THE BULLETIN OF SYMBOLIC LOGIC
Volume 15, Number 3, Sept. 2009

# IN MEMORIAM: LEON ALBERT HENKIN
1921–2006

Leon Henkin was one of the central figures of twentieth century logic. Less widely known to mathematicians were his work in mathematics education, his activities supporting underprivileged and women students, and his extensive service to the University of California, Berkeley; these aspects of his life are not treated in this obituary.

Leon was born in 1921 in Brooklyn, New York. He obtained a B.A. in mathematics and philosophy at Columbia College in 1941, and an M.A. and Ph.D. (under Alonzo Church) in mathematics at Princeton in 1942 and 1945. During World War II he worked on the Manhattan Project. After two more years at Princeton, he moved to the University of Southern California, moving to the University of California, Berkeley in 1953. He was recruited by Alfred Tarski, who was beginning to build a research center in logic which is still active. Leon was promoted to Professor in 1958. He remained at UC Berkeley until his retirement in 1991. During his years in Berkeley he was chairman or acting chairman of the department several times, and also was chairman of the Group in Logic and Methodology of Science twice. On the national level, he was active in the Association for Symbolic Logic, serving as president for three years. He was sole Ph.D. advisor for two students in mathematics, co-advisor for eight more, and sole Ph.D. advisor for five students in mathematics education.

He is best known for his proof of the completeness of first-order logic. This was in his Ph.D. thesis, published in The Journal of Symbolic Logic in 1949; see [2] in the bibliography of this paper. The proof consists in adjoining individual constants to a given first-order language, extending a given set of sentences by using those constants so as to eliminate quantifiers, and then constructing a model whose elements are equivalence classes of terms in the expanded language. This basic idea has been applied in many situations in model theory, by many authors. In particular, Henkin himself applied it to the theory of types [3]. Instead of allowing higher order variables to range over all objects of the appropriate type, one allows the range to be a specified class of such objects. This allows a completeness theorem still to be established. This idea has been important in the theory of higher order logic. He also used the above basic idea to a generalization of $\omega$-consistency


1079-8986/09/1503-0004/$1.60

[10] and $\omega$-completeness [21], to results concerning the interpolation theorem [34], and to representation theorems for cylindric algebras [15]. He also contributed to the beginnings of modern model theory in a series of articles. An example is the basic theorem that a structure $A$ can be embedded in a model of an axiomatizable class of structures if and only if every finitely generated substructure of $A$ can be so embedded; see [6]. In an entirely different direction, his paper [24] on infinitely long formulas was a milestone in the development of such languages. He showed how to make sense of quantifiers on a formula that are not well-ordered, or not even linearly ordered. In fact, he defined a general kind of quantifier in which a variable depends only on certain preassigned other variables. This is expressed rigorously by means of possibly infinite rank Skolem functions. The most famous of these quantifiers is

$$\begin{matrix} \forall x\ \exists y \\ \forall a\ \exists b \end{matrix}\varphi(x, y, a, b)$$

Using Skolem functions, this is defined to mean $\varphi(x, f(x), a, g(a))$. Hintikka's independence friendly logic can be considered as an elaboration of this idea.

A problem which Henkin formulated in JSL, vol. 17, has led to an important field of research in logic. The problem runs as follows:

> If $\Sigma$ is any standard formal system adequate for recursive number theory, a formula (having a certain integer $q$ as its Gödel number) can be constructed which expresses the proposition that the formula with Gödel number $q$ is provable in $\Sigma$. Is this formula provable or independent in $\Sigma$?

This problem was solved by Löb in JSL, vol. 20; the answer is that the formula is provable. This has led to the treatment of the provability predicate as a modal operator.

Henkin devoted a large portion of his later research to the theory of cylindric algebras; see [15], [16], [20], [26], [37], [38], [39], [41], [43], [45], [47], [49], [50], and [51]. These are algebras with operations $c_i, d_{ij}$ added to those of Boolean algebras to algebraically express quantifiers and equality. Here $i, j < \alpha$; the ordinal $\alpha$ is the *dimension* of the algebras. Cylindric algebras are equationally defined, by finitely many schemas. Concrete cylindric algebras, corresponding to fields of sets for Boolean algebras, are algebras of sets of sequences, corresponding to satisfaction sets for formulas. It turned out that the schemas defining cylindric algebras do not characterize the concrete notion, unlike the case of Boolean algebras. Henkin proved several sufficient conditions for representability as concrete algebras; the dimension-complemented algebras are the most striking. The *dimension* of an element $x$ is the set of all $i$ such that $c_i x \neq x$. In the "standard" case, with $\alpha = \omega$ the dimensions are finite, corresponding to the finite length of formulas.

An algebra is *dimension-complemented* if the dimensions are all different from $\alpha$. Another central concept of his is neat embedding, which gives a simple algebraic characterization of representable algebras, at the expense of expanding the signature of the algebras. This notion turned out to be central in the theory of representations, and it is still the subject of investigation. Henkin also systematically developed some unusual constructions of non-representable algebras, and an extensive classification of set algebras of various kinds. Together with his student Diane Resek he developed the notion of relativized cylindric algebras. These are algebras obtained from cylindric algebras by taking all subelements of some elements; they are not themselves cylindric algebras. These algebras are a subject of current research.

In several of his publications he went into the philosophy of mathematics, taking a nominalistic position; see [13], [22], [28], [30], and [33].

Several of his publications, beginning in 1960, are concerned with elementary concepts, leading up to his later preoccupation with mathematical education. See [25], [29], [32], [40], and [46]. He was, indeed, an excellent teacher, as this author can attest from experience as an undergraduate and graduate student taking courses from him. He obtained awards for this aspect of his professional life.

He will be remembered by logicians not only for his technical contributions, but also for his continuous engagement in organizing meetings and other aspects of mathematical life. He was a very outgoing and congenial person to work with, both with students and with colleagues.

J. Donald Monk

## Ph.D. students

1. Carol Karp (1959) **Languages with expressions of infinite length.**
2. Charles Howard (1965) (co-chairman with William Craig; unofficial co-advisor J. D. Monk) **An approach to algebraic logic.**
3. Peter Krauss (1966) (co-chairman with Dana Scott) **Probability logic.**
4. Michel Jean (1968) (co-chairman with William Craig) **Pure structures.**
5. Mohamed Amer (1969) (co-chairman with William Hanf) **Boolean algebras of sentences of $\omega$-order logic.**
6. Leonard Lipner (1970) (co-chairman with William Craig) **Some aspects of generalized quantifiers.**
7. Daniel Demaree (1970) (unofficial advisor J. D. Monk) **Studies in algebraic logic.**
8. Daniel Gallin (1972) (co-chairman Dana Scott) **Intensional and higher-order modal logic.**
9. Diane Resek (1974) **Some results on relativized cylindric algebras.**
10. Nitsa Hadar (1975) **Children's conditional reasoning.**

11. David Ferguson (1980) **The language of mathematics—how calculus students cope with it.**

12. Hadas Rin (1982) **Linguistic barriers to students' understanding of definitions in a college mathematics course.**

13. Francoise Tourniaire (1984) **Proportional reasoning in grades three, four, and five.**

14. Edgar G. K. Lopez-Escobar (1984) (co-chairman with Dana Scott) **Infinitely long formulas with countable quantifier degrees.**

15. Philip Treisman (1985) **A study of the mathematics performance of black students at the University of California, Berkeley.**

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