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2009 EUROPEAN SUMMER MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC LOGIC COLLOQUIUM '09

Sofia, Bulgaria
July 31–August 5, 2009

Logic Colloquium '09, the 2009 European Summer Meeting of the Association for Symbolic Logic, was hosted by the Sofia University from July 31 to August 5, 2009. Major funding was provided by the Association for Symbolic Logic (ASL), the Bulgarian National Science Fund of the Ministry of Education and Science, and the U. S. National Science Foundation (NSF).

The success of the meeting was due largely to the hard work of the Local Organizing Committee under the leadership of its Chair, Alexandra Soskova. The other members were Dimitar Dobrev, Dimitar Guelev, Lyubomir Ivanov, Stela Nikolova, Solomon Passy, Dimitar Shiyachki, Mariya Soskova, Mitko Yanchev, and Anton Zinoviev. The Program Committee consisted of Samson Abramsky (Oxford), Klaus Ambos-Spies (Heidelberg), Joan Bagaria (University of Barcelona, Chair), Fernando Ferreira (Lisbon), Martin Goldstern (Vienna), Erich Graedel (Aachen), Ehud Hrushovski (Israel), Tapani Hyttinen (Helsinki), Yiannis Moschovakis, (UCLA), Margarita Otero (Madrid), Stewart Shapiro (Ohio State), Ivan Soskov (Sofia), and W. Hugh Woodin (Berkeley).

The main conference topics were: *Computability Theory*, *Logic and Category Theory*, *Model Theory*: *New Directions in Classification Theory*, *Philosophical Logic*, *and Set Theory*. The program included the Gödel lecture, three tutorial courses, twelve invited plenary lectures, and twenty invited lectures in five special sessions. There were eighty contributed papers and two hundred and eight participants from thirty-seven countries. Thirty-five students and recent Ph.D's were awarded ASL travel grants, and seven obtained awards from the ASL's NSF grant. The registration fees of twenty-five Bulgarian students were covered by local sponsors.

A satellite meeting, the *Workshop on Computability Theory* 2009, organized by the Logic Group of the Sofia University, took place at the same venue following the Logic Colloquium (August 6–7).

The Gödel lecture was delivered by Richard A. Shore (Cornell University), with the title: *Reverse mathematics*: *the playground of logic*.

The following tutorial courses were delivered:

Ulrich Kohlenbach (Technische Universität Darmstadt), *Applied proof theory: proof interpretations and their use in mathematics*.
Andre Nies (University of Auckland), *Applying randomness to computability*.
Ralf Schindler (Universität Münster), *The evolution of inner models*.

The following invited plenary lectures were presented:

Elisabeth Bouscaren (Université Paris-Sud 11), *Ranks in model theory and definable groups*.
S. Barry Cooper (University of Leeds), *Definability in the real universe*.

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Thierry Coquand (University of Gothenburg), *Infinite objects in constructive mathematics*.
Ilijas Farah (York University), *Ultrapowers of operator algebras*.
Valentina S. Harizanov (George Washington University), *Four notions of degree spectra*.
Stephan Kreutzer (Oxford University), *Algorithmic meta-theorems: upper and lower bounds*.
David W. Kueker (University of Maryland), *Abstract elementary classes*.
Benjamin Miller, *Forceless, ineffective, powerless proofs of descriptive set-theoretic dichotomy theorems*.
Itay Neeman (University of California Los Angeles), *Forcing with ultrafilters*.
Dana S. Scott (Carnegie Mellon University), *Mixing modality and provability*.
Katrin Tent (University of Munster), *Computable functions on the reals*.
Jouko Väänänen (University of Amsterdam), *Dependence logic*.

The proceedings of Logic Colloquium '09 will be published in a special issue of the Annals of Pure and Applied Logic.

More information about the meeting can be found at the conference webpage, `http://lc2009.fmi.uni-sofia.bg/`.
Abstracts of invited and contributed talks given in person or by title by members of the Association follow.

For the Program Committee
JOAN BAGARIA

Abstract of the 20th Annual Gödel Lecture

► RICHARD A. SHORE, *The Gödel Lecture, Logic Colloquium '09, Reverse mathematics: the playground of logic*.
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The enterprise of calibrating the strength of theorems of classical mathematics, primarily in terms of the (set existence) axioms needed to prove them, was begun by Harvey Friedman in the 1970's. It is now called Reverse Mathematics as, to prove that some set of axioms is actually necessary to establish a given theorem, one reverses the standard paradigm by proving that the axioms follow from the theorem (in some weak base theory). The original motivations for the subject were foundational and philosophical. It has become a remarkably fruitful and successful endeavor supplying a framework for both the philosophical questions about existence assumptions and foundational or mathematical ones about construction techniques needed to construct objects that the theorems assert exist.

There is one common base theory and four standard systems over it. Most theorems of classical mathematics have turned out to be equivalent to one of these systems. We will briefly describe this state of affairs and an alternative view of the calibration system based on computability theoretic measures. We will also see that more recent work has provided examples that fall outside the standard systems at both the bottom and top. The examples are drawn from combinatorics and various areas of logic. The techniques employed come from all the branches of mathematical logic: proof theory, model theory, set theory and recursion theory.

Abstracts of invited tutorials

► ULRICH KOHLENBACH, *Applied proof theory: Proof interpretations and their use in mathematics*.
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This tutorial gives an introduction to an applied form of proof theory that has its roots in G. Kreisel's pioneering ideas on 'unwinding proofs' but has evolved into a systematic activity only during that last 10 years. The general approach is to apply proof theoretic techniques (notably specially designed proof interpretations) to concrete mathematical proofs with the aim of extracting new quantitative results (such as effective bounds) as well as new qualitative uniformity results from (prima facie ineffective) proofs. During the last years logical metatheorems have been developed which guarantee the extractability of highly uniform bounds from large classes of proofs in nonlinear analysis. 'Uniform' here refers to the fact that the bounds are independent from parameters in abstract classes of spaces as long as some local bounds on certain metric distances are given. The classes of structures covered include metric, hyperbolic (in the sense of Kirk and Reich), δ-hyperbolic (in the sense of Gromov), CAT(0), normed, uniformly convex spaces as well $\mathbb{R}$-trees (in the sense of Tits). To achieve uniformity in the absence of compactness it is crucial to exploit the fact that the proofs in question do not make use of any separability assumptions. Applications to various parts of mathematics have led to new results in approximation theory, nonlinear analysis, metric fixed point theory, geodesic geometry, ergodic theory and topological dynamics.

PART I outlines the general program of this kind of applied proof theory motivated by examples from different parts of mathematics. We will emphasize the close relation between this program and T. Tao's concept of 'finitary' analysis.

PART II develops some of the main logical metatheorems that guarantee the extractability of uniform bounds from large classes of proofs.

PART III surveys some of the most interesting applications to nonlinear analysis, ergodic theory and topological dynamics.

The tutorial does not require specific proof-theoretic prerequisites.

[1] J. AVIGAD, P. GERHARDY, AND H. TOWSNER, *Local stability of ergodic averages*, to appear in ***Transactions of the American Mathematical Society***.

[2] E. M. BRISEID, *A rate of convergence for asymptotic contractions*, ***Journal of Mathematical Analysis and Applications***, vol. 330 (2007), pp. 364–376.

[3] P. GERHARDY, *Proof mining in topological dynamics*, ***Notre Dame Journal of Formal Logic***, vol. 49 (2008), pp. 431–446.

[4] P. GERHARDY AND U. KOHLENBACH, *General logical metatheorems for functional analysis*, ***Transactions of the American Mathematical Society***, vol. 360 (2008), pp. 2615–2660.

[5] U. KOHLENBACH, *Some logical metatheorems with applications in functional analysis*, ***Transactions of the American Mathematical Society***, vol. 357 (2005), no. 1, pp. 89–128.

[6] ———, ***Applied proof theory: Proof interpretations and their use in mathematics***, Springer Monographs in Mathematics, Springer Heidelberg-Berlin, 2008.

[7] U. KOHLENBACH AND L. LEUŞTEAN, *Mann iterates of directionally nonexpansive mappings in hyperbolic spaces*, ***Abstract and Applied Analysis***, vol. 2003 (2003), no. 8, pp. 449–477.

[8] ———, *Asymptotically nonexpansive mappings in uniformly convex hyperbolic spaces*, to appear in ***Journal of European Mathematical Society***.

[9] ———, *A quantitative mean ergodic theorem for uniformly convex Banach spaces*, to appear in ***Ergodic Theory and Dynamical Systems***.

[10] U. KOHLENBACH AND P. OLIVA, *Proof mining in L_1-approximation*, ***Annals of Pure and Applied Logic***, vol. 121 (2003), pp. 1–38.

[11] T. TAO, *Soft analysis, hard analysis, and the finite convergence principle*, essay posted May 23, 2007, appeared in *T. Tao, Structure and randomness: Pages from year one of a mathematical blog. American Mathematical Society, 298 pp., 2008*.

► ANDRÉ NIES, *Applying randomness to computability*.
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We study the complexity and randomness aspects of sets of natural numbers. Traditionally, computability theory is concerned with the complexity aspect. However, computability-theoretic tools can also be used to introduce mathematical definitions of randomness of a set; further, once defined, these notions can be studied by considering their interplay with the complexity aspect of a set.

There is also an interaction in the converse direction: concepts and methods from randomness enrich computability theory.

The first tutorial treats the interaction from computability to randomness. The second and third tutorials cover the converse interaction, which is in the focus of recent research. Most of the results can be found in [5].

A lowness property of a set B specifies a sense in which B is close to being computable. One of the most striking examples of applying randomness in computability is the discovery of lowness properties defined in a random-theoretic way. Surprisingly, classes with very different definitions were shown to coincide.

The first property defined in this way was lowness for Martin–Löf randomness: each ML-random set is already ML-random relative to B [4]. This property was shown to be equivalent to various other lowness properties, such as a base for ML-randomness, and being low for weak 2-randomness. In a different vein, it is also equivalent to K-triviality [6], a property that expresses being far from random. Before these coincidences were proven, each of the classes was studied separately. In particular, researchers showed the existence of a promptly simple set in the class. The cost function method arose to give a general framework for these constructions.

Recent research centers on subclasses of the K-trivials. The following is a purely computability-theoretic lowness property: B is strongly jump traceable if there is a c.e. set of possible values for $J^B(x)$ that is finite of small size [2]. Nonetheless, in [1] it was proved that the c.e. strongly jump traceable sets form a proper subideal of the c.e. K-trivials. Recent work [3] suggests that the sets in this class are close to being computable because, in some sense, many ML-random oracles compute them. In [3] we prove a number of coincidences of strong jump traceability (for c.e. sets) with properties that formalize being computed by many ML-random sets.

We also discuss the possibility of natural ideals properly in between the strongly jump-traceable and the K-trivial c.e. sets. We look at reducibilities weaker than Turing that induce the lowness properties above. For instance $B \leq_{LR} C$ means that every C-random set is also B-random [6]. The least LR degree consists of the sets that are low for ML-randomness. In both areas open questions abound.

Finally, we will take a look at analogs of the foregoing results for "higher" randomness notions defined in terms of effective descriptive set theory.

There will be handouts and informal exercise sessions to make the material accessible to students.

[1] P. CHOLAK, R. DOWNEY, AND N. GREENBERG, *Strong jump traceability I: the computably enumerable case*, ***Advances in Mathematics***, vol. 217 (2008), pp. 2045–2074.

[2] S. FIGUEIRA, A. NIES, AND F. STEPHAN, *Lowness properties and approximations of the jump*, ***Annals of Pure and Applied Logic***, vol. 152 (2008), pp. 251–66.

[3] N. Greenberg, D. Hirschfeldt, and A. Nies, *Characterizing strong jump traceability via randomness*, to appear.

[4] A. Kučera and S. Terwijn, *Lowness for the class of random sets*, ***The Journal of Symbolic Logic***, vol. 64 (1999), pp. 1396–1402.

[5] A. Nies, ***Computability and randomness***, Oxford Logic Guides, Oxford University Press, 2009, 444 pages.

[6] ———, *Lowness properties and randomness*, ***Advances in Mathematics***, vol. 197 (2005), pp. 274–305.

► RALF SCHINDLER, *The evolution of inner models*.
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In mathematics, things evolve through stimulation by the right problems, spontaneous insights, and tedious verifications. Only the best methods and fittest classes of structures survive. The tutorial discusses the significance of inner model theory from historical and mathematical perspectives. It will be explained which natural questions led and lead to the production of mice (the building blocks of inner model theory) of greater and greater complexity, and why mice are necessary, useful, and exciting objects to study. Starting with Gödel, inner model theory has been used successfully for manufacturing (sometimes spectacular) results whose statements do not mention concepts of inner model theory but for which no "direct" proofs are in sight. The tutorial will present key ideas on tools, theorems, and open questions (concerning the construction of mice and core models, covering properties, combinatorial analyses, complexity issues, and their applications) in a way to be accessible to every logician.

Abstracts of invited plenary talks

► ELISABETH BOUSCAREN, *Ranks in model theory and definable groups*.
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In classical algebraic geometry over algebraically closed fields, algebraic dimension plays a prominent role.

In model theory many different versions of dimensions or "ranks" can be associated to the family of definable sets. These notions all coincide in the particular case of algebraic geometry, but this is usually not the case.

We will illustrate the interest of these different ranks by looking at groups definable in theories of fields with operators.

► S. BARRY COOPER, *Definability in the real universe*.
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A basic problem for many scientific areas, and between areas, is extracting computational content from the interface between local and global phenomena. Related concepts are those of phase transitions, emergence, strange attractors, supervenience, decoherence, and nonlocality. We argue that one can bring precision and clarity to the discussion of such topics via the mathematical theory of definability and invariance. And modelling physical interactions in terms of Turing reductions over the reals one can begin to close the gap between process and algorithm, and enable an analysis of the computability of many previously puzzling aspects of nature.

In particular, current physical theory contains a number of gaps, hypotheses and speculative theories—as involved in many worlds interpretations of quantum decoherence—which can benefit from such a mathematical analysis. And at the other end of the spectrum, even the concept of supervenience, which has provided workspace for a number of approaches to reconciling physical and mental relations, can be given a clarifying mathematical content.

The setting for all this will depend on an appropriately general modelling of causal structure in nature, and will be related to standard computability-theoretic notions.

► THIERRY COQUAND, *Infinite objects in constructive mathematics*.
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We survey some recent works in constructive mathematics, mainly in algebra. These works deal with the status of infinite objects, using ideas from point-free topology in an essential way. They can be seen as a partial realization of Hilbert's program for some part of abstract algebra and functional analysis.

► ILIJAS FARAH, *Ultrapowers of operator algebras*.
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The use of ultrapowers in the study of operator algebras can be traced back at least to 1954. In the recent years logic was applied to resolve some long-standing open problems about the structure of the ultrapowers of C^*-algebras and von Neumann algebras. In set theory, this progress led to the discovery of a new type of ultrafilter on $\mathbb{N}$. In model theory it precipitated a curious characterization of stable theories.

This is a joint work with N. Christopher Phillips and Juris Steprāns and with Bradd Hart and David Sherman.

► VALENTINA S. HARIZANOV, *Four notions of degree spectra*.
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In computable mathematics we are concerned with computability theoretic complexity of mathematical objects and constructions. Often, Turing and other degrees are used to measure such complexity, including how information can be encoded into the isomorphism type of a structure. We will give an overview of some earlier results and recent developments in our investigation of degree spectra of countable structures, degree spectra of relations on structures, and automorphism degree spectra, as well as degree spectra of spaces of orders on computable groups.

The Turing degree spectrum of a countable structure $\mathcal{A}$ is the set of all Turing degrees of the atomic diagrams of the isomorphic copies of $\mathcal{A}$. The Turing degree spectrum of an additional relation R on a computable structure $\mathcal{B}$ is the set of Turing degrees of the images of R under all isomorphisms from $\mathcal{B}$ onto computable structures. While many degree spectra of relations have upper bounds under Turing reducibility, Knight proved that the degree spectrum of every automorphically nontrivial structure is closed upward in the Turing degrees. On the other hand, Turing degree spectra of structures relate to the degree spectra of relations via spectrally universal structures, which we studied with R. Miller, Csima, and Montalbán. The automorphism Turing degree spectrum of a computable structure is the set of Turing degrees of its nontrivial automorphisms. Together with R. Miller and Morozov, we obtained results which distinguish the automorphism degree spectrum from the previous two notions.

Finally, we consider linear orderings of a fixed computable group $\mathcal{G}$, which are invariant under the group operation, and investigate the Turing degree spectrum of all such orders on $\mathcal{G}$.

This is joint work with Dabkowska, Dabkowski, and Togha.

► STEPHAN KREUTZER, *Algorithmic meta-theorems: upper and lower bounds*.
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In 1990, Courcelle proved a fundamental theorem stating that every property of graphs definable in monadic second-order logic can be decided in linear time on any class of graphs of bounded tree-width. This theorem is the first of what is today known as algorithmic meta-theorems, that is, results of the form: every property definable in a logic L can be decided efficiently on any class of structures with property P.

Such theorems are of interest both from a logical point of view, as results on the complexity of the evaluation problem for logics such as first-order or monadic second-order logic, and from an algorithmic point of view, where they provide simple ways of proving that a problem can be solved efficiently on certain classes of structures.

Following Courcelle's theorem, several meta-theorems have been established, primarily for first-order logic with respect to properties of structures derived from graph theory. (See [1, 2] for recent surveys.) In this talk I will present recent developments in the field and illustrate the key techniques from logic and graph theory used in their proofs.

So far, work on algorithmic meta-theorems has mostly focused on obtaining tractability results for as general classes of structures as possible. The question of finding matching lower bounds, that is, intractability results for monadic second-order or first-order logic with respect to certain graph properties, has so far received much less attention. Tight matching bounds, for instance for Courcelle's theorem, would be very interesting as they would give a clean and exact characterisation of tractability for MSO model-checking with respect to structural properties of the models. In the second part of the talk I will present a recent result in this direction showing that Courcelle's theorem can not be extended much further to classes of unbounded tree-width.

[1] M. GROHE, *Logic, graphs, and algorithms*, (T. Wilke J. Flum, E. Grädel, editors), ***Logic and automata—History and perspectives***, Amsterdam University Press, 2007.

[2] S. KREUTZER, *Algorithmic meta-theorems*, to appear. Preprint available at CoRR abs/0902.3616, http://arxiv.org/abs/0902.3616, 2008.

► DAVID W. KUEKER, *Abstract elementary classes*.
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An abstract elementary class is a class of structures for some vocabulary together with a substructure relation satisfying certain conditions, including closure under unions of chains and a Lowenheim–Skolem property. They were introduced by S. Shelah as a general framework for categoricity results. We will survey some recent work, in particular results relating abstract elementary classes to classical infinitary logics, including sufficient conditions for axiomatizability.

► BENJAMIN MILLER, *Forceless, ineffective, powerless proofs of descriptive set-theoretic dichotomy theorems*.
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Over the last 35 years, a variety of dichotomy theorems have been discovered whose statements are purely classical in nature, yet whose proofs require somewhat sophisticated techniques from mathematical logic. We will discuss an elementary approach to proving and generalizing these theorems.

► ITAY NEEMAN, *Forcing with ultrafilters*.
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In 1989 Woodin and others asked whether, for a singular cardinal κ of cofinality ω, the tree property at κ^+ implies the singular cardinal hypothesis at κ. At the time, the only models in which SCH failed were obtained by singularizing a regular cardinal where the GCH fails, and the question was intended to test whether this was the only way. Later results by Gitik–Magidor showed that there are other ways, yet the test question itself persisted. It has since become a motivator for several results on square principles, considered possible intermediaries on the way from the tree property to the SCH.

We settle the question in this talk. We show that the tree property at κ^+ does not imply SCH at κ. The model where the tree property holds and SCH fails is obtained by *forcing with ultrafilters*. We give a survey of this technique and of related results, starting with some of the most well known instances. The talk is self contained and accessible to a general audience in mathematical logic.

► DANA S. SCOTT, *Mixing modality and probability*.
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Orlov first [1928] and Gödel later [1933] pointed out the connection between the Lewis System S4 and Intuitionistic Logic. McKinsey and Tarski gave an algebraic formulation and proved completeness theorems for propositional systems using as models topological spaces with the interior operator corresponding to the necessitation modality. Earlier, Tarski and Stone had each shown that the lattice of open subsets of a topological space models intuitionistic propositional logic. Expanding on a suggestion of Mostowski about interpreting quantifiers, Rasiowa and Sikorski used the topological models to model first-order logic. After the advent of Solovey's recasting of Cohen's independence proofs as using Boolean-valued models, topological models for modal higher-order logic were studied by Gallin and others. (This very, very brief history does not attempt to acknowledge legions of other researchers and investigations of logics other than S4.) For Boolean-valued logic, the complete Boolean algebra Meas([0,1])/Null of measurable subsets of the unit interval modulo sets of measure zero gives every proposition a probability. Perhaps not as well known is the observation that the measure algebra also carries a nontrivial S4 modality defined with the aid of the sublattice Open([0,1])/Null of open sets modulo null sets. This sublattice is closed under arbitrary joins and finite meets in the measure algebra, but it is not the whole of the measure algebra. Consideration of this model of modality brings up several questions:

(1) What completeness results can be expected in the first-order case?

(2) How does this model differ from models used by Montague and Gallin for higher-order logic?

(3) In employing this model to interpret notions of extensional and intensional functions, what revision of the definition of a topos is appropriate?

(4) What kind of definition of random real number should be chosen to go along with the inherent probability?

(5) Will the measure-preserving automorphisms of the modal measure algebra give us a connection between properties of the logic and the results of Ergodic Theory?

► KATRIN TENT, *Computable functions on the reals*.
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We introduce a new notion of computable function on the reals and prove some basic properties. We give two applications, first a short proof of Yoshinaga's theorem that periods are elementary (they are actually low). We also show that the low real numbers form a real closed field which is closed under exponentiation and some other special functions. For holomorphic functions we prove a local-global principle.

► JOUKO VÄÄNÄNEN, *Dependence logic*.
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Dependence is a common phenomenon, wherever one looks: ecological systems, astronomy, human history, stock markets—but what is the logic of dependence? In this talk I will outline a systematic logical study of dependence, starting from the basic concept of functional dependence in database theory. Dependence logic does not obey the Law of Excluded Middle, as its semantic power is the same as that of existential second order logic. In fact, there is a somewhat unexpected connection to intuitionistic logic (joint work with S. Abramsky). Even very basic model theoretic properties of dependence logic on infinite domains depend on large cardinals and deep set-theoretical facts. On the other hand, on finite domains dependence logic provides an alternative language for non-deterministic polynomial time queries.

The basic new ingredient of dependence logic over and above first order logic is the new form of atomic formulas called dependence atoms. An example is $=(x, y, z)$ with the meaning "z depends on and only on x and y". Note that we can think of x, y and z as features of individuals, as in the sentence "The salary (z) is determined by the rank (x) and number of years of employment (y)".

One of the guiding principles is that dependence does not manifest itself in a single event or observation. Instead, we use semantics, due to W. Hodges ([3]), where the basic concept is a set (or "team") of observations. Examples of such sets are a set of chess games between Susan and Max, a set of records of stock exchange transactions of a particular dealer, a set of possible histories of mankind written as decisions and consequences, etc. As these examples show, the basic intuition is a two-person game. In a game a play is built up from the choices of the players. By looking at many plays we can learn about the players and discern what kind of dependences the moves of the players exhibit. The semantics of disjunction is characteristic: a set satisfies $\phi \vee \psi$ if the sets splits into two parts, one of which satisfies ϕ and the other satisfies ψ. The splitting reflects a player choosing in some plays ϕ and in some ψ.

Dependence logic is just one example of adding the concept of dependence to logic. This opens up possibilities to apply logic to situations, such as Arrow's Paradox of social choice, where interaction rather than plain truth is at stake.

[1] L. HENKIN, *Some remarks on infinitely long formulas*, ***Infinitistic methods*** (***Proceedings of the Symposium on Foundations of Mathematics, Warsaw, 1959***), Pergamon, Oxford, 1961, pp. 167–183.

[2] JAAKKO HINTIKKA, ***The principles of mathematics revisited***, Cambridge University Press, Cambridge, 1996, Appendix by Gabriel Sandu.

[3] WILFRID HODGES, *Compositional semantics for a language of imperfect information*, ***Logic Journal of the IGPL***, vol. 5 (1997), no. 4, pp. 539–563 (electronic).

[4] JOUKO VÄÄNÄNEN, ***Dependence logic***, vol. 70 London Mathematical Society Student Texts, Cambridge University Press, Cambridge, 2007.

Abstracts of invited talks in the Special Session on Computability Theory

► SERIKZHAN BADAEV, *Computable numberings in the hierarchies*.
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We consider computable numberings of families of the sets from any given level of well-known hierarchies such as arithmetical, hyperarithmetical, analytical and the Ershov hierarchies. Informally, a computable numbering is identified with a sequence of sets which has an algorithmic procedure of computation uniform in that level. The set of all computable numberings of a given family $\mathcal{A}$ is preordered by the reducibility relation: a numbering α is *reducible* to a numbering β if $\alpha = \beta \circ f$ for a computable function f. In a very usual way, this preorder induces a quotient structure on the collection of the computable numberings of $\mathcal{A}$. This structure is an upper semilattice, and in the theory of numberings it is called Rogers semilattice of $\mathcal{A}$. For the classical case of families of computably enumerable sets, Rogers semilattices were studied intensively by the fSU mathematicians since the 60s, and beginning the last decade they are studied in many countries for more general classes of sets. This is because the Rogers semilattice of a family may be treated as a mathematical model of all computations of the family in the whole with respect to their computable transformations of one into another. We intend to give the current state of art in this area compared with the classical case. For the most part the results were obtained by the speaker due to joint research with his students and many specialists from different countries.

► GEORGE BARMPALIAS, JOSEPH S. MILLER, AND ANDRÉ NIES, *Randomness notions and partial relativization*.
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Martin–Löf randomness has been criticized for not being strong enough to appropriately formalize our intuition of a random set. For instance, left-c.e. sets, like Ω, and superlow sets can be Martin–Löf random. On the other hand, it is the randomness notion that interacts best with computability theoretic concepts. Many examples of such interaction are given in [1] (see beginning of Chapter 4 for an overview).

This work serves two purposes, the second being the principal:

(1) We study randomness notions between Martin–Löf randomness and 2-randomness.
(2) We provide some new interactions of these randomness notions with computability theoretic concepts.

Purpose (1). We consider Martin–Löf randomness, Schnorr randomness relative to $0'$, weak randomness relative to $0'$, and weak 2-randomness. We study the computational complexity and provide various separations of these classes. In particular, we show that within the Martin–Löf random sets, weak randomness relative to any oracle can be separated from weak 2-randomness.

The notions of randomness we study are displayed in Table 1, together with the symbols for them. The following implications hold:

$$\mathsf{ML}[\emptyset'] \Rightarrow \mathsf{SR}[\emptyset'] \Rightarrow \mathsf{W2R} \Rightarrow \mathsf{Kurtz}[\emptyset'] \cap \mathsf{ML} \Rightarrow \mathsf{ML} \tag{1}$$

None of the implications in (1) can be reversed.

Purpose (2). We provide some new interactions of the randomness notions in Table 1 with computability theoretic concepts.

Given two classes $\mathcal{M}$ and $\mathcal{N}$, define High$(\mathcal{M}, \mathcal{N})$ to be the class containing all oracles A

Martin–Löf randomness	ML
weak randomness relative to $\emptyset'$	$\mathsf{Kurtz}[\emptyset']$
weak 2-randomness	$\mathsf{W2R}$
Schnorr random relative to $\emptyset'$	$\mathsf{SR}[\emptyset']$
2-randomness	$\mathsf{ML}[\emptyset']$

TABLE 1. Randomness notions and the symbols used to denote them.

such that $\mathcal{M}^A \subseteq \mathcal{N}$. For instance, $\mathrm{High}(\mathsf{ML}, \mathsf{SR}[\emptyset'])$ is the set of oracles A that are computationally complex in the sense that each set that is Martin–Löf random in A is already $\mathsf{SR}[\emptyset']$. The results are summarized in the following table. We prove the characterizations in (a)–(d) and observe (f).

(a)	$A \in \mathrm{High}(\mathsf{ML}, \mathsf{Kurtz}[\emptyset'])$	$\emptyset'$ is non-d.n.c. by A
(b)	$A \in \mathrm{High}(\mathsf{ML}, \mathsf{W2R})$	
(c)	$A \in \mathrm{High}(\mathsf{ML}, \mathsf{SR}[\emptyset'])$	$\emptyset'$ is c.e. traceable by A
(d)	$A \in \mathrm{High}(\mathsf{W2R}, \mathsf{ML}[\emptyset'])$	A is u.a.e. dominating
(e)	$A \in \mathrm{High}(\mathsf{ML}, \mathsf{ML}[\emptyset'])$	
(f)	$A \in \mathrm{High}(\mathsf{Kurtz}, \mathsf{ML})$	impossible

TABLE 2. Highness classes with respect to randomness notions and their equivalent computability-theoretic characterizations.

Some of the properties on the right column of Table 2 are obtained by partial relativization, indicated with the preposition "by", from standard notions. This means that we only relativize certain components of the notions, rather than all of them as in complete relativization. For example, we say that Y is c.e. traceable *by* A if there is a computable function h such that for each function $f \leq_T Y$ there is an A-c.e. trace for f with bound h. Recall that a sequence of sets (T_i) is a trace for a function f if $f(n) \in T_n$ for all $n \in \mathbb{N}$. Also, (T_i) has bound h if $|T_n| < h(n)$ for all $n \in \mathbb{N}$.

Let $\mathsf{DNC}[A]$ be the class of diagonally non-computable functions relative to A. That is, the functions g such that $g(e) \neq \Phi_e^A(e)$ for all e such that $\Phi_e^A(e)\downarrow$ (where Φ_e is the e-th Turing functional). We say that Y is non-d.n.c. by A if Y does not compute any function in $\mathsf{DNC}[A]$.

[1] ANDREÉ NIES, ***Computability and randomness***, Oxford University Press, 2009, 444 pp.

► DENIS HIRSCHFELDT, *Reverse mathematics of model theory*.
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I will survey some results on the proof-theoretic and computability-theoretic analysis of model-theoretic principles, using them as illustrations of the sorts of questions this kind of analysis can answer.

► ANDREY S. MOROZOV, *Some aspects of computability over the reals*.
Sobolev Institute of Mathematics, Siberian Branch of the Russian Academy of Sciences, 4 Acad. Koptyug Avenue 630090, Novosibirsk, Russia.
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First we discuss the relationship between the property of effective openennes of sets of reals, which is a key notion in the study of computability without equality test, and Σ-definability and give some numeration theorems which could be considered as arguments showing that the computability over the reals between equality test cannot be immediately reduced to Σ-definability. Then we give a series of results on the complexities of index sets for a wide series of properties of Σ-definable sets of reals. In the third part of the talk we give an overview of the author's results on the presentation of algebraic structures over the reals.

Abstracts of invited talks in the Special Session on Logic and Category Theory

► STEVE AWODEY, *Type theory and homotopy theory*.
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In recent research it has become clear that there are deep and fascinating connections between the intensional type theory of Per Martin–Löf and homotopy theory, via the modern approach to the latter in terms of Quillen model categories, as well as the theory of higher dimensional categories. This talk will survey some of these developments.

► ANDREJ BAUER, *On fixed points in the effective topos*.
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The effective topos is a matehematical universe in which computability and classical set theory mix into a unique set of features. The theory of fixed points in the effective topos is schizophrenic. On one hand the topos contains objects enjoying the fixed-point property for every endomap and recursive domain equations have solutions, but on the other it contains chain-complete lattices which falsify Tarski's fixed point theorem for monotone maps, as well as the Bourbaki–Witt theorem for progressive maps.

► RICHARD BLUTE, *From linear logic to differential categories*.
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The basic categorical structure underlying the proof theory of Girard's linear logic is a symmetric monoidal closed category equipped with a comonad. Categories of vector spaces, possibly equipped with extra structure, are fundamental examples.

The goal of differential linear logic, introduced by Ehrhard and Regnier, is to extend the usual syntax and semantics of linear logic to include a differential operator. The corresponding categories are called differential categories and were introduced by Blute, Cockett and Seely. Typical examples are categories of vector spaces with sufficient normed or topological structure to define a notion of smooth map and to allow for their differentiation.

In this talk, we introduce both the syntax and categorical semantics of this new logic, as well as discuss examples arising from functional analysis.

► COLIN S. MCLARTY, *Single-sorting, and first order axioms for the category of categories as foundation*.
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The category of categories as a foundation for mathematics (CCAF) can be formalized as s single-sorted first order theory. Yet in some ways a multi-sorted version is more natural, and it has been suggested that the theory itself somehow depends on a background of model theory or second order logic. We will explore the conceptual issues and argue for two related conclusions. A multi-sorted formulation is natural. But the different sorts should not be taken as having any different "mode of being" from one another. And in particular categories and functors are not to be understood as "second order" in any sense. The CCAF axioms are not only formally first order but are properly understood as being so, both in that all the entities are posited at one logical level, and in that CCAF has an effectively enumerable consequence relation.

Abstracts of invited talks in the Special Session on Model Theory, new directions in classification theory

► ANTONGIULIO FORNASIERO, *Tame structures*.
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Let K be a definably complete expansion of an ordered field: that is, every definable bounded subset of K has a least upper bound in K. Important examples of definably complete structures are: o-minimal structures, expansions of the real line, and ultra-products of the above structures. For definably complete structures, we can study various notions of "tameness", which generalize o-minimality; "tame" but not definably complete structures (e.g., weakly o-minimal structures) are outside the scope of this talk.

The first notion is local o-minimality: we ask o-minimality only around each point of K. Much of the theory of o-minimal structures can be generalized to locally o-minimal ones; ultra-products of o-minimal structures are locally o-minimal.

There is a dichotomy in further generalizing local o-minimality:

1. either we ask that the open core of K (that is, the reduct generated by the open definable sets) is locally o-minimal;
2. or we ask that K is d-minimal: that is, every definable subset X of K is the union of an open set and N discrete sets, where the natural number N does not depend on the parameters of X.

Finally, we consider dense pairs of d-minimal structures: while the theory is quite similar to the o-minimal situation, we cannot apply the machinery of lovely pairs, because, if K is d-minimal but not o-minimal, then the algebraic closure inside K does not satisfy the Exchange Property. We conjecture that such a dense pair has d-minimal open core.

All structures considered will be definably Baire: that is, the union of a definable increasing family of nowhere dense subsets of K is not all of K; this allows us to use techniques from the theory of Baire spaces.

► ALF ONSHUUS, *On local stability*.
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Work by Haskell, Hrushovski and Macpherson about stable domination in algebraically closed valued fields, showed applications of understanding and using the stable-like "pieces" (types or sorts) within a particular theory. This is a deep break from traditional stability theory (and general classification theory) since the main idea is not to find dividing lines between different theories anymore but instead one tries to find well behaved parts within a particular model. This approach found further applications in joint work with Assaf Hasson where we were able to understand much of the structure of a structure interpretable in an o-minimal theory by understanding independently the stable and the unstable types.

In this talk we will give an introduction to local stability together with the applications in o-minimal theories and in algebraically closed valued fields and explore possible ideas to find definitions for local dependence and local simplicity and properties that one would like to have.

► YEVGENIY VASILYEV, *Linearity and pairs of geometric structures*.
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A theory T is called geometric if in all of its models algebraic closure satisfies the exchange property, and T eliminates the quantifier $\exists^\infty$. The class of geometric theories includes both o-minimal theories (extending DLO) and strongly minimal theories, where there is a clear dividing line between linear and non-linear structures, with linearity characterized by various equivalent conditions (e.g., the CF property and non-interpretability of infinite fields in the o-minimal case, 1-basedness and local modularity in the strongly minimal case). Situation is more complicated in the SU-rank 1, thorn rank 1 and geometric cases, e.g., in the SU-rank 1 case local modularity is stronger than 1-basedness.

We analyze linearity-like conditions using lovely pairs of geometric structures, a common generalization of lovely (generic) pairs of supersimple SU-rank 1 structures [3] and dense pairs of o-minimal structures [2]. We call a geometric theory weakly locally modular, if in a lovely pair (M, P) of its models the pregeometry $(M, acl(- \cup P(M)))$ of the localization of acl at $P(M)$ is modular. This property coincides with 1-basedness in the SU-rank 1 case, and with the CF property in the o-minimal case, and implies that the geometry of $(M, acl(- \cup P(M)))$ is a disjoint union of projective geometries over division rings and/or a trivial geometry. It is shown in [3, 4] that for an SU-rank 1 theory T, T_P is supersimple of SU-rank 1, 2 or ω. Gareth Boxall [1] has established superrosiness of T_P for a superrosy theory T of thorn-rank 1. We show that if, in addition, T is weakly locally modular, then T_P has thorn-rank ≤ 2. Using the Trichotomy theorem we show that for an o-minimal theory T extending DLO, T_P is superrosy of thorn-rank 1, 2 or ω.

This is a joint work with Alexander Berenstein.

[1] GARETH BOXALL, *Superrosiness and imaginaries in lovely pairs of geometric structures*, preprint, 2008.

[2] LOU VAN DEN DRIES, *Dense pairs of o-minimal structures*, ***Fundamenta Mathematicae***, vol. 157 (1998), pp. 61–78.

[3] EVGUENI VASSILIEV, *Generic pairs of SU-rank 1 structures*, ***Annals of Pure and Applied Logic***, vol. 120 (2003), pp. 103–149.

[4] ———, *On pseudolinearity and generic pairs*, ***Mathematical Logic Quarterly***, to appear.

Abstracts of invited talks in the Special Session on Philosophical Logic

► PHILIPPE BALBIANI AND TINKO TINCHEV, *Complete axiomatizations of modal logics for region-based theories of space*.
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The aim of our talk is to give new kinds of propositional modal logics suitable for reasoning about regions in region-based theories of space. Their language contains Boolean variables and standard Boolean operations needed for constructing Boolean terms interpreted by regions in region-based models of space. It also contains the symbol C for the contact relation and the symbol $\leq$ for the part-of relation. Atomic formulas are of the form aCb and $a \leq b$ where a and b are Boolean terms. Complex formulas are built from atomic ones by means of propositional connectives. We use two kinds of semantics. The relational semantics is based on Kripke frames whereas the topological semantics is based on topological spaces. We give axiomatization and completeness theorems with respect to both relational and topological semantics for several important RCC-like logics. Among them are the systems BRCC-8 studied by Wolter and Zakharyaschev and the propositional version of GRCC of Li and Ying. All of the considered RCC-like logics are decidable. If we identify them with the set of their theorems, then each one is equivalent to one of the systems BRCC-8. The systems like BRCC-8 have been known before only with respect to their topological semantics. The fact that they are complete both to the relational semantics and the topological semantics is quite remarkable. It shows that the logics of discrete (relational) and indiscrete (topological) theory of space are in a sense indistinguishable. Relational semantics is more simple for obtaining meta-mathematical results because of its similarity with the semantics of ordinary modal logic and the variety of existing methods and tools in this area. The relationship between pointless and point-based aspects of the presented logical formalisms is reflected as follows. In the syntax of our logics we have regions but no points and points are used in the semantics for modelling regions and defining the relations between regions. The reconstruction of the points in the syntax is in the canonical constructions used for the completeness theorems. In the relational semantics, the points are the ultrafilters in the corresponding canonically defined Boolean algebras. In the topological semantics, we imitate and adapt some techniques known from the theory of proximity spaces where the abstract points are some generalizations of ultrafilters.

► ROMAN KONTCHAKOV, *Spatial logics with connectedness predicates*.
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We consider quantifier-free spatial logics, designed for qualitative spatial representation and reasoning in AI, and extend them with the means to represent topological connectedness of regions and restrict the number of their connected components. We investigate the computational complexity of these logics and show that the connectedness constraints can increase complexity from NP to PSpace, ExpTime and, if component counting is allowed, to NExpTime.

► ISTVÁN NÉMETI AND PÉTER NÉMETI, *Back and forth between logic and relativity theory. Part I*.
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This talk is strongly related to the talk of Hajnal Andréka. In the two talks together, we build up relativity theories (special, general, cosmological) as theories in the sense of mathematical logic, carefully staying all the time in the framework of first order logic. Among others, we intend to provide an easily understandable, logic based introduction to not only special, general and cosmological theories of relativity, but intend to give insight (for the logically minded) to the most exotic and recent developments related to the theory ranging from the recent acceleration of the expanding universe (e.g., exotic matter) through wormholes, timewarps and observational evidence for huge rotating black holes.

Not only logic will be applied to relativity but also applications of general relativity to logic will be mentioned.

A novelty is that we will try to keep the transition to general relativity (GR) from special relativity (SR) simple, streamlined, logically transparent and illuminating. We will introduce Einstein's Principle of Equivalence (EP) and will "derive" GR from SR+EP in a logical way. In more detail, we will build up SR as a purely first-order logic (FOL) theory. Then, by using EP, we build GR on top of SR as a strictly "logical" extension. Both GR and SR will be streamlined, easy to understand theories of FOL.

We will be careful to build up our theories in a "bottom up" way starting from simple observation oriented axioms with clear tangible operational meanings in a step by step manner, in each step assuming only what is needed. In particular, we will avoid assuming, say, the whole of ZFC set theory as part of our axiomatization.

In analogy with the foundation of mathematics (FOM), we aim for elaborating a logic based foundation for GR and its variants. As a guiding line for this, we use the success story of logic in FOM. Besides foundation, we also elaborate a logic based conceptual analysis for the versions of relativity (SR, GR, cosmology etc.) being studied. Typically, we look at some famous predictions of relativity and ask ourselves the question why these predictions are believed. We will answer this question by finding those (FOL) axioms of the theory which are responsible for the predictions in question.

We will also indicate how the first steps towards Einstein's famous insight $E = mc^2$ follows purely logically from the simple geometric axioms of our version of SR. Connections with the Einstein Equation will be mentioned, if time permits.

More detail is available on István Németi's homepage. The reported work is joint with Judit X. Madarász [1], Gergely Székely [2], and Renáta Tordai.

[1] J. X. MADARÁSZ, ***Logic and relativity (in the light of definability theory)***, Ph.D. Dissertation, Eötvös Loránd University, Budapest, 2002.

[2] G. SZÉKELY, ***First-order logic investigation of relativity theory with an emphasis on accelerated observers***, Ph.D. Dissertation, Eötvös Loránd University, Budapest, 2009.

► FRANK WOLTER, *Mathematical logic for life science ontologies*.
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We discuss how concepts and methods introduced in mathematical logic can be used to support the engineering and application of formal life science terminologies. Specifically, we will consider the notion of conservative extensions and uniform interpolation. The required applications of mathematical logic are not straightforward and we argue that such terminologies provide a new and rich family of logical theories that wait to be explored by logicians.

Abstracts of invited talks in the Special Session on Set Theory

► ASSAF RINOT, *Diamond, non-saturation, and weak square principles*.
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We report on results from [1] and [2] concerning the effect of weak square principles to guessing principles. Let Refl_λ denote the assertion that every stationary subset of $\{\alpha < \lambda^+ \mid \mathrm{cf}(\alpha) = \mathrm{cf}(\lambda)\}$ reflects. A corollary to the results that we shall discuss in our talk is the following.

THEOREM. *For a singular cardinal λ*:

1. $\mathrm{GCH} + \mathrm{Refl}_\lambda + \square^*_\lambda \Rightarrow \diamondsuit^*_{\lambda^+}$;
2. $\mathrm{GCH} + \mathrm{Refl}_\lambda + \mathrm{SAP}_\lambda \not\Rightarrow \diamondsuit^*_{\lambda^+}$;
3. $\mathrm{GCH} + \mathrm{Refl}_\lambda + \mathrm{SAP}_\lambda \Rightarrow \diamondsuit_S$ *for every stationary* $S \subseteq \lambda^+$;
4. $\mathrm{GCH} + \mathrm{Refl}_\lambda + \mathrm{AP}_\lambda \not\Rightarrow \diamondsuit_S$ *for every stationary* $S \subseteq \lambda^+$.

In addition, we prove that SAP_λ (and hence $\square^*_\lambda$) implies that $\mathrm{NS}_{\lambda^+} \restriction S$ is non-saturated for every $S \subseteq \lambda^+$ that reflects stationarily often. We prove that the failure of a guessing principle introduced by Džamonja and Shelah is equivalent to the failure of Shelah's strong hypothesis. We also provide two (negative) answers to a question of König, Larson and Yoshinobu; one in the presence of GCH, and one in its absence.

[1] M. GITIK AND A. RINOT, *The failure of diamond on a reflecting stationary set*, preprint, 2009.

[2] A. RINOT, *A relative of the approachability ideal, diamond and non-saturation*, preprint, 2009.

► GRIGOR SARGSYAN, *The core model induction*.
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The core model induction is a technique invented by Hugh Woodin that can be used to evaluate consistency strengths of combinatorial statements. In this talk, the author will outline how "external" core model induction works (as apposed to "internal" core model induction such as the core model induction in $L(R)$) and will explain some recent applications of it.

► DIMA SINAPOVA, *Exploring singular cardinal combinatorics*.
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The relationship between the Singular Cardinal Hypothesis, Jensen's square principle, very good scales and large cardinals is important in singular cardinal arithmetic and in understanding how much the universe resembles L. Jensen showed that square holds in L. On the other hand, weak square fails above a supercompact, and implies that every scale is good.

There have also been results about singular cardinals that are not relative consistency results. Using PCF theory Shelah showed that if $2^{\aleph_n} < \aleph_\omega$ for every $n < \omega$, then $2^{\aleph_\omega} < \aleph_{\omega 4}$. Scales are a central concept in PCF theory and are very useful in exploring the tension between combinatorial principles like square and the re ection properties in the presence of large cardinals.

We will discuss relative consistency results about the relationship between these principles in the context of forcing and large cardinals.

► MARTIN ZEMAN, *Computing bounds for consistency strength*.
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I will discuss some combinatorial principles at small/medium size cardinals and ways of coumputing lower bounds for their consistency strength.

Abstracts of contributed talks

► A. ABAJYAN AND A. CHUBARYAN, *Proof complexity of hard-determinable formulas in $R(lin)$*.
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The system $R(lin)$ (Ran Raz and Iddo Tzameret) is an extension of propositional resolution system by allowing it to operate with disjunctions of linear equations instead of clauses. The authors proved that well-known hard tautologies (PHP^m_n, Clique$_{n,k}$, Tseitin tautologies) have polynomial-size $R(lin)$-proofs. Earlier by second author of this abstract the concept of hard-determinable formulas was introduced and it was proved, that the proof complexity of such formulas has exponential lower bounds in "weak" proof systems, but the property of hard-determinability is insufficient for obtaining a superpolynomial lower bound of Frege proof complexities. The above mentioned hard formulas are not hard-determinable and therefore of great interests is the investigation of $R(lin)$-proof complexities just for hard-determinable formulas.

Let φ be a propositional formula, let $P = \{p_1, p_2, \dots, p_n\}$ be the set of the distinct variables of φ and let $P' = \{p_{i_1}, p_{i_2}, \dots, p_{i_m}\}$ $(1 \leqslant m \leqslant n)$ be some subset of P.

Given $\sigma = \{\sigma_1, \dots, \sigma_m\} \in E^m$, the conjunct $K^\sigma = \left\{p_{i_1}^{\sigma_1}, p_{i_2}^{\sigma_2}, \dots, p_{i_m}^{\sigma_m}\right\}$ is called φ-*determinative* if assigning σ_j $(1 \leqslant j \leqslant m)$ to each p_{i_j} we obtain the value of φ (0 or 1) independently of the values of the remaining variables.

The minimal possible number of variables in a φ-determinative conjunct is denoted by $d(\varphi)$.

Let φ_n $(n \geqslant 1)$ be a sequence of minimal tautologies and $|\varphi_n|$ be the size of φ_n. If for some n_0 there is a constant c such that $\forall n \geqslant n_0$ $\left(d(\varphi_n)\right)^c \leqslant |\varphi_n| < \left(d(\varphi_n)\right)^{c+1}$, then the formulas $\varphi_{n_0}, \varphi_{n_0+1}, \varphi_{n_0+2}, \dots$ are *hard-determinable*.

Let

$$\varphi_n = \bigvee_{(\sigma_1,\dots,\sigma_n)\in E^n} \mathop{\&}_{j=1}^{2^n-1} \bigvee_{i=1}^{n} p_{ij}^{\sigma_i} \qquad (n \geqslant 1).$$

It is not difficult to see, that the formulas $\varphi_3, \varphi_4, \dots$ are hard-determinable.

We prove the following statement

THEOREM. *There is a polynomial-size $R(lin)$ refutation of $\neg\varphi_n$.*

► KUANYSH ABESHEV, SERIKZHAN BADAEV, AND MANAT MUSTAFA, *Families without minimal numberings*.
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Computable numberings of families of sets from any given level of the arithmetical or the analytical hierarchy as well as the Ershov hierarchy are usually identified with the uniform sequences of sets from that level. More precisely, if $\{\mathcal{H}_n\}_{n\in\omega}$ is one of these hierarchies then a sequence $A_0, A_1, A_2, \dots$ of subsets of ω is uniform in $\mathcal{H}_n$ if the set $\{\langle x, k\rangle : x \in A_k\}$ is in $\mathcal{H}_n$. Now, if we denote each set A_k of this sequence by $\alpha(k)$ then we get a computable numbering $\alpha\colon \omega \mapsto \mathcal{A}$. Here, the family $\mathcal{A}$ consists of all sets of that sequence. The set of all computable numberings of $\mathcal{A}$ is preordered by the reducibility relation of numberings: a numbering α is reducible to a numbering β if $\alpha = \beta \circ f$ for some computable function f. Minimal numberings of $\mathcal{A}$ are exactly those which are minimal in this preorder. It is easy to see that any finite family of sets from each mentioned above hierarchy has a numbering which is reducible to any numbering of the family.

Theorems on the existence of the families without computable minimal numberings are the analogs of well known speed up theorems. Families of c.e. sets without computable minimal numberings were built in [3], [1]. Contrary to this, every infinite family of sets from the levels above level one in the arithmetical hierarchy has infinitely many computable pairwise incomparable minimal numberings, [2]. We do not know whether there exist infinite families of sets from any level of the analytical hierarchy which have no computable minimal numberings.

We prove that, for every finite level of the Ershov hierarchy, there exists a computable family of the sets from this level which has no computable minimal numbering. And we conjecture that the same holds for the infinite levels of the Ershov hierarchy.

[1] S. A. BADAEV, *On minimal enumerations*, ***Siberian Advances in Mathematics***, vol. 2 (1992), no. 1, pp. 1–30.

[2] S. A. BADAEV AND S. S. GONCHAROV, *On Rogers semilattices of families of arithmetical sets*, ***Algebra and Logic***, vol. 40 (2001), no. 5, pp. 283–291.

[3] V. V. V'YUGIN, *On some examples of upper semilattices of computable enumerations*, ***Algebra and Logic***, vol. 12 (1973), no. 5, pp. 287–296.

► ARTHUR W. APTER, STEPHEN C. JACKSON, AND BENEDIKT LÖWE, *Cofinality and measurability of the first three uncountable cardinals*.
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In ZFC, small cardinals such as $\aleph_1$, $\aleph_2$, and $\aleph_3$ cannot be measurable, as measurability implies strong inaccessibility; they cannot be singular either, as successor cardinals are always regular. But both of the mentioned results use the Axiom of Choice: in the Feferman–Lévy model, $\aleph_1$ has countable cofinality, in Jech's model, $\aleph_1$ is measurable, and in models of AD, both $\aleph_1$ and $\aleph_2$ are measurable and $\mathrm{cf}(\aleph_3) = \aleph_2$. Is it possible to control these properties simultaneously for the three cardinals $\aleph_1$, $\aleph_2$ and $\aleph_3$?

In this talk, we investigate all possible patterns of measurability and cofinality for the three mentioned cardinals. Combinatorially, there are exactly 60 ($= 3 \times 4 \times 5$) such patterns, of which 13 are impossible for trivial reasons. We reduce the remaining 47 patterns to eight *base cases* that we prove to be consistent relative to large cardinals. The consistency proofs heavily rely on the existence (assuming AD) of a cardinal κ such that the triple $(\kappa, \kappa^+, \kappa^{++})$ satisfies the strong polarized partition property. This is a generalization of an unpublished theorem of Kechris from the 1980s.

► CAN BAŞKENT AND ROHIT PARIKH, *Towards multi-agent subset space logic*.
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Subset space logic (SSL) is a bimodal epistemic logic with a geometrical semantics as opposed to the unimodal topological models or Aumann structures [1]. The original presentation of that logic is for a single agent [4]. There have been several suggestions for multi-agent SSL in the literature [2]. What makes multi-agent version rather difficult is the fact that admissible sets are not specified in the original construction. In this work, we present a multi-agent version of SSL by making use of the knowledge structures [3]. We first extend them to a multi-modal setting then modify them for multi-agent setting. We construct a cascade of *subset structures* inductively and give the equivalence proof with the usual semantics. Furthermore, we conjecture the completeness and decidability of our axiomatic system.

[1] ROBERT J. AUMANN, *Agreeing to disagree*, ***The Annals of Statistics***, vol. 4 (1976), no. 6, pp. 1236–1239.

[2] CAN BAŞKENT, *Topics in subset space logic*, ***Technical Report MoL-2007-05***, Institute for Logic, Language and Computation, Universiteit van Amsterdam, 2007.

[3] Ronald Fagin, Joseph Y. Halpern and Moshe Y. Vardi, *A model-theoretic analysis of knowledge*, ***Journal of the Association for Computing Machinery***, vol. 38 (1991), no. 2, pp. 382–428.

[4] Rohit Parikh, Lawrence S. Moss and Christopher Steisnvold, *Topology and epistemic logic*, ***Handbook of Spatial Logics***, (Aiello, Pratt–Hartmann and van Benthem, editors), Springer, 2007, pp. 299–342.

[5] Rohit Parikh and Ramaswamy Ramanujam, *A knowledge based semantics of messages*, ***Journal of Logic, Language and Information***, vol 12 (2003), no. 4, pp. 453–467.

► LAURENT BIENVENU AND ROD DOWNEY, *Kolmogorov complexity and Solovay functions*.

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Solovay [2] proved that there exists a computable upper bound f of the prefix-free Kolmogorov complexity function K such that $f(x) = K(x)$ for infinitely many x. In this talk, we will consider the slightly more general class of computable functions f such that $K(x) \leq f(x) + O(1)$ for all x and $f(x) \leq K(x) + O(1)$ for infinitely many x, which we call Solovay functions. We show that Solovay functions present interesting connections with algorithmic randomness notions such as Martin–Löf randomness and K-triviality.

[1] Laurent Bienvenu and Rod Downey, *Kolmogorov complexity and Solovay functions*, ***26th International Symposium on Theoretical Aspects of Computer Science, STACS 2009 Proceedings***, (Susanne Albers and Jean-Yves Marion, editors), vol. 09001, Schloss Dagstuhl–Leibniz–Zentrum für Informatik, Germany Internationales Begegnungs- und Forschungszentrum für Informatik (IBFI), Schloss Dagstuhl, Germany, 2009, pp. 147–158.

[2] Robert Solovay, Draft of a paper (or series of papers) on Chaitin's work, unpublished manuscript, 1975.

► JÖRG BRENDLE AND YURII KHOMSKII, *Polarized partition properties for $\boldsymbol{\Delta}^1_2$ and $\boldsymbol{\Sigma}^1_2$ sets*.

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Infinite polarized partition properties are weakenings of the Ramsey property and have been investigated by Henle, Di Prisco, Llopis and Todorcevic [1, 2, 3, 4]. Its simplest version says "for every $A \subseteq \omega^\omega$, there is a sequence $\{H_i \mid i < \omega\}$, each $H_i \in [\omega]^{<\omega}$, such that $\prod_i H_i \subseteq A$ or $\prod_i H_i \cap A = \varnothing$". In [4] it was proved that this property, as well as its parametrized version, holds for analytic sets of reals and for all sets of reals in Solovay's model, and that it is not equivalent to being Ramsey.

We study this property on the $\boldsymbol{\Delta}^1_2$- and $\boldsymbol{\Sigma}^1_2$-levels, i.e., we investigate the logical strength of the statement "all $\boldsymbol{\Delta}^1_2/\boldsymbol{\Sigma}^1_2$ sets satisfy the polarized partition property", and compare it to the strength of other well-known regularity properties on this level.

[1] Carlos A. Di Prisco, James M. Henle, *Partitions of products*, ***The Journal of Symbolic Logic***, vol. 58 (1993), no. 3, pp. 860–871.

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[3] ———, *Parametrized partitions of products of finite sets*, ***Combinatorica***, vol. 24 (2004), no. 2, pp. 209–232.

[4] Carlos A. Di Prisco and Stevo Todorcevic, *Souslin partitions of products of finite sets*, ***Advances in Mathematics***, vol. 176 (2003), pp. 145–173.

► JÖRG BRENDLE AND BENEDIKT LÖWE, *Eventually different functions and inaccessible cardinals*.
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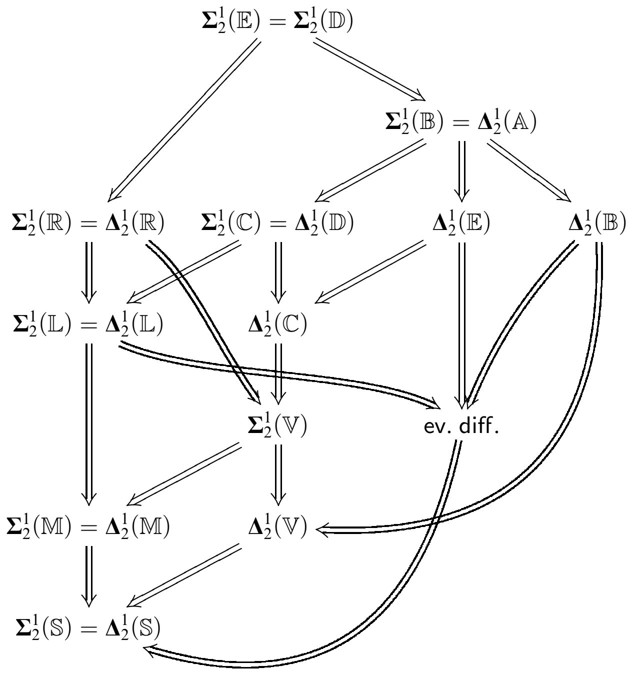
Eventually different forcing $\mathbb{E}$ is a c.c.c. forcing whose conditions generate a topology on Baire space called the *eventually different topology*. For a pointclass $\mathbf{\Gamma}$, we denote by $\mathbf{\Gamma}(\mathbb{E})$ the statement "every set in $\mathbf{\Gamma}$ has the Baire property in the eventually different topology". We prove that $\mathbf{\Sigma}^1_2(\mathbb{E})$ is equivalent to "ω_1 is inaccessible by reals" and determine the strength of $\mathbf{\Delta}^1_2(\mathbb{E})$ in the following diagram of regularity properties (where the letters $\mathbb{A}$, $\mathbb{B}$, $\mathbb{C}$, $\mathbb{D}$, $\mathbb{E}$, $\mathbb{L}$, $\mathbb{M}$, $\mathbb{M}$, $\mathbb{S}$, and $\mathbb{V}$ stand for Amoeba, random, Cohen, Hechler, eventually different, Laver, Miller, Mathias, Sacks, and Silver forcing, respectively, and ev. diff. stands for "for every x, there is an eventually different real over $\mathbf{L}[x]$"):



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► VEDRAN ČAČIĆ, MARKO DOKO, MARKO HORVAT, AND DOMAGOJ VRGOČ, *Changing the order of summation for series beyond ω.*
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Reordering terms in a series is a well-known method, and much is known about when it can be done without affecting the convergence or the sum of the series. If the series converges absolutely, we can add even-indexed and odd-indexed terms separately, or we can arrange terms in increasing columns of an infinite two-dimensional table and then sum the table by rows, or we can do more complicated reorderings. Can all of those methods be generalized into one general transformation, which can then be proved to work when the terms in the series satisfy some condition (e.g., all are nonnegative)? It turns out that it can be done, by considering the series whose indexes can go beyond ω, all the way up to (but not including) ω_1.

► OLIVIA CARAMELLO, *Lattices of theories.*
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We present a duality theorem asserting the existence of a bijection between the subtoposes of the classifying topos of a geometric theory $\mathbb{T}$ over a signature Σ and the closed geometric theories over Σ which are 'quotients' of the theory $\mathbb{T}$; next, we analyze how classical topos-theoretic constructions on the lattice of subtoposes of a given topos can be transferred, via the duality above, to logical constructions in the corresponding lattice of theories. We also discuss applications of the theorem in various contexts, of syntactic and semantic nature.

► FRANQUI CÁRDENAS, *Simplified morasses above a supercompact cardinal.*
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It is well known that above a supercompact cardinal κ, fails $\square_\lambda$ for all $\lambda \geq \kappa$. We investigate the case for a weaker combinatorial principle: gap-1 simplified morasses introduced by Velleman [1]. They imply a weaker version of $\square$ but it is enough weak to hold above a supercompact cardinal. We will considere the same problem with smaller cardinals (measurable or unfoldable cardinals).

[1] Dan Velleman, *Simplified morasses*, ***The Journal of Symbolic Logic***, vol. 49 (1984), no. 1, pp. 257–271.

► V. L. CHECHULIN, *About the selfconsidering semantic in the mathematical logic.*
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On the common filosophical foundation (gnoseological aspects) the nonpredicative constructions was admitted, as this discussed in 1st half XX sentry by Mirimanov. On the this foundation the theory of sets with selfconsidering was described, proved the theorem about uncontradictory of this theory, that and other results was described early [1]. The results of this theory was signified so in the logic field:

1. Resolving considerin's paradoxes (and Russel's paradox), for constructive resolving Russel's paradox set of all nonselfconsidering sets A, which constructed in iteration procedure, consider anyone selfconsidering sets,—$A = \{[x] \in M \mid x \in \emptyset$ or $(x = a, a \notin a,$

$a = V^{\alpha}(A), \alpha$ is number)}, M—the set of all sets ($M = \text{Exp}(M)$), $V(A)$—inside of set A, $V(A) = \{[x] \in M \mid x \in \emptyset$ or $(x \notin A$ and $A \notin x)\}$, (if $B \notin B$ then $V(B) = B$).

2. This set theory was completeness, but not axiomatisationed.

3. In this set theory the model of predicates calculus was described, logic constants was the $\emptyset$ and M, attitude $\in$ was implication $\Longrightarrow$, negation was constructed $\neg p$ is $(p \in \emptyset)$, the theorem about "tetrum non datur" was proved. This model was aloud complacing the objects and the logic of this relations on the one level, without Russel's unnatural logic languages hierarchy.

4. In the this set theory the fully adequate models of the lambda-calculation was described, with claimed of the main property $D \times D = D$ [2].

5. In this selfconsidering semantics the theorem about uncompleteness predicative formal systems (Goedel) was proved very shortly.

6. The problem of the cycle definitions was resolved too.

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[2] D. Barendregt, *Lambda-calculation*, NY, 1989

► AN. CHUBARYAN, ARM. CHUBARYAN, AND H. NALBANDYAN, *Efficiency of weak substitution rules*.

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We compare the proof complexities in Frege systems with different modifications of the substitution rule.

We use the generally accepted concept of Frege system, the well-known notions of proof complexities (size and lines) and the notion of polynomial equivalence (by size and by lines) of the proof systems.

Let $\mathcal{F}$ be some Frege system. The substitution Frege system $S\mathcal{F}$ consists of Frege system $\mathcal{F}$ augmented with the substitution rule with inferences of the form $\frac{A}{A\sigma}$ for any substitution $\sigma = \begin{pmatrix} \varphi_{i_1} & \varphi_{i_2} & \dots & \varphi_{i_m} \\ p_{i_1} & p_{i_2} & \dots & p_{i_m} \end{pmatrix}$, where p_{i_j} $(1 \leqslant j \leqslant m)$ are the propositional variables, φ_{i_j} $(1 \leqslant j \leqslant m)$ are the propositional formulas, and $A\sigma$ denotes the result of applying of the substitution σ to formula A. Such substitution rule allows to use the simultaneous substitution of multiple formulas for multiple variables of A without any restrictions. If the depths of formulas φ_{i_j} $(1 \leqslant j \leqslant m)$ are restricted by some fixed d, then we have d- restricted substitution rule and we denote the corresponding system by $S^d\mathcal{F}$.

We prove that

(1) given arbitrary $d_1 \geqslant 1$ and $d_2 \geqslant 1$, the systems $S^{d_1}\mathcal{F}$ and $S^{d_2}\mathcal{F}$ are polynomially equivalent (both by size and by lines),

(2) given arbitrary d, the systems $S^d\mathcal{F}$ and $S\mathcal{F}$ are polynomially equivalent by size,

(3) given arbitrary d, the minimal number of lines in a proof of tautology in $S^d\mathcal{F}$ can be exponentially larger than in $S\mathcal{F}$.

The analogous results have been obtained by first two authors for k-bounded substitution rule, which for some fixed k allows substitution for any no more than k variables at a time.

The main difference between these two weak substitution rules is the following: for every $k \geqslant 1$ Frege system with k-bounded substitution rule has exponential speed-up by lines over the Frege system, but for every $d \geqslant 1$ $S^d\mathcal{F}$ and $\mathcal{F}$ are polynomially equivalent by lines.

► JENNIFER CHUBB, JEFFRY HIRST, AND TIMOTHY MCNICHOLL, *Computable partitions of trees*.

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The study of effective versions of Ramsey's theorem began with Jockusch in 1972 [2], and the reverse mathematics of the theorem has been studied extensively. In [1], we examine an adaptation of Ramsey's theorem to trees. If linearly ordered n-tuples of nodes (n-chains) in the binary tree are colored with k colors, then there exists a monochromatic subtree isomorphic to the full binary tree. In reverse mathematics, Ramsey's theorem for n-element subsets of natural numbers follows from the tree theorem for n-chains, both of which are, for $n \geq 3$ equivalent to ACA_0 over RCA_0.

If the coloring of n-chains is computable, there is a bound on the complexity of the monochromatic subtree. In particular, if n-chains of nodes are computably colored with k colors, there is a Π^0_n monochromatic subtree. Furthermore, this bound is sharp: For any $n \geq 2$ there is a computable coloring of n-chains for which there is no Σ^0_n monochromatic subtree.

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[2] Carl Jockusch, *Ramsey's theorem and recursion theory*, ***The Journal of Symbolic Logic***, vol. 37 (1972), pp. 268–280.

► CHRIS J. CONIDIS, *Classifying model-theoretic properties*.

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In 2004 Csima, Hirschfeldt, Knight, and Soare [2] introduced nine predicates of a Turing degree A, one of which says that A computes a prime model of every complete atomic decidable theory. In general the nine predicates are of a wide variety, and come from different branches of mathematics including model theory, algebra, topology, and computability theory. In [2], the authors prove that the predicates are equivalent for Δ^0_2 sets A. We will give a complete classification [1] of the predicates in the general case when A is not necessarily Δ^0_2.

[1] C. J. Conidis, *Classifying model-theoretic properties*, ***The Journal of Symbolic Logic***, vol. 73 (2008), no. 3, pp. 885–905.

[2] B. F. Csima, D. R. Hirschfeldt, J. F. Knight, and R. I. Soare, *Bounding prime models*, ***The Journal of Symbolic Logic***, vol. 69 (2004), no. 4, pp. 1117–1142.

► LUCK DARNIÈRE AND MARKUS JUNKER, *Completions and model completions of co-Heyting algebras*.

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We have introduced a notion of (co)dimension, stollen from algebraic geometry, for every element of a distributive bounded lattice. It happens that these notions have a better (functorial) behavior when restricted to the class of co-Heyting algebras. Using it we have obtained the following results, for which preprints are available on Arxiv.

(1) A characterisation of the pro-finite-dimensional completion of any co-Heyting algebra as its Hausdorff completion for a certain pseudometric based on the codimension. We prove that this completion has some familiar analytic properties such as the convergence of every monotonic sequence on a compact subset.

(2) New lights on the remarkable algebraic properties of finitely presented co-Heyting algebras. By showing how the topology induced by the above mentioned pseudometric captures these algebraic properties, we extend them to the much larger class of precompact Hausdorff co-Heyting algebras.

(3) For each fixed integer d, a model-completion result (with two meaningful axioms) for an extension by definition of the variety of the so-called d-th slice.

(4) For each of the 6 varieties of co-Heyting algebras which are locally finite and have the amalgamation property, a simple axiomatization of their model-completion.

► RADHAKRISHNAN DELHIBABU AND CHANDRABOSE ARAVINDAN, *Belief dynamics, logic programming and non-monotonic reasoning for database updates*.
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We live in a constantly changing world and consequence our belief and knowledge on the state of the world change over time [1, 2]. This notion of change manifests itself in application such as database updates etc. This raises two major questions to be answered: *when are we sure that we carry out change rationally*? and *How this can be implemented for a specific application* so far, these question have been dealt with separately: various philosophical works on belief dynamics [4, 3] giving the postulates to satisfied by a rational changes.

so, how knowledge base dynamics can provide an axiomatic characterization for view insertion in database and to explore relationship between belief dynamics and various non-monotonic approaches.

[1] CHANDRABOSE ARAVINDAN AND PETER BAUMGARTNER, *Theorem proving techniques for view deletion in databases*, ***The Journal of Symbolic Computation***, vol. 29 (2000), no. 2, pp. 119–147.

[2] C. ARAVINDAN AND P. BANMGARTNER, *A rational and efficient algorithm for view deletion in databases*, ***Symposium on Logic Programming*** (***Port Jefferson, NY, USA***), (Jan Maluszynski, editor), MIT Press, 1997, pp. 165–179.

[3] J. W. LIOYD, ***Foundations of logic programming***, second extended edition, Springer-Verlag, 1987.

[4] PETER GÄRDENFORS, ***Knowledge in flux, modeling the dynamics of epistemic states***, MIT Press, 1988.

► ANGEL V. DITCHEV, *On least enumerations of partial structures*.
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It will be considered arbitrary partial structures. It is not supposed that either the equality or the inequality is among the predicates of the structure. It will be given a characterization of arbitrary partial structures which have a least (T- and e-) enumerations, i.e., the structures which have (T- and e-) degrees. First it will be done for structures with unary functions and predicates. The characterization will be in the terms of so called types and $\exists$-types of an elements of a structure. Roughly speaking, $\exists$-type of an element is the codes of all existential formulae which are true on that element in the structure. Then, using ideas for unary structures and predicates and so called Moschovakis' extension and extended enumeration on it, it will be given characterization in the general case, as well. It will be given some corollaries, concerning the spectrum of a given structure. It will be given several examples, among which of structures which don't have degree, but having quasi-degree, as well.

► DIMITER DOBREV, *The definition of AI in terms of multi agent systems*.
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The questions which we will consider here are 'What is AI?' and 'How can we make AI?'. Here we will present the definition of AI in terms of multi-agent systems. This means that here you will not find a new answer to the question 'What is AI?', but an old answer in a new form.

This new form of the definition of AI is of interest for the theory of multi-agent systems because it gives us better understanding of this theory. More important is that this work will help us answer the second question. We want to make a program which is capable of constructing a model of its environment. Every multi-agent model is equivalent to a single-agent model but multi-agent models are more natural and accordingly more easily discoverable.

► IVO DÜNTSCH AND IAN PRATT–HARTMANN, *Complex algebras of natural numbers*.
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Let $\mathbb{N}^+ = \langle\omega, 0, +\rangle$, $\mathbb{N}^\cdot = \langle\omega, 1, \cdot\rangle$, $\mathbb{N} = \langle\omega, 0, +, 1, \cdot\rangle$, and $\mathfrak{Cm}\,\mathbb{N}^+$, $\mathfrak{Cm}\,\mathbb{N}^\bullet$, $\mathfrak{Cm}\,\mathbb{N}$ be the respective complex algebras. These algebras are closely related to *arithmetic circuits*, introduced in [1]. We investigate the structure of these complex algebras and some of their significant subalgebras, and explore the complexity of their first order and equational theories. In particular we show that the first order theories of these algebras as well as $\mathbf{EQ}(\mathfrak{Cm}\,\mathbb{N})$ are undecidable; furthermore, the equational theories of the smallest subalgebras of $\mathfrak{Cm}\,\mathbb{N}^+$ and $\mathfrak{Cm}\,\mathbb{N}^\bullet$ are co–r.e.

[1] P. McKenzie and K. W. Wagner, *The complexity of membership problems for circuits over sets of natural numbers*, ***STACS 2003, 20th Annual Symposium on Theoretical Aspects of Computer Science,*** (***Berlin***), (H. Alt and M. Habib, editors), Lecture Notes in Computer Science, vol. 2607, Springer–Verlag, 2003, pp. 571–582.

► HORACIO FAAS, *Between geometry and arithmetic: the long journey to the calculus*.
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Visualization played an important role in ancient mathematics, what be illustrated by the discovery of irrational numbers, vg. square root of two, which even though non expressible by the then known numbers could be immediately "seen" as the diagonal of the square of side equal one. As we all know, straightedge and compass were the only tools admitted by ancient Greek geometers for the construction of their objects of inquiry. With such tools they were able to square any figure with straight sides, but the squaring of curved lines remained highly problematic. Nonetheless, Hippocrates of Chios (ca. 440 B.C.) succeeded in squaring a "lune", an area bounded by arcs of circles. This result encouraged him and other mathematicians at the time to try to resolve the squaring of the circle. But all attempts to solve the problem failed, even though the reason for this failure could only be established in 1882 by the proof of the transcendence of pi due to Lindemann.

It is often claimed that ancient Greek geometry remained attached to figures and shapes. But how fundamental was the role of geometric visualization in the modern advancement towards the calculus of areas? In my paper, I intend to show that even by the time the na?ve approach of ancient Greek geometry was abandoned, leading mathematicians working on the development of the calculus, including one of its founders, Leibniz, and later on, Cauchy were indeed inspired by visual insights.

In this paper I aim to highlight the role of visualization along the road to the calculus by discussing some aspects of Hippocrates's work and concluding with references to Leibniz's Transmutation Theorem, Cauchy's definition of the integral and Liouville's first transcendental number.

▶ MARAT KH. FAIZRAHMANOV, *Splitting and antisplitting theorems in classes of low degrees*.
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A Turing degree $\mathbf{a} = deg(A)$ is called *superlow* if $A' \leqslant_{tt} \emptyset'$. A Turing degree a is called *totally ω-c.e.* if every function $g \leqslant_T a$ is ω-c.e. It is known that each superlow degree is totally ω-c.e. Downey, Greenberg and Weber (2007) have proved that a degree $\mathbf{a}$ is totally ω-c.e. if and only if $\mathbf{a}$ does not bound a critical triple. Let $\mathbf{J}$ be the partially ordered set of least upper bounds of superlow c.e. degrees and let $\mathbf{C}$ be the upper semilattice of all c.e. degrees.

In the the following theorem the class Δ_a^{-1} is the Δ-level of the Ershov Hierarchy corresponding to the ordinal notation $a \in O$.

THEOREM 1. *For all notations $a \in O$ there is a low 2-c.e. set D, such that for all 2-c.e. sets E and F, if $E \in \Delta_a^{-1}$ and $F \in \Delta_a^{-1}$, then $D \not\equiv E \oplus F$.*

COROLLARY 2. *The low c.e. degrees and the low 2-c.e. degrees are not elementary equivalent.*

THEOREM 3. *For all superlow c.e. degrees $\mathbf{b}_0, \mathbf{b}_1, \mathbf{b}_2$ there are superlow c.e. degrees $\mathbf{a}_0, \mathbf{a}_1, \mathbf{a}_2$, such that $\mathbf{b}_0 \cup \mathbf{b}_1 \cup \mathbf{b}_2 = \mathbf{a}_0 \cup \mathbf{a}_1 = \mathbf{a}_0 \cup \mathbf{a}_2 = \mathbf{a}_1 \cup \mathbf{a}_2$.*

COROLLARY 4. *The $\mathbf{J}$ is an upper semilattice.*

THEOREM 5. *There is c.e. degree $\mathbf{b}$, such that for all c.e. degrees $\mathbf{a}_0, \mathbf{a}_1, \mathbf{a}_2$ if $\mathbf{b} = \mathbf{a}_0 \cup \mathbf{a}_1 = \mathbf{a}_0 \cup \mathbf{a}_2 = \boldsymbol{a}_1 \cup \boldsymbol{a}_2$ then exists some $i < 3$ such that $\boldsymbol{a}_i$ is not totally ω-c.e.*

COROLLARY 6. *The upper semilattices $\mathbf{C}$ and $\mathbf{J}$ are not elementary equivalent.*

▶ MICHELE FRIEND, *Some problems with Maddy's naturalism*.
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Quine made "naturalism" into a popular position in philosophy. "Naturalism" is a favouring of science. Scientific ontology and methodology are considered to be philosophically better justified than they are in other fields of enquiry. Here, "science" includes physics, chemistry and biology. As a general position in philosophy, naturalism fares quite well. However, the naturalist has an uneasy time in the company of mathematicians, logicians and philosophers of mathematics. From the stand point of most traditional philosophies of mathematics, e.g., Platonism, logicism, constructivism, formalism, structuralism; naturalism is quite wrong headed, since it places empirical sciences before mathematics.

Maddy has a way out of the embarrassment. She enlarges the scope of "science" to include mathematics. For her, science and mathematics together trump other areas of enquiry. In this way she adheres to, what she takes to be the spirit of naturalism: a preference for rigorous, honest, hard work, and a suspicion about other forms of enquiry. However, her modification of naturalism comes at a price, namely, that in doing her philosophical work, i.e., in the very act of discussing naturalism, she occupies an untenable position by her own lights, since philosophical methodology is not "favoured"! She is aware of this, and, true to her convictions, she "goes native". She does some honest and interesting mathematics. By her naturalist inspired reductio argument, philosophers should all "go native". This smacks of paradox since she gives philosophical arguments to argue her way out of making philosophical arguments. I propose to Maddy that she (1) preserve her naturalist insight that philosophers should develop a philosophy of mathematics which reflects what mathematicians actually think. I think that she will then be forced to be a pluralist. (2) Once a pluralist, she should widen the scope of "favoured enquiry" to include philosophy, since philosophy is contiguous with mathematics. I think that these modifications to Maddy's naturalism follow "naturally" from her general philosophical convictions.

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I will talk about low linear orderings with computable presentation. An X-computable linear ordering is called *low*, if $X' \leq_T \emptyset'$. C. G. Jockusch and R. I. Soare [5] proved that any noncomputable c.e. degree contains linear ordering with no computable presentation. Therefore, there exists a low linear ordering with no computable copy.

R. G. Downey, M. F. Moses [3] proved that any low discrete linear ordering has a computable copy (a linear ordering is called *discrete*, if any elemenet has both a successor and a predecessor). It is a natural to ask (R. G. Downey, [1])—is there a property P of order types which guarantees that if L is low and $P(L)$ then L has a computable presentation?

The author [4] proved that any low strongly η-like linear ordering is isomorphic to a computable one (a linear ordering L is called *strongly η-like*, if $L \cong \sum_{q \in \mathbb{Q}} f(q)$, where $|rang(f)| < +\infty$). Also the author showed that any low 1-quasidiscrete has a computable copy.

Definition. *A linear ordering is called k-quasidiscrete, if any equivalence class either is infinite or contains at most k elements, where $x \sim y$ iff there are only finitely many of z such that $x \leq_L z \leq_L y$ or $y \leq_L z \leq_L x$.*

Theorem. *Any low k-quasidiscrete linear ordering is a computable presentable ordering.*

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[1] R. G. Downey, *Computability theory and linear orders*, (Yu. L. Ershov, S. S. Goncharov, A. Nerode, and J. B. Remmel, editors), ***Handbook of recursive mathematics***, vol. 138, Studies in Logic and the Foundations of Mathematics, chapter 14. Elsevier, 1998. vol. 35 (1989), pp. 237–246.

[2] R. G. Downey and C. G. Jockusch, *Every low Boolean algebra is isomorphic to a recursive one*, ***Proceedings of the American Mathematical Society***, vol. 122 (1994), no. 3, pp. 871–880.

[3] R. G. Downey and M. F. Moses, *On choice sets and strongly nontrivial self-embeddings of recursive linear orderings*, ***Zeitschrift fur Mathematische Logik und Grundlagen der Mathematik***,

[4] A. N. Frolov, *Δ^0_2 copies of linear orderings*, ***Algebra and Logic***, vol. 45 (2006), no. 3, pp. 354–370, in Russian (pp. 201–209, in English).

[5] C. G. Jockusch and R. I. Soare *Degrees of orderings not isomorphic to recursive linear orderings*, ***Annals of Pure and Applied Logic***, vol. 52 (1991), pp. 39–61.

► ALEXANDER N. GAVRYUSHKIN, *On constructive models of theories with linear Rudin–Keisler ordering*.
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Syntactical characterisation of the class of Ehrenfeucht theories was got in [1] by Sudoplatov. It was proved that one can set any Ehrenfeucht theory by a finite pre-ordering (Rudin–Keisler pre-ordering) and a function from this pre-ordering to the set of natural numbers (there are additional inessential restrictions on these pre-ordering and function) as parameters.

Let Rudin–Keisler pre-ordering of an Ehrenfeucht theory T (denoted by $RK(T)$) is a linear ordering. "Which models of T have computable presentations?"—is the main question of the paper.

We say that a model $\mathfrak{M} \models T$ is *quasi-prime* if there is a type p of the theory T and a realisation $\bar{a}$ of p in some model of T such that $\langle \mathfrak{M}, \bar{a} \rangle$ is a prime model.

Let L_n be a linear ordering with $n+1$ elements: $L_n = \{x_0 < x_1 < \cdots < x_n\}$.

One of the main results of the paper is the next one. For all $1 \leqslant n \in \omega$ there exists an Ehrenfeucht theory T_n, $RK(T_n) \cong L_n$, all quasi-prime models of T_n have no computable presentations, there exists computably presentable model of T_n.

As a corollary of this theorem one can construct an Ehrefeucht theory with arbitrary large number of models the only computably presentable model of which is saturated one.

[1] S. V. Sudoplatov, *Complete theories with finitely many countable models*, ***Algebra and Logic***, vol. 43 (2004), no. 1, pp. 62–69.

► NORMA B. GOETHE AND NANCY BOYALLIAN, *Working tools and the special rigor of mathematical reasoning*.
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The modern conception of rigorous proof relies on a syntactic characterization of proof as sequence of sentences. Is proof only a matter of deductive logic, as the syntactic conception suggests? In the paper we argue, using material from Grosholz (2007), that this is not the case. In contrast to formal proofs, mathematical arguments serve many purposes: they establish definitions, formulate problems and explain solutions; they present procedures and display proofs formally and informally. Moreover, *real* proofs are not like formal syntactic proofs. Formalization in terms of predicate logic with its one-dimensional form of writing is only one form of representation among many. There is a reason mathematical results are typically written out differently. The different modes of presentation of arguments emerge from different traditions of inquiry that require *polyvalent mathematical discourse*. Grosholz shows that one and the same argument often requires the combination of different modes of representation: diagrams, proportions, equations, matrices, tables, schemata, natural language. Formal forms of reasoning must be surrounded by natural language that explains their use while the mathematicianŠs acquired know-how which is what permits him to engage in problem-solving, remains largely implicit. For example, arguments may require that one and the same representation be used *ambiguously* to allow for the mathematician to present a novel perspective and for the reader to follow the reasoning. Including ambiguity in argument illustrates the interconnection between fruitfulness and rigorous argument with signs.

The paper will focus on this reasoning style that has been neglected by philosophers of mathematics: at home in the tradition of analysis, such style is at odds with the stringent requirements of rigor which underlie the syntactic conception of proof that requires making explicit everything essential to reasoning, so that *in any complete proof* of a theorem *figures* and other non syntactic forms of representation are *dispensable*.

► VALENTIN GORANKO AND RUAAN KELLERMAN, *Classes and theories of trees associated with a class of linear orderings*.
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Given a set of linear orderings $\mathcal{C}$, several classes of trees associated with $\mathcal{C}$ arise naturally, defined in terms of how the paths of those trees are related to the orderings from $\mathcal{C}$. Each of these classes determines a first-order theory, thus yielding several first-order theories of trees associated with $\mathcal{C}$.

While much is known about certain specific classes and first-order theories of trees (see e.g., [4], [2], [1], [3]), no general study has been done so far of the classes of trees and their first-order theories arising from any given set of linear orderings. In the present study we analyze and completely determine the relationships between all of these classes of trees and between their corresponding first-order theories.

[1] R. Backofen, J. Rogers, and K. Vijay-Shankar, *A first-order axiomatization of the theory of finite trees*, ***Journal of Logic, Language and Information***, vol. 4 (1995), no. 1, pp. 5–39.

[2] K. Doets, ***Completeness and definability: Applications of the Ehrenfeucht game in second-order and intensional logic***, Ph.D. thesis, University of Amsterdam, 1987.

[3] V. Goranko, *Trees and finite branching*, ***Proceedings of the 2nd Panhellenic Logic Symposium*** (***Delphi***), (Phokion Kolaitis and George Koletsos, editors), 1999, pp. 91–101.

[4] J. Schmerl, *On $\aleph_0$-categoricity and the theory of trees*, ***Fundamenta Mathematicae***, vol. 94 (1977), no. 2, pp. 121–128.

► STEFAN HETZL, *On the form of witness terms*.
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Cut-elimination is an inherently non-deterministic process. At each stage one has the choice between different cuts to reduce and—for a single cut—there are different ways of reducing it. This formal non-determinism has the effect that depending on the chosen cut-elimination procedure, mathematically different normal forms may be produced. Understanding the possible span of the results of cut-elimination procedures is therefore of fundamental importance for judging proof analyses based on these methods.

The mathematical content of a cut-free proof is fully contained in the information how to instantiate which quantifiers, i.e., in the witness terms. They therefore provide adequate means for relating different cut-free proofs.

We analyze the development of terms during cut-elimination in first-order logic and Peano arithmetic. The main result is a characterization of the form of witness terms in cut-free proofs in terms of structured combinations of basic substitutions that are read off from the proof with cuts.

Based on this result, it is shown that each proof with cuts induces a regular tree grammar s.t. every witness term computable by cut-elimination can also be computed by the grammar. As a second application of this result, a class of proofs in first-order logic is shown to have only elementary cut-elimination (while cut-elimination in the worst case is non-elementary). From the algorithmic point of view, we obtain a method for computing witness terms that circumvents cut-elimination and has several advantages, it allows for example to find the shortest witness term. All of the above results also apply to proofs of Σ^0_1-formulas in Peano arithmetic.

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A *Weak Ehrenfeucht–Fraïssé game on L-structures $\mathcal{A}$ and $\mathcal{B}$ of length α*, denoted by $\mathrm{EF}^*_\alpha(\mathcal{A}, \mathcal{B})$ or shorter, is played between two players I and II on two L-structures $\mathcal{A}$ and $\mathcal{B}$, where L is a relational vocabulary. The players choose elements of the domains of the structures in α moves, and in the end of the game the player II wins if the chosen structures are isomorphic. Otherwise player I wins.

The obvious difference of this to the ordinary Ehrenfeucht–Fraïssé game is that the isomorphism can be arbitrary whereas in the ordinary EF-game it should be determined by the moves of the players. In particular this game is not closed (in the sense of Gale–Stewart [1]). In our article we answer the following questions and in the talk we discuss some of them.

- Are the games EF_ω and EF^*_ω equivalent? This was solved already by Kueker in [2] in the context of cub-subsets of power sets. (Answer: yes)
- Are the games EF_α and EF^*_α equivalent for an ordinal α? (Answer: no)
- Are the games EF_κ and EF^*_κ always equivalent for a cardinal $\kappa > \omega$? (Answer: for structures of size κ^+ no, for $\kappa = \omega_1$ and structures of size $\aleph_2$, independent of ZFC. Here we use results of [3])
- If structures are weakly α-equivalent and $\beta < \alpha$, are they necessarily weakly β-equivalent? (Answer: no)
- Is $\mathrm{EF}^*_{\omega_1}$ necessarily determined? (Answer: independent of ZFC, if the size of the structures is $\aleph_2$ and the answer is no, if the size of the structures is greater than $\aleph_2$).

[1] D. Gale and F. M. Stewart, *Infinite games with perfect information*, ***Contributions to the theory of games***, vol. 2, Annals of Mathematics Studies, no. 28, pp. 245–266. Princeton University Press, Princeton, NJ, 1953.

[2] D. W. Kueker, *Countable approximations and Löwenheim–Skolem theorems*, ***Annals of Mathematical Logic***, vol. 11 (1977), pp. 57–103.

[3] A. H. Mekler, S. Shelah and J. Väänänen, *The Ehrenfeucht–Fraïssé-game of length ω_1*, ***Transactions of the American Mathematical Society***, vol. 339 (1993), pp. 567–580, vol. 11 (1977), pp. 57–103.

► ADAM KAY, *Incompletability of formal linguistics*.
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With a simple Gödel-like coding of the vocabulary of a natural language into sets, formal linguistics can be entirely expressed within the theory of hereditarily finite sets (HF). The main definition assuring that linguistic properties are represented is of the triconditional $T \iff Q \iff \Phi$, between tree structures T, grammatical categories Q, and the formulas Φ representing the property of inclusion in some category. The triconditional assures that grammatical categories are associated with labeled finite rooted trees and logical formulas. The concept *grammatical* is then a decidable relation between expressions of a language (sets), and their structural descriptions (sets).

When the formulas Φ are added to HF as axioms the result is a grammar G. Grammatical sentences are thus analogous to true sentences in a formal logic, as usual. After giving a variety of grammar formalisms as examples and showing how the standard model $\mathfrak{S}$ of HF adapts to each grammar formalism, we show that to develop the concept of *ungrammatical* in G requires meta-grammatical categories M, defined in terms of Q. A simple diagonalization argument of Tarski's then immediately shows G (equipped with M) to be incomplete.

We conclude that formal linguistics, plus the concept *ungrammatical*, is incomplete, that the attempt to determine the formal rules underlying use of natural language is incompletable, and that the traditional goal of descriptive adequacy must be rethought.

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Shirshov suggested to treat projective planes as partial algebraic systems [2]. In the framework of this approach a *projective plane* is a structure $\langle A, (A^0, {}^0A), \cdot\rangle$ with a disjunction of A into two subsets $A^0 \cup {}^0A = A$, $A^0 \cap {}^0A = \emptyset$ and commutative partial operation "$\cdot$" which satisfy the following properties:

(1) $a\cdot b$ is defined iff $a\neq b$ and $a, b \in A^0$ (or $a, b \in {}^0A$) with the product $a\cdot b \in {}^0A$ ($a\cdot b \in A^0$ respectively);
(2) for all $a, b, c \in A$ if $a\cdot b$, $a\cdot c$, $(a\cdot b)\cdot(a\cdot c)$ are defined, then $(a\cdot b)\cdot(a\cdot c) = a$;
(3) there exist distinct $a, b, c, d \in A$ such that products $a\cdot b$, $b\cdot c$, $c\cdot d$, $d\cdot a$ are defined and pairwise distinct.

In any projective plane $\mathcal{A}$ we replace the partial operation by its graph and consider $\mathcal{A}$ as a predicate model. This allows us to apply methods of model theory and investigate the question of decidability of elementary theories [1].

In the present paper we prove that the class of symmetric, irreflexive graphs is relatively elementarily definable in the class of projective planes. Since the theory of symmetric, irreflexive graphs is hereditarily undecidable, we obtain the following results:

(1) *The class of all projective planes has hereditarily undecidable theory.*
(2) *The class of freely generated projective planes has hereditarily undecidable theory.*

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[1] YU. L. ERSHOV, I. A. LAVROV, A. D. TAIMANOV, AND M. A. TAITSLIN, *Elementary theories*, ***Russian Mathematical Surveys***, vol. 20 (1965), no. 4, pp. 35–105.

[2] A. I. SHIRSHOV AND A. A. NIKITIN, *On the theory of projective planes*, ***Algebra and Logic***, vol. 20 (1981), no. 3, pp. 330–356.

► ALEXANDER KREUZER, *Ramsey's theorem for pairs and provably recursive functions*.
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Applying the Elimination of monotone Skolem functions (see [1]), we show that instances of Ramsey's theorem for pairs and a fixed number of colors n ($\mathrm{RT}^2_n(c)$) at most cause provably primitive recursive function(al)s relative to certain weak fragments of analysis, e.g., WKL^*_0.

We also comment on ongoing work in a more general setting.

In this talk, we present a joint work with Ulrich Kohlenbach.

[1] ULRICH KOHLENBACH, *Elimination of Skolem functions for monotone formulas in analysis*, ***Archive for Mathematical Logic***, vol. 37 (1998) pp. 363–390.

[2] ALEXANDER KREUZER, *Ramsey's theorem for pairs and provably recursive functions*, to appear in ***Notre Dame Journal of Formal Logic***.

► PETR KULIKOV, *Some constructions on groups of computable automorphisms*.
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The necessary definitions can be found in [1].

Let $Aut_{rec}\mathfrak{M}$ be the group of all computable automorphisms of a computable structure $\mathfrak{M}$.

THEOREM 1. *Let G be a computable group and H be a subgroup of G such that $G \setminus H$ is c.e. Then there exists a computable model $\mathfrak{M}$ such that $H \cong Aut_{rec}\mathfrak{M}$.*

Therefore the center of a computable group can be represented as a group of computable automorphisms of a computable model.

Let $\{G_i\}_{i\in\omega}$ be an uniformly computable family of groups and $\{\theta_i\}_{i\in\omega}$ be an uniformly computable family of homomorphisms such that

$$\cdots \to G_2 \xrightarrow{\theta_1} G_1 \xrightarrow{\theta_0} G_0.$$

A sequence $(\dots g_2, g_1, g_0)$ is called *a thread* if for any $i \in \omega$ $g_i \in G_i$ and $\theta_i(g_{i+1}) = g_i$. The multiplication of threads is defined in natural way. A thread is *computable* if there exists a computable function f such that $f(i) = g_i$. The set of all computable threads with defined multiplication operation is a group called *a reverse computable limit* $\varprojlim_{rec} G_i$ of $\{G_i\}_{i\in\omega}$.

THEOREM 2. *Let $\{G_i\}_{i\in\omega}$ and $\{\theta_i\}_{i\in\omega}$ be as above. Then there exists a computable model $\mathfrak{G}$ such that $\varprojlim_{rec} G_i \cong Aut_{rec}\mathfrak{G}$.*

Let B be a computable group, A be a group of all computable automorphisms of a computable model $\mathfrak{M}$ and $Rec(A^B)$ be the set of all computable mappings $\mu\colon B \to A$. The Cartesian product $B \times Rec(A^B)$ with an operation $*$

$$(b_1, f_1) * (b_2, f_2) = (b_1 b_2, f_1^{b_2} f_2)$$

(here $f^b(x) = f(bx)$) is a group called *a computable wreath product $A \diamond B$ of A by B*.

THEOREM 3. *Let A and B be as above. Then there exists a computable model $\mathfrak{T}$ such that $A \diamond B \cong Aut_{rec}\mathfrak{T}$.*

[1] A. S. MOROZOV, *Groups of computable automorphisms*, ***Handbook of recursive mathematics***, Studies in Logic and Foundations of Mathematics, vol. 1, (Y. L. Ershov, S. S. Goncharov, A. Nerode, and J. B. Remmel, editors), Elsevier, Amsterdam; 1998, pp. 311–345.

► JUI-LIN LEE, *The classical model existence theorem in subclassical predicate logics II*.
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In [2] it is proved that there are weak subclassical predicate logics (which are classically sound but weaker than the first-order logic) which also satisfy the Classical Model Existence property (*CME* for short): Every consistent set has a classical model. In this paper we improve the result in [2] to some subclassical predicate logics with weaker propositional parts (for example, some weak extensions of the implicational linear logic *BCI*). Two types of approaches (by prenex normal form construction or by Hintikka style construction) will be considered.

We will also discuss whether there is a weakest subclassical predicate logic satisfying *CME*. (Note that in [1] it is proved that there exists a weakest subclassical propositional logic which characterizes *CME*. However, this depends on which consistency is chosen and what kind of proof rules are allowed.)

[1] JUI-LIN LEE, *Classical model existence and left resolution*, ***Logic and logical philosophy***, vol. 16 (2007), no. 4, pp. 333–352.

[2] ———, *The classical model existence theorem in subclassical predicate logics I*, ***Towards mathematical philosophy, Trends in logic, 28*** (David Makinson, Jacek Malinowski, and Heinrich Wansing, editors), Springer, 2008, pp. 178–199.

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Using a priority construction, we will prove a strong version of a theorem in Rosenstein [2]: every computably well-founded partial order has a computably well-founded ω-c.e. linear extension. Note that Rosenstein's theorem provides a construction of a computably well-founded Δ^0_2 linear extension under the same condition, using an oracle for $\emptyset'$. On the other hand, Rosenstein [2] gives a counterexample to show that there is a computably well-founded computable partial order with no computably well-founded computable linear extension. We will discuss the possibility of extending this counterexample to that of a computably well-founded d-c.e. linear extension.

Joint work with S. B. Cooper and A. Morphett.

[1] RODNEY G. DOWNEY, *Computability theory and linear orderings*, ***Handbook of Recursive Mathematics, II*** (Yu. L. Ershov, S. S. Goncharov, A. Nerode, and J. B. Remmel, editors), Elsevier, Amsterdam, Lausanne, New York, Oxford, Shannon, Singapore, Tokyo, 1998, pp. 823–976.

[2] JOSEPH G. ROSENSTEIN, *Recursive linear orderings*, ***Orders: description and roles*** (Maurice Pouzet and Denis Richard, editors), Elsevier, 1984, pp. 465–475.

► MARGARITA LEONTYEVA, *Decidable Boolean algebras of elementary characteristics* $(1, 0, 1)$.
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A computable Boolean algebra is said to be *n-constructive* if there exists an algorithm determining whether the given tuple satisfies given finite Σ_n-formula. A *strongly constructive* Boolean algebra is one for which such an algorithm exists for all formulas of the predicate calculus. Boolean algebra is *decidable* if there exists its strongly constructive isomorphic copy.

If $\mathfrak{A}$ is a Boolean algebra, then $\mathrm{At}(\mathfrak{A})$ is the set of atoms of $\mathfrak{A}$, $\mathrm{Atm}(\mathfrak{A})$ is the ideal of atomic elements, $\mathrm{Als}(\mathfrak{A})$ is the ideal of atomless elements, $\mathrm{E}(\mathfrak{A}) = \mathrm{Atm}(\mathfrak{A}) + \mathrm{Als}(\mathfrak{A})$ is the Ershov–Tarski ideal and F is Frechet ideal.

Relations between computability of $\mathrm{At}(\mathfrak{A})$, $\mathrm{Atm}(\mathfrak{A})$, $\mathrm{Als}(\mathfrak{A})$, $\mathrm{E}(\mathfrak{A})$ and decidability of Boolean algebra $\mathfrak{A}$ began to be studied in the articles of Ershov in 1964.

Let $\mathfrak{A}$ be a computable Boolean algebra of elementary characteristics (1,0,1). This means that $\mathfrak{A}/E$ is a nontrivial atomless Boolean algebra. Let S be a subset of the set $\{\mathrm{At}, \mathrm{Atm}, \mathrm{Als}, \mathrm{E}\}$. We consider a general question: if the predicates in S are computable in $\mathfrak{A}$ then can we state that $\mathfrak{A}$ is decidable? It was proved earlier that if $\mathrm{S} = \{\mathrm{At}, \mathrm{Als}\}$ then the answer is yes, and if $\mathrm{S} = \{\mathrm{At}\}$ or $\mathrm{S} = \{\mathrm{Als}, \mathrm{Atm}, \mathrm{E}\}$ then the answer is no. In theorems 1 and 2 we complete this theme.

THEOREM 1. *Let $\mathfrak{A}$ be a computable Boolean algebra of elementary characteristics* $(1, 0, 1)$. *If $At(\mathfrak{A})$ and $Atm(\mathfrak{A})$ are computable then $\mathfrak{A}$ is decidable.*

THEOREM 2. *There exists a computable Boolean algebra $\mathfrak{A}$ of elementary characteristics* $(1, 0, 1)$ *with computable $At(\mathfrak{A})$ and $E(\mathfrak{A})$ which is not decidable.*

During the work with theorem 2 the following description of Δ^0_6-computable Boolean algebras was obtained. Let $\mathrm{T} = (\mathrm{Atm} \to \mathrm{F}) + \mathrm{Atm}$, where $\mathrm{Atm} \to \mathrm{F} = \{x \mid \forall z \leq x(z \in \mathrm{F} \vee z \notin \mathrm{Atm})\}$.

THEOREM 3. $\mathfrak{A}$ *is* Δ^0_6*-computable Boolean algebra if and only if there exists a computable Boolean algebra* $\mathfrak{C}$ *such that* $\mathfrak{C}/T \cong \mathfrak{A}$.

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Following previous work by Abramsky (2007), Tulenheimo and Venema (2007) (for full references, see [1]) and ourselves [1], we develop a multi-player logic MPL_R for *rational* players. The syntax of MPL_R is as follows:

$$\phi ::= p \mid (\phi \vee_i \psi) \mid \neg_{ij}\phi,$$

where $i, j \in A$, the set of players. Formulas of MPL_R describe games, where $\vee_i$ is a choice operator for player i and negations, of the form $\neg_{ij}$, permute the roles of players i and j. A valuation assigns to each proposition letter a set of winners.

MPL_R can be seen as a generalization of two-player game semantics of propositional logic. We show that the complexity of MPL_R is linear in the general case, but if we impose some (reasonable) restrictions to the valuation function, it becomes NP-complete. Also, a completeness result for a functionally complete extension, MPL^+_R, of MPL_R will be shown. The logic MPL^+_R contains two families of negations that are the same in the classical two-player setting.

The fact that we assume rationality of the players allows us to study the logics from a game theoretical perspective. Each extensive form game can be described by a formula of MPL_R and we compare our semantics to various solution concepts from game theory. In particular, we will show that if a backward induction solution to the game exists, this will be the semantic value of its formula. We illustrate this point by analyzing some well-known games like the Centipede game within our framework.

[1] LOES OLDE LOOHUIS, ***Multi-Player Logics***, ILLC publication series, 2008, no. MoL-2008-07. url: `http://www.illc.uva.nl/Publications/ResearchReports/MoL-2008-07.text.pdf`

► ROUSSANKA LOUKANOVA, *Computational intension, denotation and propositional intention in the languages of acyclic recursion*.
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In a sequence of papers, Moschovakis developed a class of languages of recursion, which opens a new type-theoretic approach to the semantic concepts of sense and denotation. In essence, Moschovakis defined the concept of sense by an abstract, mathematical notion of algorithm and its syntactic representation by recursion terms in canonical forms. In particular, the language and theory of acyclic recursion L^{λ}_{ar}, introduced in Moschovakis [2], are intended for modeling the concepts of meaning and synonymy, from computational perspective, by targeting adequateness of computational semantics of natural languages spoken by humans (and commonly denoted by NL). The type theory of L^{λ}_{ar} has a highly expressive language, an effective reduction calculus, and elegant mathematical properties for modeling recursive computations of denotations. Independently form the development of the languages of recursion, representation of semantic underspecification is a major effort of contemporary works on semantics of NL, see Bunt [1] for an overview of the field. In particular, for efficiency and adequateness, e.g., due to lack of resolving context information, it is preferable to represent scope ambiguities as underspecified. Typically, this is achieved by meta-theoretic means. Distinctively, L^{λ}_{ar} has its own inherent facilities for modeling scope ambiguity and other semantic underspecification in NL, at the level of its own language and semantics, with respect to lexical and syntactic structure of NL expressions.

The first part of the paper introduces the syntax of L^{λ}_{ar} and its two semantic layers. The usefulness of L^{λ}_{ar} for computational semantics of NL is reviewed by rendering representative NL sentences into L^{λ}_{ar}-terms. The focus is on demonstrating the expressiveness of L^{λ}_{ar} for resolving major problems in semantics, in particular, such as: distinction of strict co-reference from anaphora–antecedent naming, and, semantic underspecification in representing quantifier scopes. The second part of the paper shows that Thomason's technique, revised by Muskens (2005), can be incorporated within an extended language of acyclic recursion. The propositions are introduced as primitives, with a corresponding, newly introduced, propositional type. A transformation operator reinforces Tarski's style of truth conditions into the object level of type theory, which gives a possibility for using L^{λ}_{ar} in applications with logic programming. Furthermore, the result is a formal notion of propositional *intention*, as a part of the denotational semantics of the extended language and a sub-layer of the notion of referential intension. The propositional intention of a propositional term distinguishes the denoted proposition P from its corresponding extension, i.e., from the set of states (possible worlds), where the proposition P is true. The result is a treatment of various modals and propositional attitudes in the extended language L^{λ}_{ar}.

[1] HARRY BUNT, *Semantic underspecification: Which technique for what purpose?*, ***Computing meaning***, vol. 3 (Harry Bunt and Reinhard Muskens, editors), Studies in Linguistics and Philosophy 83, Springer, Dordrecht, 2007, pp. 55–85.

[2] YIANNIS N. MOSCHOVAKIS, *A logical calculus of meaning and synonymy*, ***Linguistics and philosophy***, vol. 29, pp. 27–89.

[3] REINHARD MUSKENS, *Sense and the computation of reference*, ***Linguistics and Philosophy***, vol. 28, pp. 473–504.

► LARISA MAKSIMOVA, *Weak interpolation in extensions of Johansson's minimal logic*.
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A weak version of interpolation in extensions of Johansson's minimal logic is defined, and its equivalence to a weak version of Robinson's joint consistency is proved. We find some criteria for validity of WIP in extensions of the minimal logic.

Let L be a logic, $\vdash_L$ deducibility relation in L. *The weak interpolation property WIP* is defined as follows:

WIP. If $A, B \vdash_L \bot$, then there exists a formula C such that $A \vdash_L C$ and $B \vdash_L \neg C$, and all the variables of C are in both A and B.

Let L be any axiomatic extension of the minimal logic. An *L-theory* is a set T closed with respect to $\vdash_L$. An L-theory is *consistent* if it does not contain $\bot$. *The weak Robinson property WRP* is defined as follows:

WRP. Let T_1 and T_2 be two L-theories in the languages $\mathcal{L}_1$ and $\mathcal{L}_2$ respectively, $\mathcal{L}_0 = \mathcal{L}_1 \cap \mathcal{L}_2$, $T_{i0} = T_i \cap \mathcal{L}_0$. If the set $T_{10} \cup T_{20}$ in the common language is L-consistent, then $T_1 \cup T_2$ is L-consistent.

THEOREM 1. *For any* (*predicate or propositional*) *extension L of the minimal logic, WIP is equivalent to WRP.*

The language of the minimal logic J contains &, $\vee$, $\to$, $\bot$ as primitive; negation is defined by $\neg A = A \to \bot$. A formula is said to be *positive* if contains no occurrences of $\bot$. The logic J can be given by the calculus, which has the same axiom schemes as the positive intuitionistic calculus, and the only rule of inference is modus ponens. By a J-*logic* we mean an arbitrary set of formulas containing all the axioms of J and closed under modus ponens and substitution rules. We denote Int $= \mathrm{J} + (\bot \to p)$, Gl $= \mathrm{J} + (p \vee \neg p)$. A J-logic is *superintuitionistic* if it contains the intuitionistic logic Int, and *negative* if contains $\bot$.

THEOREM 2. *For any* J-*logic* L *the following are equivalent*: (1) L *has WIP*, (2) $L \cap L_1$ *has WIP for any negative logic* L_1, (3) $L \cap \mathrm{Neg}$ *has WIP.*

THEOREM 3. *Any propositional* J-*logic containing* $\mathrm{J} + \neg\neg(\bot \to p)$ *possesses WIP.*

The problem of weak interpolation is reducible to the same problem over Gl.

THEOREM 4. *For any* J-*logic* L, L *has WIP if and only if* $L + (p \vee \neg p)$ *has WIP.*

THEOREM 5. *There exists a* J-*logic, which contains* $\mathrm{Gl} = \mathrm{J} + (p \vee \neg p)$ *and does not possess the weak interpolation property.*

To prove that we consider two J-algebras $\boldsymbol{B}$ and $\boldsymbol{C}$. The universe of $\boldsymbol{B}$ consists of four elements $\{a, b, \bot, \top\}$, where $a < b < \bot < \top$. The algebra $\boldsymbol{C}$ consists of five elements $\{c, d, e, \bot, \top\}$, where $e < x < \bot < \top$ for $x \in \{c, d\}$ and the elements c and d are incomparable. Let a J-logic L_1 be the set of all formulas valid in the both algebras $\boldsymbol{B}$ and $\boldsymbol{C}$. Let

$$A(x, y) = (x \to y)\&((y \to x) \to x)\&(y \to \bot)\&((\bot \to y) \to y),$$
$$B(u, w) = ((u \to w) \to w)\&((w \to u) \to u)\&((u \vee w) \leftrightarrow \bot).$$

We prove that $A(x, y), B(u, w) \vdash_{L_1} \bot$, but this formula has no interpolant in L_1.

► JOÃO MARCOS, *Simulating negation in positive logic*.
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The first part of this talk will consider what happens when one adds a new axiomless 0-ary constant to (propositional) positive logics (cf. [2]), providing conservative extensions of them into 'logics of refutability' (cf. [3]) or of 'assertibility'. The second part will show that the resulting logics are essentially *non-truth-functional*, and then consider what happens when one adds axioms that forces this new constant to behave as the supremum or as the infimum of the corresponding algebras of values (cf. [5]). The third part will evaluate the behavior and propose adequate formal non-deterministic semantics (cf. [1]) for several unary constants defined with the help of the above 0-ary constant, and show in which circumstances such unary constants behave as *negations* or alternatively as *identity*-like connectives (cf. [4]). The final picture will display, among other things, the relations that hold between the positive fragments of both intuitionistic and classical logic, as well as Johánsson's minimal intuitionistic logic, Curry's logic of classical refutability, full intuitionistic logic and full classical logic, and also their paracomplete relatives.

[1] ARNON AVRON AND IDDO LEV, *Non-deterministic multiple-valued structures*, ***Journal of Logic and Computation***, vol. 15 (2005), pp. 241–261.

[2] HASKELL B. CURRY, *On the definition of negation by a fixed proposition in inferential calculus*, ***The Journal of Symbolic Logic***, vol. 17 (1952), no. 2, pp. 98–104.

[3] ———, ***Foundations of Mathematical Logic***, McGraw-Hill, 1963.

[4] JOÃO MARCOS, *On negation: Pure local rules*, ***Journal of Applied Logic***, vol. 3 (2005), no. 1, pp. 185–219.

[5] ———, *What is a non-truth-functional logic?*, ***Studia Logica***, vol. 92 (2009), no. 2, pp. 215–240.

► MARIA DA PAZ N. MEDEIROS, *A reviewed syntactic proof of Gödel interpretation of intuitionist logic into S4*.
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In 1933 Gödel introduced an interpretation of Heyting's propositional calculus into a system equivalent to S4 modal logic, called Σ, and did the following conjecture: *a formula holds in Heyting's calculus if and if its translation is proved in* Σ [1]. Gödel's main aim was to give an interpretation of the propositional intuitionistic logic that was also significant from a non intuitionistic point of view.

In 1948, the first semantic proof of this conjecture is known, but only in 1968 a syntactic proof was outlined by Prawitz and Malmnäs in [3].

We will present one more syntactic proof to Gödel's conjecture, taking into account the recent results and approaches about S4 modal system in natural deduction, particularly using the structural properties of modal inference rules introduced by Medeiros in [2].

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▶ JOSÉ M. MÉNDEZ, GEMMA ROBLES, AND FRANCISCO SALTO, *Paraconsistency and consistency understood as the absence of the negation of any implicative theorem*.

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As is stated in its title, in this paper consistency is understood as the absence of the negation of any implicative theorem. Then, we define the basic logic adequate to this concept of consistency in the ternary relational semantics with a set of designated points, negation being modelled with the Routley Star (see., e.g., [1]). Next, a series of logics extending this basic one is defined. All logics in this paper are paraconsistent, but none of them is relevant.

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▶ HEIKE MILDENBERGER AND SAHARON SHELAH, *Proper translation*.

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We continue our work on weak diamonds [1]. We show that $2^{\omega} = \aleph_2$ together with the weak diamond for covering by slaloms, the weak diamond for covering by meagre sets, the weak diamond for covering by null sets, and "all Aronszajn trees are special" is consistent relative to ZFC. We iterate alternately forcings specialising Aronszajn trees without adding reals (the NNR forcing from [2, Ch. IV]) and $< \omega_1$-proper forcings adding reals. We show that over a tower of elementary submodels there is a sort of a reduction ("proper translation") of our iteration to the c.s. iteration of simpler iterands. If we use only Sacks iterands and NNR iterands, this allows us to guess the values of Borel functions into small trees and thus derive the weak diamonds.

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[2] SAHARON SHELAH, ***Proper and improper forcing***, 2nd Edition, Springer, 1998.

► RUSSELL MILLER, *Survey of degree spectra of high$_n$ and non-low$_n$ degrees*.

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The high$_n$ Turing degrees are those whose n-th jump computes $\emptyset^{(n+1)}$; the non-low$_n$ degrees are those with n-th jump $\not\leq_T \emptyset^{(n)}$. These well-known classes of degrees are all upwards-closed under Turing reducibility, so it is natural to search for structures with spectra equal to various of these classes. We give a survey of recent results on this topic, by the speaker and many other authors, and also consider the same question for spectra of relations on computable structures. Linear orders turn out to be of particular interest.

► TAKAKO NEMOTO, *Determinacy of Wadge classes in the Baire space and simple iteration of inductive definition*.

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In [2], we introduced determinacy schemata motivated by Wadge classes in descriptive set theory and investigated the strength of them in the Cantor space.

In this talk, we consider the strength of $\mathsf{Sep}(\Delta^0_2, \Sigma^0_2)$ determinacy, one of these determinacy schemata, in the Baire space, by comparison with Σ^1_1-TID (Σ^1_1 *Transfinite Inductive Definition*) defined as follows: For a given wellordering $(W, \prec)$ and a sequence $\langle \Psi_w : w \in W\rangle$ of Σ^1_1 operators along $(W, \prec)$, there exists a sequence $\langle Y^w : w \in W\rangle$ such that, for every $w \in W$, Y^w is a fixed point of Ψ_w starting from $\bigcup_{z \prec w} Y^z$. Here, Y is said to be a *fixed point of an operator* Ψ *starting from* $X \subseteq \mathbb{N}$ if there exist an ordinal γ and a sequence $\langle Y_\alpha : \alpha < \gamma\rangle$ such that

- $Y_0 = X$;
- $Y_\alpha = \bigcup_{\beta<\alpha} Y_\beta \cup \Psi(\bigcup_{\beta<\alpha} Y_\beta)$;
- $Y = \bigcup_{\alpha<\gamma} Y_\alpha$;
- $\Psi(Y) = Y$.

We prove that, over RCA_0, Σ^1_1-TID implies $\mathsf{Sep}(\Delta^0_2, \Sigma^0_2)$ determinacy, which asserts the determinacy of games equivalent both to $\Sigma^0_2 \wedge \Pi^0_2$ games and to $\Sigma^0_2 \vee \Pi^0_2$ games, in the Baire space. We also consider the converse.

[1] M. O. MEDSALEM AND K. TANAKA, *Weak determinacy and iterations of inductive definitions*, ***Computational prospects of infinity Part II: Presented Talks***, (Chitat Chong, Qi Feng, Theodore A. Slaman, W. Hugh Woodin, and Yue Yang, editors), Lecture Notes Series, Institute for Mathematical Sciences, National University of Singapore, vol. 15, World Scientific, 2008, pp. 333–354.

[2] T. NEMOTO, *Determinacy of Wadge classes and subsystems of second order arithmetic*, ***Mathematical Logic Quarterly***, vol. 55 (2009), no. 2, pp. 154–176.

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► ROLAND SH. OMANADZE AND ANDREA SORBI, *s-Reducibility and immunity properties*.

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We proceed our investigation of immunity properties of s-degrees, initiated in [1], in which it is shown that the complete s-degree $\deg_s(\overline{K})$ does not contain any Δ_2^0 hyperhyperimmune set, where $\overline{K}$ is the complement of the halting set. We show that neither the immune nor the hyperimmune s-degrees are upwards closed since there exist Δ_2^0 s-degrees $\mathbf{a} \leq_s \mathbf{b}$ such that $\mathbf{a}$ is hyperimmune, but $\mathbf{b}$ is immune free. This contrasts with the fact that the immune and hyperimmune e-degrees are upwards closed, [2]. We also show that there is no hyperhyperimmune Π_2^0 set A such that $\overline{K} \leq_{\hat{s}} A$, where $\leq_{\hat{s}}$ denotes the finite-branch version of s-reducibility; this gives as a particular case a result already proved in [1], that $\deg_{\hat{s}}(\overline{K})$ is hyperhyperimmune free.

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► SERGEY OSPICHEV, *Some properties of computable numberings in various classes in difference hierarchy*.

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In this work are considered computable numberings [4] of families from various classes Σ_α^{-1} in difference hierarchy [2], where α is computable ordinal number.

It is shown that there are no computable numbering of the family of all sets from class Δ_α^{-1}, where α is computable ordinal number.

DEFINITION. Numbering $\{\nu_n\}_{n\in\omega}$ is called ω-computable, if a set $\{< m, n > | m \in \nu_n\}$ is in class Δ_ω^{-1}.

In work is annonced THEOREM. There is a ω-computable minimal numberings of the family of all sets from class $\bigcup\limits_{n\in\omega} \Sigma_n^{-1}$ in difference hierarchy.

In work [3] were proved that for all finite classes in difference hierarchy Σ_n^{-1} there is minimal Friedberg numbering of the family of all sets from Σ_n^{-1}.

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► LUIZ CARLOS PEREIRA, EDWARD HERMANN HAEUSLER, AND MARIA DA PAZ N. MEDEIROS, *Constructive results in fragments of classical first order logic*.

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In 1933 Gödel proved that we cannot distinguish classical logic from intuitionistic logic with respect to their theorems in the fragment $\{\wedge, \neg\}$. Although the fragments $\{\forall, \neg, \wedge\}$ and $\{\exists, \wedge, \neg\}$ are sufficient to establish a distinction between classical logic and intuitionistic logic, there are several constructive results that can be proved in these fragments. For example, we can prove that negation is constructively involutive in the fragment $\{\forall, \neg, \wedge\}$ and that every classical theorem of the form $\exists x A(x)$ with $A(x)$ quantifier free is intuitionistically provable in the fragment $\{\exists, \wedge, \neg\}$. The aim of the present paper is to show that, as a consequence of these constructive results we can obtain (1) a proof-theoretical argument for the decidability of classical monadic predicate logic and (2) a proof that the logic of categorical propositions is constructive.

► ALEXANDRA REVENKO, *The complexity of automatic partial orders*.
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The theory of automatic structures found wide application in logic. Numerous decidability problems was solved by using this theory. We here consider an automatic structure as a relational structure whose domain and relations are recognized by finite string atomata. It seemed that these structures must be easy from some complexity points of view. But it was showed that they aren't. For example, Khoussainov, Nies and others showed that the complexity of the isomorphism problem for automatic structures is Σ_1^1-complete [2]. Also Khoussainov and Minnes showed that for every ordinal $\alpha \leq \omega_1^{CK} + 1$ there exists an automatic structure such that the Scott rank of it is α [1]. Then it is wishful to prove good complexity results at least for specific classes of automatic structures.

We studied automatic oderings. And the class of automatic partial orderings isn't easy. It was showed that also for any given $\alpha \leq \omega_1^{CK} + 1$ there is an automatic partial order of Scott rank equal or greater than α.

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► GEMMA ROBLES, *Paraconsistent logics included in Lewis' S4*.
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Let $\vDash$ be a relation of consequence, be it defined either semantically or proof-theoretically. As is known, a logic S is *paraconsistent* iff the ECQ ('E contradictione quodlibet') rule $A \wedge \neg A \vDash B$ is not a rule of S (cf. [3]).

It is well known that Lewis' modal logics are not paraconsistent, fact that did not disturb at all this great logician. Far from it, Lewis vindicates the validity of the ECQ rule in a famous proof ([2], p. 250) currently known as the Lewis' proof or the Lewi's argument (cf. [1], §16.1). This proof essentially leans on the disjunctive syllogism. The aim of this paper is not to discuss this proof, but the following. Let $S4_+$ be the positive fragment of Lewis' S4, we define a series of paraconsistent logics included in $S4_+$ extended with the double negation axioms, the contraposition axioms, the principle of non-contradiction and the disjunctive syllogism.

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► PAUL SHAFER, *The first order theory of the Medvedev lattice is third order arithmetic*.
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For sets $\mathcal{A}, \mathcal{B} \subseteq \omega^\omega$, we say $\mathcal{A}$ *Medvedev reduces* to $\mathcal{B}$ ($\mathcal{A} \leq_M \mathcal{B}$) if there is a Turing functional Φ such that Φ^f is total and in $\mathcal{A}$ for all f in $\mathcal{B}$, and we say $\mathcal{A}, \mathcal{B} \subseteq \omega^\omega$ are *Medvedev equivalent* ($\mathcal{A} \equiv_M \mathcal{B}$) if $\mathcal{A} \leq_M \mathcal{B}$ and $\mathcal{B} \leq_M \mathcal{A}$. The *Medvedev lattice* is the degree structure $(\mathcal{P}(\omega^\omega)/\equiv_M, \leq)$, where $\leq$ is the ordering $\leq_M$ induces on equivalence classes. Similarly, $\mathcal{A}$ *weakly reduces* to $\mathcal{B}$ ($\mathcal{A} \leq_w \mathcal{B}$) if for every f in $\mathcal{B}$ there is g in $\mathcal{A}$ with $f \geq_T g$. *Weak equivalence* ($\equiv_w$) and the *Muchnik lattice* $(\mathcal{P}(\omega^\omega)/\equiv_w, \leq)$ are defined analogously to Medvedev equivalence and the Medvedev lattice. We code the standard model of third order arithmetic directly into the Muchnik lattice. This result, combined with the definability of the Muchnik lattice in the Medvedev lattice and the definability of the Medvedev lattice in true third order arithmetic, proves the following theorem: the first order theory of the Muchnik lattice, the first order theory of the Medvedev lattice, and the third order theory of true arithmetic are recursively isomorphic. This approach is different from the one taken by Lewis, Nies, and Sorbi at CiE 2009.

► STANISLAV SMERDOV, *On the question of consistence of the semantic μ-prediction*.
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First we define a probability on the set of all ground sentences according to [2]. Allow literals (not only atoms) to appear in a classical logic programming structures of rule, fact and query; here probabilities of ground instances are defined as conditional.

$$\mathtt{Rule}_{\mathrm{L}}^{\mu} \rightleftharpoons \{\mathrm{C} \in \mathtt{Rule}_{\mathrm{L}} \mid \textit{for some ground } \theta \text{ probability of } \mathrm{C}\theta \textit{ is determined}\};$$
$$\underline{\mu}(\mathrm{C}) \rightleftharpoons \inf\{\mu(\mathrm{C}\theta) \mid \theta \textit{ is a ground substitution and } \mathrm{C}\theta \in \mathtt{Rule}_{\mathrm{L}}^{\mu}\}, \textit{ where } \mathrm{C} \in \mathtt{Rule}_{\mathrm{L}}^{\mu}.$$

Binary relation $\mathrm{C}_1 \rhd \mathrm{C}_2$ between $\mathrm{C}_1 = (\mathrm{A}_1 \Leftarrow \mathrm{B}_1, \dots, \mathrm{B}_n)$, $\mathrm{C}_2 = (\mathrm{A}_2 \Leftarrow \mathrm{D}_1, \dots, \mathrm{D}_m)$ in $\mathtt{Rule}_{\mathrm{L}}^{\mu}$ takes place iff there exist a substitution θ such that $\{\mathrm{B}_1\theta, \dots, \mathrm{B}_n\theta\} \subseteq \{\mathrm{D}_1, \dots, \mathrm{D}_m\}$, $\mathrm{A}_1\theta = \mathrm{A}_2$ and C_1 is not a variant for C_2; in this case we say "C_1 *is more general than* C_2". Rules from $\mathtt{Rule}_{\mathrm{L}}^{\mu}$ which can't be generalized without decreasing the conditional probability $\underline{\mu}(\cdot)$ are called *μ-laws*; denote corresponding set by $\mathtt{GLaw}_{\mathrm{L}}^{\mu}$.

$\mathtt{Data}(\mathfrak{B})$ is a set of actual facts for 1-st order model $\mathfrak{B}$. A process of SLD-inference with probabilistic estimations is replaced with *semantic μ-prediction* (or *P-prediction*) from the set of μ-laws which are valid for accessible data. Note: this concept has common features with semantic approach in programming (see [1] for analogies). As a result we obtain *the best prediction μ-law* for every ground literal, if semantic μ-prediction is determined. All necessary definitions are natural extension of those introduced in [3]. For each best rule $\mathrm{C} = \mathrm{A} \Leftarrow \mathrm{B}_1, \dots, \mathrm{B}_n$ used in prediction of some H we consider all $\mathrm{C}\theta$ such that θ is a ground substitution, $\{\mathrm{B}_1\theta, \dots, \mathrm{B}_n\theta\} \subseteq \mathtt{Data}(\mathfrak{B})$, $\mathrm{A}\theta = \mathrm{H}$ and $\{\mathrm{B}_1\theta, \dots, \mathrm{B}_n\theta\}$ is μ-concurred (set of literals S is called *μ-concurred* iff $\mu\left(\bigwedge_{A\in S}A\right) \neq 0$). Resulted set of described ground instances (over all literals) is denoted by $\mathtt{Prdct}_{\mathrm{L}}^{\mu,0}$.

THEOREM 1. *Let some ground atom* H *be semantically μ-predicted by ground instance* $\mathrm{C}_{pos} \in \mathtt{Prdct}_{\mathrm{L}}^{\mu,0}$ *of the rule* $\mathrm{C}_1 \in \mathtt{GLaw}_{\mathrm{L}}^{\mu}$ $(\mathrm{C}_{pos} = \mathrm{C}_1\theta_{pos})$, *while* $\neg$H—*by* C_{neg} $(\mathrm{C}_{neg} = \mathrm{C}_2\theta_{neg})$. *Then the set of atoms from premises of* C_{pos} *and* C_{neg} *is not μ-concurred.*

$$\Gamma_{\mathfrak{B}} = \left\{\mathrm{B}_1 \wedge \cdots \wedge \mathrm{B}_n \to \mathrm{A} | \mathrm{A} \Leftarrow \mathrm{B}_1 \wedge \cdots \wedge \mathrm{B}_n \in \mathtt{Prdct}_{\mathrm{L}}^{\mu,0}\right\} \cup \{\mathrm{A} | (\mathrm{A} \Leftarrow) \in \mathtt{Data}(\mathfrak{B})\}$$

THEOREM 2. *Let* $\mathtt{Data}(\mathfrak{B})$ *be μ-concurred. Then minimal theory containing* $\Gamma_{\mathfrak{B}}$ *is logically consistent*.

[1] S. S. GONCHAROV, YU. L. ERSHOV, AND D. I. SVIRIDENKO, *Semantic programming*, ***10th World Congress Information Processing 86*** (Dublin), vol. 10, Amsterdam, 1986, pp. 1093–1100.

[2] J. Y. HALPERN, *An analysis of first-order logics of probability*, ***Artificial Intelligence***, vol. 46, 1990, pp. 311–350.

[3] E. E. VITYAEV, *The logic of prediction*, ***Mathematical Logic in Asia, Proceedings of the 9th Asian Logic Conference*** (Novosibirsk, Russia), (S. S. Goncharov, R. Downey, and H. Ono, editors), World Scientific, Singapore, 2006, pp. 263–276.

► TEODOR J. STĘPIEŃ AND ŁUKASZ T. STĘPIEŃ, *On the consistency of Peano's Arithmetic System*.

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§1. *Terminology*. Let $\to, \sim, \wedge, \equiv$ denote connectives of implication, negation, conjuction and equivalence, respectively . Next Sp denotes the set of all well-formed formulas of Peano's Arithmetic System. R_{Sp} denotes the set of all rules over Sp. For any $X \subseteq Sp$ and for any $R \subseteq R_{Sp}$, $Cn(R, X)$ is the smallest subset of Sp containing X and closed under the rules of R. The couple $\langle R, X\rangle$ is called a system, whenever $R \subseteq R_{Sp}$ and $X \subseteq Sp$. Next r_o denotes Modus Ponens, r_+ denotes the generalization rule and $R_{o+} = \{r_o, r_+\}$ (see [1]). We use $\Rightarrow, \neg, \&, \Leftrightarrow, \forall, \exists$ as metalogical symbols. L_2 and A_r denote the set of all logical axioms and the set of all specific axioms in Peano's Arithmetic System, respectively (see [2]). Hence $\langle R_{o+}, L_2 \cup A_r\rangle$ is Peano's Arithmetic System.

The traditional consistency:

DEFINITION. $\langle R, X\rangle \in Cns^T \Leftrightarrow (\neg\exists\alpha \in Sp)[\alpha \in Cn(R, X) \ \ \& \sim \alpha \in Cn(R, X)]$

§2. *The main result*.

THEOREM. $\langle R_{0+}, L_2 \cup A_r\rangle \in Cns^T$.

PROOF. (elementary and combinatorial, see [3]).

[1] W. POGORZELSKI, ***The classical calculus of quantifiers***, PWN, Warszawa, 1981 (in Polish).

[2] H. RASIOWA, ***Introduction to contemporary mathematics***, PWN, Warszawa 1977 (in Polish).

[3] J. VON NEUMANN, *Die formalistische Grundlegung der Mathematik*, ***Erkenntnis***, vol. 2 (1931), pp. 116–121.

► SERGEY SUDOPLATOV, *The Lachlan problem*.

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A generic construction realizing basic characteristics of Ehrenfeucht theories, i.e., of complete first order theories with finitely many but more then one pairwise non-isomorphic countable models, is stated. On the basis of that construction as well as of the Hrushovski—Herwig generic construction, a solution of the Lachlan problem on existence of stable Ehrenfeucht theories is found.

► TINKO TINCHEV, *Universal fragments of some region-based theories of space*.
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Let $\mathbb{T} = (T, \tau)$ be a topological space. It is well known that the regular closed subsets of $\mathbb{T}$ form a Boolean algebra, $RC(\mathbb{T})$, under inclusion with top element $1 = T$ and bottom element $0 = \emptyset$. A number of predicates in $RC(\mathbb{T})$ have a natural geometrical meaning, for example, the predicates *k-contact*, $k \geq 2$, defined by $C_k(a_1, \dots, a_k)$ iff $a_1 \cap \dots \cap a_k \neq \emptyset$. So, the structure $\mathfrak{A}_{\mathbb{T}} = \langle RC(\mathbb{T}), \{C_k\}_{k<\omega}\rangle$ is a model for the first-order language $\mathfrak{L}$ extending the language of the Boolean algebras with the predicates symbols C_k, $k \geq 2$. In this way any class K of topological spaces determines a theory Γ_K. We give an axiomatization of the universal fragment of Γ_K in the cases K is the class of all topological spaces, the class of all connected topological spaces and the singletons $\mathbb{R}^n$, $n \geq 1$. We treat these universal fragments as modal logics and prove the completeness theorems with respect to the finite Kripke structures and show that they are not strong complete.

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Let $\mathbb{ZFI}2C$ is a intuitionistic two-sorted set theory with variables of sort 0 are variables on natural numbers, and variables of sort 1 are set variables.

Axioms of $\mathbb{ZFI}2C$ consist of usual axioms and schemata of Heyting predicate calculus (HPC), all usual axioms of Heyting Arithmetic (HA), and all usual Zermelo–Fraenkel axioms for set theory including Exstensionality, Collection as Substitution axiom, and transfinite induction as Foundation axiom.

We consider also the additional principle DCS (Double Complement of Sets).

We use some modifications of formalized realizabilities from [1] and proved the following (for T is either $\mathbb{ZFI}2C$ or $\mathbb{ZFI}2C + DCS$):

1. For T: Disjunction Property(DP); Numerical Existensional Property (EP_ω); Curch Rule (CR); Markov Rule (MR); Uniformization Rule (UR).

All these properties are proved with set parameters.

Each combination of the following extra axioms can be added to T with preserving of results (i)–(iii) and (v): Church Thesis CT, Markov Principle M, Uniformization Principle UP, and Independence of Permisses IP.

2. For $T + ECT$ (where ECT is a Extended Church Thesis): Disjunction Property (DP) and EP_ω; the conservativity of $T+ECT$ over T w.r.t. class of all negative formulas; $T+ECT = T + \{R\varphi \equiv \varphi \mid \varphi \text{ is a formula of } T\}$ for a variant of Kleene realizability R.

3. For $T + ECT + M$: the conservativity of $T + ECT + M \vdash \varphi$ over T w.r.t. class of all negative formulas; relative consistency of T w.r.t. $T + ECT + M$; DP and EP_ω for $T + ECT + M$.

4. For $T + nCT + P$: the conservativity of $T + nCT + P \vdash \varphi$ over T w.r.t. class of all negative formulas; relative consistency of $T + nCT + P$ over T; DP and EP_ω for $T + nCT + P$.

[1] M. BEESON, *Continuity in intuitionistic set theories*, ***Logic ColloquiumŠ78.*** North-Holland Publishing Company, 1979, pp. 1–52.

► MICHAEL A. WARREN, *On the homotopy theory of Martin–Löf complexes*.
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Martin–Löf complexes are defined as algebras for a monad, which acts by freely adjoining cells in accordance with the rules of intensional type theory, on the category of globular sets. In this talk we study the homotopy theory of such complexes and address the question whether they can be used to model homotopy types.

This is joint work with S. Awodey and P. Hofstra.

► MARS M. YAMALEEV, *Splitting properties in 2-c.e. degrees*.
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A Turing degree $\mathbf{a}$ is splittable in a class of degrees $\mathcal{C}$ avoiding an upper cone of a degree $\mathbf{d}$ if there exist degrees $\mathbf{x}_0, \mathbf{x}_1 \in \mathcal{C}$ such that $\boldsymbol{a} = \mathbf{x}_0 \cup \mathbf{x}_1$, $\mathbf{x}_i < \mathbf{a}$ and $\mathbf{d} \not\leq \mathbf{x}_i$ for $i = 0, 1$. The following theorem presents sufficient conditions for a properly 2-computably enumerable (2-c.e.) degree to be splittable in $\mathbf{D}_2$ avoiding the upper cone of another properly 2-c.e. degree.

THEOREM 1. *Let* $\mathbf{a}$ *and* $\mathbf{d}$ *be properly* 2*-c.e. degrees such that* $\mathbf{0} < \mathbf{d} < \mathbf{a}$ *and there are no c.e. degrees between* $\mathbf{a}$ *and* $\mathbf{d}$. *Then the degree* $\boldsymbol{a}$ *is splittable in* $\mathbf{D}_2$ *avoiding the upper cone of* $\boldsymbol{d}$.

Theorem 1 holds when $\mathbf{d}$ is a $\boldsymbol{\Delta}_2^0$-degree, which does not contain c.e. sets. Theorem 2 states that the well-known bubble (see [1]) can be constructed in low 2-c.e. degrees.

THEOREM 2. *There exist low noncomputable* 2*-c.e. degrees* $\mathbf{b} < \mathbf{a}$ *such that for any* 2*-c.e. degree* $\mathbf{v} \leq \mathbf{a}$ *either* $\mathbf{v} \leq \mathbf{b}$ *or* $\mathbf{b} \leq \mathbf{v}$.

As a consequence we obtain the following: the partial orders of m-low c.e. and m-low 2-c.e. degrees are not elementarily equivalent for any $m \geq 1$. Also, I will talk about a link between splitting properties (Theorem 1) and the bubbles and how the link could be uniformly adapted to higher levels of the Ershov's hierarchy.

[1] M. M. ARSLANOV, I. SH. KALIMULLIN, AND S. LEMPP, *On Downey's conjecture*, ***The Journal of Symbolic Logic***, to appear (`http://www.math.wisc.edu/~lempp/papers`)

► MOSTAFA ZAARE, *Preservation theorems for intuitionistic first-order logic*.
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There are several ways for defining the notion *submodel* for Kripke models of intuitionistic first-order logic. In our approach a Kripke model $\mathcal{A}$ is a submodel of a Kripke model $\mathcal{B}$ if they have the same frame and for each two corresponding worlds A_α and B_α of them, A_α is a (classical) submodel of B_α. We introduce some intuitionistic formula classes, including the classes $\mathcal{U}$ and $\mathcal{E}$ of intuitionistic universal and existential formula classes, and prove analogues of the well-known classical preservation theorems for them.

We also define some other notions like *elementary submodel* and *union of chain* for Kripke models and investigate their properties.

This is a joint work with Morteza Moniri.

[1] S. M. BAGERI AND M. MONIRI, *Some results on Kripke models over an arbitrary fixed frame*, ***Mathematical Logic Quarterly***, vol. 49 (2003), no. 5, pp. 479–484.

[2] B. ELLISON, J. FLEISCHMANN, D. MCGINN, AND W. RUITENBURG, *Kripke submodels and universal sentences*, ***Mathematical Logic Quarterly***, vol. 53 (2007), no. 3, pp. 311–320.

[3] M. Moniri and M. Zaare, *Preservation theorems for Kripke models*, ***Mathematical Logic Quarterly***, vol. 55 (2009), no. 2, pp. 177–184.

[4] A. Visser, *Submodels of Kripke models*, ***Archive for Mathematical Logic***, vol. 40 (2001), no. 4, pp. 277–295.

► MAXIM ZUBKOV, *Strongly η-representable sets and limitwise monotonic functions*.
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We consider strongly η-representable sets. An infinite set $A = \{a_0 < a_1 < \dots\}$ is called *η-representable*, if there is a computable linear order L of the order type $\eta + a_0 + \eta + a_1 + \eta + \dots$, where η is the order type of rationals $\mathbb{Q}$. R. Downey [1] stated the question of a description of Turing degrees with strongly η-representable sets. For details see [1] and [2]. We proved that any Turing degree contains a strongly η-representable set iff it contains a range of a $\mathbf{0}'$-limitwise monotonic and pseudo increasing on $\mathbb{Q}$ function. A function F is *$\mathbf{0}'$-limitwise monotonic*, if there is a $\mathbf{0}'$-computable function $f\colon \mathbb{Q}\times\omega \to \omega$ such that $F(x) = \lim_s f_s(x,s)$ and $f(x,s) \le f(x,s+1)$. A function F is *pseudo increasing on* $\mathbb{Q}$, if it is increasing on the support of F. A support of F is $\{q \in \mathbb{Q} \mid F(x) > 1\}$.

[1] R. G. Downey, *Computability theory and linear orderings*, ***Handbook of computable algebra***, vol. 2 (1998), Elsevier, Amsterdam, pp. 823–976.

[2] K. Harris, *η-representations of sets and degrees*, ***The Journal of Symbolic Logic***, vol. 73 (2008), pp. 1097–1121.

Abstracts of talks presented by title

► JOHN CORCORAN, *Defining 'logical' in first-order metalogic*.
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Without appropriate definitions of 'logical', logical truth and logical falsehood have been called *logical* properties; 2-place logical consequence and logical independence have been called *logical* relations. Let L be a first-order language. A *formal* 1-1 function f on L's vocabulary carries each non-logical constant to one in the same syntactic category leaving every other character fixed. Any formal f extends to L: for each sentence s, if the n-th character in s is k, the n-th character in the sentence fs is fk. A sentence p is *formally similar* to (or *in the same logical form* as) a sentence q if, for some formal f, fp is an alphabetic variant of q. Any [extended] formal f extends to [ordered] couples (p,q) of sentences: $f(p,q) = (fp, fq)$. Formal similarity extends to couples: (p,p^*) is *formally similar* to (q,q^*) if, for some formal f, fp and fp^* are respectively alphabetic variants of q and q^*. A property (set) or a relation (set of couples) closed under formal similarity is *formal*. A sentence p is *contentually similar* to a sentence q if they are logically equivalent. *Contentual similarity* extends to ordered pairs: (p,p^*) is *contentually similar* to (q,q^*) if p and p^* are logically equivalent to q and q^* respectively. A property (set), or a relation (set of couples), closed under contentual similarity is *contentual*. Each "logical" property and relation mentioned above is both formal and contentual. This paper explains the historical and philosophical adequacy of my proposal to define a property or relation to be *logical* if it is both formal and contentual. This lecture sharpens discussions in this Bulletin, vol. 7 (2001), pp. 105–106, and vol. 10 (2004), p. 445.

► JOHN CORCORAN AND GEORGE BOGER, *Paradox and antinomy*.
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The ambiguous words 'paradox' and 'antinomy' have vague meanings—admitting twilight zones of borderline cases. See [2]. Both words are used as parts of *relational* expressions and as common nouns or *sortals*. Following [1], this paper takes 'paradox' relationally as in: "Russell's argumentation was a *paradox for* Frege but not a *paradox for* later logicians". A given argumentation is a *paradox for* a given person iff the person takes following its chain-of-reasoning to be a deduction of a conclusion the person disbelieves from premises the person believes. This approximates senses used by logicians and it agrees with dictionaries: cf. [2], definition 2c. In relational senses, 'paradox per se' is incoherent just as 'consequence per se' is. An argumentation that is a paradox for certain persons is sometimes elliptically and misleadingly called 'a paradox' or 'a genuine paradox' by *them* and 'an apparent paradox' by *those for whom it is not a paradox*. As mentioned, 'paradox' and 'antinomy' are also used non-relationally as non-elliptical sortal common nouns as in: "Kant believed he discovered antinomies". This paper reserves 'antinomy' for such a non-relational sense. An argumentation is an *antinomy* iff its chain-of-reasoning *derives* an *unbelievable* conclusion from premises that are *indubitable*. This definition presupposes absolute idealized notions of *derivation*, *belief*, and *doubt*. Reference to particular persons is inappropriate. Only consequences are derivable from premises in absolute idealized senses. Stylistic variants of 'antinomy' include 'genuine paradox' or 'absolute paradox'. This paper argues against the existence of antinomies, even if suitable absolute notions are accepted.

[1] John Corcoran, *Argumentations and logic*, ***Argumentation***, vol. 3 (1989), pp. 17–43.
[2] ***Merriam–Webster collegiate dictionary***, 11th ed., Merriam–Webster, Springfield, MA, 2008.

▶ JOHN CORCORAN, NEWTON DA COSTA, AND LUIZ LOPES DOS SANTOS, *Aristotelian direct deductions: metatheorems*.

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Informally, a *direct deduction* of a conclusion c from a given premise set P is a finite list beginning with members of P, ending with c, and such that each member after those from P is "derived" from previous members. List members are *categorical sentences*: *sAp*, *sEp*, *sIp*, and *sOp*, traditionally expressing "Every s is a p", "No s is a p", "Some s is a p", and "Some s is not a p", respectively, with s and p non-empty distinct terms such as "square" and "polygon". Different formal *systems* have different rules for "deriving". Aristotle proposed more than one. A system Σ is [strongly] *complete* if every conclusion c that follows logically from a given premise set P is derivable from P by means of a deduction in Σ. Aristotle seemed to claim completeness for his direct deductions. This paper reports complementary results obtained separately. Corcoran contributed negative results including: (1) the system $\mathbf{\Sigma A}$ using all rules adopted by Aristotle is incomplete and (2) no conclusion c (except premises) that depends essentially on an O-premise in a set P is directly derivable from P in $\mathbf{\Sigma A}$. This means that neither *sEp* nor *sIp* is so derivable from the pair composed of *sOp* and *sAp*: Aristotle's direct deduction systems are *paraconsistent* in a suitable sense. Filling the void Aristotle left, da Costa and Lopes dos Santos supplied positive results including a strong completeness proof for their system $\mathbf{\Sigma N\Lambda}$ which adds one "inconsistency trivialization" rule to ten others with consistent premise sets.

► SEAN COX, *Consistency strength of nonregular ultrafilters*.
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A *nonregular ultrafilter* is a weak version of a countably complete ultrafilter which arose from classic questions in model theory about cardinalities of ultrapowers. In particular, if an ultrafilter U on ω_1 has the property that $|^{\omega_1}\omega/U| = \omega_1$, then U must be nonregular.

Nonregularity is a weakening of countable completeness. Although in ZFC there is never a countably complete ultrafilter over a cardinal like ω_n, it is consistent relative to large cardinals that there is a nonregular ultrafilter on ω_n ($n \geq 1$). For $n = 1$, such an ultrafilter can be obtained starting with ω many Woodin cardinals (see [4]). For $n = 2$, the known upper bounds are higher, in the realm of huge cardinals (see [3] and [2]).

The best-known lower bound for the consistency strength of a nonregular ultrafilter on ω_1 is a stationary limit of measurable cardinals, due to Deiser and Donder [1]. This was an improvement on previous work by Ketonen and Donder/Jensen/Koppelberg. I will discuss my extensions of this work, particularly for nonregular ultrafilters on ω_2.

[1] Oliver Deiser and Dieter Donder, *Canonical functions, non-regular ultrafilters and Ulam's problem on* ω_1, ***The Journal of Symbolic Logic***, vol. 68 (2003), no. 3, pp. 713–739.

[2] Matthew Foreman, *An* $\aleph_1$*-dense ideal on* $\aleph_2$, ***Israel Journal of Mathematics***, vol. 108 (1998), pp. 253–290.

[3] M. Foreman, M. Magidor, and S. Shelah, *Martin's maximum, saturated ideals and nonregular ultrafilters. II*, ***Annals of Mathematics***, vol. 127 (1988), no. 3, pp. 521–545.

[4] W. Hugh Woodin, ***The axiom of determinacy, forcing axioms, and the nonstationary ideal***, de Gruyter Series in Logic and its Applications, 1999.

► VALERY KHAKHANYAN, *The relations between Uniformization Principle and Church Thesis in the set theory with intuitionistic logic*.
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Let CT and CT! be Church Thesis with choice (strong) and with uniqueness (weak) respectively. Let U and U! be principles strong and weak uniformization. Let ZFI2C + DCS (all postulates here or about are in [1] or [2]) be the set theory with intuitionistic logic and two kinds of variables (natural and set) and with collection scheme. It is known that: (a) ZFI2C+ CT! $\vdash$ U!; (b) ZFI2C + DCS + CT! $\nvdash$ CT; (c) ZFI2C-extensionality + DCS + CT $\nvdash$ U. We proved that (d) ZFI2C + DCS + CT $\nvdash$ U. Corollaries: (e) ZFI2C + DCS + U! $\nvdash$ U; (f) ZFI2C + DCS + CT! $\nvdash$ U.

I am going to speak also about some properties of models of realizability types which were used to prove (b)–(d) and to discuss some open problems.

[1] V. H. Hahanjan, *The comparative strength of variants of Church Thesis at the level of set theory*, ***Soviet Math. Dokl.***, vol. 21 (1980), no. 3, pp. 894–898.

[2] ———, *The consistency of intuitionistic set theory with formal mathematical analysis*, ***Soviet Math. Dokl.***, vol. 23 (1980), no. 1, pp. 46–50.

► MD. AQUIL KHAN AND MOHUA BANERJEE, *A quantified modal logic for rough sets*.
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A structure of the form $(U, \{R_i\}_{i\in N})$, where N is an initial segment of $\aleph$ representing N 'sources' and R_i is an equivalence relation on U representing the knowledge base of the i^{th} source, was considered in [3] to formally study the behavior of rough sets [4] in a multiple-source scenario. In this paper, we extend our study to consider structures of the form $(U, \{R_P\}_{P \subseteq_f N})$, called *multiple-source approximation systems for groups* (denoted $MSAS^G$), where R_P represents the combined knowledge base of the finite group P of sources. R_P is an equivalence relation on U satisfying (i) $R_P = \cap_{i \in P} R_i$ and (ii) $R_\emptyset = U \times U$. A quantified propositional modal logic $LMSAS^G$, different from modal logic with propositional quantifiers [1] and modal predicate logic, is proposed with semantics based on $MSAS^G$'s. The language has a set PV of propositional variables, and a set T of terms built with countable sets of constants and variables and a binary function symbol $\star$. Formulae are got through the scheme: $p\,|\neg\alpha|\alpha\wedge\beta|A\alpha|[t]\alpha|t = s|\forall x\alpha$, where $p \in PV$, $t, s \in T$, and A is the global modal operator. Thus quantification ranges over modalities. The semantics is defined with the help of a function which maps a term t to a finite subset of N, $*$ being translated as union of sets. The function determines which equivalence relation is to be used to evaluate a modality involving a term t. A sound and complete axiomatization is given and some decidability problems are addressed. It is found that the modal systems $B, S5$ and epistemic logics $S5_n^D$ [2] are embedded in $LMSAS^G$. It is also observed that $S5_n^D$ cannot replace $MSAS^G$ to serve our purpose. The semantics of $S5_n^D$ considers a finite and fixed number of agents, thus giving a finite and fixed number of modalities in the language. But in the case of $LMSAS^G$, the number of sources is not fixed, and could also be countably infinite. So, unlike the case of epistemic logics, it is not possible here to refer to all/some sources using only the connectives $\wedge, \vee$. The quantifiers $\forall, \exists$ are used to achieve the task.

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► GIAN ARTURO MARCO, *The dynamics of reference: Symmetry and semantics.*
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In this paper we outline a mathematical foundation of the notion of reference, ubiquitous to various areas of mathematics, logic, philosophy of language and natural language semantics.

In particular, we introduce the notion of "orbital reference": this rises in the realm of automorphism group actions on the space of interpretations for a given countable language.

In this setting the classical notion of reference unravels its functional structure: namely, it appears as a variety of definable point-re-identification maps within a suitable space of recognition (i.e., a Polish G-space X , where X is a Polish space and G is a closed group of transformations acting continuosly on X [1]). The complexity of the orbital reference for a given referring term t heavily depends on the algebraic and topological properties of the group stabilizer of the term t. One of the main consequences of our approach is that the Fregean notion of compositionality, accordingly with the referential variety, spreads out into a variety of patterns of compositionality. Such a variety of patterns rests upon the complexity spectrum of the invariant complete codes reducing the orbital spaces induced by the group actions.

Examples from Model Theory, like coordinatization for relatively categorical theories over a predicate, give evidence for the "independence" of our notion of orbital reference both from the Russellian notion of ostensive denotation and from the Kripkean concept of "rigid designator". It is just on its going beyond the denotational character that our concept of orbital reference turns out to be of crucial interest in the determinacy of semantic games for anaphoric co-reference: as pointed out by G. Sandu [2], the demonstrative, deictic notion of reference is unfit for the determinacy of semantic games on anaphora whenever the anaphoric link occurs within the scope of a negation, We define a point-reidentification game that settles the problem of determinacy of the coreferential anaphoric link in negative contexts.

Last, but not least, we give evidence also for the fact that "rigidity" is not a property intrinsic to the pure denotational "nature" of reference: rather it derives from the complete left invariant metric attribute of the automorphism group of a given structure.

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Synchronization protocols, especially those that handle multiway synchronization, have been an active area of research for several years. Although a number of mechanisms to achieve multiway synchronization have been proposed [1, 2]—some of the synchronization aspects of protocols have been shown to contain subtle errors [4, 3]. Thus it is desirable to have a logic to reason about the correctness of such protocols. We develop a logic for the formal analysis of binary and multiway synchronization protocols, with and without the preemption feature.

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► CYRUS F. NOURANI, *Functorial projective set models*.
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The author had defined specific realizabilities on positive formulas since 2005. Let $L_{P,\omega}$ be the positive fragment obtained from the Kiesler fragment. Define the category $\mathbf{L}_{P,\omega}$ to be the category with objects positive fragments and arrows the subfoumual preorder on formulas. Consider the infinitary counterpart to Robinson consistency, fragment consistency since the author's AMS–ASL, San Diego, January 1997.

PROPOSITION. *The interim sets realizing the interim models on the infinitary consistency on a tower form a projection to the set realizing the limit piece model.*

Let us define a discrete topology on the Keisler fragment K, on $L_{\omega_1,\omega}$. Let M be the infinite product copies on K. Give K the product toplogy. Let $F = \{K, K^2, \dots\}$.

OBSERVATIONS. (i) *A subset of elements of F from a topological space with a pointset topology.*

(ii) *M is hommorphic to its product with itself.*

DEFINITION. *If $A \subseteq M^{n+1}$ the fragment projection of A is* $\{\langle \alpha_1, \dots, \alpha_n, \beta\rangle \in A)\}$.

THEOREM 1. *On Fragment consistency theorem, consider the interim fragment models M_i. $M \models \Sigma$ iff $\forall i M_i \models$ a projection on fragments sets that positively locally realize Σ_i, $\Sigma_i \subset \Sigma$ is the set modeled at the i-th tower iteration.*

DEFINITION. *Let us say that model $\mathcal{R}$ is positively saturated if for every subset X of $\mathcal{R}$ of cardinality $< \kappa$, every type is positively locally realized in $\mathcal{R}_\kappa$.*

COROLLARY. The interim models M_i, theorem 1, are positively ω-saturated at the interim language fragment.

Remark 1. The above might be lifted to local realizability on arbitrary formulas.

Remark 2. Theorem 1 is a projective compactness that might ad insight to an ultraproduct natural transformation.

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Our goal is to present *Categorial Morphology*, a tool originally proposed by Hoeksema (1985). Basically, it is an attempt to adapt Categorial Grammar(CG), a tool used regularly in synthatic analysis that regards linguistics relations as mathematical functions, to word formation in natural languages. In this model, with few primitive categories, (for example, N for nouns and S for sentences) we can arrive recursively at all categories of lexical entries. With this logical tool it is not necessary to arbitrarily postulate categories to expressions because its recursivity allows new categories to be established based on the others. In CG, the compositional visualization of complex structures—even if they are phrases, sentences or words—is always relevant and clear because we take simples elements to create complex ones (bottom-up view). Since CG is a tool employed on syntactic analyses, its use for dealing with morphological analysis brings a high theoretical gain, since both syntax and morphology can be dealt with the same rule types. More than that, a successful employment of CG to morphology shows a new logical perspective to regard this area, besides GC is nothing but a kind of functional application of predicate logic.

Although Hoeksema (1985) proposal leads this work in our semantic representation, we will basically apply Carpenter's (1997) and Bayer (1997) proposals. We believe that, concerning mainly the transparence of argumental inheritance of each morpheme, the CG format presented by Carpenter is more adequate. Concerning the semantic structures, we adopt Bayer's (1997) approach because his treatment of eventive nouns (namely the question about eventive nouns's nature: they are second order objects or not) seems more relevant to our work.

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We study some model theoric properties of cuts in models of the fragments of bounded arithmetic, specially S_2^i and T_2^i. In correspondence with polynomial induction we have defined a new notion of cut and concluded some overspill properties.Constructing special substructures in these models and some extensions are of consequences of our results.

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► DENIS I. SAVELIEV, *Completeness in infinitary logic*.

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The Completeness Theorem, like most of theorems of the usual (finitary) logic $\mathcal{L}_{\omega,\omega}$, fails in many infinitary logics $\mathcal{L}_{\kappa,\lambda}$. E.g., Scott's undefinability theorem states that $\mathcal{L}_{\omega_1,\omega_1}$ refutes Completeness even in a strong sense: the set of its valid formulas is not definable in the set of all its formulas by some its formula, while the set of its provable formulas is. The same holds for any successor cardinal instead of ω_1. The natural question is to determine which logics $\mathcal{L}_{\kappa,\lambda}$ satisfy Completeness. We show that in fact Completeness is equivalent to Compactness; any consistent theory in $\mathcal{L}_{\kappa,\lambda}$ has a model if and only if κ is strongly compact, and any consistent theory in $\mathcal{L}_{\kappa,\lambda}$ using at most κ non-logical symbols has a model if and only if κ is weakly compact.

► MAURICIO SIMÃŲES CAMILO HERNANDES, *A classical logic characterization*.

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The objective of this work is to show a classical logic characterization. For that purpose we investigate some implications when we characterize the logic consequence by the existence of a polinomial root in $\mathcal{Z}_2$.

Afterwards we study extentions of this result for many-valued logics and during this process we founded what can be a classical logic characterization.

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An *And/Or formula* such as $((X_1 \vee \overline{X}_2) \wedge X_3) \vee (\overline{X}_1 \wedge X_3)$ is a Boolean formula formed from literals using binary $\wedge$ and $\vee$ connectives (and brackets). Its *size* m is the number of occurrences of literals. ($m = 5$ in the example.) Suppose the variables are drawn from among $X_1, \dots, X_n$. Let T_m denote the total number of And/Or formulas of size m in these n variables, and $T_m(True)$ be the number of these which are *tautologies*. A natural definition of the probability of a tautology is

$$P_n(True) = \lim_{m \to \infty} \frac{T_m(True)}{T_m}.$$

A second natural notion of probability is defined by generating a formula by means of a *Galton–Watson random branching process*. Throw a fair coin. If it is *heads*, throw a fair $2n$-sided die to choose a literal and then stop. If it is *tails*, throw the coin again to choose $\wedge$ or $\vee$ as the principal connective; then repeat the process to construct the left and right subformulas. Let $\pi_n(True)$ be the probability that the formula generated is a tautology.

THEOREM 1 (With Danièle Gardy [1]). *$\pi_n(True) < P_n(True)$ for all n.*

THEOREM 2. *$\pi_n(True) \sim \dfrac{1}{4n}$ and $P_n(True) \sim \dfrac{3}{4n}$ as $n \to \infty$.*

The probability that a random formula defines other simple Boolean functions such as a literal (as in the example above) can also be analysed.

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Mutually-inversistic logic unifies classical logic, Aristotelian logic, ancient Chinese logic, relevance logic, modal logic, classical inductive logic, many-valued logic, dialectical logic, Boolean algebra, nonmonotonic logic, paraconsistent logic, fuzzy logic. It is the unification of mathematical logic and philosophical logic, the unification of ancient logic and modern logic, the unification of Chinese logic and western logic, the unification of deductive logic and inductive logic, the unification of two-valued logic and many-valued logic, the unification of crisp logic and fuzzy logic. Mutually-inversistic logic intercrosses universal logics constructed by Professor Huacan He. Some of the operators in mutually-inversistic logic are special cases of universal logics, others are not. The author is the first to propose unified logic. In 1990, in reference [1] the author proposes unified logic, about 10 years earlier than Professor Huacan He of China, Professor Jean-yves Beziau of Switzerland, Professor Brady Ross of Australia to propose universal logics.

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