusses the benefits which flow from them. It first shows that cut elimination holds for any regular hypersequent calculus, i.e., any calculus constituted by the identity axioms, the cut rule and structural rules which satisfy the substitutivity condition (cuts can be moved upwards) as well as logical rules which satisfy both substitutivity and reductivity (in cases in which both premises introduce the cut-formula, applications of cut can be replaced by cuts on its subformulae). Having established that cut-free regular calculi possess the subformula property the authors supply conservative extension and decidability results (the latter for hypersequents which are built up from contexts which are sets rather than multisets).

Next, cancellation elimination is shown to hold for a slightly modified hypersequent calculus (with cancellation) for Abelian Logic which implies cut elimination for the original calculus for that logic. An algebraic completeness proof for the modified calculus (without cancellation) is presented which appeals to so-called lattice-ordered abelian groups, a corollary of completeness being that the equational theory of these groups is decidable.

Finally, density elimination is established, first for calculi with and then for those without weakening. The result is first shown for all single-conclusion regular calculi with weakening. The proof rests on the idea of p-regularity preservation in the derivations (no occurrences of the same atom p on both sides of the same sequent) which is guaranteed by locality constraints on the rules. Next, density elimination is shown to hold for multiple-conclusion regular calculi with the weakening rule. In this case the notion of p-regularity has to be generalized in order to avoid inconsistency. The proof of density admissibility for calculi without weakening rests on a preliminary substitution which makes non-p-regular sequents p-regular (by modifying non-p-regular single-conclusion sequents in such a way that the left-hand p is deleted and the right-hand p replaced by the multiplicative truth constant). Interestingly, it is observed that density elimination for Gentzen systems implies the eliminability of the density rule for the corresponding Hilbert system which, in view of the fact that Hilbert systems with density are complete with respect to standard algebras (Ch. 3), implies completeness for Hilbert systems without density with respect to these algebras. This is certainly an observation which helps to justify the authors' conviction that hypersequent calculi are useful for the study of fuzzy logics. An open question is whether density elimination can be obtained for logics such as Ł and P.

Ch. 6 develops novel sequent (resp. hypersequent) calculi for the three "fundamental fuzzy logics": G, Ł and P. In Ch. 5 it has been shown that cut elimination, decidability and standard completeness hold for the hypersequent system for G, this chapter proposes a multiple-conclusion sequent calculus for this logic which is more efficient for proving theorems as it, obviously, does not contain external contraction (so the size of the sequents does not double) and as it contains special decomposition rules for the logical connectives which, in contrast to the hypersequent calculus for G, are all invertible (so no backtracking is needed). Completeness is shown purely semantically (i.e., by direct appeal to valuations) rather than by density elimination as in the case of the hypersequent version and which immediately gives standard completeness for the Hilbert system for G. Next, a multiple-conclusion hypersequent calculus for Ł is developed which is based on a definition of a semantic criterion for validity rather than on an interpretation of hypersequents as formulae and which contains non-standard rules for implication (e.g., the abelian rule for implication on the left) and for multiplicative conjunction. Again completeness and decidability are shown without appeal to syntactic eliminations. The sequent calculus for Ł is special. It rests on the idea of representing differences between sequents in terms of implicational formulae. The logical rules of Ł are particularly artificial. The completeness proof for this calculus simulates the proof for its hypersequent counterpart using a labeling method. In order to weaken the impression of artificiality, the authors suggest three interpretations of the Gentzen system for Ł: an interpretation in terms of (i) an embedding into Abelian Logic, (ii) McNaughton functions, and (iii) Giles's games (which are inspired by material Lorenzen-style dialogue games). The last interpretation does perhaps most to help ordinary intuitions.

The calculus for P rests on a non-standard interpretation of hypersequents as formulae which allows to assimilate validity in P to validity in Ł. Completeness for P is shown, as in previous cases, in terms of valuations rather than via cut elimination. Using previously established materials standard completeness and decidability can be obtained from this result.

Uniformity and efficiency are the topics of Ch. 7. The chapter begins with a non-standard interpretation of hypersequents. Relational (or r-)hypersequents generalize hypersequents in that they may contain two rather than a single kind of sequent. This introduction of additional structure makes it possible to define uniform logical rules for the fundamental logics and to distinguish them only by means of structural rules. To make proof search more focused, methods from goal-directed proof theory which is originally due to the third author are imported and adjusted (e.g., by adding a rule for combining databases and goals) so as to obtain goal-directed r-hypersequent systems for these logics. (Only their implicational fragments are considered.) These systems reduce implicational to atomic queries which may be checked for validity by other methods (graphs, linear programming methods). It is interesting to observe how changes in success and restart rules (for historical loop-checking) affect changes in logics. Finally, complexity theorems are established. In particular, the validity problems for Ł, G, and P are co-NP-complete.

Ch. 8 is devoted to first-order fuzzy logic. It first defines a core first-order language and an algebraic semantics for it which takes the value of a universally (existentially) quantified formula to be undefined in case the infimum (supremum) of the instances of its immediate subformula exists in the domain of the structure for the algebra. The authors proceed to define first-order extensions of the Hilbert systems from Ch. 3 which crucially contain an axiom scheme for infinite distributivity which allows to prove completeness (by the Henkin method) with respect to linearly ordered algebras. Completeness is also obtained with respect to dense and standard algebras. Next, soundness and completeness are shown for first-order hypersequent calculi with respect to first-order Hilbert systems by adapting the matching method of Ch. 4. These calculi syntactically distinguish free and bound variables (only free variables occur freely, only bound variables are bound) to simplify the proofs. Cut elimination is established for them as well as their conservativity with respect to their propositional versions. Also density is shown to hold for these calculi which gives standard completeness for several first-order Hilbert systems. Herbrand's theorem (the validity of an existential formula is equivalent to the validity of propositional formulae) is shown to hold for these systems by appeal to a special theorem which says that for any prenex formula which is derivable in a hypersequent calculus there is a derivation of that formula in that calculus in which applications of the rules for the connectives precede applications of the quantifier rules. Herbrand's theorem is then used to show, via Skolemization for fuzzy logics, that the validity of a prenex formula reduces to the validity of propositional formulae. The chapter concludes with proofs of an "approximate" Herbrand theorem for first-order Ł (which rests on the idea that an invalid propositional formula can to a certain degree come close to being valid) and, building on this, of Skolemization for that logic. In contrast to the preceding chapters the proofs tend to be sketchy.

The final Ch. 9 glimpses at topics such as modality, propositional quantifiers, non-commutative logics, finite-valued as well as comparative logics and discusses several open problems for the proof-theoretic approach. An intriguing difficulty is that no Gentzen system is (presently) available for Hájek's eminent Basic Logic.

In view of this and other open problems the investigation of fuzzy logics in terms of sequent and hypersequent calculi appears to be programmatic in important respects. However, the work carried out so far is impressive and the fact that there are things which need to be understood better make the topic attractive. This monograph is the natural place to begin self study of the proof theory for fuzzy logics.

(Some typos: p. 84 (Example 4.35, first derivation, right branch): $A \Rightarrow B \mid B \Rightarrow B$ should be $B \Rightarrow A \mid B \Rightarrow B$; p. 84 (same example, second derivation): $(\vee \Rightarrow)$ should be $(\Rightarrow \wedge)$; p. 86 (Example 4.41, left branch): (ID) should be $(\Rightarrow \mathrm{e})$; p. 146: $\min(1, 1 - x + y)$ should be $\max(0, x + y - 1)$; p. 164: $(Ga \Rightarrow \Delta)$ should be $(\Gamma \Rightarrow \Delta)$; p. 171: "$(\Rightarrow \odot)_{\mathsf{P}}$ of GP" should be "$(\Rightarrow \odot)_{\mathsf{A}}$ of GP"; p. 173: "$(\to\Rightarrow)$ rule fails" should be "$(\to\Rightarrow)_c$ rule fails"; p. 209: "$\mathsf{LIND}_T^{L\forall}$-valid" should be "$\mathsf{LIND}_T^{L\forall}$-invalid"; p. 210: "$\inf_{i \in I} \geq b \to c$" should presumably be "$(\inf_{i \in I} a_i) \geq b \to c$"; p. 218 (and subsequent pages): two superscripts (S and F) for the indication of Skolem forms of prenex formulae are one too much here; pp. 247–8 (Example 9.33): the first elements in the displayed matrices should be in braces.)

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APOSTOLOS DOXIADIS, CHRISTOS H. PAPADIMITRIOU, ALECOS PAPADATOS, AND ANNIE DI DONNA. ***Logicomix.*** Bloomsbury, New York, 2009, 347 pp.

Logicomix is a graphic novel that tells the story of some of the major events of Bertrand Russell's life (up to 1939) and the 'quest' for the foundation of mathematics in the period between 1900 and 1939. The narrator is Russell himself but at various points in the graphic novel the authors also make their appearance by giving their take on the significance of the events being narrated by Russell. Altogether there are ten sections. Six sections concern the major events recounted by Russell covering in order: Russell's childhood at Pembroke Lodge; 2. Russell's studies in Cambridge and his love for Alys (his first wife); 3. Travels to Germany; 4. The discovery of the paradoxes, ***Principia Mathematica***, and the break up with Alys; 5. Russell, Wittgenstein, and the ***Tractatus***; 6. Gödel's incompleteness. In addition, there are three sections (Overture, Entracte, Finale) in which the authors reflect upon the events being narrated and draw various alleged connections between madness and logic and the analogies between the quest for the foundations of mathematics and Aeschylus' ***Oresteia***. The last section, called "Notebook", aims at setting the historical record straight concerning the cast of characters in the story (Russell, Wittgenstein, Whitehead, Cantor, Frege, Hilbert, Gödel, etc.) and about the technical details of the results mentioned in the course of the narration (predicate calculus, completeness theorem, Russell's paradox, incompleteness theorem, etc.). The need for the "Notebook" was felt on account of the fact that the graphic novel admittedly takes liberties with the real course of events. The authors acknowledge that their reconstruction of Russell's life contains many fictional elements such as the alleged meetings Russell would have had with Frege and Cantor. Such departures from reality can be fruitful for narrative purposes. However, in some cases they are objectionable on account of the false image they convey of the field as a whole. Among such distortions are the portrayal of Frege as a rabid paranoid anti-semite and the constant refrain of the alleged causal link between logic and madness. While these aspects will certainly attract more readers (and ***Logicomix*** has been enormously successful in attracting a wide readership), I see no evidence whatsoever that mathematical logicians are affected by mental illness in ways that differ from the rate at which mental illness affects the general population. From the conceptual point of view, some of the major ideas about the foundations of mathematics are conveyed with reasonable accuracy although there are some serious errors that occur in the description of Wittgenstein's picture theory of language (where words are said to picture what they denote) and in the emphasis the authors put on certainty in Russell's foundational program after 1906 (contrary to the inductivist position that Russell himself holds from that point onward). If a second edition of the graphic novel is forthcoming, the authors should certainly revise parts of the "Notebook", which contains some mistakes and inaccuracies. For instance, the entry 'incompleteness theorem' confuses semantic with syntactic completeness; the entry on 'predicate calculus' gives the misleading impression that one cannot have first-order quantification over sets; the entries on 'Frege', 'Hilbert' and 'von Neumann' also contain alleged 'historical' facts for which there is no evidence. (I have discussed all of the above at length, and provided evidence for my critical remarks, in an essay review that can be downloaded at `https://philosophy.berkeley.edu/file/509/logicomix-review-january-2010.pdf`)

Logicomix is great fun to read. The story is engaging and the drawings captivating. The authors have managed to convey to the non-specialist some of the intellectual and human adventure of Russell's life and of the debate on the foundations of mathematics in the early part of the twentieth century. Readers of ***The Bulletin of Symbolic Logic*** will find ***Logicomix*** to provide much needed respite from their daily work. They will find much to enjoy and much to disagree with: but they will not remain indifferent.

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The search for diamonds.

SAHARON SHELAH. *Middle diamond*. ***Archive for Mathematical Logic***, vol. 44 (2005), pp. 527–560.

SAHARON SHELAH. *Diamonds*. ***Proceedings of the American Mathematical Society***, vol. 138 (2010), no. 6, pp. 2151–2161.

MARTIN ZEMAN. *Diamond, GCH and weak square*. ***Proceedings of the American Mathematical Society***, vol. 138 (2010), no. 5, pp. 1853–1859.

The Continuum Hypothesis (CH) implies, and is in fact equivalent to, the existence of an enumeration $\{A_\alpha : \alpha < \omega_1\}$ of *all* bounded subsets of ω_1. Given a set $A \subseteq \omega_1$, and an ordinal $\alpha < \omega_1$, we say that the above enumeration *predicts* A at α, if $A \cap \alpha = A_\alpha$. Jensen's diamond principle, $\diamondsuit$, is the strengthening of CH asserting the existence of an enumeration $\{A_\alpha : \alpha < \omega_1\}$ of all bounded subsets of ω_1 such that every subset $A \subseteq \omega_1$ gets predicted at some limit $\alpha < \omega_1$.

Now, consider the following generalization. Given a stationary set $S \subseteq \kappa$, $\diamondsuit_S$ asserts the existence of an enumeration $\{A_\alpha : \alpha \in S\}$ such that for every $A \subseteq \kappa$, the set $\{\alpha \in S : A_\alpha = A \cap \alpha\}$ is stationary. This concept has been introduced by R. B. Jensen in the late 60's of the last century; Jensen proved that $\diamondsuit_S$ holds in Gödel's constructible universe for every stationary $S \subseteq \kappa$ and every regular uncountable cardinal κ, and introduced the very first $\diamondsuit$-based construction of a complicated combinatorial object—a Souslin tree.

Since then, these principles have drawn a considerable attention, and have been used to solve problems, not just in logic, but also in real analysis, group theory, and topology.

Motivated by the utility of diamonds, the community began to study the validity of these principles, as well as weaker variants. We give a concise survey that is nowhere near complete: Already in the 1970's it was known that $\diamondsuit_{\omega_1}$ is equivalent to several seemingly-weaker statements (Devlin, Kunen), that GCH $+\neg\diamondsuit_{\omega_1}$ is consistent (Jensen), that GCH $+\diamondsuit_{\omega_1}+\neg\diamondsuit_S$ for some stationary $S \subseteq \omega_1$ is consistent (Shelah), that $\diamondsuit_\kappa$ holds for every measurable cardinal κ (Kunen), that $\diamondsuit_{2^\omega}$ holds if 2^ω is real-valued measurable (Ketonen), and that GCH implies $\diamondsuit_{\kappa^+}$ for every uncountable cardinal κ (Jensen, Gregory, Shelah). In the 1980's, Woodin established the consistency of $\neg\diamondsuit_\kappa$ for a Mahlo cardinal κ, while Shelah, dealing with successor cardinals, established the consistency of GCH $+\neg\diamondsuit_{T_\kappa}$ for a regular uncountable κ and $T_\kappa := \{\alpha < \kappa^+ : \mathrm{cf}(\alpha) = \mathrm{cf}(\kappa)\}$, as well as, GCH $+\neg\diamondsuit_S$ for a singular cardinal κ and a non-reflecting stationary subset $S \subseteq T_\kappa$. In the 1990's, Hauser, improving an earlier result of Woodin, showed that $\diamondsuit_{\mathrm{Reg}(\kappa)}$ may consistently fail for indescribable cardinals κ, where $\mathrm{Reg}(\kappa) := \{\alpha < \kappa : \mathrm{cf}(\alpha) = \alpha\}$, and Shelah proved that $2^\kappa = \kappa^+$ entails $\diamondsuit_{\kappa^+}$ for every cardinal $\kappa \geq \beth_\omega$.

The three papers under review continue this line of research. We commence with describing the main results of these papers, and subsequently, we shall be discussing the leading arguments of the involved proofs.

The main result of *Diamonds* is as follows: For a given subset $S \subseteq \kappa^+$ and an uncountable cardinal κ, if $S \setminus T_\kappa$ is stationary, then $2^\kappa = \kappa^+$ implies $\diamondsuit_S$. In particular, $2^\kappa = \kappa^+$ implies $\diamondsuit_{\kappa^+}$ for every uncountable cardinal κ.

As $\diamondsuit_{\kappa^+}$ yields an enumeration witnessing that $2^\kappa = \kappa^+$, Shelah's theorem provides a complete understanding of the relation between the value of 2^κ, and the validity of diamond over stationary subsets of $\kappa^+ \setminus T_\kappa$, thus, concluding a 40 year old search for such an understanding(!).

But there are further questions. More specifically, the preceding theorem leaves the following questions untouched:

1. The mathematical question: For a singular cardinal κ satisfying $2^\kappa = \kappa^+$, how large is the class of subsets of T_κ on which diamond may fail? In particular, is GCH $+\neg\diamondsuit_{T_\kappa}$ consistent?
2. The metamathematical question: Can one find a non-trivial prediction principle for κ^+ which is provable even assuming $2^\kappa > \kappa^+$?

One of the standard ways of attacking the first question is to study sufficient conditions that impose a link between elements of $\check{T}_\kappa := \kappa^+ \setminus T_\kappa$ (on which, diamond holds), to elements of T_κ (on which, we would like to establish diamond). Of course, reflection of stationary sets yields a link of this kind, and indeed, in the second and third papers under review, instances of reflection are shown to entail a partial answer to the first question.

In *Diamonds*, Shelah proves that for every singular cardinal κ of uncountable cofinality, $2^\kappa = \kappa^+$ implies $\diamondsuit_{T_\kappa}$ provided that κ^+ weakly reflects at $\mathrm{cf}(\kappa)$. In *Diamond, GCH and weak square*, Zeman, improving a theorem from an early paper of Shelah (*Diamonds, uniformization*, JSL XLIX 1022), proves that for every singular cardinal κ, if $2^\kappa = \kappa^+$ and $\square^*_\kappa$ holds, then $\{S \subseteq T_\kappa : \diamondsuit_S \text{ fails}\}$ omits any set that reflects stationarily often.

Next, let us deal with the second, metamathematical, question.

One of the earliest attempts to provide an answer to this type of question may be found in a paper by Devlin and Shelah (*A weak form of $\diamondsuit$ which follows from $2^{\aleph_0} < 2^{\aleph_1}$*, ***Israel Journal of Mathematics*** XXIX 239). To describe their result, let us first point out that $\diamondsuit_\kappa$ is equivalent to the existence of an enumeration of functions $\{g_\alpha : \alpha < \kappa\}$ such that for every function $f : \kappa \to 2$, the set $\{\alpha < \kappa : g_\alpha = f \restriction \alpha\}$ is stationary. Thus, in their paper, Devlin and Shelah introduce the following prediction principle:

(Φ) For every coloring $F : \bigcup_{\alpha<\omega_1} {}^\alpha 2 \to 2$, there exists a coloring $g : \omega_1 \to 2$, such that for every function $f : \omega_1 \to 2$, the set $\{\alpha < \omega_1 : g(\alpha) = F(f \restriction \alpha)\}$ is stationary.

They named this principle by *weak diamond*, and proved that it follows from $2^{\aleph_0} < 2^{\aleph_1}$. So, not only that weak diamond follows outright from CH, but, more importantly, it is a reminiscent of diamond which is consistent with $\neg$ CH.

Skipping a vast body of work concerning generalizations (as well as applications) of the weak diamond, we now arrive to Shelah's paper, entitled *Middle diamond*—a paper in which a striking answer to the metamathematical question is given.

Let us say that κ has the (θ, χ)-*middle diamond*, if there exists an enumeration $\{g_\alpha : \alpha < \kappa\}$ such that for every function $f : \kappa \to \chi$, the set of all $\alpha < \kappa$ satisfying:

- $\mathrm{cf}(\alpha) = \theta$, and
- there exists a club subset $c_\alpha \subseteq \alpha$, such that $g_\alpha = f \restriction c_\alpha$,

is stationary.

Nota bene that the above clubs c_α could be of a rather small size, and this is why it is not a-priori impossible for κ^+ to enjoy middle diamond while satisfying $2^\kappa > \kappa^+$.

Indeed, in the first paper under review, Shelah introduces middle diamond and establishes that many instances of it are consequences of ZFC. To exemplify: Suppose that μ is a limit of strong limit cardinals, and $\chi < \mu$; then for every regular cardinal $\kappa > \mu$ which is not strongly inaccessible, the set $\{\theta < \mu : \kappa$ has the (θ, χ)-middle diamond$\}$ contains a non-empty final segment of regular cardinals.

In particular, this shows that for every successor cardinal $\kappa > \beth_{\omega_1}$, there exists a sequence of local clubs $\langle c_\alpha : \alpha < \kappa \rangle$, and an enumeration $\{g_\alpha : \alpha < \kappa\}$, such that for every function $f : \kappa \to 2$, the set $\{\alpha < \kappa : g_\alpha = f \restriction c_\alpha\}$ is stationary.

We now turn to discuss some of the proofs from the papers under review. For this, it is helpful to establish a jargon. We do so by revisiting the following well-known claims:

- ► Kunen's theorem (***Set Theory, An Introduction to Independence Proofs***): $\diamondsuit_{\kappa^+}$ is equivalent to the existence of a matrix $\{A^i_\alpha : (\alpha, i) \in \kappa^+ \times \kappa\}$ such that $\{\alpha < \kappa^+ : \exists i < \kappa\ (A^i_\alpha = A \cap \alpha)\}$ is stationary for every $A \subseteq \kappa^+$.
- ► Shelah's *club guessing* theorem (***Cardinal Arithmetic***): If κ is an uncountable cardinal, then there exists a collection $\{C_\alpha : \alpha < \kappa^+\}$ with $\sup(C_\alpha) = \alpha$ for all $\alpha < \kappa^+$, such that $\{\alpha < \kappa^+ : C_\alpha \subseteq E \cap \alpha\}$ is stationary for every club subset $E \subseteq \kappa^+$. (So, unlike diamond, where all subsets of κ^+ gets predicted, here, we settle for *guessing* just *closed and unbounded* subsets of κ^+.)

The idea of Kunen's proof is to use some coding of the sets A^i_α, together with projection maps π_i, in such a way that allows to argue that for some $i < \kappa$, $\{\pi_i(A^i_\alpha) : \alpha < \kappa^+\}$ successfully predicts all subsets of κ^+. Indeed, if the latter fails, then we may collect κ many independent counterexamples—one to each i—and then unify them into a single encoded set whose projections are these counterexamples. By the hypothesis, this encoded set gets predicted stationarily often, hence by Fodor's lemma it gets predicted on some fixed $i < \kappa$, and in particular, its i_{th} projection gets predicted, contradicting the choice of the i_{th} counterexample.

The idea of the club guessing proof is to start with an arbitrary potential guessing sequence $\{C_\alpha : \alpha < \kappa\}$, together with an operation $\mathrm{g}\ell_D$, and to recursively construct a decreasing sequence of clubs $\langle D_i : i < \kappa \rangle$, in such a way that allows to argue that for some $i < \kappa$, $\{\mathrm{g}\ell_{D_i}(C_\alpha) : \alpha < \kappa^+\}$ successfully guesses all club subsets of κ^+. As in Kunen's proof, towards a contradiction, we collect κ many counterexamples. However, here, we do so recursively, where at stage $i+1$ of the recursion, we first improve our potential guessing sequence to make it correctly guess the i_{th} counterexample, and then we pick a new counterexample, witnessing that the just constructed sequence still fails to guess all clubs. The contradiction we meet here is that a potential guessing sequence cannot be improved "forever".

Now, the proof of the main theorem from *Diamonds* may be viewed as a combination of the above two. The outline of the proof is as follows. Since $S \cap \check{T}_\kappa$ is statinoary, we may pick a matrix indexed by $S \times \kappa$ that has a very rough, yet non-trivial, predicting feature, commonly dubbed as *hitting*. By $2^\kappa = \kappa^+$, we can fix for each $i < \kappa$, a certain projection map $\mathrm{cd}_i : \kappa^+ \to \kappa^+$. Then, by an argument a-la Kunen, we find some $i < \kappa$, such that the projection cd_i of the i_{th} column alone enjoys the very same hitting feature of the whole matrix. Next, we run a club guessing type of argument, using this projected column as our initial potential predicting sequence, which we keep improving recursively via counterexamples. Finally, a second projection argument is invoked, establishing that the recursive improvement process cannot be carried endlessly.

It is worth mentioning that the above argument is not really limited to stationary subsets of $\check{T}_\kappa$. In fact, this proof applies to any stationary set (or just normal filters, concentrating on a set) that admits a hitting matrix. Recalling that Džamonja and Shelah showed that T_κ admits a hitting matrix whenever κ^+ weakly reflects at $\mathrm{cf}(\kappa)$, it should now be clear how to prove the other result from the *Diamonds* paper.

While weak reflection is used to *pump up* diamond from $\check{T}_\kappa$ to T_κ, in Zeman's paper, we go in the inverse direction. The outline is as follows. If $S \subseteq T_\kappa$ reflects stationarily often, then it necessarily reflects within $\check{T}_\kappa$. So, utilizing the coherence property of $\square^*_\kappa$, it is possible to *pull* the arguments that work successfully for $\check{T}_\kappa$, *down* to the set S.

Zeman's paper commences with the introduction of a new principle, denoted $\bigcirc_\kappa(S)$, which has been isolated from a simplified presentation, due to P. Komjáth, of the second paper under review. Then it is proved that $\bigcirc_\kappa(S)$ implies $\diamondsuit_S$ for every stationary $S \subseteq \kappa$ and every regular uncountable cardinal κ (including the case that κ is inaccessible!). And, finally, it is proved that $2^\kappa = \kappa^+$ together with $\square^*_\kappa$ entails $\bigcirc_{\kappa^+}(S)$ provided that $S \subseteq T_\kappa$ reflects stationarily often.

We now turn to describe the structure of the first paper under review.

In Section 1 of *Middle diamond*, a sufficient condition for the validity of middle diamond over cardinals of the form $\mathrm{cf}(2^\mu)$, is given. Of course, if $2^\mu = \mu^+$, then we have natural projection maps that enable us to run a Kunen-type of argument, but how do we diagonalize otherwise?

Shelah's solution to this problem is in the form of a newly introduced principle, denoted $\mathrm{Sep}(\mu, \chi)$, which commutes nicely with certain projection maps, $\mathrm{cd}_i\colon 2^\mu \to 2^\mu$. The *main claim* of this section is: If $\lambda = 2^\mu$, $\kappa = \mathrm{cf}(\lambda)$ and some stationary $S \subseteq \{\alpha < \kappa\colon \mathrm{cf}(\alpha) = \theta\}$ carries a reasonably-coherent local clubs sequence, $\overline{c} = \langle c_\alpha\colon \alpha \in S\rangle$, then $\mathrm{Sep}(\mu, \chi)$ together with an arithmetic hypothesis involving λ and θ, yields the following relative of weak diamond:

($\Phi_{\overline{c},\lambda,\chi}$) For every coloring $F\colon \bigcup_{\alpha\in S} {}^{(c_\alpha)}\lambda \to \chi$, there exists a coloring $g\colon S \to \chi$, such that for every function $f\colon \kappa \to \lambda$, the set $\{\alpha \in S\colon g(\alpha) = F(f \restriction c_\alpha)\}$ is stationary.

The proof of the latter may be viewed as a fine, yet natural, abstraction of the Kunen argument, where each ingredient of the original argument admits a subtle counterpart that allows to overcome the unrelaxed working conditions.

Apparently, at this point of the paper, the value of the main claim is not completely understood; for instance, it is unclear whether its hypothesis is commonly satisfiable, and whether its assertion entails that κ has the (θ, χ)-middle diamond. Fortunately, the rest of the paper is devoted to clarifying these aspects.

At the second part of Section 1, sufficient conditions for $\mathrm{Sep}(\mu, \chi)$ to hold, and for $\Phi_{\overline{c},\lambda,\chi}$ to imply middle diamond, are provided. In addition, Shelah addresses the above-mentioned arithmetic hypothesis "involving λ and θ" by recalling results from his seminal paper *The generalized continuum hypothesis revisited*, ***Israel Journal of Mathematics*** CXVI 285.

In Section 2, the existence of a stationary subset of $\{\alpha < \kappa\colon \mathrm{cf}(\alpha) = \theta\}$ that carries a "reasonably-coherent" local clubs sequence is studied. For a cardinal κ which is the successor of a regular cardinal, the existence of such a stationary set has already been established in Shelah's paper *Reflecting stationary sets and successors of singular cardinals*, ***Archive for Mathematical Logic***, XXXI 25. As for cardinals which are not of the above form, the existence of such a stationary set is obtained here as a corollary to a generalization of the Engelking-Karłowicz theorem, whose proof occupies a large portion of this section.

By the end of the paper, it is understood that the hypothesis of the main claim is indeed commonly satisfiable, and that (θ, χ)-middle diamond is derivable from $\Phi_{\overline{c},\lambda,\chi'}$ for a large enough χ'. Thus, the only gap from our original goal is that the main claim applies to cardinals of the form $\mathrm{cf}(2^\mu)$, while we wanted to yield middle diamond over cardinals of additional patterns. For this, the paper is concluded with two theorems that address the question of lifting up the middle diamond from cardinals of the form $\mathrm{cf}(2^\mu)$, to arbitrary regular cardinals.

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S. Feferman. *Reflecting on incompleteness*. ***The Journal of Symbolic Logic***, vol. 56 (1991), no. 1, pp. 1–49.

W. N. Reinhardt. *Some remarks on extending and interpreting theories with a partial predicate for truth*. ***Journal of Philosophical Logic***, vol. 15 (1986), no. 2, pp. 219–251.

V. Halbach and L. Horsten. *Axiomatizing Kripke's theory of truth*. ***The Journal of Symbolic Logic***, vol. 71 (2006), no. 2, pp. 667–712.

H. Friedman and M. Sheard. *An axiomatic approach to self-referential truth*. ***Annals of Pure and Applied Logic***, vol. 33 (1987), no. 1, pp. 1–21.

V. Halbach. *A system of complete and consistent truth*. ***Notre Dame Journal of Formal Logic***, vol. 35 (1994), no. 3, pp. 311–327.

Much of the literature on theories of truth can be traced back to Kripke's seminal paper, *An outline of a theory of truth*. Kripke's aim was to provide an account of self-applicable truth which avoided the inconsistencies highlighted by Tarski's theorem. This involved abandoning the naive approach that truth was a total concept and passing to notions of partial truth where there are formulae that are neither declared true nor false. Let $\mathcal{L}_T = \mathcal{L} \cup \{T\}$ denote the language of arithmetic augmented by an additional unary predicate symbol T. A partial $\mathcal{L}_T$-structure of truth over the natural numbers is a triple $\langle \mathbb{N}, S_1, S_2\rangle$ with $S_1, S_2 \subseteq \mathbb{N}$. Satisfaction for such a structure is defined as usual for the connectives and atomic formulae with the following additions: $\langle \mathbb{N}, S_1, S_2\rangle \models T(t)$ if $t^{\mathbb{N}} \in S_1$, and $\langle \mathbb{N}, S_1, S_2\rangle \models {\sim}T(t)$ if $t^{\mathbb{N}} \in S_2$. The partiality of the structure arises from the fact it need not satisfy $T(t) \vee {\sim}T(t)$ for every term t. Kripke, however, provides a canonical method to *close off* partial models and obtain classical models; one simply drops the interpretation S_2 and sets $\langle \mathbb{N}, S_1\rangle \models {\sim}T(t)$, if $t^{\mathbb{N}} \notin S_1$. A *fixed point model* is a partial model $\langle \mathbb{N}, S_1, S_2\rangle$ where the pair S_1, S_2 is a fixed point of the induced monotone function $\Phi_{\mathbb{N}}\colon \langle X_1, X_2\rangle \mapsto \langle\{\ulcorner A\urcorner\colon \langle \mathbb{N}, X_1, X_2\rangle \models A\}, \{\ulcorner A\urcorner\colon \langle \mathbb{N}, X_1, X_2\rangle \models {\sim}A\}\rangle$. Such models validate a sentence A iff $A \in S_1$, and ${\sim}A$ iff $A \in S_2$. Moreover, they validate the Tarskian bi-conditional for all sentences that obtain a truth value. These sentences, which Kripke called *significant*, play a fundamental rôle in Kripke's treatment of truth.

Kripke's approach is purely model-theoretic and there have been attempts to formalise his methods. Feferman, in *Reflecting on incompleteness* [***The Journal of Symbolic Logic***, vol. 56 (1991), no. 1, pp. 1–49] employs Kripke's techniques to characterise what "*ought* to be accepted *if* one has accepted the given basic notions [of a theory]" (p. 44, Feferman's italics) in a manner that harnesses the informal notion of acceptance without making explicit use of ordinals and notation systems. Thus, he formulates a number of operations to obtain new theories from old. The first of these of interest is the *ordinary truth theory* associated with a theory S, labelled $\mathsf{Tr}(S)$: a new predicate T_0 is added to S along with axioms expressing that T_0 behaves as a truth predicate for the language of S and all axiom schemata are extended to include the augmented language. Iteration of the underlying operation, $S \mapsto \mathsf{Tr}(S)$, up to an ordinal α yields what has since become known as the ramified hierarchy of truth, RT_α; one has an hierarchy of predicates, $\{T_\beta\colon \beta < \alpha\}$, each a truth predicate for the language encompassing the predicates of smaller index. The author remarks that RT_α and the ramified hierarchy at level α, RA_α, are inter-translatable, though the proof Feferman refers to is missing from the paper. The reduction of RA_α to RT_α, however, is relatively straightforward and a proof of the converse reduction can, essentially, be gleaned from the author's arguments in section 4.3. A direct proof using Feferman's techniques can be found in section V.18 of Halbach's book ***Axiomatische Wahrheitstheorien*** [Akademie Verlag, Berlin, 1996].

By replacing the basic truth predicate of $\mathsf{Tr}(S)$ with self-applicable notions of truth Feferman is able to define stronger theories of truth. The (*weak*) *reflective closure of* S, $\mathsf{Ref}(S)$, extends S with predicates T and F for truth and falsity. The predicates are defined to behave compositionally on the language $\mathcal{L}_S \cup \{T, F\}$ and hence allow self-applicability. Furthermore all axiom schemata of S are extended to also incorporate the new predicates. The theory $\mathsf{Ref}(S)$ is classical and starting from certain models of S, Feferman shows how one may construct closed-off Kripke models of $\mathsf{Ref}(S)$ by interpreting $F(A)$ as $T(\sim A)$. Thus, if $\mathsf{Ref}(S) \vdash T(A)$, A is valid in all fixed point models of S and it is in this sense that truth and falsity in $\mathsf{Ref}(S)$ can be viewed as partial. If S is formulated as a schematic theory with a free predicate symbol P and a rule of the form P-Subst: 'from $A(P)$ infer $A(B)$ for any formula B', Feferman also considers the *strong reflexive closure of* $S(P)$, $\mathsf{Ref}^*(S(P))$, in which the truth predicate also applies to P and one has a suitably modified form of P-Subst. The strong reflexive closure forms a significantly stronger theory than the weak version. The main theorem of the paper is that $\mathsf{Ref}(\mathsf{PA}) \equiv \mathsf{RA}_{<\epsilon_0}$ and $\mathsf{Ref}^*(\mathsf{PA}(P)) \equiv \mathsf{RA}_{<\Gamma_0}$, where $\equiv$ denotes the relation of proof-theoretic equivalence and Γ_0 denotes the so-called Feferman–Schütte ordinal of predicativity. Of particular interest is the embedding of $\mathsf{RA}_{<\epsilon_0}$ into $\mathsf{Ref}(\mathsf{PA})$. Feferman makes use of what he refers to as the *determinate* sentences in $\mathsf{Ref}(\mathsf{PA})$, those which are either true or false but not both, to enable him to form infinite conjunctions: given a list of sentences $A_0, A_1, \dots$ and a primitive recursive function f such that $f(n) = \ulcorner A_n \urcorner$, he constructs (the code of) a sentence $\bigwedge f$ such that $T(\bigwedge f) \leftrightarrow \forall n\, T(A_n)$ is derivable in $\mathsf{Ref}(\mathsf{PA})$ provided every A_n is determinate. In Kripke models where the two predicates are disjoint, the determinate sentences are precisely the significant ones. It is perhaps unsurprising then that the schema $T(A) \leftrightarrow A$ is derivable in $\mathsf{Ref}(S)$ for all determinate sentences A. From this observation it becomes straightforward to show the jump hierarchy and hence also the ramified hierarchy up to ϵ_0 can be interpreted within the reflective closure of PA.

Key to the motivation behind introducing the notions of reflective closure is their naturalness. Feferman argues that the axioms added to S to form $\mathsf{Ref}(S)$ "correspond directly to the informal notion of grounded truth and falsity, which is the simplest notion of partial self-applicable truth predicates. Moreover, the use of classical logic and the full schematic principles in $\mathsf{Ref}(S(P))$ accords with informal reasoning" (p. 42). Another important factor for Feferman is the view that the reflective closure is a general operation and can be applied to a number of differing theories. He suggests applying it to, for example, Howard's theory of the countable ordinals, the theory of real numbers and fragments of set theory. In the case of set theory, set-theoretical reflection principles that are expressible with the aid of a truth predicate become candidates for inclusion as further axioms, providing a notion of reflective closure specific to set theory.

Related to Kripke's notion of significance is Gödel's idea that concepts such as 'meaningfully applicable' need not be meaningful when applied to arbitrary arguments. By interpreting 'A is meaningfully applicable to x' as '$A(\bar{x})$ is significant', Reinhardt, in his paper *Some remarks on extending and interpreting theories with a partial predicate for truth* [***Journal of Philosophical Logic***, vol. 15 (1986), no. 2, pp. 219–251], explores the possibility of Kripke's theory serving as an implementation of Gödel's suggestion. As there are sentences A for which the statement 'it is significant that A is significant' does not hold in Kripke's theory, Reinhardt's interpretation accords with Gödel's informal view that the concept 'meaningful' is partial.

Reinhardt's main proposal is to view the Kripke-Feferman theory KF, defined as $\mathsf{Ref}(\mathsf{PA})$ plus the axiom stating that truth and falsity do not intersect, as an axiomatisation of Kripke's theory. KF is not itself sound with respect to fixed point models but its internal theory $\mathsf{IKF} = \{A \colon \mathsf{KF} \vdash T(A)\}$ is; thus the author recommends taking $\mathsf{KF} \vdash T(A)$ as a sufficient condition to declare A to be true. Reinhardt notes that his proposal has a number of desirable properties for an informal theory of truth and significance; for example, its formalisation, $\mathsf{Prov}_{\mathsf{KF}}(\ulcorner T(\dot{x})\urcorner) \rightarrow T(x)$, is significant and can be safely admitted as true: adding $T(\ulcorner \forall x[\mathsf{Prov}_{\mathsf{KF}}(\ulcorner T(\dot{x})\urcorner) \rightarrow T(x)]\urcorner)$ to KF results in a stronger, but consistent, theory and a weaker notion of truth. The proposal is explored further by considering variants of the liar sentence and their consequences. His analysis shows that many statements about non-significance are inconsistent with KF, spreading some light on the nature of significance and non-significance within Kripke's theory. These include the naive 'it is always significant that A is significant' and the more plausible 'if it is provable in KF that A is not significant, it is significant that A is not significant'. With this interpretation, however, KF behaves merely as an instrument for generating the 'real' theorems of interest, which all lie within IKF. Reinhardt is clearly aware of this and suggests embarking on an analogue of Hilbert's programme, the objective being, he writes, "to justify the use of nonsignificant sentences entirely within the framework of the significant sentences" (p. 225).

With this in mind Reinhardt, in *Remarks on significance and meaningful applicability* [in ***Mathematical logic and formal systems***, (de Alcantara, editor), Lecture Notes in Pure and Applied Mathematics, vol. 94 (1985), pp. 227–242], asks whether, for every $A \in \mathsf{IKF}$ there is a proof $A_0, A_1, \ldots, A_n$ of A, such that $A_i \in \mathsf{IKF}$ for each $i < n$. This question is answered negatively by Halbach and Horsten in the third paper under review, *Axiomatizing Kripke's theory of truth* [***The Journal of Symbolic Logic***, vol. 71 (2006), no. 2, pp. 667–712]. It is shown that for any such proof of $A \in \mathsf{IKF}$, in fact $\mathsf{PA} \vdash A$ and thus Reinhardt's goal cannot be obtained. The result does not preclude IKF from having a natural axiomatisation, but does demonstrate the reliance on the 'ideal' statements of KF to prove the 'real' theorems of IKF. The classical nature of KF provides a number of undesirable features for an axiomatisation of Kripke's theory, not least of which is the discrepancy between its internal and external theories. The authors therefore reject IKF and set themselves the task of determining a natural axiomatisation of Kripke's theory which, they conclude, should be formulated directly in partial logic. Halbach and Horsten are not the first to consider formal systems of partial logic; the theory they propose is related to systems of Scott and Blamey and formulated in a Gentzen-style two-sided sequent calculus. This is not the place to explain in detail their system, but it can be recognised as essentially classical logic with, in place of any introduction or elimination rules for negation, a rule of *modus tollens*, sequents for double negation elimination and the curious λ and $\sim\!\lambda$-sequents: $\lambda \Rightarrow \sim\!\lambda$ and $A, \sim\!A \Rightarrow \lambda$ respectively, where λ denotes the liar sentence $\lambda \leftrightarrow \sim\!T(\lambda)$. The idea behind the $\sim\!\lambda$-sequents is as sound replacements for the unsound *ex falso quodlibet* sequents $A, \sim\!A \Rightarrow \emptyset$. These peculiar sequents are, however, never utilised and appear unnecessary for the analysis. On top of the logic are added the axioms of arithmetic and sequents corresponding to the axioms of compositional truth used in KF, forming the system PKF. Halbach and Horsten show that Kripke fixed point structures form natural models of PKF and, moreover, the soundness argument can be formalised within a sub-theory of KF with restricted induction. By a result due to Cantini this sub-theory has the same proof-theoretic strength as $\mathsf{RA}_{<\omega^\omega}$, and thus the authors obtain an upper bound on the strength of PKF.

Although formulated in partial logic, PKF behaves classically on its truth-free sub-language and thus one may query its proof-theoretic strength in relation to classical theories. The authors demonstrate, in fact, that PKF behaves classically on the sub-language of $\mathcal{L}_T$ corresponding to $\mathsf{RT}_{<\omega^\omega}$, and thus the embedding of $\mathsf{RT}_{<\omega^\omega}$ into PKF is established with relative ease. This result also acts as a response to an objection made by Feferman that partial logic is unsuitable for everyday reasoning in mathematics: "We do not propose to carry out mathematics in partial logic", write Halbach and Horsten, "Only when the problematic truth predicate is added, [do] we have to give up classical reasoning" (p. 706). The result also raises an interesting question: Why do KF and PKF differ so much in their content when each appears to provide a natural axiomatisation of Kripke's theory of truth? The authors suggest that KF instead axiomatises "large parts of its metatheory. If the resources for distinguishing between truth, falsity and indeterminateness are dropped and a thoroughly partial system is adopted, proof-theoretic strength is lost" (p. 707).

Required reading for almost anyone in the subject is Friedman and Sheard's paper, *An axiomatic approach to self-referential truth* [***Annals of Pure and Applied Logic*** vol. 33 (1987), no. 1, pp. 1–21]. It stands out in the literature on axiomatic truth in its aim to provide a philosophically neutral analysis of theories of self-applicable truth. This is a delicate undertaking and the authors are careful to avoid passing any personal judgements on the behaviour of truth: "We are not solving a problem in philosophy" they remark, "but rather a problem in logic with a philosophical motivation" (p. 2). The authors isolate twelve axioms, axiom schemata and rules of inference involving a predicate T (together referred to as *Optional Axioms*) for further analysis. Each Optional Axiom expresses a property one might intuitively (or naively) expect to hold if T is to be interpreted as 'truth'. These range from direct weakenings of the inconsistent Tarski bi-conditional to principles stating that truth is consistent, complete or commutes with the first-order quantifiers. By adjoining different subsets of the Optional Axioms to a weak base theory of truth conservative over PA, the authors obtain their possible theories. To call each one a 'theory of truth', however, could be misleading: "One may reasonably take the attitude that what we are really exploring is the axiomatic properties of concepts which lie somewhere between 'provability' (which is well understood and somehow insufficient) and full 'truth' (which is mysterious and perhaps inherently unstable)" (p. 3). The main result of the paper, that over the base theory there are exactly nine maximal consistent subsets of the Optional Axioms, is presented in the form of a sequence of charts illustrating the various sources of inconsistency as well as groups of consistent principles.

The proof of the theorem splits into two parts: constructing a model for each of the nine theories in question and proving that any subset of the Optional Axioms not covered by the first part is inconsistent. Concerning the first half of the proof, it is the theories containing both the rule of inference T-Elim ('*from* $T(A)$ *infer* A') and the axiom U-Inf, stating the predicate commutes with the universal quantifier, that have the most interesting model constructions. The theory named D, for example, is further axiomatised by the rule T-Intro ('*from* A *infer* $T(A)$') and by principles stating that the truth predicate commutes with all connectives and quantifiers. The authors show truth in D can be interpreted as truth in the stable limit of Belnap and Gupta's revision theory of truth. The theory H, however, which contains the schema $T(A) \to A$, and the axioms T-Rep, $T(x) \to T(T(x))$, and U-Inf requires the truth predicate to be interpreted by a notion of derivability in ω-logic which fulfils certain 'self-satisfiability' criteria. The second half of the proof begins with what can only be described as a thorough analysis of the liar sentence; the inconsistency of sixteen differing subsets of the Optional Axioms is directly computed from the liar sentence over the base theory. This mass of raw computation is enough to answer the question of consistency for all subsets except those containing the quadruple T-Intro, T-Elim, T-Rep and T-Del, $T(T(x)) \to T(x)$. To deal with such theories the authors turn to Löb's theorem. They show that T-Intro and T-Rep suffice to derive a schematic form of Löb's theorem, and by further assuming T-Del and T-Elim one achieves a contradiction.

To conclude the paper two consistent subsets of the Optional Axioms are analysed for their proof-theoretic content. Although the subsets considered are not maximal consistent their analysis is of independent interest. One subset, while expressively rich, is shown to be a conservative extension of Peano Arithmetic, whereas another seemingly simple subset, H without the axiom T-Rep, proves the same arithmetical statements as the theory of Bar Induction, a theory proof-theoretically equivalent to the theory of one inductive definition.

Halbach's paper, *A system of complete and consistent truth* [***Notre Dame Journal of Formal Logic***, vol. 35 (1994), no. 3, pp. 311–327], takes a closer look at the theory D introduced by Friedman and Sheard. Halbach begins by providing a more natural axiomatisation of D, labelled FS, which eliminates the need for the base theory. Thus FS is described as PA augmented by a self-applicable truth predicate commuting with all logical connectives and quantifiers, and closed under the rules T-Intro and T-Elim. The first result of the paper is that FS essentially axiomatises the finite iterations of Belnap and Gupta's revision theory of truth: if FS_0 denotes PA and FS_{n+1} denotes the set of theorems of FS derivable with at most n

applications of each of T-Intro and T-Elim, Halbach proves that the ω-models of FS_n are exactly those standard $\mathcal{L}_T$-structures that are the image of n applications of the revision rule for truth. Moreover, each such ω-structure has associated with it a largest natural number k such that it is an ω-model of FS_k, thus showing FS is ω-inconsistent. As the author notes, McGee's theorem can be applied directly to FS to deduce its ω-inconsistency, but this would not offer the finer grained results Halbach obtained regarding its approximations. The second result is that FS is proof-theoretically equivalent to $\mathsf{RT}_{<\omega}$. In his reduction of $\mathsf{RT}_{<\omega}$ to FS the author does not make use of the rule T-Elim, and thus determines that the rule is unnecessary for deriving the truth-free theorems of FS. A later result by Sheard shows that likewise T-Intro is not required to derive the truth-free consequences of FS; it is unknown, however, whether either rule is superfluous in its axiomatisation.

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MATTHIAS SCHIRN. *On translating Frege's die Grundlagen der Arithmetik*. ***History and Philosophy of Logic***, vol. 31 (2010), no. 1, pp. 47–72.

In 1984, Ignacio Angelelli came to the conclusion that the "English translations of Frege's texts on abstraction are disastrous" (cf. "Frege and Abstraction", ***Philosophia Naturalis*** 21, p. 470). This evaluation referred in particular to the first English translation of Frege's ***Grundlagen*** by John Austin in 1950. A complete re-translation—as undertaken by Dale Jacquette (Prentice Hall 2007)—seems to be necessary and desirable. But as the article under consideration points out, the new translation fulfils neither the requirement of better terminological accuracy (than Austin's) nor the demand for stylistic elegance as already established by Austin. Instead, Austin's version is, according to Schirn, for the most part still "more polished and more idiomatic than Jacquette's" (p. 50).

Schirn's paper is far more than only a critical review essay on the recent translation of Frege's Grundlagen by Jacquette. In addition, it proposes a number of constructive suggestions for further translations as well as for contemporary anglophone investigations using Frege's texts. Nevertheless, the major work of Schirn's review essay consists in a thorough correction of Jacquette's mistakes ranging from misunderstandings and shortcomings in his Introduction and Critical Commentary to the choice of a—partially—inappropriate terminology, to the use of different renderings of a characteristic Fregean phrase up to the loss of important words and entire sentences. Schirn's paper is organized as follows: Whereas in sections 2 and 3 (pp. 51–58) he puts Jacquette's views on Frege's semantic terminology and the logicist definition of the cardinality operator in the right perspective, the two following paragraphs (pp. 58–62) shed light on the important phrases "the natural series [sequence] of numbers" and "ascription of number", which go straight to the core of Frege's philosophy of arithmetic. Sections 6 and 7 (pp. 62–71) deal with the remedy of further unfortunate choices of words, suggestions for the improvement of literal accuracy and some delicate omissions of text. The analyzed inaccuracies and errors reach from simple but meaning destroying mistranslations like 'to put together' instead of 'put on a par' for 'zusammenstellen' (cf. pp. 62f) or 'much more' instead of 'rather' for 'vielmehr' (cf. pp. 64f), too literal translations of passages (cf. pp. 68f) up to the deletion of whole but indispensable sentences (cf. pp. 69f). Moreover—as Schirn points out by way of example in his introduction—the attached bibliography is "unbalanced and not carefully compiled" (p. 50).

Some of Jacquette's mistakes make for an outright reversion of Frege's intended meaning. This reminds one loosely of a prominent and, by Neo-Fregeans, often borrowed passage from Frege's correspondence (letter to Russell, 28th of July 1902) which is affected by the addition of a negation where there is none in the German: "But the difficulties here are not [sic!] the same" (***Philosophical and Mathematical Correspondence***, p. 141). A remarkable number of such mistakes (including the just named example) are already made explicit and corrected by Michael Kremer in his likewise enlightening review of Jacquette's translation (***Notre Dame Philosophical Reviews*** 2008.01.07). But, whereas Kremer still favours Austin's translation and treats it as a "marvellous rendering" of Frege's ***Grundlagen***, Schirn does not recognize either translation as terminologically accurate and stylistically elegant. His final judgment "It is time for a truly reliable English translation of Frege's masterwork" (p. 71) is unequivocal and, in the light of his analysis, well justified. Schirn's analysis is much more thoroughgoing than Kremer's and considers, parallel to Jacquette, not only Austin's translation but also the whole range of Frege's relevant publications. His manifold corrections and improvements are addressed to both full translations. (Michael Beaney's translation of the Introduction and selected paragraphs of Frege's ***Grundlagen*** (cf. ***The Frege Reader***, Blackwell 1997, pp. 84ff.) are broadly excluded from this critique.)

Due to his sensible recommendations for both a terminologically precise and more suitable and smoother English version of Frege's ***Grundlagen***, Schirn's paper is a valuable source for Frege scholars.

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