THE BULLETIN OF SYMBOLIC LOGIC
Volume 16, Number 2, June 2010

REVIEWS

The Association for Symbolic Logic publishes analytical reviews of selected books and articles in the field of symbolic logic. The reviews were published in ***The Journal of Symbolic Logic*** from the founding of the JOURNAL in 1936 until the end of 1999. The Association moved the reviews to this BULLETIN, beginning in 2000.

The Reviews Section is edited by Steve Awodey (Managing Editor), John Baldwin, John Burgess, Mark Colyvan, Anuj Dawar, Marcelo Fiore, Noam Greenberg, Hannes Leitgeb, Roger Maddux, Ernest Schimmerling, Carsten Schürmann, Kai Wehmeier, and Matthias Wille. Authors and publishers are requested to send, for review, copies of books to *ASL, Box 742, Vassar College, 124 Raymond Avenue, Poughkeepsie, NY 12604, USA.*

In a review, a reference "JSL XLIII 148," for example, refers either to the publication reviewed on page 148 of volume 43 of the JOURNAL, or to the review itself (which contains full bibliographical information for the reviewed publication). Analogously, a reference "BSL VII 376" refers to the review beginning on page 376 in volume 7 of this BULLETIN, or to the publication there reviewed. "JSL LV 347" refers to one of the reviews or one of the publications reviewed or listed on page 347 of volume 55 of the JOURNAL, with reliance on the context to show which one is meant. The reference "JSL LIII 318(3)" is to the third item on page 318 of volume 53 of the JOURNAL, that is, to van Heijenoort's *Frege and vagueness*, and "JSL LX 684(8)" refers to the eighth item on page 684 of volume 60 of the JOURNAL, that is, to Tarski's *Truth and proof*.

References such as 49***5*** or 280***1*** are to entries so numbered in *A bibliography of symbolic logic* (the JOURNAL, vol. 1, pp. 121–218).

ECKART MENZLER-TROTT. TRANSLATED BY CRAIG SMORYŃSKI AND EDWARD GRIFFOR. ***Logic's lost genius: The life of Gerhard Gentzen.*** History of mathematics, vol. 33. American Mathematical Society, Providence, RI, 2007, xxii+441 pp.

This book is a translation of an unpublished revised edition of ***Gentzens Problem. Mathematische Logik im nationalsozialistischen Deutschland. Mit einem Essay von Jan von Plato*** (Birkhäuser Verlag, Basel, 2001). The first edition was reviewed in this journal by Volker Peckhaus (BSL IX 508). It contains four appendices: two essays, "Gentzen and Geometry" (Appendix A) and "Hilbert's Programme" (Appendix B), by the principal translator, Craig Smoryński, "Three Lectures by Gerhard Gentzen" (Appendix C), translated by Smoryński, and "From Hilbert's Programme to Gentzen's Programme", by Jan von Plato. The latter is von Plato's translation and revision of his essay in the original edition. Smoryński's discussion of Hilbert's program is a refreshing and provocative addition to the literature on that subject. Of the three lectures of Gentzen in Appendix C, two of them are also translated by M. E. Szabo in his *The Collected Papers of Gerhard Gentzen* (JSL XXXVII 752): "The concept of infinity in mathematics" and "The current situation in research in the foundations of mathematics". The other one, "The concept of infinity and the consistency of mathematics" is a short paper that was delivered to a mathematical congress in Paris in 1937 and was not included in Szabo's collection.

The first chapter concerns Gentzen's (1909-1945) life up through his final examinations (the Abitur) in school in 1928. His early family life, in Greifswald in north-east Germany,

1079-8986/10/1602-0005/$2.80

seems to have been comfortable and happy: his mother was a teacher and his father a lawyer. Gentzen had one sibling, a sister Waltraut who was born in 1911 and to whom by all accounts he was very close. At the outset of World War I in 1914 the father was called up to serve in the home guard. His health broke down and he died in 1919. The remaining family then moved further north to Stralsund to live with Gentzen's paternal grandmother, where they lived less comfortably and, from the recorded recollections of Waltraut it seems, less happily. He did very well in his final examinations: out of the fourteen categories in which he was graded (which includes religion, physical education and handwriting), he received four grades of 'very good', including mathematics, the rest being 'good's or, in the case of music, 'satisfactory'. (He was later reported by friends in Prague to have confessed to finding music "extremely unpleasant noise.") Clearly the age of grade inflation hadn't begun.

Chapters 2 and 3 comprehend the years 1928–1942 and so cover the period of all of Gentzen's published scientific papers with the exception of his posthumously published paper in which he proved that multiple instances of mathematical induction in a deduction in arithmetic can be replaced by a single one and which was written in 1944. After some semesters successively at the universities of Greifswald, Göttingen, Munich and Berlin, he returned to Göttingen in 1931, where he remained as student and then assistant until 1943, except for a period from September 1939 to June 1942 of military service. The paper containing his direct proof that ϵ_0 is the proof-theoretic ordinal of arithmetic was submitted in 1942, a month after his discharge from military service, and was his habilitation thesis, which was submitted in 1939.

Chapter 4 is a lengthy discussion of the complex interactions, in particular in the field of mathematical logic, among mathematicians, the mathematical establishment and the Nazi party in general during the years the latter was in power. Roughly, the issue concerned whether the axiomatic conception of mathematics, in which logic plays a major role, was sufficiently German or whether, to the contrary, it was the product of an alien race or culture. The defenders of the axiomatic conception certainly had some advantage with Hilbert the leading proponent of that conception; but what is surprising is the extent to which some of the defenders accepted the terms of the debate. Gentzen entered the Nazi teachers union in 1936 and joined the Nazi Party in 1937, perhaps to further his chances for an academic career.

Chapters 5 and 6, covering the period from 1942 until 1945, detail Gentzen's discharge from the army and his subsequent life and death in Prague. With the exception already noted, no work of his survives from this period as far as is known, except for some stenographic notes from his time in Prague. These have not yet been edited and published. He is reported to have been thinking about the consistency of analysis until his death in prison, but so far no trace of his work at this time has been found.

The author's story is seriously disjointed and the translation, although entirely readable and a heroic effort on the part of Craig Smoryński (it was his first effort at translation), contains many mistakes. If you are going to dip into it, be prepared to be puzzled, surprised, distressed or even outraged. But, having said this, let me also say that the book has a remarkable charm whose source, I think, is the author's dual role as story-teller and Greek chorus, offering his own, often acute and often surprising, take on the events he relates. It is a near certainty that, having once dipped, you will dip again and again. However it is not a definitive biography of Gentzen: I expect that the author would agree that it would best be described as *notes towards* such a biography: there is in particular an enormous amount of source material which would be a solid foundation for a biography. But especially in connection with Gentzen's scientific work, in an area about which the author had no previous knowledge, there are too many gaps and too many misconceptions. (The author would perhaps not agree with the latter.)

The Introduction begins by defining 'structural proof theory' to be the study of "the general structure and properties of mathematical proofs" and then goes on to state that it was invented by Gentzen. That is surely an overstatement: proof theory for classical first-order and higher-order logic was first introduced by Hilbert in his lectures of 1917–18, in which the properties of completeness, independence (and consistency) and the decision problem were introduced. The author notes that Gentzen read Hilbert and Ackermann's 1928 write-up of the 1917–18 lectures in ***Grundzüge der Theoretischen Logik***. (In a quote at the beginning of the Introduction, it is claimed that Hilbert's work in logic was in the tradition of Frege, Peano and Russell. That of course is just false: none of those previous logicians had the conception of a formalism of logic that is independent of any particular interpretation or model but rather applies to all models.) Also Gödel's completeness theorem for first-order logic preceded all of Gentzen's work in proof theory, as did Herbrand's dissertation. What *is* true in connection with (structural) proof theory is, first, that Gentzen expanded the subject to include the proof theory of the intuitionistic logic, codified by Heyting in 1930. Neither the epsilon calculus of Hilbert nor the method of Herbrand expansion (both of which were developed before Heyting's formalization of intuitionistic logic) is adequate for that system. Moreover, secondly, the codifications of proofs that Gentzen invented for this purpose have turned out to be extraordinarily fruitful: his system of natural deduction is indeed very natural for constructive reasoning and (very naturally) extends to the Curry-Howard type theory that today plays an important role in the foundations of mathematics and in computer science. It, and certain variants of it, are also the basis of programs for theorem proving. And his sequent calculus, although more suited to classical logic (with its duality principle), can by suitable restriction also represent constructive reasoning. One should note though that the idea of the sequent calculus owes something to Paul Hertz. Also, it would be wrong to think that Gentzen's ideas entirely dominate proof theory today. For example, the techniques used in 'proof mining' depend heavily on the so-called double negation interpretation and on Gödel's functional interpretation of intuitionistic systems.

Another misconception frequently expressed in the book (and surprisingly in an excerpt quoted from a letter from Ackermann) is that Gentzen's proof that, assuming consistency, induction up to ϵ_0 is not deducible in arithmetic is a proof of the incompleteness of arithmetic that does not depend on Gödel's arithmetization. Incompleteness means that there is a *sentence of first-order* arithmetic that is not decided in that system. Induction up to ϵ_0 is not expressed by such a sentence.

A question that remains inadequately answered concerns the relation between Gentzen and Herbrand. Gentzen's return to Göttingen in 1931 overlapped with Herbrand's visit there, but the author is unable to tell us whether or not they ever met. What influence did Herbrand's thesis have on Gentzen's work? Gentzen presumably saw some form of Herbrand's dissertation and he saw a copy of the letter that Herbrand had sent to Gödel from Berlin. (The author mistakenly describes this as a letter from Gödel to Herbrand: Herbrand was of course dead when Gödel's only letter to him arrived at his home.) We are told that Gentzen "learned a lot from Herbrand" and that, together with Bernays, he simplified and gave more elegant formulations of some of his results, but it would be useful to have more details and documentation for this. In particular, the author refers in this connection to Gentzen's *Hauptsatz*, from which 'Herbrand's Theorem' easily follows. But what did Bernays have to do with that?

Another topic upon which more details would be desirable is the background of Gentzen's consistency proof for first-order arithmetic. There is much in the book about its reception, but not enough about its creation. For example, the original version of the proof was submitted in 1935 to ***Mathematischen Annalen***, but was subsequently withdrawn. The author states that the withdrawal was in response to criticisms imparted to Gentzen by Bernays, but it is not clear who was ultimately responsible for the criticisms. They seem to have originated from

IAS in Princeton while Bernays was visiting there. Gödel was also visiting there for a short while in the autumn and von Neumann and Weyl were there, too. The author reports Kreisel's speculation that the criticism ultimately came from Gödel and von Neumann. Perhaps, but the only criticism that is known to have been made, as it was later reported by Bernays in "On original Gentzen consistency proof for number theory" was that the proof used the Fan Theorem, which states that every well-founded finitely branching tree has a uniform finite bound on the length of its paths, and, in a letter to Gödel in 1970, Bernays reports that *he* had this misconception of the proof at the time. One would guess from the letter that Gödel had not been involved in making this criticism of the original version. In any case, Bernays resurrected the original version from his files and provided Szabo with the parts of the original proof that were replaced in the published version for an appendix in Szabo's collection and then he presented the original proof mentioned above, contributed to the Conference on Intuitionism and Proof Theory at Buffalo in 1968 and published in ***Intuitionism and Proof Theory*** (edited by A. Kino, J. Myhill and R. Vesley and published by North-Holland in 1970). Actually, the proof did implicitly employ the Bar Theorem, from which Brouwer proved the Fan Theorem. The Bar Theorem states essentially that one can assign ordinals to the nodes of a well-founded tree such that higher nodes have smaller ordinals. Its use, without more discussion than Gentzen provided, should have been at least questioned. (It is hard to see how anyone could have thought that it was the Fan Theorem that is involved: the tree of reductions of a sequent $\Gamma \Rightarrow \forall x\, A(x)$ has the infinity of immediately succeeding nodes $\Gamma \Rightarrow A(n)$ for each n.) In the published version, Gentzen eliminated the use of the Bar Theorem by explicitly assigning the ordinals ($< \epsilon_0$) to the well-founded trees of reductions in question. But how did Gentzen come by the use of the Bar Theorem in his original proof? Was he simply making the constructively invalid inference from well-foundedness of a tree to proof by induction on its subtrees? In Bernays's presentation, although it is emphasized that the reductions should be understood to be free choice sequences, there is no explicit recognition of where the Bar Theorem is being employed. In connection with the origin of Gentzen's proof, Gödel's notes for his lecture at Zilsel's are of interest. (Also see the editors' note to the Gödel lecture notes in volume III of the ***Collected Papers***.) Whether or not Gödel originally thought that Gentzen was using the Fan Theorem, he is clear in these notes that it was the Bar Theorem, which he calls "Souslin's schema," that is employed.

In footnote 31 to Chapter 2, the author raises but does not discuss the role played by Bernays concerning "Gentzen's ideas". Elsewhere, he notes that Bernays directed Gentzen to the ideas of Paul Hertz concerning systems of sentences, which were the source of Gentzen's first published paper and influenced his development of the sequence calculus and, as mentioned above, he notes that Gentzen and Bernays worked together over Herbrand's dissertation. But his portrayal of Bernays is not entirely accurate. First, it is his view that Bernays was never committed to Hilbert's Program: "Bernays never considered himself a supporter of Hilbert's Programme, at least philosophically" (p. 101) and that he failed to fully appreciate Gentzen's achievements: "To be sure one can ask why Bernays didn't have greater confidence in Gentzen's work, instead of viewing him only from the Hilbert-Bernays point of view" (p. 102) There is good evidence that, in spite of his disavowal of a strict "formalist" standpoint in a letter to Gödel in 1941, Bernays was a co-creator, if not the principal one, of the epistemological basis of Hilbert's program, i.e., of the finitist point of view. One simply cannot read Bernays's "The philosophy of mathematics and Hilbert's proof theory" (1930) with understanding and believe that Bernays had reservations about the program. Secondly, the author seems to believe that Bernays's reference to Gentzen as a member of Hilbert's school both undervalued his originality and the importance of his results and either attributed to him philosophical concerns that were foreign to his interests or burdened him with such concerns. This all seems very far from the truth. The author states several times that Gentzen was an 'independent thinker' or an 'intellectual independent'. So was Isaac

Newton, who wrote "If I have seen further, it is by standing on the shoulders of giants." Bernays appreciated long before Gentzen began work on the consistency problem that methods would be needed that go beyond those that he and Hilbert had contemplated. The question was: what new methods were to be accepted in such proofs and on what grounds? This was the 'philosophical' question and Gentzen explicitly recognized that it was crucial to answer it and indeed devoted a good deal of effort to arguing that induction up to ϵ_0 is suitably finitist. We have noted Bernays's early blunder in thinking that the original proof required the Fan Theorem; but there are no grounds for thinking that he was critical of Gentzen's consistency proof using transfinite induction. It is true that Bernays wished to have the consistency proof for arithmetic cast in terms of Hilbert's approach. When he wrote volume II of ***Grundlagen der Mathematik***, he chose to present Kalmar's unpublished proof of the consistency of arithmetic rather than Gentzen's. Now that is very likely explained by the fact that Kalmar was using the axiomatic formalization of logic and arithmetic that had already been developed in Volume I of the work. But there is no mistaking in Bernays's correspondence with Ackermann, quoted by the author, Bernays's desire for Ackermann to obtain a proof of consistency by the epsilon-substitution method using induction up to ϵ_0 (which Ackermann finally accomplished in his "*Zur Widerspruchsfreiheit der Zahlentheorie*" in 1940). But it is unfair to take this very natural desire to see if it could be done his way as an instance of Bernays's lack of confidence in the work of Gentzen or of attempting to dim the light on him. Incidentally, in connection with the consistency proof, there is a puzzling quote from a reply of 4 July 1936 by Gentzen to Bernays: "What particularly interested me was that you had already envisaged the possibility of carrying out a consistency proof via transfinite induction up to ϵ_0." It is one of many instances in which one would like to have seen or at least have had described some of the other side of the correspondence. Another such case is a quote from another reply by Gentzen to Bernays of 11 August 1935. "I believe that your worries about my consistency proof are unfounded."

The photos of Gentzen from the first edition of the book are not included in the second. There is only the inset photo on the cover, of him playing ping-pong and grinning with glee—in none of the other photos does he smile—and a background photo of him on the cover, taken at the wedding of friends in Prague, in which his face has fallen in and shrunk to a death's head. Perhaps it is in part the result of imagining life in Germany under the Nazis and in part reflecting on Gentzen's tragic end, but for the reviewer the overall impression of the color of his life from reading this book is grey. But read the author's insightful summary, entitled "Upshot" on pp. 269–271.

The titles of the two editions immediately present a puzzle—in fact, three puzzles: why did the author change the title in the second edition of the book and what exactly is either title intended to convey? The obvious reference of "Gentzens Problem" would seem to be what is called by that description in the text: "And herewith Gentzen's problem from now on was named: to find the consistency proof for analysis." (p. 86 of the first and p. 99 of the second edition.) But what has that to do with the subtitle "Mathematische Logik im nationalsozialistischen Deutschland"? Is it possible that the author is referring (also?) to the problem of pursuing a career in logic in Nazi Germany? The title of the new edition also contains some ambiguity. The author explicitly recognizes that Gentzen's contributions to logic have not been 'lost', i.e., forgotten. Certainly logic lost a genius with Gentzen's early death in the sense that, whatever he might have made of the consistency problem for analysis, he was certain to have contributed much more of value to logic had he lived longer and regained his health. But what again does this have to do with in the other theme of the book (p. xviii), namely clarifying "Gentzen's position as a mathematician under National Socialism"? Is the meaning of the title rather (or also) that Gentzen, perhaps in his effort to solve his career problem, lost himself? I suppose that I could have written the author and asked him what he meant; but I might have been disappointed by the answer and, anyway, I like the ambiguity.

The reviewer has had and will have a fine time with this book. I have not read through every page in sequence. But the story isn't told in sequence and, anyway, it has been a pleasure to open it to a random section and brace for enlightenment, surprise or outrage, whatever the author chose to serve up there. With due respect to John Crossley in his review of this work (***Australasian Journal of Logic***, vol. 6 (2008), pp. 83–86), his judgement that it is not a book for bedtime reading is precisely wrong.

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NINO B. COCCHIARELLA AND MAX A. FREUND. ***Modal logic. An introduction to its syntax and semantics.*** Oxford University Press, Oxford, 2008, xi + 268 pp.

This book consists of a preface, ten chapters, an afterword, a bibliography, and an index. The material is presented in a rigorous mathematical style. Proofs of important results are given in detail, some other proofs are relegated to the numerous exercises.

The first introductory chapter is in a good sense old-fashioned. It focuses on methodological issues by, among other things, specifying an extension of von Neumann–Bernays–Gödel set theory as the metalanguage, using concatenation theory to define the notion of one expression being part of another expression, and explaining the so-called logistic method. The completeness of classical propositional logic is proved using Lindenbaum's Lemma. The second chapter presents the axiomatic proof-theory of propositional modal logics. Classical, regular, and normal modal logics are introduced. Among the normal modal logics, the systems K, KT, KB, $S4$, $S4.2$, $S4.3$, and $S5$ are dealt with. A special feature is the treatment of matrix semantics in Chapter 3. Dugundji's Theorem stating that no subsystem of $S5$ has a finite characteristic matrix is proved. A topic that, as it seems, has not been considered in a textbook on modal logic before is the Carnapian extension of $S5$ discussed in Chapter 4 under the name L_{at}, cf. Georg Gottlob, *Review of a Carnapian extension of S5*, in (Ewa Orłowska, editor), ***Logic at work. Essays dedicated to the memory of Helena Rasiowa***, Physica Verlag, 1999. The system L_{at} is presented as the logic of logical necessity according to logical atomism. Logically possible worlds are represented by classical truth value assignments, and a formula $\Box\varphi$ is true at a possible world iff φ is true at *all* assignments. Chapter 5 relates L_{at} to $S5$, which is characterized with respect to a 'secondary' semantics. The interpretation of the necessity operator is restricted to universal quantification over a non-empty subclass of the class of all assignments. The secondary semantics thus bears similarity to the Henkin semantics for second-order logic. At this point, a reference to the general frame semantics for normal modal logics would have been in place. Chapter 6 introduces models with a binary accessibility relation. A thorough treatment is given of the relational semantics of all the normal modal logics from Chapter 2 (and some more). Modal first-order logic with necessary identity (i.e., including $(x = y) \to \Box(x = y)$) is the subject of Chapters 7 and 8. In Chapter 7, two kinds of universal and particular quantifiers are introduced, possibilist quantifiers $\forall$ and $\exists$ which quantify over possibly existing objects, and actualist quantifiers $\forall^e$ and $\exists^e$ quantifying over actually existing objects. Lindenbaum's Lemma is extended to maximally consistent sets satisfying various notions of ω-completeness. The distinction between a primary semantics based on logical atomism and a secondary semantics is taken up again in Chapter 8. Given a first-order language $\mathcal{L}$ and a non-empty set D, the class of "all logically possible worlds" is defined as the class of all first-order models for $\mathcal{L}$ with individual domain D. A formula $\Box\varphi$ in the modal first-order language without the actualist quantifiers is true at a possible world under an assignment iff φ is true at *every* possible world under that assignment. The primary semantics validates the modal thesis of anti-essentialism, which is a modal first-order formula stating that if an open formula φ is necessarily true of some individuals in a given domain, then it is necessarily true of any individuals in that domain. Monadic modal predicate logic (without actualist quantifiers) is shown to be decidable and complete w.r.t. the primary semantics, whereas in the presence of a single binary predicate already the logic is not recursively enumerable. Moreover, it is demonstrated that first-order $S5$ (without actualist quantifiers) is complete with respect to the secondary semantics and that this semantics does not validate the thesis of anti-essentialism. Next, the actualist quantifiers are introduced and the secondary semantics for the full language with both pairs of quantifiers is presented.

A unique feature of the book is the development of second-order actualist and possibilist modal logic in Chapters 9 and 10. Actualism is extended to considering second-order quantification over existence-entailing concepts. First, in Chapter 9, the axiomatic proof-theory of second-order modal logic with first and second-order identity is defined. The second-order extensions of the smallest normal modal propositional logic are defined in two versions, a possibilist version in the language with both possibilist and actualist quantifiers, and an actualist version in the language without the possibilist quantifiers. Notions of second-order ω-completeness are defined and Lindenbaum's Lemma is proved for maximally consistent sets that are ω-complete in the defined sense. In Chapter 10 three secondary semantical frameworks are characterized in which the n-ary second-order quantifiers range over a proper subset of the powerset of set of all n-tuples from an underlying individual domain.

The book has some mildly idiosyncratic features. One is the application of the 'logistic' methodology. Moreover, the definitions, lemmata, and theorems are numbered consecutively. As a result the main text ends with Theorem 737.

The afterword is meant to survey various developments in modal logic broadly conceived. The survey leaves out a reference to an important monograph in deontic logic, viz. John Horty, ***Agency and deontic logic***, Oxford University Press, 2001, and to a central monograph in epistemic logic, viz. Ronald Fagin, Joseph Halpern, Yoram Moses and Moshe Vardi, ***Reasoning about knowledge***, MIT Press, 1995. Moreover, the authors refer to the ***Nordic Journal of Philosophical Logic*** as one of the journals in which the reader can get a wider view of modal logic. This journal ceased to exist in 2002.

The book is intended as a textbook on modal logic for students, researches, and professionals in philosophy, mathematics, linguistics, and computer science. Therefore, it would have been appropriate and useful to include in the references some other relevant volumes on modal logic, such as, e.g., Patrick Blackburn, Maarten de Rijke, and Yde Venema, ***Modal logic***, Cambridge University Press, 2001; (Patrick Blackburn, Johan van Benthem, and Frank Wolter, editors), ***Handbook of modal logic***, Elsevier, Amsterdam, 2006; Alexander Chagrov and Michael Zakharyaschev, ***Modal logic***, Oxford University Press, 1997; Melvin Fitting and Richard L. Mendelsohn, ***First-order modal logic***, Kluwer Academic Publishers, Dordrecht, 1998; Marcus Kracht, ***Tools and techniques in modal logic***, North-Holland, Amsterdam, 1999.

In summary, this is a highly recommendable book, its main distinguishing characteristics being (i) the presentation of a semantics for logical necessity in the framework of logical atomism and (ii) a detailed treatment of actualist and possibilist second-order modal logic. As the authors rightly point out, "the second-order modal logics described here are important subjects that students and others in philosophy, mathematics, and computer and information sciences should know about and be familiar with in some detail".

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Katalin Bimbó and J. Michael Dunn. ***Relational semantics of nonclassical logical calculi.*** CSLI Lecture Notes, no. 188. CSLI Publications, Stanford University, 2008, x + 382 pp.

This monograph, written by two well known contributors to philosophical and algebraic logic, is devoted to a very general theory of algebraic structures arising from the study of nonclassical logics. The motivation for the theory stems on the one hand from the theory of Boolean algebras with operators, expounded in two classical papers by Jónsson and Tarski JSL XVIII 70, and on the other by completeness proofs for modal and relevance logics with respect to relational semantics. The authors say: "The main purpose of our investigation is to provide semantics for nonclassical logics in a uniform way" (p. 10).

Chapter 1, entitled *Boolean gaggles*, starts with an exposition of the family of normal modal propositional logics and their semantics, in the style of Kripke. Completeness is proved as usual using the canonical frame construction. The main concern of the chapter, however, is the structures of the title, for which the algebraic versions of these modal logics provide motivation. The theory of Jónsson and Tarski deals with Boolean algebras equipped with multi-place operations $O(\vec{x})$ satisfying the conditions $O(\vec{y}, 0, \vec{z}) = 0$ (normality) and $O(\vec{y}, a \vee b, \vec{z}) = O(\vec{y}, a, \vec{z}) \vee O(\vec{y}, b, \vec{z})$ (additivity), where $\vec{y}$ and $\vec{z}$ are vectors of elements from the underlying Boolean algebra.

Boolean gaggles generalize these conditions to cases where, for instance, an operation could distribute over $\vee$ in its first place, and over $\wedge$ in its second; as an example, consider classical implication $\supset$, for which we have $((a \vee b) \supset c) = (a \supset b) \wedge (a \supset c)$, as well as $(a \supset (b \wedge c)) = (a \supset b) \wedge (a \supset c)$. Each such operation is assumed to have an associated "distribution type." Boolean gaggles can perhaps be most easily understood as notational variants of Boolean algebras with operators, in the style of Jónsson and Tarski. Given a normal, additive operation $O(x_1, \dots, x_k)$ on a Boolean algebra, we can define a new operation $\pm O(\pm x_1, \dots, \pm x_k)$, where $\pm a$ is either a or $\neg a$. The new operation satisfies the conditions defining a Boolean gaggle, and furthermore, any Boolean gaggle arises in this way.

Chapter 2, entitled *Gaggles: distributive lattices with families of operations*, generalizes the ideas of the previous chapter by considering distributive lattices with families of operations that have distribution types (gaggles), and gives representation theorems for them extending the representation theorems of Jónsson and Tarski. The motivation here comes not from modal logic, but from relevance logics; the syntax, ternary relational semantics and completeness results for some of these is discussed in detail.

In Chapter 3 the ideas of the previous chapter are generalized to nondistributive lattices with families of operations with distribution types. The motivation here comes from linear logic, and the logics *BCI* and *BCK*. Chapter 4, *Semi-lattice gaggles*, discusses algebras of logic where the underlying structure is a semi-lattice, rather than a lattice; the notion of distribution type is generalized to a "partial distribution type" to accommodate the absence of meet or join. Chapter 5, on "partial gaggles" is inspired by calculi of Lambek, as well as certain weak relevance logics; a notion of abstract residuation replaces the concept of residuation type. Chapter 6 discusses the addition of constants to various logics, including constants corresponding to combinators, as in the "structurally free logics" of Dunn and Meyer.

Chapter 7, entitled *Inductively defined operations*, discusses logics containing operations representing some kind of iteration, in particular Kleene logic, an axiomatic calculus corresponding to Kleene algebras, and the action logic with conjunction invented by Dexter Kozen. Ternary relational semantics and completeness proofs are provided in both cases. Chapter 8 is devoted to the topic of canonicity. A normal modal logic is *canonical* if it is validated by its canonical frame. After sketching the topics of canonicity and first-order definability in modal logic, the authors give some positive results on canonicity in their framework.

The lengthy Chapter 9 is devoted to the topic of duality. The model here is the well known duality between the category of Boolean algebras and the dual category of totally disconnected compact spaces and continuous maps. The authors describe a duality theory for Boolean gaggles, and then extend it to distributive gaggles, basing their construction on the duality theory for distributive lattices due to Hilary Priestley, given in terms of ordered topological spaces. A further extension is provided to algebras where the underlying lattice is not necessarily distributive, though here the extension is not as elegant or informative, because of the less satisfactory duality theory for general lattices.

The final chapter 10 is devoted to heterogeneous topics, including a discussion of gaggle semantics for intuitionistic logic, and the general notion of a Galois logic. The monograph concludes with a useful appendix on ordered sets, lattices and Boolean algebras.

The authors have packed a great deal of interesting and useful information into their book. However, it is not at all easy to read. The striving for maximum generality, together with the frequent absence of motivation, makes for a difficult book, even for somebody who knows the area fairly well. The decision to eschew any application of the representation and duality theories adds to the difficulty. Nevertheless, this is a significant contribution to the literature, and a logician who is willing to persevere with it will learn a lot of interesting facts.

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J. Zapletal. ***Forcing idealized.*** Cambridge Tracts in Mathematics, vol. 174. Cambridge University Press, 2008, vi+ 314 pp.

At the time I was a young postdoc, so about the same time when Jindřa Zapletal was finishing his Ph.D., the accepted wisdom of the junior community of set theorists working in the set theory of the reals was that large cardinals were a fancy useless invention one better steer clear off. The accepted wisdom of the ones working in large cardinals was that the set theory of the reals consisted of infinitely many intellectually isomorphic ways to juggle infinitely many cardinal invariants between the values of $\aleph_1$ and $\aleph_2$. Zapletal's work over the years, and culminating in this book, proved them all wrong by discovering a deep connection between these two parts of set theory.

To start with, this is a book about forcing of the form P_I, consisting of the positive Borel sets of a σ-ideal I on a Polish space, ordered by inclusion. Such a forcing adds a single generic point which is exactly the intersection of all sets in the generic filter (Proposition 2.1.2). Examples are easy to think of—say random reals—but it is rather surprising to realise that in the presence of suitable large cardinals, a very large class of forcings is of this kind (e.g., Proposition 2.1.8). A basic concept here is that under large cardinal assumptions 'somewhat definable' ideals behave as if they were indeed definable, so what is known about Borel σ-ideals can be extrapolated to larger classes of ideals. How large the class depends on how large the cardinal. An interesting class of forcing that emerges through this type of consideration are universally Baire forcings and in this class large cardinal assumptions give striking dichotomy results and characterisations. For example if we assume LC=large cardinals *and* CH, then every universally Baire forcing is either proper or collapses $\aleph_1$ below some condition. The previous theorem is connected with the old question of how close is the notion of properness to saying that the forcing preserves $\aleph_1$, or stationary subsets of ω_1. It is known that these notions are not equivalent. However, Zapletal's result which says that under LC a universally Baire forcing P is proper iff no condition makes $([P]^{\aleph_0})^V$ nonstationary basically says that morally speaking properness is equivalent to preserving stationary sets, if we are happy to remain within the class of forcings which are reasonably definable and regular.

To quote from the book 'a careful review of this book will reveal that it is full of dichotomies and the dichotomies are really the driving force behind most arguments'. The results quoted above are dichotomies, and they are obtained by using more general dichotomies which under the large cardinal assumptions apply to universally Baire forcing. The choice of the name 'universally Baire' is not a coincidence, there is a strong connection with the known regularity properties of universally Baire sets of reals under large cardinal assumptions, although the connection is a little too technical to go into here. Universally Baire forcing is defined in Definition 2.1.7, and the basic idea is that the forcing conditions and the generic code a universally Baire set.

The book considers a myriad of properties 'idealized forcing' might have, adding this kind of real or that kind of real, how to deal with cardinal invariants, where do the classical forcings fit in, everything that a set theorist of the reals might ask. There is then naturally a lot about iterations, preservation theorems, even ill-founded iterations, games, applications. The book starts by stating that it is not a textbook. True. It is hard to read, for various reasons, the depth of the subject not being the least important one. The book is an excellent reference book and an excellent research monograph as well. Those who persevere will be rewarded not just with some good maths but with some good fun too: "What pentagram is to heavy metal, Cichoń's diagram is to set theory".

The research presented in this book has already had an excellent audience, including some young researchers working in the field, most promisingly a recent Wrocław graduate Marcin Sabok whose work has continued the themes investigated in the book.

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JOHN HARRISON. ***Handbook of practical logic and automated reasoning.*** Cambridge University Press, Cambridge, UK, 2009, xix + 681 pp.

This book is an extensive overview of automated reasoning methods for classical first-order logic. The author follows a rather unusual presentation style, where "pure logic and automated theorem proving are explained in a closely intertwined manner", and "automated theorem proving methods are explained with reference to actual concrete implementations ... " (page xi). The implementations are done in the functional programming language OCaml. This style of presentation certainly has its drawbacks. In particular, in the descriptions of most concepts and methods, the author often starts by explaining in very detailed and concrete terms how to deal with various preliminary steps before proceeding to the main algorithms, which can be a bit of a distraction on a first reading, when one is trying to get an intuitive understanding of a particular concept or method. For instance, in the beginning of Chapter 2 on propositional logic, one needs to go through a lengthy discussion on representations of formulae in OCaml and issues such as parsing and printing before one gets to the definiton of satisfiability, validity, etc. But once one gets past these details, the main ideas are usually well explained, and can often be understood without having to understand completely the corresponding functional programs.

There are considerable omissions in the book, e.g., techniques based on higher-order logics, modal logics, substructural or constructive logics are not covered. This is quite understandable since automated reasoning is such a huge area of research that including too many topics will only serve to confuse beginners, for whom this book is intended. The focus on classical logic is appropriate given that classical theorem proving is a mature area with many important results already established decades ago and have been applied successfully in computer science. There is also a heavy emphasis on model theoretic arguments in most chapters. The only exception is Chapter 6, where a brief account of natural deduction and sequent calculus is given, and where a Hilbert-style axiomatic system is used in the construction of an interactive theorem prover.

Although this book is intended as “a first introduction to the field” (of automated reasoning) and to “logical reasoning” (page xi), it is not an easy read for those not already familiar with classical logic and functional programming. As such, for teaching purpose, this book is best used in conjunction with other introductory books in logic. The materials in Chapter 2 and Chapter 3 will make an excellent supplementary reading for an advanced undergraduate or graduate level course on automated reasoning. Overall this is an excellent book that provides a wide-ranging view on automated reasoning techniques for classical logic. The author achieves a good balance between providing good intuition and rigour in presenting the selected materials. Its breadth will make sure that even an expert in the area will find something useful in the book.

The book consists of seven main chapters and three appendices. The appendices provide minimal mathematical and functional programming backgrounds needed to understand the proofs and the codes in the main chapters. An overview of selected materials from each chapter is given below. Each chapter ends with suggestions for further reading and exercises. I did not spot any typos other than those that were already discovered by the author and others. A list of these typos can be found in the website given in the book.

Chapter 1: Introduction. This gives a light overview of the history of logic and symbolic computation, and discusses the issues of parsing and printing syntactic expressions.

Chapter 2: Propositional logic. This is a comprehensive overview of propositional logic. It covers the standard subjects in propositional logic, but also contains discussions on methods used in modern SAT solvers, e.g., the ‘clause learning’ technique, and interesting applications of propositional logic.

Chapter 3: First-order logic. This chapter introduces the basic notions in first-order logic and an overview of the main automated theorem proving procedures for first-order logic. The underlying idea is to reduce unsatisfiability of a first-order formula to propositional unsatisfiability of finitely many of its instances. This relies on three key results: Skolemization, the Herbrand Theorem, and the compactness theorem, all of which are proved. The refutation procedures given include the analytic tableaux, and various improvements of the resolution method. Several metatheorems are discussed, notably, the Löwenheim–Skolem theorem.

Chapter 4: Equality. This chapter is concerned with models of first-order logic in which the equality symbol ‘$=$’ is interpreted as equality. The author first discusses the congruence closure algorithm, which is used to decide whether a ground equation follows from a set of ground equations E. The more general case where E consists of universally quantified equations can be decided by viewing E as *rewrite rules*. The subsequent sections are basically an overview of rewriting theory, covering notions such as confluency, termination (of a rewriting system), and the Knuth–Bendix procedure for finding a confluent rewrite system for an equational theory.

Chapter 5: Decidable Problems. This is an extensive collection of decision procedures for restricted fragments of classical logic. An interesting result is a complete characterisation of decidability of classes of formulae in prefix normal form, according to its quantifier prefix. A significant part of the chapter deals with decision problems under certain theories, including Presburger arithmetic, theories of reals and complex numbers. This is followed by a discussion on a general method for combining decision procedures, the Nelson–Oppen method, for combining decision procedures for disjoint theories, which sees an interesting use of Craig’s interpolation theorem.

Chapter 6: Interactive Theorem Proving. This chapter describes an approach to interactive theorem proving based on the so-called Edinburgh LCF framework. In this approach, proofs of logical statements are built from a “logical kernel”, i.e., a small set of inference rules. The role of a proof assistant is confined to making sure that each inference step uses only instances from the kernel. An interactive theorem prover for first-order logic is constructed following this LCF approach, using a Hilbert system for first-order logic as the logical kernel.

Chapter 7: Limitations. This chapter discusses the limit of automation, covering classic results such as Tarski's theorem on undefinability of arithmetic truth, Church's theorem on undecidability of first-order logic and Gödel's first incompleteness theorem. Interestingly, the author shows that Gödel's first incompleteness theorem can actually be proven mechanically once the essential lemmas (mainly, Carnap's fixpoint theorem) have been established.

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STEVEN GIVANT AND PAUL HALMOS. ***Introduction to Boolean algebras.*** Undergraduate Texts in Mathematics. Springer, 2009, xiv + 574 pp.

This is an excellent and much-needed comprehensive undergraduate textbook on Boolean algebras. It contains a complete and thorough introduction to the fundamental theory of Boolean algebras. Aimed at undergraduate mathematics students, the book is, in the first authors words, "a substantially revised version of Paul Halmos' ***Lectures on Boolean algebras***." It certainly achieves its stated goal of "steering a middle course between the elementary arithmetic aspects of the subject" and "the deeper mathematical aspects of the theory" of Boolean algebras.

Chapters 1–28 concentrate on the algebraic aspects, while Chapters 29–45 concentrate on the topological aspects of Boolean algebras. The book starts with Boolean rings in Chapter 1, introducing Boolean algebras in Chapter 2 by abstracting properties from power set algebras. Chapter 3 shows the natural correspondence between Boolean algebras and Boolean rings. All subsequent chapters focus on Boolean algebras. Chapters 1–4 are quite rigorous and logical, but void of examples of Boolean algebras save for power set algebras and the one and two element Boolean algebras. In Chapter 5, set algebras are introduced. Though I would like more examples to be given earlier, this approach is reasonable, given the importance of a solid logical foundation for the rest of the book and the possible lack of background in other branches of mathematics students taking such a course may have. Chapters 6–21 cover the fundamentals of Boolean algebras such as order, infinite operations, an introduction to topology and regular open algebras, homomorphisms, finite Boolean algebras, ideals and filters, and other topics leading up to Stone's Representation Theorem in Chapter 22. The authors prove Stone's Representation Theorem via 2-valued homomorphisms, which is elegant but a step removed from the ultrafilter construction, which makes the use of the Boolean Prime Ideal Theorem more obvious. Chapters 23–28 cover canonical extensions, complete homomorphisms and complete ideals, completions, products, isomorphisms of factors, and free algebras. The approach in Chapter 25 to constructing the completion of a Boolean algebra $\mathbb{B}$ uses complete ideals of $\mathbb{B}$, thereby sidestepping the use of topology.

The second half of the book focuses more on topological aspects of Boolean algebras, especially the duality between Boolean algebras and Boolean spaces. Chapter 29 begins this half of the book with Boolean σ-algebras. In this chapter, notions such as Borel sets, meager sets, compactness, and Hausdorff spaces are introduced, and basic theorems involving these notions are covered. From there, Chapters 30 and 31 cover the countable chain condition and measure algebras. Here, construction of a measure algebra from a normalized measure on a Boolean σ-algebra and the facts that measure algebras are complete and satisfy the countable chain condition are proved, while the weak (ω, ω)-distributive law is not mentioned. Chapters 32 and 33 involve Boolean spaces (especially Cantor spaces) and continuous functions on topological spaces. The next five chapters focus on duality between Boolean algebras and Boolean spaces, including duality for ideals, homomorphisms, subalgebras, and completeness. Chapters 39–41 cover Boolean σ-spaces and Boolean measure spaces, including the representation of σ-algebras as homomorphic images of σ-fields via the Loomis–Sikorski

Theorem. The final four chapters cover incomplete Boolean algebras, duality for products, sums of algebras, and isomorphisms of countable factors.

Although there are 45 chapters, the entirety of the book could conceivably be covered in a year-long course. Each chapter is short, ranging from 3 to 17 pages, averaging around 7 pages. Likely a class could cover 1–3 chapters per week. The first 28 chapters would make a nice self-contained one-semester course. A diagram in the introduction of the book outlines a possible self-contained one-semester course concentrating on the topological aspects of Boolean algebras. Using this diagram, it is also possible to construct self-contained one-quarter or two-quarter long courses. The book contains numerous problems at the end of each chapter. The harder problems are marked "Harder." The authors mention that there is an instructor's manual with complete solutions to aid instructors in choosing appropriate homework problems.

The book is written for undergraduate students who already have skills in proving theorems. However, since the proofs are so detailed and clear, it could work well as a text for a second or even first course involving substantial proofs. For this reason, it would also make a great book for a student doing independent study. The text is somewhat informal in the sense that sometimes proofs appear in the prose rather than under the heading, "Proof," but it is always clear when this is being done. Though the book starts with an introduction to Boolean rings, knowledge of group theory or rings is not a prerequisite for using this book.

The only challenge I can see to using this book for a course is that it is slow to develop examples of Boolean algebras—the only examples of a Boolean algebra appearing before Chapter 5 are power set algebras and the one and two element Boolean algebras. The first few chapters concentrate on the axiomatic and algebraic aspects of Boolean algebras from a logically rigorous angle. Though pedagogically sound, this may present a motivational challenge for some students at the beginning of such a course.

In summary, ***Introduction to Boolean algebras*** is a gem of a text which fills a long-standing gap in the undergraduate literature. It combines the best of both worlds by rigorously covering all the fundamental theorems and topics of Boolean algebra while at the same time being easy to read, detailed, and well-paced for undergraduate students. It is my most highly recommended text for undergraduates studying Boolean algebras.

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Robin Milner. ***The space and motion of communicating agents.*** Cambridge University Press, 2009, 212 pp.

Robin Milner's ***The space and motion of communicating agents*** gives an excellent account of a decade of research in bigraphical reactive systems. The long-term goal is to provide a theoretical foundation for ubiquitous computing systems. The short-term goal, which has served as a focus for developing the theory and is prioritized in the book, is to provide a meta-model that unifies existing theories of processes.

The book is a natural successor of Milner's books ***Communication and concurrency*** and ***Communicating and mobile systems: the pi-calculus***. They collect and present in a similar style a decade of research in the calculus of communicating systems (CCS) and the pi-calculus respectively. Differently from the previous work, the theory of bigraphs is founded on a basic, yet rewarding use of category theory.

The Prologue of the book describes the fundamental thesis of the bigraph meta-model and argues for its generality. It requires no prior knowledge of concurrency and category theory and is recommended reading for anyone interested in exploring the foundations for ubiquitous computing systems.

The subsequent material is divided into three parts.

Part I, entitled *Space*, covers the static structure of bigraphs and introduces the basic categorical constructs used in the theory. A bigraph is defined as the combination of two graphs, a place graph and a link graph, representing respectively nested, co-located places and links between such places. Bigraphs constitute the arrows of a symmetric (partially) monoidal (pre)category in which composition represents "nesting in a context" and the monoidal product represents "parallel composition". The static structure for a bigraph process language is then defined in the meta-model by providing a signature consisting of a set of controls labelling the places and the link arity for each control. The link arity of a control determines the number of links connected to each place labelled by that control. As exemplified in the book, the signature for the finite fragment of CCS has three controls: send, get, and alt. The send and get controls each has arity one. Thus, places labelled by one of these controls are connected to a single link (representing the communication channel). The control alt has arity zero and is used to represent the non-deterministic choice construct in CCS.

To readers familiar with process calculi, the graphical representation given in the first two chapters may be a bit difficult to get hold on. Similarly, the use of pre-categories may appear too concrete to readers with a background in category theory. However, the approach chosen is more accessible outside the narrow group of experts in concurrency and category theory, which is essential if the theory is to be picked up by researchers and engineers working in the broader area of ubiquitous computing.

Relative pushouts (RPOs) and idem pushouts (IPOs) are two key categorical constructs in the theory of bigraphs. They were introduced by Milner and his former PhD student James J. Leifer to give a more general account of the notion of "minimal context allowing reaction" originally due to Peter Sewell. The existence of RPOs and IPOs in the pre-category of concrete bigraphs makes it possible for any bigraph process language to give a universal definition of a minimal contextual labelled transition system (lts) for which bisimulation is a congruence. Defining an lts semantics for which bisimulation is a congruence has hitherto often been a painful and non-trivial task for the numerous process calculi, which have been proposed and studied. The drawback of the general approach is that it in some situations results in more complex labels than the ad hoc approaches, which makes the general approach less tractable to automatic verification.

Another important concept is that of sortings. A sorting reduces the collection of bigraphs for a signature to those satisfying the structural (or grammatical) constraints of a particular process language. This is exemplified for finite CCS and condition-event Petri Nets.

Part II is entitled *Motion* and covers the dynamic structure of bigraphs. It does so by taking a step back to the more general notion of reactive systems originating in the work by Milner and Leifer and defining bigraphical reactive systems as a special case of wide reactive systems. It is then described how the existing models and theories of bisimulation congruences for CCS and condition-event nets can be recovered from the theory of bigraphs using the RPO/IPO constructs.

The theory described in the first two parts has reached a high degree of stability. Using Milner's own words, it has been developed to the point that it can begin to underlie experiments with real systems, and form the basis for theories that deal with more subtle notions of ubiquitous computing systems. The road ahead is exciting and leaves many interesting areas to be explored further. Some of these areas are touched upon in the last part of the book, including investigations of binding, stochastic reaction rules, and a notion of *growth* allowing for non-linear (infinite) resources. Hereto comes ongoing research around Europe, e.g., at IT University of Copenhagen, University of Udine, Paris 7, and University of Nottingham. Topics being researched include a refinement of the RPO/IPO approach to provide lts semantics more tractable to automatic verification/model checking, experiments providing bigraphical process languages for real systems in ubiquitous computing and systems biology, and finally to develop a theory of constructions on and mappings between bigraph process languages. The latter would be essential to fulfill Milner's vision of a tower of process languages able to explain ubiquitous computing at different levels of abstraction.

In conclusion, the book provides a solid basis for the further research in providing a theoretical foundation for ubiquitous computing systems. It will be of great value to a number of different kinds of readers. In particular students, researchers and practitioners working on the areas of ubiquitous computing systems, process calculi and concurrency theory, and general meta frameworks for computation and deduction, such as graph rewriting and logical frameworks.

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JC BEALL. ***Spandrels of truth.*** Oxford University Press, Oxford, 2009, xiv + 154 pp.

Deflationism about truth is a thesis as simple and plausible as any one can hope for in philosophy. As Jc Beall puts it in his new research monograph, *truth* is entirely *transparent*; or again, $T(\ulcorner\alpha\urcorner)$ is always intersubstitutable with α (in non-opaque contexts). A truth predicate is an expressive device designed by language users for the purposes of abbreviation and generalization. Rather than list all the theorems of arithmetic, for example, we say that all the theorems of arithmetic are true. And that, according to the minimal conception, is the sum total of what can be fruitfully said about truth.

Of course, given a completely transparent truth predicate, a liar contradiction follows very quickly. Logically minded philosophers have been trying for some time to reconcile the obvious correctness of the transparency thesis with the obvious falsity of a liar contradiction. Beall's tack is admirably direct: He regards transparent truth as the correct account of a human-devised instrument, and its liar paradoxes as innocuous *spandrels*. (A spandrel is an inevitable, usually unintended byproduct of a design, e.g., the V shape between a human middle and ring finger, not selected for by evolution, but nevertheless there if our fingers are to be as they are.) Thus the book is an exposition of a deflationary, dialetheic, and so paraconsistent theory of truth.

Beall's brand of dialetheism is, like his truth theory, deflated. He calls himself a 'semantic dialetheist,' meaning that true contradictions do not extend beyond the parts of language touched by truth. The T-free fragments are consistent. Beall is a dialetheist because he holds that negation is exhaustive—namely, that $\alpha \vee \neg\alpha$, and similarly, by substitution, $T(\ulcorner\alpha\urcorner) \vee \neg T(\ulcorner\alpha\urcorner)$. If the liar paradox must be either true or false, then it is both. Beall goes one step further than Graham Priest, holding that falsity is truth of negation, so $\neg T(\ulcorner\alpha\urcorner)$ implies $T(\ulcorner\neg\alpha\urcorner)$. This means that truth really is fully transparent, and that bivalence holds:

$$T(\ulcorner\alpha\urcorner) \vee T(\ulcorner\neg\alpha\urcorner).$$

(Nit-picking about typos: there are some corner-quotes missing on p. 3.) Beyond this assertion of exhaustive negation, and unlike Priest, Beall holds that truth in itself has no nature for us to discuss or reveal. The bulk of the book, then, is not about truth at all, but is rather a sustained exercise in facing down the logical problems that accompany such a view. This is carried out in terms of semantics, and is a lucid primer on the topic of interpreting a class of intensional paraconsistent logics.

The slim book has five main chapters, the first three of which have essential technical appendices. Chapter 1 lays out the basic picture, stating the basic philosophical idea and semantics for the straightforward 'boolean' connectives $\wedge$, $\vee$, and $\neg$. The theory to this point is Priest's LP plus a transparent truth device, giving the theory $LPTT$. (***Spandrels*** has an abundance of acronyms, summarized in a glossary.) The chapter ends with a rehearsal of a non-triviality proof for the theory, a fixed point construction, first given by Ross Brady. Beall gets a nice result here, vindicating his semantic dialetheism: The truth-free fragment of the language is completely classical, and therefore non-trivial.

Chapter 2 turns to the serious challenge of the book, namely fixing a paraconsistent logic up to the task of modeling an inconsistent truth theory. The LP-based theory is not enough, because the material conditional $\alpha \supset \beta := \neg\alpha \vee \beta$ does not obey modus ponens. (Let α be a liar sentence. Then α and $\neg\alpha \vee \beta$ are both true, but β can be anything at all.) The constraints on a logic with a suitable ('ponenable') conditional are stringent. Beall turns to a weak relevant logic in the Australasian tradition, BX, which is basic relevant logic B plus the law of excluded middle. The conditional of BX obeys modus ponens, and avoids Curry paradoxes. This logic, plus basic axioms for truth, yield a non-trivial truth theory, $BXTT$. The non-triviality proof of chapter 1 is continued, modeling the augmented relevant theory.

Critics of dialetheism will take interest in chapter 3, because beyond the logical details, it is often felt that dialetheism falls prey to 'revenge' problems as much as any other approach to truth. Critics allege that dialetheists require but cannot express the notion 'just true', on pain of incoherence. Following a number of suggestions, Beall responds to the 'just-true' problem in the same way as Priest: by denying there is a serious problem to solve. Transparent truth already covers the needs of just-true, since the truth predicate has the resources to express 'α is true and not false.' Of course, Beall cannot *guarantee* that α is not also false, but then, neither can anyone else, classical or paraconsistent logician alike.

In chapter 4, Beall compares his theory to other non-classical approaches, in particular Kripke's seminal theory and Hartry Field's recent paracomplete approach in ***Saving truth from paradox*** (OUP 2008). As Beall himself is the first to point out, the latter comparison is dissatisfying, since it appears that there is no principled way to select one theory over the other: Beall has gluts, Field has gaps. A cost-benefit approach seems moot, because for any advantage taken by one side, there is a disadvantage in tandem. Perhaps this dialectical impasse itself speaks against completely deflationary methodology.

One particularly interesting topic that is given sustained attention in chapter 5 is how to formalize *restricted quantification*, statements of the form 'all As are Bs'. This has been a longstanding technical problem for paraconsistent theories in the relevant tradition. We find a very elegant suggestion (attributed to Greg Restall) for defining a restricted arrow $\mapsto$:

$$\alpha \mapsto \beta := (\alpha \to \beta) \vee \beta,$$

where $\to$ is the BX conditional. This definition gives some natural and expected properties. It is also obviously a non-trivializing addition to the logic, since nothing is being added. The above proposal has a great deal of potential in helping advance useful relevant theories.

Because Beall's thesis is that truth has no nature, his theory leaves a great deal unsaid. ***Spandrels*** does not, for example, explain *why* a simple device, designed by language users in order to generalise, would generate contradictions. By comparison, Priest has put forward the inclosure schema, and the liar as but one contradiction among many, but Beall demures. Beall is content with liar sentences, and only liars, as brute inconsistencies, to be accommodated at almost any cost. Because there is no deeper story, the overall object of the exercise is somewhat obscure. We are given a minimal account of truth, and a minimal logic for its non-trivial manipulation; but I am not sure what Beall takes this theory to be *for*. It is hard not to be struck at how *extremely* weak the logic BX and resulting theory $BXTT$ are. What is the adequacy criteria here? Non-triviality proofs, carried out entirely in classical metalanguage, appear to be an end in themselves. For all Beall's clarity in the details, perhaps if we could see more of $BXTT$ 'in action', logically or philosophically, the broader project of ***Spandrels*** would be easier to keep in view.

This valuable book is a fresh compilation and distillation of a large body of logico-philosophical work from the last few decades, packed with creative and novel technical ideas. It is the first time a completely deflationary truth theory has been expounded with a paraconsistent logic, and also the first time that the logic has been so well explained. The book is a great point of entry for new researchers, and reads like a prolegomena to much exciting work in non-classical truth theory to come.

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Gottfried Gabriel, Karlheinz Huelser, and Sven Schlotter. *Zur Miete bei Frege—Rudolf Hirzel und die Rezeption der stoischen Logik und Semantik in Jena.* ***History and Philosophy of Logic***, vol. 30 (2009), pp. 369–388.

This article contains really new historiographic insights on Frege including a photograph (p. 369, partially enlarged on p. 386) taken during the occasion of the jubilee ceremony of Jena University, in which Frege is sitting next to Rudolf Hirzel.

There are remarkable similarities between the original propositional logic developed by the Stoics and some of Frege's logical and semantic notions. Thus, in the history of logic it was on several occasions stated that some of Frege's views had already been *anticipated* in Stoicism. Such a claim does not entail that Frege was *influenced* by Stoic logicians, but only that we have two different and insofar independent strands leading (in different times) to comparable results. The article under review proves through circumstantial evidence that Frege was indeed influenced by Stoic logic. The "missing link" (p. 371) is the classical philologist Rudolf Hirzel who lived for more than 20 years in rented accommodation at Frege's house on the floor above Frege. Since this claim can be considered as being of the form "there is—for all we know—one x who makes y familiar with P", the causal connection has to be proven while verifying an existence claim, as well as a probability claim and the predication. This will be tackled from section 3 on and consists especially in the following steps: verifying an overall personal contact between Frege and Hirzel on questions of ancient philosophy, logic and semantics ("existence claim"); justifying that Hirzel's interpretation of Stoic epistemology is for Frege's ends more appropriate than any other and contains remarkable precursors of some of Frege's novelties ("predication"); and arguing for the thesis that Frege's novelties under consideration came up *after* he got in contact with Hirzel ("probability claim").

Prior to that, in section 2 the authors retrace some of the biographical aspects of Hirzel and can already offer some parallels to Frege's character and his biography. Among several points of interest might be the fact that both were, as pupils, acquainted with Lotze at Goettingen. So it is self-evident that their engagement in personal communication at Frege's house was supported by their common philosophical roots (p. 373). Hirzel was an outstanding philologist who was also excellently educated in Greek and Latin philosophy. In section 3 the authors argue for the thesis that Hirzel was an important interlocutor for Frege. One important piece of circumstantial evidence for justifying the thesis is the fact that Hirzel was, e.g., the source for Frege's further knowledge on Plato's dialogue "Hippias maior", to which Frege refers in 1919. The authors show (pp. 374ff.) why this reference has a distinguished character and allows the entailment in the interlocutor-thesis. In section 4 the authors sketch the lines of reception of the important Stoic concept *lekton* and its differentiation into *incomplete lekta* (predicates) and *complete lekta* (e.g., propositions, questions, requests) up to Eduard Zeller. Section 5 brings to light why Hirzel's interpretation of Stoic epistemology is—especially in contrast to Zeller's reconstruction—more appropriate for indicating similarities between the Stoic philosophy and Frege. In particular, Frege's distinction between the content of a judgment and the act of judging, an expression and its reference, and the categorical difference of saturated and unsaturated constituents has remarkable precursors in Stoic semantics (e.g., p. 384). It might be a coincidence, but after his preliminary works on expression and reference in 1886, Frege offered his analysis on sense and reference, concept and object, function and logical subject for the very first time after Hirzel had already been living in rented accommodation at Frege's house for about two years. But it is reasonable to assume that they engage in closer scientific contact from about 1890 on. For sure, Frege was, thanks to his earlier considerations about semantics, well prepared for Hirzel's reflections on Stoic philosophy. And it would surely be wrong to assume that it was Hirzel who brought Frege in an appropriate position for developing his revolutionary philosophy of language at all. But nevertheless, Hirzel might have played the important role of a catalyst while Frege completed his final works on his fundamental distinctions (cf. p. 384).

Generations of Frege scholars have visited the Forstweg 29 at Jena but it was the authors of this article who actually brought the content of the second plaque on Frege's house ("Rudolf Hirzel/Prof. d. Klass. Philologie/1886–1917") into the right/appropriate systematic light. And this is indeed a stroke of luck!

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