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2008 EUROPEAN SUMMER MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC LOGIC COLLOQUIUM '08

Bern, Switzerland
July 3–July 8, 2008

Logic Colloquium '08, the 2008 European Summer Meeting of the Association for Symbolic Logic, was hosted by the University of Bern from 3 July to 8 July 2008. The lectures were held on the University Campus.

The sponsors of the meeting were the Association for Symbolic Logic, the Hasler Foundation, the Swiss Academy of Science, the Swiss National Science Foundation, The University of Bern, Präsenz Schweiz, Beck Glatz Confiseur, Burgergemeinde Bern and Zunft zu Mittellöwen.

The success of the meeting was due largely to the hard work of the Local Organizing Committee under the leadership of its Chair, Gerhard Jäger. The other members were Luca Alberucci, Kai Brünnler, Bettina Choffat, Jürg Krähenbühl, Richard McKinley, Dieter Probst, Jürg Schmid, Daria Spescha, Thomas Strahm and Thomas Studer. The Program Committee consisted of Alessandro Berarducci (Pisa), Jacques Duparc (Lausanne), Mirna Dzamonja (East Anglia), Ali Enayat (Washington, DC), Gerhard Jäger (Bern), Piotr Kowalski (Wroclaw), Jan Krajíček (Prague), Dave Marker (Chicago), Andre Nies (Auckland), Simon Thomas (Rutgers), William Tait (Chicago), Boban Velickovic (Paris), Albert Visser (Utrecht), and Alex Wilkie (Manchester, Chair).

The main conference topics were: *Arithmetic and Computability Theory*, *Logic and Computer Science*, *Model Theory*, and *Set Theory*. The program included three tutorial courses, twelve invited plenary lectures, and sixteen invited lectures in four special sessions corresponding to the four main topics. There were about eighty contributed papers and 170 participants from twenty-eight countries. Sixteen students and recent Ph.D's were assisted by local grants (for example, by the Swiss Academy of Sciences) and a further eight by the ASL.

The following tutorial courses were delivered:

Anand Pillay (University of Leeds), *Compact spaces and definability*.
Michael Rathjen (University of Leeds), *Infinitary proof theory and its uses*.
Stevo Todorcevic (University of Toronto), *Combinatorial dichotomies in set theory*.

The following invited plenary lectures were presented:

Miklós Ajtai (IBM Almaden), *First-order definable bijections and the notion of cardinality*.
Akihiro Kanamori (Boston University), *Bernays and set theory*.
Roman Kossak (City University of New York), *Automorphisms of models of PA: the neglected cases*.
Hannes Leitgeb (University of Bristol), *On truth and probability*.
Amador Martin-Pizarro (University of Lyon), *Categoricity and the classification of strongly minimal sets*.

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Joseph S. Miller (University of Wisconsin-Madison), *Extracting information is hard*.
Jaap van Oosten (University of Utrecht), *Geometric aspects of the effective topos*.
Thomas Scanlon (University of California), *Algebraic dynamics from the model theory of difference fields*.
Stephen G. Simpson (Pennsylvania State University), *Mass problems*.
Lajos Soukup (Hungarian Academy of Sciences), *Rainbow colourings*.
Thomas Strahm (University of Bern), *Weak theories of operations and types*.
Matteo Viale (University of Vienna), *Forcing axioms, supercompact cardinals, singular cardinal combinatorics*.

There will be no proceedings of Logic Colloquium '08.

More information about the meeting can be found at the conference webpage, `http://www.lc08.iam.unibe.ch/`.

Abstracts of invited and contributed talks given in person or by title by members of the Association follow.

For the Program Committee
ALEX J. WILKIE

Abstracts of invited tutorials

► ANAND PILLAY, *Compact spaces and definability*.
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This tutorial will hopefully have some appeal to both a general audience of logicians as well as to researchers in model theory. The aim is to describe various interactions between definability in model theory and compact spaces, with reference to current research.

In the introductory first lecture I will discuss the category of definable sets (for a given first order theory), as well as notions such as hyperdefinability. I will also introduce various notions of strong type and state a conjecture relating the Lascar group of a first order theory to the complexity of Borel equivalence relations.

In the second lecture I will discuss Keisler measures, or probability measures on type spaces, and give certain consequences, for theories without the independence property, of the Vapnis–Chervonenkis theorem (uniform law of large numbers).

In the third lecture I will introduce the notion of a definable set (or group) being dominated by a compact space (or group), and describe some recent results.

The second and third lectures are related to joint work with Ehud Hrushovski, as well as being closely related to Peterzil's tutorial in LC '07.

► MICHAEL RATHJEN, *Infinitary proof theory and its uses*.
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Proof theory was the technical area of logic instituted by Hilbert in the early 20th century in order to carry out his programme: to lay to rest all worries about the foundations of mathematics once and for all by securing mathematics via an absolute proof of consistency. Strong restrictions were placed on the methods to be applied in consistency proofs of axiom systems for mathematics: namely, these methods were to be completely *finitistic* in character. Hilbert's Programme is a reductive enterprise with the aim of showing that whenever a 'real' proposition can be proved by 'ideal' means, it can also be proved by 'real', finitistic means. However, Hilbert's so-called formalism was not intended to eliminate nonconstructive existence proofs in the practice of mathematics, but to vindicate them.

In the 1920s, Ackermann and von Neumann, in pursuit of Hilbert's Programme, were working on consistency proofs for arithmetical systems, but already with the infusion of an infinitary concept. Ackermann's 1924 dissertation gives a consistency proof for a second-order version of primitive recursive arithmetic which explicitly uses a finitistic version of transfinite induction up to the ordinal $\omega^{\omega^{\omega}}$. The employment of transfinite induction on ordinals in consistency proofs came explicitly to the fore in Gentzen's 1936 consistency proof for Peano arithmetic. His proof became the paradigm for an *ordinal analysis* of a theory by assigning a *proof-theoretic ordinal* to a theory which calibrates its logical power.

Beginning in the 1950s, proof theory began to employ infinitary methods more frankly, first with the use of infinitary rules of inference (such as the omega rule) and then with the use of infinitely long formulas. At first, these rules, formulas and the resulting derivations were all countable, but eventually proof theory moved on to make use of prima facie uncountably long rules of inference and/or formulas, thence uncountably long derivations. These countably and uncountably long derivations have been applied to treat various formal systems for parts of analysis and set theory, by translating their finite derivations into corresponding infinite ones of a more purely logical character. By then applying the process of cut-elimination or other forms of normalization, the latter are transformed into detour-free derivations whose associated ordinals give a measure of complexity to the systems thus treated. For example, they can be used to classify their provably recursive functions. Cut-free derivations are easily shown not to end in a contradiction, so one also concludes consistency from such analyses. The more that such infinitary methods were employed, the closer did proof theory come to ongoing developments in recursion theory, particularly as generalized to admissible sets; in both one makes use of analogues of regular cardinals, as well as "large" cardinals (inaccessible, Mahlo, etc.).

The lectures will trace the remarkable transformation of proof theory up to developments in recent years. Notwithstanding that it employs infinitary derivations, proof theory provides finitistic reductions between theories, especially Π^0_2 conservation results. Moreover, the lectures will explore its applications in logic, combinatorics, computer science and constructivism. If time permits, we will also take up the question of where the boundaries of current ordinal-theoretic proof theory lie and what are the main obstacles to be overcome.

► STEVO TODORCEVIC, *Combinatorial dichotomies in set theory*.
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Early investigations of set-theoretic Forcing Axioms have led us to several dichotomies of combinatorial nature that are easy to understand and use. This will be a survey of some of these dichotomies with a special emphasis to their applications. We shall also expose the influence of these dichotomies on the cardinality of the continuum and its other cardinal invariants.

Abstracts of invited plenary talks

► MIKLÓS AJTAI, *First-order definable bijections and the notion of cardinality*.
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The set theoretical definition that two sets have the same cardinality involves the existence of a bijection between them. In a more algorithmic setting it is more natural if, in the definition of cardinality, we restrict our attention to bijections which are given in some constructive way. We investigate this question in the case when the sets are subsets of $\{0,1\}^n$, for a natural number n, which are the sets of zeros of a constant depth polynomial size unlimited fan-in boolean circuit (AC_0 circuit), or equivalently subsets of the set $\{1,\ldots,n\}$ which can be defined by a first-order formula on a suitable structure whose universe is $\{1,\ldots,n\}$. We restrict our attention to bijections computable in both direction by an AC_0-circuit, or equivalently bijections where, in both directions, each bit of the image can be defined by a first-order formula on a suitable structure.

We show that the notion of cardinality in this constructive world is essentially different from the set theoretical one, namely there are sets with identical cardinalities in the set theoretical sense so that there is no AC_0 computable bijection between them in either directions. The talk will also include results about AC_0-definability which are needed for the proof.

► AKIHIRO KANAMORI, *Bernays and set theory*.
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Thirty years after his passing, we take this opportunity to survey the work of Paul Bernays and its influence on modern set theory.

► ROMAN KOSSAK, *Automorphisms of models of PA: the neglected cases*.
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Automorphism groups of countable recursively saturated models of PA were studied intensively in the 1990's. In a series of papers (joint and separate) by Lascar, Kaye, Kotlarski, Schmerl and the author, the important notion of arithmetic saturation was isolated and a series of results characterizing arithmetic saturation in group theoretic terms were proved. The notions involved include: generic automorphisms, maximal automorphisms, the small index property, maximal open subgroups, and the cofinality of the group. Most of this material is presented in [1]. Much less has been done concerning models which are recursively saturated and not arithmetically saturated (we do not even have a good name for this class of models), and even less concerning short recursively saturated models. I will give a survey of this entire area of research and I will include some more recent results concerning the neglected cases, in particular the problem of extendabability of automorphisms to cofinal extensions.

This last topic is joint work with Henryk Kotlarski.

[1] Roman Kossak and James Schmerl, ***The structure of models of Peano Arithmetic***, Oxford Logic Guides 50, Oxford University Press, 2006.

► HANNES LEITGEB, *On truth and probability*.
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We will give an overview of how probability theory can be studied from a logical point of view. On the probabilistic side, this leads us to an investigation of (unary) absolute probability measures and (binary) conditional probability measures on propositional or first-order languages. In particular, we will show (i) how the standard axioms of probability may be justified in terms of "closeness to the truth", (ii) how logical truth can be pinned down by purely probabilistic means, (iii) how conditional probability measures on formulas may be represented as limits of ratios of absolute probability measures on formulas, (iv) what a probabilistic semantics for conditional logic looks like, (v) how a probabilistic theory of truth for semantically closed languages can be developed, and finally (vi) how models for higher-order probabilities, probabilistic reflection principles, and type-free probability can be constructed.

(Most parts of this talk will be based on unpublished or forthcoming material.)

► AMADOR MARTIN-PIZARRO, *Categoricity and the classification of strongly minimal sets*.
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Strongly minimal sets, i.e., irreducible sets of dimension one, were first identified as the principal component of any uncountably categorical structure by Michael Morley in his proof of Łos' uncountable categoricity conjecture. They were subsequently recognized to be the main building blocks of any structure of finite Morley rank; Boris Zilber conjectured that they fall essentially into one of three well-known types. This was refuted by an ingenious construction of Hrushovski, which is the principal tool of creating exotic strongly minimal sets. The talk will be mostly oriented to an audience not familiar with Model Theory, based on examples to illustrate the results exhibited.

► JOSEPH S. MILLER, *Extracting information is hard.*
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Can randomness—or more technically, "information"—be effectively extracted from a semi-random source? A special case of this question was answered by von Neumann in 1951. He described a simple method for extracting an unbiased random sequence from the flips of a biased coin. A more general form of the question was asked by Reimann and Terwijn in the context of algorithmic randomness, so we will start with an introduction to Kolmogorov complexity, effective Hausdorff dimension, and Martin-Löf randomness. Kolmogorov complexity measures the information content of a finite binary string. Informally, the complexity of a string is defined to be the length of the shortest program that generates it. A closely related notion, effective (Hausdorff) dimension, measures the information density of an infinite binary sequences. We can now formulate the question in terms of effective dimension: is there a sequence that has effective Hausdorff dimension $1/2$—in other words, a half-random sequence—that does not compute a sequence of higher effective dimension? As it turns out, such sequences exist. We will discuss this and related results.

► THOMAS SCANLON, *Algebraic dynamics from the model theory of difference fields*.
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In algebraic dynamics, one studies the behavior of iterated applications of rational functions while the model theory of difference fields concerns the structure of definable sets in existentially closed difference fields. I will discuss recent work in which problems in algebraic dynamics have been expressed in terms of the model theory of difference fields and thereby resolved or at least clarified.

► STEPHEN G. SIMPSON, *Mass problems*.
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Kolmogorov 1923 proposed to view intuitionistic logic as a "calculus of problems" (Aufgabenrechnung). This is essentially the famous BHK interpretation of intuitionism. Medvedev 1955 introduced mass problems as a rigorous elaboration of Kolmogorov's proposal. A *mass problem* is a set of reals. If P is a mass problem, the *solutions* of P are the elements of P. We say that P is *solvable* if there exists a computable solution of P. We say that P is *weakly reducible* to Q if each solution of Q can be used as a Turing oracle to compute some solution of P. A *weak degree* is an equivalence class of mass problems under mutual weak reducibility. Let $\mathcal{D}_w$ be the lattice of weak degrees. There is an obvious, natural embedding of the Turing degrees into $\mathcal{D}_w$, obtained by identifying the Turing degree of a real with the weak degree of the singleton set consisting of that real. Muchnik 1963 observed that $\mathcal{D}_w$ is a model of intuitionistic propositional calculus. Since 1999 I have been studying the sublattice $\mathcal{P}_w$ consisting of the weak degrees of nonempty effectively closed sets in Euclidean space. I have discovered that there is a natural embedding of the recursively enumerable Turing degrees into $\mathcal{P}_w$. Moreover, I have discovered that $\mathcal{P}_w$ contains a variety of specific, natural, weak degrees which are closely related to various foundationally interesting topics. Among these topics are reverse mathematics, algorithmic randomness, algorithmic information theory, hyperarithmeticity, diagonal nonrecursiveness, almost everywhere domination, subrecursive hierarchies, resource-bounded computational complexity, effective Hausdorff dimension, and Kolmogorov complexity. Recently I have applied $\mathcal{P}_w$ to the study of 2-dimensional symbolic dynamics. The purpose of this talk is to introduce $\mathcal{P}_w$ and to survey what is known about it.

► LAJOS SOUKUP, *Rainbow colourings*.
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Given a function $f:[X]^n \to X$ a subset Y of X is a *rainbow subset for* f provided $f \restriction [Y]^n$ is one-to-one. A typical question is the following old problem of Erdős: *Assume that the colouring* $f:[\omega_1]^2 \to 3$ *establishes the negative partition relation* $\omega_1 \nrightarrow [\omega_1]^2_3$. *Is there a rainbow triangle for* f?

We survey some old and new results and we also present some new theorems. E.g., we show that if a colouring c establishes $\omega_2 \nrightarrow [(\omega_1 : \omega)]^2_\omega$ then c establishes this negative partition relation in each Cohen-generic extension of the ground model, i.e., this property of c is Cohen-indestructible. This result yields a negative answer to a question of Erdős and Hajnal: it is consistent that GCH holds and there is a colouring $c:[\omega_2]^2 \to 2$ establishing $\omega_2 \nrightarrow [(\omega_1 : \omega)]^2_2$ such that some colouring $g:[\omega_1]^2 \to 2$ can not be embedded into c.

It is also consistent that 2^{ω_1} is arbitrarily large, and there is a function g establishing $2^{\omega_1} \nrightarrow [(\omega_1, \omega_2)]^2_{\omega_1}$ but there is no uncountable g-rainbow subset of 2^{ω_1}.

We also show that if GCH holds then for each $k \in \omega$ there is a k-bounded colouring $f:[\omega_1]^2 \to \omega_1$ and there are two c.c.c. posets $\mathcal{P}$ and $\mathcal{Q}$ such that

$$V^{\mathcal{P}} \models \text{“}f \text{ c.c.c.-indestructibly establishes } \omega_1 \nrightarrow^* [(\omega_1;\omega_1)]_{k-bdd}\text{,”}$$

but

$$V^{\mathcal{Q}} \models \text{“}\omega_1 \text{ is the union of countably many } f\text{-rainbow sets.”}$$

► THOMAS STRAHM, *Weak theories of operations and types*.
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The theories of operations and types addressed in this talk are formulated in the spirit of Feferman's explicit mathematics. It has turned out that this framework provides a natural axiomatic basis for studying notions of abstract computability, especially from a proof-theoretic perspective. Our emphasis in this talk is on proof-theoretically weak systems and their relationship to complexity classes and weak arithmetics.

In the first part of our talk, we are concerned with so-called type-free applicative theories, which form the operational core of systems of explicit mathematics. In an applicative universe of discourse, all objects may be thought of as operations or rules, which can freely be applied to each other: self-application is meaningful, though not necessarily total. We survey results on various weak applicative theories whose provably total operations coincide with the functions computable in polynomial time, linear space, and polynomial space. Our systems can be seen as natural applicative analogues of systems of bounded arithmetic.

It is a distinguished advantage of our applicative theories that they allow for a very intrinsic and direct discussion of higher type issues, since higher types arise naturally in our combinatory setting. Moreover, due to the fact that the untyped language does not a priori restrict the class of functionals which can be expressed, it makes perfect sense to consider the class of higher type functionals which are provably total in a given applicative system. We will present some proof-theoretic aspects of higher type complexities in our setting; in particular, we will address the Melhorn–Cook–Urquhart basic feasible functionals.

The second part of the talk will be devoted to extensions of the first-order applicative framework by various means of typing disciplines. We study weak explicit typing and naming in the spirit of explicit mathematics. In particular, we discuss weak type systems with a restricted form of elementary comprehension whose provably terminating operations coincide with the polytime functions.

An interesting alternative of extending the applicative core by classes or types is by means of a partial (self-referential) truth predicate. We discuss results of Cantini who has studied substantial extensions of weak first-order applicative theories by partial truth, an axiom of choice and a positive uniformity principle.

► JAAP VAN OOSTEN, *Geometric aspects of the effective topos*.
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The effective topos, discovered by Martin Hyland in 1979, is a generalization to full higher-order type theory of Kleene's realizability interpretation of 1945. It is one coherent mathematical universe in which also recursive analysis, in the words of Hyland, "finds its natural home". Moreover, various classically inconsistent theories which did not originate in recursive interpretations of mathematics have been shown to admit models in the effective topos, such as synthetic domain theory, algebraically compact categories and Aczel's Constructive Set Theory (augmented with several nonclassical axioms).

In this talk, we shall explore to what extent constructions known from abstract homotopy theory can be performed in the effective topos, and which properties of the category of simplicial sets find analogues in it.

► MATTEO VIALE, *Forcing axioms, supercompact cardinals, singular cardinal combinatorics*.
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I will give a survey on my work over the combinatorics at the successor of a singular cardinal in these two different scenarios:

— the singular cardinal is larger than a strongly compact cardinal κ and of cofinality smaller than κ,
— A strong forcing axiom like Martin's maximum or the proper forcing axiom PFA holds.

Under these assumptions I will show that a variety of combinatorial objects which I call covering matrices are useful tools to attack many of the classical problems on successor of singular cardinals, for example we can obtain proofs of Solovay's theorem that the singular cardinal hypothesis SCH holds above a strongly compact cardinal, or that PFA implies SCH, or a recent characterization by me and Assaf Sharon of the structure of the approachability ideal for successors of singular cardinals of the type described above, etc. ... Further information is available at: `www.logic.univie.ac.at/~matteo`

Abstracts of invited talks in the Special Session on Computability and Arithmetic

► BARBARA F. CSIMA, *Degree spectra of almost computable structures*.
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A countable algebraic structure is almost computable if almost every Turing degree can compute a copy of the structure; in other words, if the degree spectrum of the structure has measure 1 under the standard measure on the Cantor space. We give examples of almost computable structures the complements of degree spectra of which are uncountable. Moreover, we give an example of a structure whose degree spectrum coincides with the hyperimmune degrees.

This is joint work with Iskander S. Kalimullin.

► ANTONÍN KUČERA, *Properties of PA sets and random sets*.
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We discuss methods of coding information into PA-sets and ML-random sets (e.g., how to make a given set T-reducible to some PA-set or ML-random set). Slaman and Kucera used these techniques for PA-sets to provide a characterization of ideals in the T-degrees for which there is a low T-upper bound. As a corollary, there is a PA-set which is a low T-upper bound for the class of K-trivial sets. In the case of ML-random sets, the situation is naturally more complicated and the techniques are less powerful. We survey some results in this area including new developments.

► SHAHRAM MOHSENIPOUR, *A Bezout computable nonstandard model of open induction*.
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In contrast with Tennenbaum's theorem [8] that says Peano Arithmetic (PA) has no nonstandard computable model, Shepherdson [6] constructed a computable nonstandard model for a very weak fragment of PA, called Open Induction (Iopen) in which the induction scheme is only allowed to be applied for quantifier-free formulas (with parameters). Since then several attempts have been made from both sides to strengthen Tennenbaum's and Shepherdson's theorems. From one direction one would like to find fragments of arithmetic as weak as possible with no nonstandard computable model. On the other hand we are also interested in knowing those fragments that are as strong as possible and do have a computable nonstandard model. Attempts in the first direction were culminated in the work of Wilmers [10] where it is shown that IE_1 does not have a computable nonstandard model (IE_1 is the fragment based on the induction scheme for bounded existential formulas). Our work deals with the second direction. Since Open Induction is too weak to prove many true statements of number theory (It cannot even prove irrationality of $\sqrt{2}$), a number of algebraic first order properties have been suggested to be added to Iopen in order to obtain closer systems to number theory. These properties include: Normality [2], having the GCD property [7], being a Bezout domain [3], cofinality of primes (abbreviated here as cof(prime)) and so on. We mention that GCD is stronger than normality, Bezout is stronger than GDC and Bezout is weaker than IE_1. Berarducci and Otero [1], based on earlier works of Wilkie [9], van den Dries [2] and Macintyre–Marker [3] constructed a computable nonstandard model for Iopen+Normality+cof(prime). Also Moniri [5] by using transseries, managed to generalize Shepherdson's method directly, to construct primitive recursive nonstandard models of Iopen + cof(prime) with any finite transcendance degree > 1. In [4] we succeeded to strengthen Berarducci–Otero's construction by combining their method with that of Smith [7] (which is itself a generalization of Macintyre–Marker's work to the GCD and Bezout case) and obtained a nonstandard computable model of Iopen+GCD+ cof(prime). In this talk, we go one step further by bringing all of these materials together (Smith's chains, Berarducci–Otero's computable construction and Moniri's transseries) to produce a computable nonstandard model of Open Induction which is Bezout and has cofinal primes.

[1] A. Berarducci and M. Otero, *A recursive nonstandard model of normal open induction*, ***The Journal of Symbolic Logic***, vol. 61 (1996), pp. 1228–1241.

[2] Lou van den Dries, *Some model theory and number theory for models of weak systems of arithmetic*, ***Model theory of algebra and arithmetic***, Lecture Notes in Mathematics, vol. 834, Springer-Verlag, Berlin, 1980, pp. 346–362.

[3] Angus Macintyre and David Marker, *Primes and their residue ringes in models of open induction*, ***Annals of Pure and Applied Logic***, vol. 43 (1989), no. 1, pp. 57–77.

[4] Shahram Mohsenipour, ***A recursive nonstandard model for open induction with GCD property and confinal primes***, Lecture Notes in Logic, vol. 26 (2006), pp. 227–238

[5] Mojtaba Moniri, *Recursive models of open induction of prescribed finite transcendence degree* > 1 *with cofinal twin primes*, ***Comptes Rendus Mathématique. Académie des Sciences Paris, Serie I***, vol. 319 (1994), pp. 903–908.

[6] J. C. Shepherdson, *A non-standard model for a free variable fragment of number theory*, ***Bulletin de l'Académie Polonaise des Sciences***, vol. 12 (1964), pp. 79–86.

[7] S. Smith, *Building discretely ordered Bezout domain and GCD domains*, ***Journal of Algebra***, vol. 159 (1993), pp. 191–239.

[8] S. Tennenbaum, *Non-Archimediam models for arithmetic*, ***Notices of the American Mathematical Society***, vol. 6 (1959), p. 270.

[9] A. J. Wilkie, *Some results and problems on weak systems of arithmetic*, ***Logic Colloquium '77***, pp. 285–296, North-Holland, Amsterdam–New York, 1978.

[10] George Wilmers, *Bounded existential induction*, ***The Journal of Symbolic Logic***, vol. 50 (1985), no. 1, pp. 72–90.

► NEIL THAPEN, *The provably total search problems of bounded arithmetic*.
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We give natural combinatorial principles GI_k, based on sequences of k-turn games, which are complete for the class of NP search problems provably total at the kth level T_2^k of the bounded arithmetic hierarchy and hence characterize the $\forall\Sigma_1^b$ consequences of T_2^k. Our argument uses a translation of first order proofs into large, uniform propositional proofs in a system in which the soundness of the rules can be witnessed by polynomial time reductions between games.

Abstracts of invited talks in the Special Session on Logic and Computer Science

► M. DEZANI-CIANCAGLINI, *Logical semantics*.
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Stone dualities allow to describe special classes of topological spaces by means of (possibly finitary) partial orders. Typically, these partial orders are given by the topology, a basis for it, or a subbasis for it. The seminal result is the duality between the categories of Stone spaces and that of Boolean algebras (see [23]). Other very important examples are the descriptions of *Scott domains* as *information systems* [27] and the description of *SFP domains* as *pre-locales* [1]. It is worthwhile to mention also Martin-Löf's domain interpretation of intuitionistic type theory [24].

Intersection types can be viewed also as a restriction of the domain theory in logical form, see [1], to the special case of modelling pure lambda calculus by means of ω-algebraic complete lattices. Intersection types have been used as a powerful tool both for the analysis and the synthesis of λ- models, see *e.g.*, [3], [8], [2], [17], [16], [25], [21], [26]. On the one hand, intersection type disciplines provide finitary inductive definitions of interpretation of λ-terms in models. On the other hand, they are suggestive for the shape the domain model has to have in order to exhibit certain properties, [9], [22], [13], [14], [28], [15].

More recently intersection (together with union) types have been used to build *fully abstract* models of extensions of the λ-calculus including parallel features [4], [11], [12], [5], of Higher-Order Processes [19], [20], [18], of the π-calculus [10] and of ambient calculi [7], [6].

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► KAZUSHIGE TERUI, *Towards a logical foundation of computational complexity*.
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Traditional theories of computational complexity are built on concrete machine models. As a result, the nature of explanation often tends to be procedural (e.g., via transformation of machines). Aiming at a more algebraic account (e.g., via logical isomorphisms), we propose a new foundation based on logic and functional computation (in the paradigm of Curry–Howard correspondence).

The underlying framework is ludics of J.-Y. Girard, that is a pre-logical system upon which ordinary logics are to be built and analyzed. Due to its monistic nature, data and machines are both represented by abstract proofs, and the notion of acceptance is replaced by a fundamental notion of orthogonality. Space efficient composition of algorithms (which is usually explained by a peculiar construction of machines) is very naturally achieved by adopting a pointer-based normalization procedure (known as Krivine's abstract machine).

After introducing the basic formalism, we shall illustrate how some basic complexity theoretic phenomena (e.g., space compression) are explained by logical properties (e.g., focalization) and logical isomorphisms.

► YDE VENEMA, *Logic and automata: a coalgebraic perspective*.
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A long and respectable tradition in theoretical computer science, going back to the work of Büchi and Rabin, links the research fields of automata theory and logic. This link becomes particularly strong for automata operating on (potentially) *infinite* objects like streams, trees or transition systems. An interesting phenomenon in this area is that most (but not all) key results hold for word and tree automata alike, and that many can even be formulated and proved for automata that operate on yet other objects such as trees of unbounded branching degree, or labelled transition systems.

The aim of the talk is to show that these results can be formulated and proved at the more general abstraction level of *coalgebra*.

Universal Coalgebra is an emerging mathematical theory of state-based evolving systems. Words, trees and transition systems are all examples of coalgebras of a certain type, which is formally given as a functor F on the category Set (with sets as objects and functions as arrows). We introduce the concept of an F-automata, a device that operates on coalgebras of type F. The criterion under which such an automaton accepts or rejects a pointed coalgebra is formulated in terms of an infinite two-player graph game. Extending Moss' coalgebraic logic with fixpoint operators, we also introduce a language of coalgebraic fixed point logic for coalgebras, and we provide a game semantics for this language.

Finally we show that some of the central *results* in automata theory can be generalized to the abstraction level of coalgebras and thus lay out the foundations of a universal theory of automata. As examples of such results, we will see that the class of recognizable languages of coalgebras is closed under taking unions, intersections, and projections. We also prove that if a coalgebra automaton accepts some coalgebra it accepts a finite one of bounded size. Many of these results are based on an explicit construction which transforms a given alternating F-automaton into an equivalent nondeterministic one, of bounded size.

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► TING ZHANG, *Knuth–Bendix order and its decidability*.
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Several kinds of orderings, such as polynomial orderings, lexicographic path orderings and Knuth–Bendix orderings, are widely used in term rewriting and theorem proving. In term rewriting, these orderings are powerful tools in proving termination of rewriting systems. In theorem proving, they are key instruments to prune search space without compromising refutational completeness. Solving ordering constraints is therefore essential to the successful application of ordered rewriting and ordered resolution. Besides the needs for decision procedures for existential theories, situations arise in constrained deduction where the satisfiability of arbitrarily quantified formulas need be decided. Unfortunately, the first-order theory of lexicographic path orderings is undecidable, so is existential theory of polynomial orderings. This leaves an open question whether the first-order theory of Knuth–Bendix orderings is decidable (RTA problem 99). In this talk, we give a positive answer to this question using quantifier elimination on a complex structure containing term algebras and integer arithmetic. In fact, we shall show the decidability of a theory that is more expressive than the theory of Knuth–Bendix orderings.

Abstracts of invited talks in the Special Session on Model Theory

▶ MARTIN HILS, *Generalised Hrushovski constructions and generic automorphisms*.
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Hrushovski's amalgamation method is a powerful tool to construct (finite rank) structures with interesting combinatorial geometries. Ehud Hrushovski first used it in [3] to construct, a counter-example to Zilber's Trichotomy Conjecture, then to fuse two strongly minimal theories into a single one. The method later served to construct various unexpected expansions of algebraically closed fields, most recently [1] a *bad field* in characteristic 0. It is useful to divide the method into two steps: the *free amalgamation*, where a theory T_ω (usually of infinite rank) is obtained; then the *collapse* of T_ω onto the desired finite rank theory.

We will focus on generalisations of the free amalgamation method, in particular the free fusion of two simple rank 1 theories *over a common ω-categorical reduct*. In interesting cases, the resulting theory is supersimple [2]. The situation is similar to the expansion of some (stable) theory by a *generic automorphism*.

In important algebraic contexts, e.g., for algebraically closed fields, the class of models together with a generic automorphism is axiomatisable. We show that the same is true for some of the infinite rank structures obtained by Hrushovski's free amalgamation method.

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[2] M. Hils, *La fusion libre : le cas simple*, **Proceedings of the conference 'Logicum Lugdunensis'**, (special volume of the ***Journal of Institute of Mathematics in Jussieu***), to appear.

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▶ GARETH JONES, *Model completeness and o-minimality*.
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Establishing model completeness is one of the main methods for showing that an expansion of the real field is o-minimal. There are also results in the other direction, showing that certain o-minimal structures are model complete (in a reasonable language). Previous results of this type only applied to restricted functions. I shall discuss a result which applies to unrestricted functions, provided that the structure is polynomially bounded. I shall also give examples and applications.

▶ MOSHE KAMENSKY, *Internality to algebraically and differentially closed fields*.
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I will review the model theoretic notion of internality, and the associated Galois theory. I will then review some applications in the particular context of internality to theories of fields.

► KRZYSZTOF KRUPINSKI, *Some model theory of Polish structures*.
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I have introduced Polish structures in order to apply model theoretic ideas in the studies of purely descriptive set theoretic and topological objects such as Polish G-spaces or, more generally, Borel G-spaces. In particular, Polish structures generalize profinite structures introduced by Newelski. Polish structures allow us to apply ideas and techniques from model theory, descriptive set theory, topology and the theory of profinite groups.

A Polish structure is a pair (X, G) where G is a Polish group acting (faithfully) on a set X so that the stabilizers of all points are closed subgroups of G. We say that (X, G) is small if for every natural number n there are countably many orbits on X^n under G.

A simple non-profinite example of a small Polish structure is the unit circle with the full group of homeomorphisms. In fact, most natural examples of compact metric spaces with the full group of homeomorphisms are small Polish structures. More complicated examples are Hilbert cube and the pseudo-arc with the full group of homeomorphisms.

I will discuss a purely topological notion of independence, called non-meager independence, that satisfies some nice properties (e.g., symmetry, transitivity, existence of independent extensions) in small Polish structures, and so allows us to introduce basic stability-theoretic concepts and to prove fundamental results about them (e.g., Lascar inequalities). In profinite structures, this notion of independence coincides with the notion of m-independence introduced by Newelski.

In the second part of my talk I will concentrate on the structure of small compact G-groups, i.e., small Polish structures (X, G) where X is a compact group and G acts continuously on X as a group of automorphisms. I will present an example of such a group which is not solvable-by-finite. On the other hand, under a natural model theoretic assumption of 'superstability' with respect to nm-independence, each such group is solvable-by-finite, and assuming finiteness of the underlying rank, it is even nilpotent-by-finite.

I will finish with some open questions.

Abstracts of invited talks in the Special Session on Set Theory

► PANDELIS DODOS, *Descriptive Set Theory and the Geometry of Banach spaces*.
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We shall discuss some recent advances on the interaction between Banach Space Theory and Descriptive Set Theory. We will focus on classical problems in the Geometry of Banach spaces (problems going back to the beginnings of the theory) which are of the following type:

> Suppose that (P) is a certain property of Banach spaces. Suppose further that we are given a class C of separable Banach spaces such that every space in the class C has property (P). When can we find a separable Banach space Y which has property (P) and contains an isomorphic copy of every member of the given class C?

As it turns out, for most natural properties, an affirmative answer to the above problem is possible if and only if the class C is "simple". The "simplicity" of the class is measured in set theoretic terms. In particular, if the class C is analytic in a natural coding of separable Banach spaces, then we can indeed find a space Y as required above.

We will present the historical background and overview related results, starting from the seminal work of Jean Bourgain in the 80s which led, eventually, in the solution the last few years.

► TAMÁS MÁTRAI, *On a new construction of σ-ideals of compact sets*.
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We present a new construction of $\mathbf{\Pi}^0_2$ σ-ideals of compact sets. To shed light on the role of this construction, we take a tour of the recent developments of the theory of σ-ideals of compact sets. We are mainly concerned with open and recently solved problems related to regularity properties and Tukey reducibility.

► KATHERINE THOMPSON, *Obtaining inner models for global results using class forcing*.
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The forcing technique to produce outer models has been exploited to a great extent to show an enormous range of possibilities which may be true of the universe. Relative to a fixed universe, some of the same results can be true in inner and outer models, but different techniques are used. S. Friedman introduced a programme which aims to discover what questions may be answered in an inner model.

There are many consistency results achieved using forcing which are not yet known to be obtainable via inner models, since internal consistency results are harder to achieve. To obtain any consistency result, one must show that a generic filter exists. To achieve internal consistency, one needs an inner model with all the ordinals of the ambient universe, therefore one cannot restrict to a countable submodel, as is usual in set-forcing. The question then is how to build generics, if one even exists.

One class of results which are studied in this context are called "global properties". These are statements which are true throughout the universe (e.g., at every regular cardinal). Global properties are achieved using class forcing. This adds an extra challenge to building generics as all antichains of the forcing which exist in the universe must be considered.

In this talk, we will examine the techniques used to build such generics and the properties of the forcings which are sufficient to utilise these methods.

► TODOR TSANKOV, *Positive definite functions on equivalence relations*.
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We consider positive definite functions on countable equivalence relations, in particular ones arising from subequivalence relations. We discuss various applications, including ones to a recent co-inducing construction of Epstein and orbit equivalence as well as percolation on Cayley graphs of property (T) groups.

This is joint work with A. Ioana and A. Kechris.

Abstracts of contributed talks

► BAHAREH AFSHARI AND MICHAEL RATHJEN, *Theories of iterated positive induction*.
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In 1963, G. Kreisel [5] initiated the study of formal theories featuring inductive definitions. Subsystems of the theories of iterated inductive definitions (ID_n) such as the fixed point theories $\hat{ID}_n$ where investigated by Aczel and Feferman in connection with Hancock's conjecture about the strength of Martin-Löf type theories with universes. Another interesting type of theory lying between $\hat{ID}_n$ and the usual ID_n is ID_n^*. To illustrate this in the case $n = 1$, in contrast to $\hat{ID}_1$, ID_1^* has an induction principle for the fixed points but it is restricted to formulas in which other fixed points occur only positively. Results about the theories ID_n^* were obtained by Friedman, Feferman [4], and Cantini [2]. However, they did not settle the proof-theoretic strength of the theories ID_n^*. I would like to talk about our recent results revealing the strength of these theories.

Added in proof: After submission of this abstract the authors became aware of the work of Dieter Probst who obtained the same results earlier [6].

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[2] Andrea Cantini, ***A note on a predicatively reducible theory of elementary iterated induction***, Bollettino U.M.I., pp. 413–430, 1985.

[3] ——— *On the relation between choice and comprehension principles in second order arithmetic*, ***The Journal of Symbolic Logic***, vol. 51 (1986), pp. 360–373.

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[5] Georg Kreisel, *Generalized inductive definitions*, ***Technical report***, Stanford University, 1963.

[6] Dieter Probst, *The proof-theoretic analysis of transfinitely iterated quasi least fixed points*, ***The Journal of Symbolic Logic***, vol. 71 (2006), pp. 721–746.

[7] Michael Rathjen, ***Auwahl und Komprehension in Teilsystemen der Analysis***, M.Sc. thesis, University of Münster, Germany, 1985.

[8] Kurt Schütte, ***Proof theory*** Springer-Verlag, Berlin, Heidelberg, 1977.

[9] Helmut Schwichtenberg, *Proof theory: Some applications of cut-elimination*, ***Handbook of mathematical logic***, pp. 868–895, North-Holland, 1977.

[10] Stephen G. Simpson, ***Subsystems of second order arithmetic***, Springer-Verlag, Berlin, Heidelberg, 1999.

► PAVEL ALAEV, *Ideals in computable Boolean algebras with distinguished ideals*.
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We consider computable Boolean algebras with a finite number of distinguished ideals (computable I-algebras), i.e., structures of the form $(\mathfrak{A}^*, I_1, \dots, I_\lambda)$, where $\mathfrak{A}^*$ is a computable Boolean algebra and $I_1, \dots, I_\lambda$ are computable ideals in $\mathfrak{A}^*$. A relation R is said to be intrinsically computable (c.e., ...) if for every isomorphism $f : \mathfrak{A} \to \mathfrak{B}$ onto a computable structure $\mathfrak{B}$ the image $f(R)$ is computable (c.e., ...), and to be relatively intrinsically computable if for every isomorphism $f : \mathfrak{A} \to \mathfrak{B}$ onto an arbitrary structure $\mathfrak{B}$ with elements in ω the image $f(R)$ is computable with respect to the diagram $D(\mathfrak{B})$.

We consider the problem of describing intrinsically computable ideals in $\mathfrak{A}$. This problem can be considered as related to investigating decidable theories. A complete algebraic description is found for relatively intrinsically computable, c.e. and co-c.e. ideals in $\mathfrak{A}$, and also for intrinsically computable ideals in the case $\lambda \leqslant 2$.

Let the set $L_1, \dots, L_m$ denote all finite intersections $\bigcap_{t\in T} I_t$, $\{0\}$ and $\mathfrak{A}$. Let $R \subseteq \mathfrak{A}$ be an ideal. We say that R is a Σ-ideal if $R = \sum_{s\in S} L_s$ or $\{0\}$; R is a Π-ideal if $R = \bigcap_{(t,s)\in X}(L_t \to L_s) \cap L_n$, and R is a Δ-ideal if R is a Σ-ideal and a Π-ideal simultaneously.

Here $L_1 + L_2 = \{x + y \mid x \in L_1, y \in L_2\}$, $L_1 \to L_2 = \{x \in \mathfrak{A} \mid \forall y \leqslant x (y \in L_1 \to y \in L_2)\}$, $a_1, \dots, a_n \mid 1$ means that $a_1 + \dots + a_n = 1$ and $a_i \cdot a_j = 0$ for $i \neq j$, and $\widehat{a}$ means the restriction of $\mathfrak{A}$ to the set $\{x \in \mathfrak{A} \mid x \leqslant a\}$.

THEOREM 1. *An ideal R is relatively intrinsically c.e. in $\mathfrak{A}$ if and only if there exists a set $a_1, \dots, a_n \mid 1$ such that $R \cap \widehat{a}_i$ is a Σ-ideal in $\widehat{a}_i$ for all $i \leqslant n$.*

THEOREM 2. *An ideal R is relatively intrinsically co-c.e. in $\mathfrak{A}$ if and only if there exists a set $a_1, \dots, a_n \mid 1$ such that for each $i \leqslant n$ at least one of the following holds:*
(a) $R \cap \widehat{a}_i$ is a Π-ideal in $\widehat{a}_i$;
(b) $R \cap \widehat{a}_i$ is a Σ-ideal and a maximal ideal in $\widehat{a}_i$ simultaneously.

COROLLARY. *An ideal R is relatively intrinsically computable in $\mathfrak{A}$ if and only if there exists a set $a_1, \dots, a_n \mid 1$ such that for each $i \leqslant n$ at least one of the following holds:*
(a) $R \cap \widehat{a}_i$ is a Δ-ideal in $\widehat{a}_i$;
(b) $R \cap \widehat{a}_i$ is a Σ-ideal and a maximal ideal in $\widehat{a}_i$ simultaneously.

THEOREM 3. *Let $\mathfrak{A} = (\mathfrak{A}^*, I_1, I_2)$ be a computable I-algebra with two distinguished ideals. An ideal R is intrinsically computable in $\mathfrak{A}$ if and only if R is relatively intrinsically computable.*

► BENNO VAN DEN BERG, *Realizability in algebraic set theory*.
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The aim of algebraic set theory [2] is to provide a uniform categorical semantics for set theories of different kinds (classical or constructive, predicative or impredicative, well-founded or non-well-founded, etc.). In this talk we show how realizability methods for constructive set theories like **IZF** and **CZF** fit into the framework of algebraic set theory. This allows us to simultaneously recover known realizability interpretations of Friedman, McCarty and Rathjen, and introduce some unfamiliar ones. (The contents of this talk are based on [1], which is the second in a series of papers on algebraic set theory, written together with Ieke Moerdijk.)

[1] B. VAN DEN BERG and I. MOERDIJK, *Aspects of predicative algebraic set theory II: realizability*, arXiv:0801.2305, accepted for publication in Theoretical Computer Science.

[2] A. JOYAL and I. MOERDIJK, ***Algebraic set theory***, London Mathematical Society Lecture Note Series, vol. 220, Cambridge University Press, 1995.

► ACHIM BLUMENSATH AND BRUNO COURCELLE, *Interpretation degrees for guarded second-order logic*.
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We study guarded second-order interpretations between classes of finite structures. The main result is a complete description of the resulting hierarchy. It is linear of order type $\omega + 3$. Each level can be characterised in terms of a suitable variant of tree-width. Canonical representatives of the various levels are: the class of (i) all trees of height n, for $n < \omega$; (ii) all paths; (iii) all trees; and (iv) all grids.

► DOUGLAS CENZER AND ALI DASHTI, *Computable symbolic dynamics*.
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We investigate computable subshifts. Π^0_1 subshifts are constructed in $2^{\mathbb{N}}$ that have different and incomparable Medvedev degrees.

[1] D. CENZER, Π^0_1 *classes in computability theory*, ***Handbook of recursion theory***, (E. Griffor, editor), North Holland, Studies in Logic and Foundations of Mathematics 140, 1999, pp. 37–85.

[2] D. CENZER, S. A. DASHTI and J. L. F. KING, *Effective symbolic dynamics*, ***Computability and Complexity in Analysis, Siena, June*** 2007, (R. Dillhage, T. Grubb, A. Sorbi, K. Weihrauch and N. Zhong, editors), Springer, 2007, pp. 79–89.

[3] D. LIND and B. MARCUS, ***An introduction to symbolic dynamics and coding***, Cambridge University Press, 1995.

► MICHIEL DE SMET AND ANDREAS WEIERMANN, *Phase transitions for weakly increasing sequences*.

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Motivated by the classical Ramsey for pairs problem in reverse mathematics we investigate the recursion-theoretic complexity of certain assertions which are related to the Erdös–Szekeres theorem. We show that resulting density principles give rise to Ackermannian growth. We then parameterize these assertions with respect to a number-theoretic function f and investigate for which functions f Ackermannian growth is still preserved. Let d be a natural number and A_d be the dth approximation of the Ackermann function A. We show that Ackermannian growth is preserved for $f(i) = i^{\frac{1}{A^{-1}(i)}}$, but not for $f(i) = i^{\frac{1}{A_d^{-1}(i)}}$.

[1] PETER A. CHOLAK, CARL G. JOCKUSCH, and THEODORE A. SLAMAN, *On the strength of Ramsey's theorem for pairs*, ***The Journal of Symbolic Logic***, vol. 66 (2001), no. 1, pp. 1–55.

[2] PAUL ERDÖS and GEORGE SZEKERES, *A combinatorial problem in geometry*, ***Compositio Mathematica***, vol. 2 (1935), pp. 463–470.

[3] STEPHEN G. SIMPSON, ***Subsystems of second order arithmetic***, Perspectives in Mathematical Logic, Springer-Verlag, Berlin, 1999, xiv+445.

► JUSTUS DILLER, *Functional interpretations of classical systems*.

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Classical theories can (and should) be formalized without mention of existential quantification and disjunction. Then, as Peano Arithmetic *PA* is Dialectica interpreted in Gödel's theory T of primitive recursive functionals, its neutral finite type version PA^ω is Diller–Nahm interpreted in $T^c_\wedge$, an extension of T by a bounded universal quantifier. Also Kripke–Platek set theory $KP\omega$ is Diller–Nahm interpreted in Burr's bounded theory $T^c_\in$ of constructive set functionals. It is only for the interpretation of Aczel's Constructive Set Theory that we need a new translation of the existential quantifier.

► NATASHA DOBRINEN AND SY-DAVID FRIEDMAN, *Homogeneous iteration and measure one covering relative to* HOD.

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Assuming that there is no inner model with a Woodin cardinal, Steel constructs a certain inner model, K^c, from which the 'true' core model K for a Woodin cardinal is obtained. An important lemma in the derivation of K from K^c is the following, due to Steel: If κ is a measurable cardinal with a normal measure μ, then $\{\alpha < \kappa \colon \alpha^+ = (\alpha^+)^{K^c}\}$ has μ-measure one. This is called measure one covering relative to K^c [2].

Since the inner model K^c is contained in HOD, the universe of hereditarily ordinal definable sets, it is natural to ask whether measure one covering might fail relative to HOD. Indeed, we prove that, relative to a hyperstrong cardinal, it is consistent that measure one covering fails relative to HOD; that is, it is consistent that there is a measurable cardinal κ with a normal measure μ such that the set $\{\alpha < \kappa \colon \alpha^+ = (\alpha^+)^{\mathrm{HOD}}\}$ has μ-measure zero. Along the way, we prove a preservation theorem for iterations of homogeneous forcings [1].

[1] NATASHA DOBRINEN and SY-DAVID FRIEDMAN, *Homogeneous iteration and measure one covering relative to HOD*, to appear in ***Archive for Mathematical Logic***.

[2] JOHN STEEL, ***The core model iterability problem***, Lecture Notes in Logic, Springer-Verlag, 1996.

► DAMIR D. DZHAFAROV AND CARL G. JOCKUSCH, JR., *Ramsey's theorem and cone avoidance*.

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The computability theoretic content of Ramsey's Theorem in combinatorics has been the subject of much study. Jockusch [3] asked whether every computable 2-coloring of pairs admits an infinite homogeneous set which does not compute $0'$, and whether every computable 2-coloring of pairs admits a low_2 infinite homogeneous set.

Seetapun [4] affirmatively answered the first by showing that for any sequence $C_0, C_1, \dots$ of noncomputable sets, every computable 2-coloring of pairs admits an infinite homogeneous set H with $C_i \not\leq_T H$ for all i. Cholak, Jockusch, and Slaman [1] affirmatively answered the second question. In turn, they asked whether for any given noncomputable set C, every computable 2-coloring of pairs admits a low_2 infinite homogeneous set H with $C \not\leq_T H$.

In this paper, we obtain an affirmative answer to this question by two different methods. The first combines control of the first jump with a new proof of Seetapun's result which is considerably more straightforward than both the original and the subsequent simplification of it due to Hummel and Jockusch [2]. The other controls the second jump directly and yields the following stronger theorem: every computable 2-coloring of pairs admits a pair of low_2 infinite homogeneous sets whose degrees form a minimal pair.

[1] PETER A. CHOLAK, CARL G. JOCKUSCH, JR., and THEODORE A. SLAMAN, *On the strength of Ramsey's theorem for pairs*, ***The Journal of Symbolic Logic***, vol. 66 (2001), no. 1, pp. 1–55.

[2] TAMARA L. HUMMEL, *Effective versions of Ramsey's theorem: avoiding the cone above* $\mathbf{0}'$, ***The Journal of Symbolic Logic***, vol. 59 (1994), no. 4, pp. 1301–1325.

[3] CARL G. JOCKUSCH, JR., *Ramsey's theorem and recursion theory*, ***The Journal of Symbolic Logic***, vol. 37 (1972), no. 2, pp. 268–280.

[4] DAVID SEETAPUN and THEODORE A. SLAMAN, *On the strength of Ramsey's theorem*, ***Notre Dame Journal of Formal Logic***, vol. 36 (1995), no. 4, pp. 570–582.

► PHILIP EHRLICH, *Three infinitesimalist theories of continua*.

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Paul du Bois-Reymond [1], Giuseppe Veronese [8], and Charles Sanders Peirce (see [5] for references) all proposed theories of continua that make use of infinitesimals. Du Bois-Reymond's and Veronese's theories were given precision by Hausdorff [6] and Levi-Civita [7], respectively, and in [5] we provided a formal replacement for the purported continuum of Peirce. In [3], [4] and [5] we have suggested that whereas the standard continuum $\mathbb{R}$ of real numbers should be regarded as an arithmetic continuum modulo the Archimedean axiom, Conway's ordered field *No* of surreal numbers may be regarded as a sort of absolute arithmetic modulo NBG. Here we show how the (aforementioned formalizations of the) theories of du Bois-Reymond, Veronese, and Peirce along with $\mathbb{R}$ are naturally situated in *No*.

[1] P. du Bois-Reymond, ***Die allgemeine Functionentheorie I***, Verlag der H. Laupp'schen Buchhandlung, 1882.

[2] J. H. Conway, ***On numbers and games***, Academic Press, 1976.

[3] P. Ehrlich, *Universally extending arithmetic continua*, ***Le Continu Mathematique, Colloque de Cerisy***, (Hourya Sinaceur and Jean-Michel Salanskis, editors), Springer-Verlag France, 1992, pp. 168–177.

[4] ———, *The absolute arithmetic continuum*, ***Synthese***, (forthcoming).

[5] ———, *The absolute arithmetic continuum and its Peircean counterpart*, ***New essays on Peirce's mathematical philosophy***, (Matthew Moore, editor), Open Court, (forthcoming).

[6] F. Hausdorff, *Die Graduierung nach dem Endverlauf*, ***Abhandlungen der mathematische und physikische Klasse der königlich sächsischen Gesellschaft der Wissenschaften***, vol. 31, 1909, pp. 295–335.

[7] T. Levi-Civita, *Sui Numeri Transfiniti, Atti della Reale Accademia dei Lincei*, Classe di scienze fisiche, matematiche e naturali, ***Rendiconti, Roma serie Va***, vol. 7, 1898, pp. 91–96, 113–121.

[8] Giuseppe Veronese, ***Fondamenti di geometria a più dimensioni e a più specie di unità rettilinee esposti in forma elementare***, Tipografia del Seminario, Padova, 1891.

► RACHEL EPSTEIN, *Prime models of computably enumerable degree*.
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We examine the computably enumerable (c.e.) degrees of prime models of complete atomic decidable (CAD) theories. Every CAD theory has a prime model, but not necessarily a computable prime model. Previous research has dealt with the Δ_2 degrees of the elementary diagrams of prime models, but the c.e. degrees remained unexplored. We show that many results for the Δ_2 degrees can be strengthened to hold for the c.e. degrees. The results for Δ_2 degrees used finite extension oracle constructions, while for the c.e. case, we use infinite injury arguments. Let $D^e(\mathcal{M})$ denote the elementary diagram of the model $\mathcal{M}$. We show that every CAD theory T has prime models $\mathcal{M}$ and $\mathcal{M}'$ such that the degrees of $D^e(\mathcal{M})$ and $D^e(\mathcal{M}')$ form a minimal pair of low c.e. degrees. This strengthens Csima's result that every CAD theory has a minimal pair of low prime models. Next we prove that if a CAD theory T has a prime model $\mathcal{M}$ such that $D^e(\mathcal{M})$ has c.e. degree $\mathbf{c}$, then we can push down the degree to get a prime model $\mathcal{M}'$ of T such that $D^e(\mathcal{M}')$ has c.e. degree $\mathbf{b} < \mathbf{c}$. This pushdown theorem holds for the c.e. degrees but not for the Δ_2 degrees where $\mathbf{c}$ may have minimal degree. We also give a density theorem which states that for any c.e. degrees $\mathbf{d} < \mathbf{c}$ where $\mathbf{c}$ is not low$_2$ ($\mathbf{c}'' > \mathbf{0}''$), then every CAD theory has a prime model $\mathcal{M}$ such that $D^e(\mathcal{M})$ is between $\mathbf{d}$ and $\mathbf{c}$ and has the lowest possible jump degree $\mathbf{d}'$.

► ALESSANDRO FACCHINI AND LUCA ALBERUCCI, *The modal μ-calculus hierarchy over restricted classes of transition systems*.
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In this talk we discuss the strictness of the modal μ-calculus hierarchy on some restricted classes of transition systems. First, we show that the hierarchy is strict over reflexive frames and, by proving the finite model theorem for reflexive systems, the same results holds on finite models.

Then, we prove that the hierarchy collapses to the alternation-free fragment over transitive systems. This is done by giving an explicit syntactical translation of the full modal μ-calculus into the alternation-free fragment which preserves logical equivalence for finite systems. By proving a finite model theorem this translation can be extended to all transition systems. Moreover, we verify that if symmetricity is added to transitivity, the hierarchy collapses to the purely modal fragment. In this case this is done by giving an explicit syntactical translation of the full modal μ-calculus into the modal fragment, too. The results of this talk are presented in [1].

[1] LUCA ALBERUCCI and ALESSANDRO FACCHINI, *The modal μ-calculus hierarchy over restricted classes of transition systems*, submitted.

► HENRIK FORSSELL, *First-order logical duality*.

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Stone duality can be seen as a 'syntax-semantics' duality for propositional logic by regarding Boolean algebras as Lindenbaum–Tarski algebras of propositional theories and the corresponding Stone spaces of ultrafilters as spaces of models. Representing the syntax of first-order theories by Boolean coherent categories, we generalize Stone duality to first-order logic by constructing a duality between Boolean coherent categories and certain 'semantical' topological groupoids. The heart of the construction is a representation of the topos of coherent sheaves on a Boolean coherent category as equivariant sheaves on the topological groupoid of set-valued coherent functors and invertible natural transformations; a representation which allows us to recover the syntax of a first-order theory from its models and model-isomorphisms. Compared to the well-known duality between Boolean pretopoi and ultragroupoids constructed by Michael Makkai [1], this construction is more geometrical in that it uses topology and sheaves rather than structure based on ultraproducts. Moreover, it specializes to the classical Stone duality for Boolean algebras.

[1] M. MAKKAI, *Duality and definability in first order logic*, ***Memoirs of the American Mathematical Society***, vol. 503, 1993.

► GUNTER FUCHS, JOEL HAMKINS, AND JONAS REITZ, *Set theoretic geology*.

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We investigate some new kinds of inner models. The basic idea is to invert forcing: Instead of looking at forcing extensions of the universe, we look at grounds of the universe. A ground is an inner model of which the universe is a forcing extension. We refer to the the intersection of all grounds of V as the mantle. The generic mantle of V is the intersection of all grounds of all set forcing extensions of V. We show that the generic mantle is always an inner model of Zermelo–Fraenkel set theory, and we prove that every model of ZFC is the mantle and generic mantle of another model of ZFC. Other kinds of inner models result from iterating the process of forming mantles, which gives the inner mantles, and from intersecting the

HODs of generic extensions, which gives what we call the generic HOD. The latter is a ZFC model. Many fundamental questions touching upon the basics of forcing remain open.

► MARÍA GABRIELA FULUGONIO, *Gottlob Frege:* Begriffsschrift *and impredicativity*.
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In the spirit of the 19th mathematical project of arithmetization analysis, Frege aims still a further goal: to demonstrate that arithmetic can be properly expressed by pure logic concepts. Thus, in 1879 shows up the *Begriffsschrift* and with it the very first complete first-order calculus with identity axiomatization. Although the importance of the *Begriffsschrift* in the history of logic is quite difficult to be exaggerated, it is well known that it remained mostly underappreciated, when even not understood at all, and that this annoyed Frege deeply. Even though, Frege himself seems to ignore that he had in the young Austrian Benno Kerry (1858–1889) the sharpest reader of the days. Nowadays, we know of Benno Kerry, basically, as the one who inspired (1892) "On Concept and Object". But just going through the eight articles conforming the series that Frege quotes in his famous 1892 article reveals we are in front of an authentic philosopher, very well informed on the 19th century's problematic on foundations of mathematics. In particular, Kerry goes into Frege's work in his II and III articles and accuses his logicist reduction of the induction principle of being circular.

In my work I aim to show that such criticism is an antecedent of the (today known as) "impredicativity problem" and that it is also an antecedent of Russell's Vitiosus Circulus Principle, which was formulated in 1908 as an eventual justification of his type theory. With such a purpose on mind, I offer in I an analysis of the fregean definition of succession; in II, I present Kerry's criticisms and, in III, Russell's comments about it in his "Appendix A" of (1903) *Principles of Mathematics*. I leave IV for conclusions.

► PHILIPP GERHARDY, *Proof mining in topological dynamics*.
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"Proof mining" is the activity of extracting additional data from proofs in mathematics and computer science. Proof mining roughly falls into two parts: on the one hand, proving general metatheorems that classify theorems and proofs from which one can extract interesting additional information, on the other hand, carrying out case studies. In this talk, we will focus on the latter discussing two topological proofs of van der Waerden's Theorem and their computational content.

van der Waerden's Theorem [3] states that for any finite colouring of the integers one colour contains arbitrarily long arithmetic progressions, where an arithmetic progression of length k is a sequence $a, a + b, a + 2b, \dots, a + (k - 1)b$ for integers a, b. Equivalently, for integers q, k there is an N such that any q-colouring of $[0, N]$ admits a monochromatic progression of length k. One may then ask for the growth rate of N in q and k.

The bounds given by van der Waerden in his original combinatorial proof are of Ackermann complexity, while one may show an exponential lower bound on $N(q, k)$. There is a theorem in topological dynamics equivalent to van der Waerden's theorem, the Multiple Birkhoff Recurrence Theorem. We analyse two different proofs of the Multiple Birkhoff Recurrence Theorem, the original one by Furstenberg and Weiss [1] and a variant by Girard [2], and discuss the relationship between the bounds extracted from these proofs and van der Waerden's original combinatorial treatment. A paper in which the results of this analysis appear has been accepted for publication.

[1] H. Furstenberg and B. Weiss, *Topological dynamics and combinatorial number theory*, ***Journal d'Analyse Mathématique***, vol.34 (1978), pp. 61–85.

[2] J.-Y. Girard, ***Proof theory and logical complexity, Volume I***, Bibliopolis, Naples, 1987.

[3] B. L. van der Waerden, *Beweis einer Baudetschen Vermutung*, ***Nieuw Archief voor Wiskunde***, vol. 15 (1927), pp. 212–216.

► VALENTINA HARIZANOV, RUSSELL MILLER, AND ANDREI MOROZOV, *Automorphism spectra and tree-definability*.
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The *automorphism spectrum* of a computable structure $\mathcal{A}$ is the set of Turing degrees of nontrivial automorphisms of $\mathcal{A}$. Thus it measures the complexity of the symmetries of $\mathcal{A}$. We present several results about which sets of Turing degrees can be automorphism spectra of computable structures, and in particular, for which single degrees $\boldsymbol{d}$ a computable structure can have automorphism spectrum $\{\boldsymbol{d}\}$. This condition is equivalent to $\boldsymbol{d}$ being *tree-definable*, i.e., to the existence of a computable subtree $T \subseteq \omega^{<\omega}$ such that there is exactly one path p through T and $\deg(p) = \boldsymbol{d}$.

► CHRIS IMPENS AND SAM SANDERS, *Dangerous axioms of nonstandard arithmetic*.
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In Foundations of Mathematics, understanding theories of arithmetic can be greatly enhanced by studying their nonstandard extensions. The usual nonstandard tools (Transfer, Saturation, Overspill) are very valuable in this respect, but may have unwanted consequences or even make the resulting theories incompatible with initial metamathematical considerations. We assess both positive and negative implications, and extract contributions to Reverse Mathematics from a variety of examples, taken from nonstandard analysis and from logic and phase transitions.

The authors wish to thank Professor Ulrich Kohlenbach (Technische Universität Darmstadt) and Professor Andreas Weiermann (University of Ghent) for their valuable advice.

► BERNHARD IRRGANG, *Applications of higher-dimensional forcing*.
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I will report on my project to develop a theory of higher-dimensional forcing iterations. Classical iterated forcing with finite support as introduced by Solovay and Tennenbaum works with continuous, commutative systems of complete embeddings which are indexed along a well-order. The idea of my approach is not to consider a linear system of embeddings but a higher-dimensional system indexed along a simplified morass. Results are:

(1) There exists a ccc forcing of size ω_1 which adds an ω_2-Suslin tree.

(2) There exists consistently a 0-dimensional Hausdorff space with spread ω_1 and size $exp(exp(\omega_1))$.

Moreover, it yields new proofs for:

(3) There exists consistently a chain $\langle X_\alpha \mid \alpha < \omega_2 \rangle$ such that $X_\alpha \subseteq \omega_1$, $X_\beta - X_\alpha$ is finite

and $X_\alpha - X_\beta$ has size ω_1 for all $\beta < \alpha < \omega_2$ (P. Koszmider).

(4) $\omega_2 \not\to (\omega; 2)_\omega$ is consistent (S. Todorcevic).

These results were first proved with S. Todorcevic's method of ordinal walks.

[1] B. IRRGANG, *Morasses and finite support iterations*, accepted for the ***Proceedings of the American Mathematical Society***.

[2] ———, *Morasses and finite support iterations II*, submitted for the ***Annals of Pure and Applied Logic*** .

► ALEKSANDER IWANOW, *Topological completions of ω-categorical groups*.
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A subgroup H of an infinite group G is called *inert* in G if for every $g \in G$, $H \cap gHg^{-1}$ is of finite index in H. We have found that any finite subgroup of an ω-categorical group is contained in an infinite residually finite inert subgroup.

Let G be countably categorical and H be an infinite residually finite inert subgroup of G. Let $X = \{gK : K$ is commensurable with H and $g \in G\}$. Then the action of G on X by left multiplication defines an embedding of G into $Sym(X)$ such that the closure of G in $Sym(X)$ is a locally compact group $\bar{G}$ and the closure of any $K < G$ commensurable with H is a compact subgroup of $\bar{G}$. We study how properties of $\bar{G}$ are connected with model theoretic properties of G. In particular we study measurable subsets of $\bar{G}$. Our main results describe the cases when $\bar{G}$ has uncountable cofinality and when $\bar{G}$ is ω-categorical.

► IGNACIO JANÉ, *Cantor's two views of ordinal numbers*.
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When in *Grundlagen* (1883) Cantor extended the sequence of the positive integers beyond the finite, he described the finite and the transfinite numbers as being obtained by means of two generating principles. The first principle provides the immediate successor of any generated number, while the second yields the limit of any definite totality of generated numbers with no largest element. From this characterization it immediately follows that no definite totality contains all numbers. This consequence was essential for Cantor's claim that the extended number sequence is absolutely infinite.

Although numbers were not introduced as ordinals, Cantor pointed out that each well-ordered set is isomorphic to the set of predecessors of a unique number under the generating order. He took this number (*Zahl*) as the ordinal (*Anzahl*) of that set. Soon after *Grundlagen* Cantor enhanced the ordinal role of numbers by redefining them as order-types of well-ordered sets. This new characterization, he said, is *purely mathematical*. From then on, Cantor disregarded the generating conception of number and kept to the ordinal definition.

Perhaps the ordinal way to numbers is purely mathematical. But discarding generation barred a natural path to block the set-theoretical paradoxes. Cantor explained them away as resting on a confusion between sets and inconsistent multiplicities. The elements of the latter, he said, do not coexist, but he failed to convince even Hilbert and Dedekind of the soundness of this distinction. One paradox involved the totality of all ordinals. Had Cantor kept to his original characterization of numbers, he could have explained what their lack of coexistence means and why their totality is not a set. In the *Grundlagen* setting, the ordinals do not all coexist because the two generating principles have no closure, i.e., because there is no definite totality closed under them both.

► REINHARD KAHLE, *Understanding functional self-application*.
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While functional self-application, i.e., the possibility of a function to be its own argument, is usually not considered in mathematics, it turned out that this concept can be meaningfully integrated in logical systems and programming languages. In mathematical logic it was first studied in the context of *combinatory logic* introduced by SCHÖNFINKEL in 1920 [7] and later on developed by CURRY [3, 4], as well as by CHURCH [1, 2] in the twin theory of λ *calculus*. It is an essential feature of type-free functional programming languages, but also of logical frameworks like FEFERMAN's systems of *explicit mathematics* [5, 6]. Moreover, from the theoretical point of view, self-application can be used to prove standard *undefinability results*.

In this talk we show how functional self-application can be considered as a special case of *diagonalization*. In this perspective, it looses all its mystery; in particular, the *recursion-theoretic* reading of functional self-application allows to understand it as a well-known concept, used not only in logic, but implicitly also in every modern computer.

[1] ALONZO CHURCH, *A set of postulates for the foundations of logic*, ***Annals of Mathematics*** (2), vol. 33 (1932), no. 2, pp. 346–366.

[2] ———, *A set of postulates for the foundations of logic*, (second paper), ***Annals of Mathematics*** (2), vol. 34 (1933), no. 4, pp. 839–864.

[3] HASKELL CURRY and ROBERT FEYS, ***Combinatory logic***, vol. I, North-Holland, 1958.

[4] HASKELL CURRY, J. ROGER HINDLEY, and JONATHAN SELDIN, ***Combinatory logic***, vol. II, North-Holland, 1972.

[5] SOLOMON FEFERMAN, *A language and axioms for explicit mathematics*, ***Algebra and logic*** (J. Crossley, editor), Lecture Notes in Mathematics, vol. 450, Springer, 1975, pp. 87–139.

[6] ———, *Constructive theories of functions and classes*, ***Logic Colloquium 78*** (M. Boffa, D. van Dalen, and K. McAloon, editors), North-Holland, 1979, pp. 159–224.

[7] MOSES SCHÖNFINKEL, *Über die Bausteine der mathematischen Logik*, ***Mathematische Annalen***, vol. 92 (1924), pp. 305–316. German original of [8].

[8] ———, *On the building blocks of mathematical logic*, ***From Frege to Gödel: A source book in mathematical logic, 1879–1931*** (Jean van Heijenoort, editor), Harvard University Press, 1967, pp. 355–366. English translation of [7].

► ANASTASIA KARPENKO, *Weak interpolation property in $NE(K4)$*.
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A *normal modal logic* is any set of modal formulas containing all the classical tautologies and the axioms $\Box(A \to B) \to (\Box A \to \Box B)$ and closed under the rules of modus ponens, necessitation and substitution. The logic $K4$ is defined by adding one axiom scheme $\Box A \to \Box\Box A$. By $NE(L)$ we denote the set of all normal modal logics containing L.

If $\mathbf{p}$ is a list of propositional variables, let $A(\mathbf{p})$ denote a formula whose all variables are in $\mathbf{p}$. Let L be a logic, $\vdash_L$ a consequence relation associated with L. Suppose that $\mathbf{p}, \mathbf{q}, \mathbf{r}$ are disjoint lists of propositional variables. We define the *weak interpolation property*:

WIP: If $A(\mathbf{p}, \mathbf{q}), B(\mathbf{p}, \mathbf{r}) \vdash_L \bot$, then there exist a formula $C(\mathbf{p})$ such that $A(\mathbf{p}, \mathbf{q}) \vdash_L C(\mathbf{p})$ and $C(\mathbf{p}), B(\mathbf{p}, r) \vdash_L \bot$.

Larisa Maksimova proved that WIP is equivalent to some weak variant of amalgamation. A class V has the *amalgamation property* if it satisfies the condition:

AP: For any $\mathcal{B}, \mathcal{C} \in V$ with a common subalgebra $\mathcal{A}$, there exist an algebra $\mathcal{D}$ in V and monomorphisms $\delta\colon \mathcal{B} \to \mathcal{D}$ and $\varepsilon\colon \mathcal{C} \to \mathcal{D}$ such that $\delta(x) = \varepsilon(x)$ for all $x \in \mathcal{A}$.

It is well known that there is a duality between normal modal logic and varieties of modal algebras. The K4 logic is defined by the variety of transitive modal algebras. For a given logic L, its associated variety $V(L)$ is determined by identities $A = \top$ for A provable in L.

For a class V of algebras, $FG(V)$ denotes the class of finitely generated algebras of V. An algebra is said to be *simple* if it has exactly two congruences.

THEOREM 1 [1]: For any normal modal logic L the following are equivalent:
(1) L has WIP;
(2) the class $Sim(V(L))$ of simple algebras of $V(L)$ has AP;
(3) the class $FG(Sim(V(L)))$ of finitely generated and simple algebras of $V(L)$ has AP.
By V_0 we denote a two-element transitive modal algebra with $\Box x = 1$.
For an arbitrary $n > 0$ let us define a topoboolean algebra V_n. The algebra V_n has n atoms $a_1, \dots, a_n$, and $\Box x = \begin{cases} 1, & \text{if } x = 1; \\ 0, & \text{if } x \neq 1. \end{cases}$.
PROPOSITION: $FG(Sim(V(K4))) = \{V_n \mid n \geq 0\}$.
THEOREM 2: Let L be a logic over $K4$. Then L has WIP iff one of the following conditions is valid:
(1) $V_n \in V(L)$ for all $n > 0$;
(2) $V_3 \notin V(L)$.
THEOREM 3: WIP is decidable over $K4$.

[1] L. MAKSIMOVA, *On a form of interpolation in modal logic*, ***Logic Colloquium 2005, The Bulletin of Symbolic Logic***, vol. 12, no. 2 (2006), p. 340.

► LESZEK ALEKSANDER KOŁODZIEJCZYK, *The Σ_1 collection principle over finite fragments of bounded arithmetic*.
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Let T_2^n be the usual Buss-style theory axiomatized by the induction scheme for Σ_n^b formulae. We show that if T_2^n does not prove the weak pigeonhole principle for PV_n functions (functions from the n-th level of the polynomial hierarchy), then the Σ_1 collection principle $B\Sigma_1$ is not finitely axiomatizable over T_2^n. The proof is based on the construction of a model of $T_2^n + \neg WPHP(PV_n)$ in which a given finite fragment of $B\Sigma_1$ holds, but full $B\Sigma_1$ would imply that the polynomial hierarchy "almost collapses" in a way which is incompatible with the failure of $WPHP$.

Completely analogous results hold with the theory S_2^n in place of T_2^n.

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I will summarize my results about completeness, model completeness and decidability of tame fields relative to their value groups and residue fields, and a corresponding result about separably tame fields. Tame (or separably tame, resp.) fields are henselian valued fields K for which the ramification field of the extension $K^{sep} \mid K$ is algebraically closed (or separable-algebraically closed, resp.); here, K^{sep} denotes the separable-algebraic closure of K. The class of tame fields includes the class of Kaplansky fields, for which relative completeness, model completeness and decidability have been shown by Ershov and, independently, Ziegler. In contrast to these fields and all other classes of valued fields for which such results had been known, the maximal immediate extensions of tame fields are in general not unique. To balance this loss of an important tool, I had to prove much more advanced theorems about valued fields with residue fields of positive characteristic.

All tame fields are perfect, and all algebraically maximal perfect fields of positive characteristic are tame. Since algebraic maximality is necessary to obtain good model theoretic results, the model theory of tame fields represents the model theory of perfect valued fields in positive characteristic.

The model theory of tame fields has nice applications to the theory of places of algebraic function fields in arbitrary characteristic.

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Logics J, JD, JT, J4, JD4, and LP are explicit counterparts of modal epistemic logics K, D, T, K4, D4, and S4 respectively (see [2, 1] for more details). An upper bound of Σ_2^p for the satisfiability problem in these justification logics was announced in [3]. The algorithm was essentially a propositional tableau procedure (though not presented as such) followed by a check on possible contradictions among justifications. The former part is traditionally NP while the latter was shown to be co-NP, thus yielding Σ_2^p as the total complexity.

Recently an omission was found in the proof for two of the six logics: JD and JD4. Restoring the upper bound for JD called for the use of prefixed tableaux. Below we provide the tableau-style formulation of the first part of the original algorithm for J, JT, J4, and LP as well as a new tableau algorithm for JD that help establish the Σ_2^p upper bound for satisfiability problems in all these 5 logics (note the integer prefixes used for JD).

J, J4	$\dfrac{T\ s{:}G}{T\ *(s,G)}$	$\dfrac{F\ s{:}G}{F\ *(s,G)}$
JT, LP	$\dfrac{T\ s{:}G}{\begin{array}{l}T\ G\\ T\ *(s,G)\end{array}}$	$\dfrac{F\ s{:}G}{F\ *(s,G)\ \mid\ F\ G}$
JD	$\dfrac{n\ T\ s{:}G}{\begin{array}{r}n\ T\ *(s,G)\\ n{+}1\ T\ G\end{array}}$	$\dfrac{n\ F\ s{:}G}{n\ F\ *(s,G)}$

Whether the same bound holds for JD4 remains at this point an open problem.

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[3] Roman Kuznets, *On the complexity of explicit modal logics*, ***Proceedings of CSL 2000*** (Fischbachau, Germany), (Peter Clote and Helmut Schwichtenberg, editors), Lecture Notes in Computer Science, vol. 1862, Springer, 2000, pp. 371–383.

► GRAHAM E. LEIGH AND MICHAEL RATHJEN, *An ordinal analysis of theories of truth*.
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We augment the first-order language of Peano Arithmetic with a unary predicate T with $T(x)$ intended to mean "x is the Gödel number of a true sentence of arithmetic". In [2] Friedman and Sheard constructed a list of twelve axioms and rules of inference concerning the predicate T, each expressing some desirable property of truth, and classified all subsets of the list as either consistent or inconsistent. This gave rise to a collection of nine theories of truth, two of which have been treated in the literature (see [2], Halbach [3], Sheard [4] and Cantini [1] for more details). We uncover the proof-theoretic strength of the remaining seven and in the process construct a proof-theory of truth allowing the systems to be subject to an ordinal analysis.

[1] A. Cantini, *A theory of formal truth arithmetically equivalent to ID_1*, ***The Journal of Symbolic Logic***, vol. 55 (1990), no. 1, pp. 244–259.

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[3] V. HALBACH, *A system of complete and consistent truth*, ***Notre Dame Journel of Formal Logic***, vol. 35 (1994), no. 3, pp. 311–327.

[4] M. SHEARD, *Weak and strong theories of truth*, ***Studia Logica***, vol. 68 (2001), pp. 89–101.

► LARISA MAKSIMOVA, *Interpolation and related properties in semilattice based varieties*.
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Let us fix an arbitrary signature consisting of functional symbols and constants and including $\wedge$. We consider varieties of algebras, where $\wedge$ is a greatest lower bound; as usual, $x \leq y \iff x \wedge y = x$. We define a variant of interpolation property and find its algebraic equivalent.

There is a close connection between different versions of interpolation and Beth properties in varieties of algebras and amalgamation and epimorphisms surjectivity. One can find the definitions of amalgamation property AP, super-amalgamation SupAP, strong amalgamation StrAP, restricted amalgamation RAP, strong epimorphisms surjectivity SES and of their algebraic equivalents, namely, Robinson property ROB* (originated from H. Ono 1986), restricted interpolation IPR, projective Beth property PBP in ([1],[2]). We recall the definition of Super-amalgamation property:

(SupAP) For any $\mathbf{A}, \mathbf{B}, \mathbf{C} \in V$ such that $\mathbf{A}$ is a common subalgebra of the algebras $\mathbf{B}$ and $\mathbf{C}$, there exist an algebra $\mathbf{D}$ in V and monomorphisms $g\colon \mathbf{B} \to \mathbf{D}$ and $h\colon \mathbf{C} \to \mathbf{D}$ such that $g(x) = h(x)$ for all $x \in \mathbf{A}$ and, moreover,

$$g(x) \leq h(y) \iff (\exists z \in \mathbf{A})(x \leq z \text{ and } z \leq y),$$
$$g(x) \geq h(y) \iff (\exists z \in \mathbf{A})(x \geq z \text{ and } z \geq y).$$

(WSupAP) arises from (SupAP) by changing "monomorphisms" to "homomorphisms".

Let $\mathbf{x}, \mathbf{y}, \mathbf{z}$ be pairwise disjoint lists of variables. We formulate *Inequalities interpolation principle*:

(IIP) Let $\Gamma(\mathbf{x}, \mathbf{y})$ and $\Delta(\mathbf{x}, \mathbf{z})$ satisfy the condition $\Gamma'(\mathbf{x}) = \Delta'(\mathbf{x})$, where $\Gamma'(\mathbf{x}) = \{\alpha(\mathbf{x}) \mid \Gamma(\mathbf{x}, \mathbf{y}) \models_V \alpha(\mathbf{x})\}$ and $\Delta'(\mathbf{x}) = \{\alpha(\mathbf{x}) \mid \Delta(\mathbf{x}, \mathbf{z}) \models_V \alpha(\mathbf{x})\}$. If $\Gamma(\mathbf{x}, \mathbf{y}), \Delta(\mathbf{x}, \mathbf{z}) \models_V u(\mathbf{x}, \mathbf{y}) \leq v(\mathbf{x}, \mathbf{z})$, then there is a term $t(\mathbf{x})$ such that $\Gamma(\mathbf{x}, \mathbf{y}) \models_V u(\mathbf{x}, \mathbf{y}) \leq t(\mathbf{x})$ and $\Delta(\mathbf{x}, \mathbf{z}) \models_V t(\mathbf{x}) \leq v(\mathbf{x}, \mathbf{z})$.

THEOREM 1. *For any variety V*:
(i) *IIP is equivalent to WSupAP*;
(ii) *V has SupAP iff it has AP and WSupAP.*

PROPOSITION 2. (i) *IIP* $\Rightarrow$ *PBP*; (ii) *WSupAP* $\Rightarrow$ (*SES & RAP*).

PROPOSITION 3. *Let a variety V satisfy the condition*: *there are a term $\varepsilon(x, y)$ and a constant* $\mathbf{e}$ *such that $x = y \models_V \mathbf{e} \leq \varepsilon(x, y)$ and $\mathbf{e} \leq \varepsilon(x, y) \models_V x = y$. Then IIP $\Rightarrow$ AP, and so IIP, SupAP and WSupAP are equivalent*.

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[2] L. MAKSIMOVA, *Restricted interpolation and projective Beth property in equational logic*, ***Algebra and Logic***, vol. 42 (2003), pp. 712–726.

► MACIEJ MALICKI, *Isometry groups of separable metric spaces*.
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The talk will be concerned with representing groups as full isometry groups of metric spaces so that nice properties of the group are reflected by nice properties of the metric space. It is easy to see that if X is a Polish metric space, that is, a complete metric space whose metric induces a separable topology, then the isometry group of X, $\mathrm{Iso}(X)$, considered with the pointwise convergence topology, is Polish. Again, it is an easy observation that $\mathrm{Iso}(X)$ is compact provided that X is compact. It was proved in [1] that $\mathrm{Iso}(X)$ is locally compact if X is proper, that is, if all closed balls of (X, d) are compact. A natural question arises whether the converses to these facts hold.

In [1], Gao and Kechris showed that every Polish group is indeed isomorphic to the isometry group of some Polish space. Then Melleray [3] found a simpler proof of their result and used it to prove that every compact group is isomorphic to the isometry group of a compact space. In [2], we provide the last missing piece of the picture by showing that every locally compact Polish group is the isometry group of a proper Polish space. This solves a problem posed by Gao and Kechris in [1, p. 76].

If time permits, I will comment on the tools used in the proof of this result, and say a couple of words about ultrametric spaces and their isometry groups.

This is a joint work with Slawek Solecki.

[1] S. Gao and A. Kechris, *On the classification of Polish metric spaces up to isometry*, ***Memoirs of the American Mathematical Society***, vol. 161 (2003), no. 766.

[2] M. Malicki and S. Solecki, *Isometry groups of separable metric spaces*, ***Mathematical Proceedings of the Cambridge Philosophical Society***, to appear.

[3] J. Melleray, *Compact metrizable groups are isometry groups of compact metric spaces*, ***Proceedings of the American Mathematical Society***, vol. 136 (2008), pp. 1451–1455.

► JOSÉ M. MÉNDEZ, GEMMA ROBLES, AND FRANCISCO SALTO, *Semantics for relevant logics plus the disjunctive syllogism*.

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As is known, the disjunctive syllogism (d.s) (i.e., the rule if $\vdash A$ and $\vdash \neg A \vee B$, then $\vdash B$) is not a rule of relevant logics. It is, however, an admissible rule in them. The simplest proof of its admissibility is maybe the "normalization" procedure due to Routley and Meyer (cf. [1], [3]). The idea is to define "normal models" to prove that d.s holds in normal models, and finally, to show that any theorem of the relevant logic under consideration is valid in any model iff it is valid in normal models.

The aim of this paper is to provide a semantics for d.s and not only a proof of its admissibility, in particular, to provide a (Routley–Meyer) semantics for any logic including Contractionless Ticket Entailment TW (cf. [2]) plus d.s but without t and $\circ$.

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[3] ———, *The semantics of Entailment IV*, in R. Routley, et al., ***Relevant logics and their rivals***, vol. 1, pp. 407–424, Ridgeview Publishing Co., Atascadero, CA, 1982.

▶ RYSZARD MIREK, *Weak systems of relevance logic*.
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My aim is to examine the weaker systems of relevant logic. There are at least two approaches to the problem. First of all, it is possible to restrict a validity of some theorems. Thus, in the logic S of Martin and Meyer we have just two axioms, it means:

$$(\beta \to \gamma) \to [(\alpha \to \beta) \to (\alpha \to \gamma)] \quad \text{(suffixing)},$$

$$(\alpha \to \beta) \to [(\beta \to \gamma) \to (\alpha \to \gamma)] \quad \text{(transitivity)},$$

and the rule of modus ponens. The next example is Brady's logic DJ^d in which we have a weaker form of transitivity:

$$[(\alpha \to \beta) \wedge (\beta \to \gamma)] \to (\alpha \to \gamma),$$

and the *law of contraction*:

$$[(\alpha \to (\alpha \to \beta)] \to (\alpha \to \beta)$$

is not valid.

However, this is not the only possible approach. We can investigate even weaker logics which have no theorems and are characterised only by rules of deducibility. In this way D. M. Gabbay introduced systems $\vdash_0$ and $\vdash_1$ and Font with Rodriguez the deductive system *WR*. The well known system *R* is an axiomatic extension of *WR*. We can say that *WR* is the deductive system determined by the following conditions:

(1) $\vdash_{WR}$ is finitary,
(2) *WR* has no theorems,
(3) For every $\alpha, \alpha_1, \ldots, \alpha_n \in Fm, \{\alpha_1, \ldots, \alpha_n\} \vdash_{WR} \alpha$ if and only if $\vdash_R (\alpha_1 \wedge, \ldots, \wedge \alpha_n) \to \alpha$.

Following this idea it is possible to introduce other deductive systems like *WRM*, *WE* etc. Such systems are not protoalgebric and since every algebraizable deductive system is also protoalgebric, it follows that they are not algebraizable either.

[1] ROSS BRADY, ***Relevant logics and their rivals***, vol. II, Ashgate, 2003.

[2] JOSEP MARIA FONT and GONZALO RODRIGUEZ, *Algebraic study of two deductive systems of relevance logic*, ***Notre Dame Journal of Formal Logic***, vol. 35 (1994), no. 3, pp. 369–397.

▶ ELENA NOGINA, *Logic of strong provability and explicit proofs*.
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Strong provability *F is true and provable in Peano Arithmetic* is known [3] to be propositionally axiomatized by Grzegorczyk's logic Grz, which postulates are classical logic axioms and rules, $\Box(F\to G)\to(\Box F\to\Box G)$, $\Box F\to\Box\Box F$, $\Box F\to F$, $\Box(\Box(F\to\Box F)\to F)\to F$, and Rule of Necessitation: $\vdash F \Rightarrow \vdash \Box F$. Artemov's Logic of Proofs LP [2] axiomatizes proof operators *t is a proof of F in Peano Arithmetic*. The postulates of LP are the ones of the classical propositional logic, $t{:}(F\to G) \to(s{:}F\to(t\cdot s){:}G)$, $t{:}F\to F$, $t{:}F \to\, !t{:}(t{:}F)$, $s{:}F\to(s+t){:}F$ and $t{:}F\to(s+t){:}F$, and the rule of inferring $c{:}A$ whenever A is an axiom and c is a proof constant.

Joint logics of proofs and provability have been studied in a variety of languages [1, 4, 5, 6, 7]. In this paper we found the arithmetically complete logic of strong provability and explcit proofs GrzA in the joint language of Grz and LP. GrzA consists of axioms and rules of Grz and LP together with axioms $t{:}P \rightarrow \Box P$ and $\neg t{:}P \rightarrow \Box \neg t{:}P$. We show that GrzA is complete with respect to the arithmetical provability interpretation, supplied with a corresponding Fitting epistemic semantics and enjoys some sort of a finite model property, which also yields the decidability of GrzA for any given finite specification of constants.

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[5] ———, *On logic of proofs and provability*, ***The Bulletin of Symbolic Logic***, vol. 12 (2006), no. 2, pp. 356.

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[7] T. Yavorskaya (Sidon), *Logic of proofs and provability*, ***Annals of Pure and Applied Logic***, vol. 113 (2001), no. 1–3, pp. 345–372.

► ANVAR NURAKUNOV AND MICHAŁ STRONKOWSKI, *A new proof of Pigozzi theorem*.

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In universal algebra Baker theorem states that if a variety (equationally definable class) $\mathcal{V}$ in finite language is generated by finite number of finite algebras and all algebras in $\mathcal{V}$ have distributive lattices of congruences then $\mathcal{V}$ is strictly elementary [1]. Pigozzi generalized the above theorem to quasivarieties (quasi-equationally definable classes) [3], which was a significant improvement. Probably the easiest proof of Baker theorem was given by Baker and Wang [2]. It is based on the notion of definable principal subcongruence. We introduce the notion of definable relative principal subcongruence what allows us to carry over the proof to the quasivariety case. As the result we obtain an easy (probably the easiest) proof of Pigozzi theorem.

[1] K. A. Baker, *Finite equational bases for finite algebras in congruence-distributive equational classes*, ***Advances in mathematics***, vol. 24 (1977), pp. 207–243.

[2] K. A. Baker and J. Wang, *Definable principal subcongruences*, ***Algebra Universalis***, vol. 47 (2002), pp. 145–151.

[3] D. Pigozzi, *Finite basis theorem for relatively congruence-distributive quasivarieties*, ***Transactions of the American Mathematical Society***, vol. 331 (1988), no. 2, pp. 499–533.

► LUIZ CARLOS PEREIRA, EDWARD HERMANN HAEUSLER, VASTON GONÇALVES DA COSTA, AND WAGNER SANZ, *Revisiting Peirce's rule in natural deduction*.

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The introduction and elimination rules for material implication in natural deduction are not complete with respect to the implicational fragment of classical logic. A natural way to complete the system is through the addition of a new natural deduction rule corresponding to Peirce's formula $(((A \to B) \to A) \to A)$:

$$\begin{array}{c} [(A \to B)] \\ \vdots \\ \dfrac{A}{A} \end{array}$$

E. Zimmermann [6] has shown how to extend Prawitz' normalization strategy to Peirce's rule: applications of Peirce's rule can be restricted to atomic conclusions. The aim of the present paper is to extend Seldin's normalization strategy to Peirce's rule by showing that every derivation Π in the implicational fragment can be transformed into a derivation Π' such that no application of Peirce's rule in Π' occurs above applications of $\to$-introduction and $\to$-elimination. As a corollary of Seldin's normalization strategy we obtain a weak form of Glivenko's theorem for the fragment $\to$.

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► DAVID PIERCE, *Induction and recursion*.
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My main purpose is to point out, and to ask why it is, that certain foundational matters are commonly misrepresented or misunderstood.

Dedekind [1, II.130] makes an observation overlooked by Peano [4] and others: A set with an initial element and a successor-operation may admit proof by induction without admitting inductive or rather *recursive* definition of functions.

Landau [3, Preface for the Teacher] confesses to having confused induction with recursion. Henkin [2] works out the distinction. Yet the confusion continues to be made, even in textbooks intended for students of mathematics and computer science who ought to be able to understand the distinction. Textbooks also perpetuate related confusions, such as suggestions that induction and 'strong' induction (or else the 'well-ordering principle') are logically equivalent, and that either one is sufficient to axiomatize the natural numbers.

Landau implicitly (and Henkin explicitly) shows that addition and multiplication can be defined by induction alone. But the argument takes some work. (Strictly, the argument requires that these operations are being defined on a *set*; but one can avoid this assumption.) If one thinks that the recursive definitions of addition and multiplication—$n + 0 = n$, $n + (k + 1) = (n + k) + 1$, $n \cdot 0 = 0$, $n \cdot (k + 1) = n \cdot k + n$—are *obviously* justified by induction alone, then one may think the same for exponentation, with $n^0 = 1$, $n^{k+1} = n^k \cdot n$. However, while addition and multiplication are well-defined on $\mathbb{Z}/(n)$ (which admits induction), exponentiation is not; rather, we have $(x, y) \mapsto x^y : \mathbb{Z}/(n) \times \mathbb{Z}/(\phi(n)) \to \mathbb{Z}/(n)$. This is one example to suggest that getting things straight may make a pedagogical difference.

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[2] LEON HENKIN, *On mathematical induction*, ***The American Mathematical Monthly***, vol. 67 (1960), no. 4, pp. 323–338

[3] EDMUND LANDAU, ***Foundations of Analysis***, Chelsea, 1966

[4] GUISEPPE PEANO, *The principles of arithmetic, presented by a new method*, ***From Frege to Gödel***, (Jean van Heijenoort, editor), Harvard, 1967

► RODRIGO PODIACKI AND WALTER CARNIELLI, *First-order logics of formal inconsistency*.
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The Logics of Formal Inconsistency (LFIs) were proposed in [1] as a general family of paraconsistent logics which can internalize the notions of consistency and inconsistency at the level of object-language. Though a large variety of such logics having different syntactic features is developed in [1], it remains at the propositional level. Our present purpose is to extend these logics to a first-order predicate calculi. Besides their first-order axiomatization, we'll show an interpretation of the calculi in terms of quasi-structures (or pre-structures) which permit a paraconsistent approach to their semantics. Moreover, this approach makes possible to exhibit a Henkin-type [2] construction according to which the completeness of a wide family of first-order LFIs can be established.

[1] W. A. CARNIELLI, M. E. CONIGLIO, and J. MARCOS, *Logics of formal inconsistency*, (D. Gabbay and F. Guenthner, editors), ***Handbook of philosophical logic***, vol. 14, pp. 15–107, Kluwer Academic Publishers, 2nd edition, 2007.

[2] L. HENKIN, *The completeness of the first-order functional calculus*, ***The Journal of Symbolic Logic***, vol. 14, no. 3, pp. 159–166, 1949.

► GEMMA ROBLES, *On some of the different faces of "Das Absurde"*.
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According to Ackermann (cf. [1]), "Das Absurde" is the meaning of the propositional falsity constant he introduces in his system Π' in order to define the modalities in it. Ackermann remarks that he introduces the modalities similarly as Johansson introduced negation in the "Minimalkalkül" (cf. [2]). Ackermann is, arguably, the first author presenting a logical system in which the falsity constant is not equivalent to *some* class of contradictions. Be it as it might be, we shall generally use Ackermann's term "Das Absurde" to refer to the meaning of the falsity constant.

The aim of this paper is to investigate some of the different faces of "Das Absurde". In particular, we shall define the minimal logics (and their extensions) in which "Das Absurde" is equivalent to:

1. Any contradiction
2. The negation of a theorem
3. The argument of a negation theorem
4. The negation of any self-identity
5. An unqualified falsehood.

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[1] W. ACERMANN, *Begründung einer strenger Implikation*, ***The Journal of Symbolic Logic***, vol. 21 (1956), pp. 113–128.

[2] I. JOHANSSON, *Der Minimal Kalkül, ein reduzierte intuitionistischer Formalismus*, ***Compositio Mathematica***, pp. 119–136, 1936.

► MARCIN SABOK, *σ-continuity and the related forcing*.
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Given a σ-ideal I of Borel sets in a Polish space we usually associate with I a forcing notion $\mathbb{P}_I = \mathrm{Bor}/I$. On the other hand, we say that a Borel function $f: X \to Y$ is σ-continuous if countably many (arbitrary) sets X_n can be found such that they cover X and $f \restriction X_n$ is continuous for each n. A particularly simple example of a Borel function which is not σ-continuous is the function P found by Pawlikowski. A σ-ideal of sets on which P is σ-continuous gives rise to a forcing notion (called Steprāns forcing). I will talk about this forcing and establish some its basic properties (e.g., continuous reading of names, Axiom A, etc.).

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We consider modal logics where the underlying logic is non-classical and modalities come in adjoint pairs, but are not (in general) closure operators. Despite absence of negation and implication, and of axioms corresponding to the characteristic axioms of (e.g.) **T**, **S4** and **S5**, such logics are useful, as shown in [1], for encoding and reasoning about information and misinformation in interactive multi-agent systems.

The semantics of such logics can be determined by algebras (e.g., lattices or quantales) with agent-indexed families of adjoint pairs of operators.

We present here cut-free sequent calculi for such algebras (and thus for the logics). The calculi exploit structural operators in the style of Belnap [2], as illustrated in [3] for classical modal logics and in [4] for residuated monoids. Cut-admissibility, and thus the completeness of the calculi, is shown by constructive syntactic methods, as in [5].

[1] A. BALTAG, B. COECKE, and M. SADRZADEH, *Epistemic actions as resources*, ***Journal of Logic and Computation***, vol. 17 (2007), no. 3, pp. 555–585.

[2] N. D. BELNAP, JR, *Display logic*, ***Journal of Philosophical Logic***, vol. 11 (1982), pp. 375–417.

[3] K. BRÜNNLER, *Deep sequent systems for modal logic*, ***Advances in Modal Logic*** (***Noosa, Queensland, Australia***), (G. Governatori, I. M. Hodkinson and Y. Venema, editors), vol. 6, College Publications, 2006, pp. 107–119.

[4] M. MOORTGAT, *Multi-modal linguistic inference*, ***Logic Journal of Interest Group on Propositional Logic***, vol. 17 (1995), no. 3, pp. 555–585.

[5] A. S. TROELSTRA and H. SCHWICHTENBERG, ***Basic proof theory***, Cambridge University Press, 2001.

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We show for a wide class of ordinal functions F, if there exist parameters $\beta_1, \dots, \beta_n$ such that $2^{\aleph_\alpha} = \aleph_{F(\alpha,\beta_1,\dots,\beta_n)}$ for all α, then all the $\beta_1, \dots, \beta_n$ are finite.

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Σ^1_n absoluteness for a forcing means that the same Σ^1_n facts about reals in the ground model are true in the ground model and in the generic extension. A related property is that there are no new equivalence classes of thin $\utilde{\Pi}^1_{n-1}$ and $\utilde{\Sigma}^1_{n-1}$ equivalence relations (equivalence relations with no perfect set of pairwise inequivalent reals) in the generic extension. We use an idea from [1] to show both properties for reasonable forcing if the right iterable models with Woodin cardinals exist. This can be applied to $\utilde{\Sigma}^1_2$ c.c.c. forcing assuming projective determinacy.

[1] MATTHEW FOREMAN and MENACHEM MAGIDOR, *Large cardinals and definable counterexamples to the continuum hypothesis*, ***Annals of Pure and Applied Logic***, vol. 76 (1995), no. 1, pp. 47–97.

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Martin-Löf type theory (MLTT) is based on the dependent function type and (inductively defined) algebraic types. In order to model concepts like interaction or object-orientation in MLTT in a direct way, it is useful to add (coinductively defined) weakly final coalgebras to MLTT. We introduce formation, introduction, elimination, and equality rules for weakly final coalgebras in MLTT. We will show that guarded induction is nothing but an informal description of the introduction rules for weakly final coalgebras.

We investigate the duality between algebraic and coalgebraic types in those rules: For algebraic types the introduction rules are simple and predicative, the elimination rules involve some degree of impredicativity. There is a large variety of possible elimination rules, all of which are derived from the principle of having a least set closed under the introduction rules. For coalgebraic types, the elimination rules are simple and predicative, whereas the introduction rules involve some degree of impredicativity. There is a large variety of possible introduction rules, all of which are derived from the principle of having the largest set allowing the elimination principle.

We introduce a model of the extension of MLTT by weakly final coalgebras, and investigate the implications for meaning explanations, namely the need for types, the meaning of which is given by an elimination principle.

We will then show that bisimulation is an example of a dependent weakly final coalgebra. We demonstrate that proofs by guarded induction of bisimulation form a much more intuitive way of proving bisimulation properties than the usual proofs based on the introduction of a bisimulation relation.

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The notion of *Turing degree spectrum* of a countable structure $\mathfrak{A}$ is introduced by Richter as the set of all Turing degrees of the presentations of $\mathfrak{A}$. Soskov [2] initiated the study of the properties of the degree spectrum as a set of enumeration degrees. He considered *all* enumerations of the structure, not only the injective ones. The benefit of this definition is that every degree spectrum is upwards closed with respect to total enumeration degrees. The *co-spectrum* of $\mathfrak{A}$ is the set of all lower bounds of the elements of the degree spectrum of $\mathfrak{A}$. The degree spectra behave with respect to their co-spectra very much like the cones of the enumeration degrees $\{\mathbf{x} \mid \mathbf{x} \geq \mathbf{a}\}$ behave with respect to the ideals $\{\mathbf{x} \mid \mathbf{x} \leq \mathbf{a}\}$. Further properties true of the degree spectra but not necessarily true of all upwards closed sets are: the minimal pair theorem for the degree spectrum and the existence of quasi-minimal degree for the degree spectrum. These properties remain true for the relativized spectrum of a structure with respect to finitely many structures, introduced in [1].

We relativize Soskov's approach to degree spectra by considering multi-component spectra, i.e., a degree spectrum with respect to a given sequence of sets of natural numbers. We study this under the ω-enumeration reducibility. It is a uniform reducibility between sequences of sets of natural numbers, introduced and studied in [4, 3]. The notion of ω-degree spectrum generalizes the notion of relative spectrum. The ω-co-spectrum is the set of ω-enumeration degrees which are lower bounds of the elements of the ω-spectrum.

We prove that some properties of the degree spectrum such as the minimal pair theorem and the existence of quasi-minimal degree are true for the ω-degree spectrum. We give an explicit form of the elements of the ω-co-spectrum of a structure by means of recursive Σ_k^+ formulae.

[1] A. A. SOSKOVA, *Relativized degree spectra*, ***Journal of Logic and Computation***, vol. 17 (2007), no. 6, pp. 1215–1233.

[2] I. N. SOSKOV, *Degree spectra and co-spectra of structures*, ***Annuaire de l'Univérsité de Sofia***, vol. 96 (2004), pp. 45–68.

[3] ———, *The ω-enumeration degrees*, ***Journal of Logic and Computation***, vol. 17 (2007), no. 6, pp. 1193–1214.

[4] I. N. SOSKOV and B. KOVACHEV, *Uniform regular enumerations*, ***Mathematical Structures in Computer Science***, vol. 16 (2006), no. 5, pp. 901–924.

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Given extended Zermelo system $\mathcal{H}_{\mathfrak{D},\Sigma}$ over domain $\mathfrak{D}$, we describe second-order language $\mathcal{L}^{\mathrm{SOL}}_{\mathfrak{D},\Sigma}$ with constants for arbitrary Boolean-valued functions over $\mathfrak{D}$ and define the *canonical expansion modulo saturation* of any formula of $\mathcal{L}^{\mathrm{SOL}}_{\mathfrak{D},\Sigma}$ having no occurrences of individual or function constants. We show that the canonical expansion of any sentence of $\mathcal{L}^{\mathrm{SOL}}_{\mathfrak{D},\Sigma}$ having no occurrences of individual or function constants is symmetric in Zermelo's sense. A converse is presented as well. The relation between second-order logic and Zermelo's theory of systems of infinitely long propositions is thereby clarified.

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The region-based theory of space goes back to Whitehead but last decade it draws additional attention from the so called qualitative spatial reasoning. It is the first-order theory of the class of contact Boolean algebras, i.e., Boolean algebras of regular closed sets in connected normal topological spaces with the binary relation C, contact, defined with $C(a,b) \iff a \cap b \neq \emptyset$. In [1] the universal fragment T of this theory is investigated as fragment L of an appropriated modal logic. Here we prove that T coincides with the universal fragment of some narrow class of structures: contact Boolean algebras of the class of topological spaces of the type (X,τ), where X is finite union of squares in $\mathbb{R}^2$ and τ is the induced topology. We demonstrate also non-canonicity of L.

[1] PH. BALBIANI, T. TINCHEV, and D. VAKARELOV, *Modal logics for region-based theories of space*, ***Fundamenta Informaticæ***, vol. 81 (2007), no. 1–3, pp. 29–82.

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Given Turing degrees $\mathbf{0} < \mathbf{b} < \mathbf{a}$ and a class of Turing degrees $\mathcal{C}$, we say that $\mathbf{b}$ *is noncuppable to* $\mathbf{a}$ *in the class* $\mathcal{C}$ if there is no degree $\mathbf{w} \in \mathcal{C}$ such that $\mathbf{w} < \mathbf{a}$ and $\mathbf{a} = \mathbf{b} \cup \mathbf{w}$; we say that $\mathbf{b}$ *is strongly noncuppable to* $\mathbf{a}$ *in the class* $\mathcal{C}$ if there is no degree $\mathbf{w} \in \mathcal{C}$ such that $\mathbf{a} \not\leq \mathbf{w}$ and $\mathbf{a} \leq \mathbf{b} \cup \mathbf{w}$.

Let $\mathbf{R}^{\mathbf{low}}$ and $\mathbf{D}_2^{\mathbf{low}}$ be the classes of all low computably enumerable (c.e.) and all low 2-c.e. degrees, respectively. We proved the following theorem.

THEOREM 1. *There exist noncomputable low c.e. degrees* $\mathbf{b} < \mathbf{a}$ *such that* $\mathbf{b}$ *is strongly noncuppable to* $\mathbf{a}$ *in the class* $\mathbf{R}^{\mathbf{low}}$ *and for any low degree* $\mathbf{w}$ *the degree of* $\mathbf{b} \cup \mathbf{w}$ *is low again*.

Together with the following result this gives that the partial orders $\mathbf{R}^{\mathbf{low}}$ and $\mathbf{D}_2^{\mathbf{low}}$ are not elementarily equivalent.

THEOREM (Cooper, Lempp, Watson [1]). *For every high c.e. degree* $\mathbf{h}$ *and for every noncomputable n-c.e.* $(n \geq 1)$ *degree* $\mathbf{b} < \mathbf{h}$ *there exists a low* 2*-c.e. degree* $\mathbf{d}$ *such that* $\mathbf{h} = \mathbf{b} \cup \mathbf{d}$.

[1] S. B. COOPER, S. LEMPP, and P. WATSON, *Weak density and cupping in the d-c.e. degrees.*, ***Israel Journal of Mathematics***, vol. 67 (1989), pp. 137–152.

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I will talk about a classification of Turing degrees with strongly η-representable sets. In particularly, I'll show that if A is $\Sigma_\omega^{-1,\emptyset}$ set then A is strongly η-representable. On the over hand $A \in \Sigma_\omega^{-1,\emptyset}$ provided that a set $A \oplus \omega$ is strongly η-representable.

An infinite set $A = \{a_0 < a_1 < \cdots\}$ is called strongly η-representable, if there is a computable linear ordering of order type $\eta + a_0 + \eta + a_1 + \cdots$.

By definition a set A is $\Sigma_\omega^{-1,\emptyset}$ if and only if there is a computable sequence of computable functions $f_i(x,s)$, such that

(1) $\chi_{A_i}(x) = \liminf_s f_i(x,s)$;
(2) $A_0 \subseteq A_1 \subseteq A_2 \subseteq \cdots$;
(3) $A = \bigcup_{i=0}^{\infty}(A_{2i+1} - A_{2i})$.

Abstracts presented by title

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In the spirit of Klein's Erlanger program logical notions have been characterized as the invariant of some suitably motivated similarity relations S over structures. Not every authors agree on a choice of a similarity relation (e.g., Tarski [3], Feferman [2], Bonnay [1]) but one might argue that any plausible candidate should abide by a constraint of *closure under definability*: every operator which is definable in a language L whose logical constants are interpreted by S-invariant operators should be S-invariant. If L is a logic, being generated by a similarity relation closed under definability amounts to a kind of functional completeness of L with respect to a target notion of logicality.

On the other hand we can define alethic extensions of a given model-theoretic logic L as the logics that one may obtain from L by adding to it the expressive power of an interpreted Tarskian truth predicate. We say that a logic L is truth-complete if every class of structures which is elementary (i.e., definable by one sentence) in an alethic extension of L is elementary in L.

Eventually, we prove the following: a logic is truth complete if and only if it is generated by a similarity relation which is closed under definability.

[1] Bonnay, *Logicality and invariance*, ***The Bulletin of Symbolic Logic***, vol. 14 (2008), no. 1, pp. 29–68.

[2] Feferman, *Logic, logics, and logicism*, ***Notre Dame Journal of Formal Logic***, vol. 40 (1999), no. 7, pp. 31–54.

[3] Tarski, *What are logical notions?*, ***History and Philosophy of Logic***, vol. 7 (1986), pp. 143–154.

► AN. CHUBARYAN AND ARM. CHUBARYAN, *On the proof complexities in the Frege systems with different substitution rules*.
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We compare the proof complexities in Frege systems with multiple substitution rule and with constant bounded substitution rule. We use the generally accepted concept of Frege system $\mathcal{F}$ as follows: it uses a finite complete set of propositional connectives and it has a finite set of schematically defined rules of inference; furthermore it must be sound and complete.

A substitution Frege system $S\mathcal{F}$ consists of a Frege system $\mathcal{F}$ augmented with the substitution rule with inferences of the form $\frac{A}{A\sigma}$ for any substitution σ, consisting of a mapping from propositional variables to propositional formulas, and $A\sigma$ denotes the result of applying the substitution to formula A, which replaces each variable in A with its image under σ. This definition of substitution rule allows to use the simultaneous substitution of multiple formulas for multiple variables of A without any restrictions.

If for any constant integer $k \geqslant 1$ we allow substitution for only no more than k variables at a time, then we have k-bounded substitution rule. The k-bounded substitution Frege system $S_k\mathcal{F}$ consists of a Frege system $\mathcal{F}$ augmented with the k-bounded substitution rule.

We use also the well-known notions of proof complexities (size or steps) and the notion of polynomially equivalence (by size or by steps) of the proof systems. We prove that

(1) for every fixed integers k_1 and k_2 the systems $S_{k_1}\mathcal{F}$ and $S_{k_2}\mathcal{F}$ are polynomially equivalent (both by size and by steps);
(2) for every fixed integer k the systems $S_k\mathcal{F}$ and $S\mathcal{F}$ are polynomially equivalent by size;
(3) there is a sequence of tautologies φ_n of the length $\theta(n)$ such that for every fixed integer k the number of steps needed to prove φ_n in $S_k\mathcal{F}$ is $\Omega(n)$, but it can be proved using only $O(\log_2 n)$ steps in $S\mathcal{F}$.

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The *alternative constituent format* displays linguistic or logical data in a succinct way that permits ready grasp of subtle contrasts. A grammatical constituent of a sentence is replaced by a sequence of alternatives. Consider the ambiguous question Q.

(Q) Is two closer to three than four?

Consider an initial sentence A as an affirmative answer.

(A) Two is closer to three than four.

We can compare this with a negative answer by replacing the constituent 'is' with the choice between 'is' and 'is not'.

(AN) Two (is * is not) closer to three than four.

We can also compare the initial ambiguous sentence A with two other unambiguous sentences that give different *readings* or *interpretations*. Replace the constituent 'four' with the choice among three alternatives: 'four', 'to four', and 'four is'. Asterisks separate choices.

(1) Two is closer to three than (four * to four * four is).

The first alternative yields the initial, elliptically and structurally ambiguous sentence; the second and third alternatives give two of its readings (second true, third false). This presents three sentences in one *alternative constituent string* or ACS.

Two constituents can be varied independently.

(2) (Zero * One* Two) is closer to three than (four * to four * four is).

Here, nine sentences are presented in one ACS: the three with 'four' are ambiguous; the three with 'to four' are true; and the three with 'four is' are false. Any number of constituents can be varied.

A constituent can be "zeroed" using a blank as in the following.

(AN1) Two is (* not) closer to three than four.

This yields the same two ambiguous alternatives as AN—emphasizing the relation of the choice to affirm and the choice not to deny.

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Domain Theory is one of approaches toward foundations for Computable Analysis [1, 4]. A weak point of it in comparison to other models, e.g., [5] is lack of a Complexity Theory.

The reason behind this problem is that Domain Theory's main objective was to model computability abstractly, where as for complexity we need more information about the process of computation, which a Domain does not provide, i.e., a Domain does not provide how good an approximation is w.r.t. total elements (*quality of approximation*). We solve this by adding a Scott-continuous function q from a Domain D to $([0, 1], \leq)$. Continuity is plausible. Total elements are those with $q(x) = 1$. The choice of q can vary, depending on what is important for the application. This is a generalization of the situation occurring in Solid Domains [2], where two measures for quality of Solid Objects are provided, depending on the application. We show that our model generalizes [2] approach to arbitrary domains, and for a suitable q, is compatible with Wierauch's approach in [5], and therefore with Ko's [3] approach.

[1] ABBAS EDALAT, *Domains for computaion In mathematics, physics and exact real arithmetic*, ***The Bulletin of Symbolic Logic***, vol. 3(4), (1997), pp. 401–452.

[2] ABBAS EDALAT and ANDRÉ LIEUTIER, *Foundation of a computable solid modelling*, ***Proceedings of the fifth ACM symposium on solid modeling and applications***, ACM, 1999, pp. 278–284.

[3] KER-I KO, ***Complexity theory of real functions***, Birkhäuser, 1991.

[4] VIGGO STOLTENBERG-HANSEN and JOHN TUCKER, *Computability on topological spaces via domain representations*, ***New computational paradigms: Changing conceptions of what is computable***, (S. Barry Cooper and Benedikt Löwe and Andrea Sorbi, editors), Springer, Publisher's address, 2007, pp. 153–194.

[5] KLAUS WEIRAUCH, ***Computable analysis, an introduction***, Texts in Theorical Computer Science, Springer, 2000.

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In "An interpretation about space and time in quantum mechanics" in Logic Colloquium 2007, we have seen that the usual distance is not defined for space and time because we apply the theory ZFU (set theory with urelements). We explained some correlations between particles.

I would have doubts about the assumptions if I did not apply them for space successfully to explain the Big Crunch and the Big Bang in http://adibjebara.blogspot.com/ As time is but another dimension, I apply ZFU to time as well.

Now, let $U\ 1 \times U_2$ be space time with U_1 space and U_2 time.

Many things are discontinuous in quantum mechanics. So, why not space and time? At the level of elementary particles, space and time are also not linearly ordered. As the experiment set up gives us meters and seconds, that means that the space and time considered by the experiment set up are the space and time at our level.

I conjecture that when we measure, we introduce, finitely, order of urelements in U_1 and U_2, in a laboratory. A distance is number of urelements in between $+\ 1$ which is converted to the measurement at our level in meters and seconds. The experiment set up changes much what exists as space and time.

The skeptics should be aware that some experiments needed interpretations and I provided them.

As a Platonist, I see some reality in Dedekind cardinals and, thus, in ZFU.

There is already an experiment where a particle goes both two separate ways. So, its space is different from ours.

There remain to find other experiments to detect that locations and times are not linearly ordered. There remains also to comment on the lack of localism.

I thank Mr. Andereas Blass and Mr. Abderazak Abadlia for their comments.

► VALERY KHAKHANIAN, *The independence of strong uniformization from Church Thesis with choice in the set theory with extensionality and intuitionistic logic*.
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Let CT and CT! be Church Thesis with choice (strong) and with uniqueness (weak) respectively. Let U and U! be principles strong and weak uniformization. Let ZFI2C + DCS (all postulates here or about are in [1] and [2]) be the set theory with intuitionistic logic and two kinds of variables (natural and set) and with collection scheme. It is known that: (a) ZFI2C + CT! $\vdash$ U!; (b) ZFI2C + DCS + CT! $\nvdash$ CT; (c) ZFI2C − extensionality + DCS + CT $\nvdash$ U. We proved that (d) ZFI2C + DCS + CT $\nvdash$ U. Corollaries: (e) ZFI2C + DCS + U! $\nvdash$ U; (f) ZFI2C + DCS + CT! $\nvdash$ U.

I am going to speak about some properties of models of realizability types which were used to prove (d).

[1] V. H. Hahanjan, *The comparative strength of variants of Church Thesis at the level of set theory*, ***Soviet Mathematics Doklady***, vol. 21 (1980), no. 3, pp. 894–898.

[2] ———, *The consistency of intuitionistic set theory with formal mathematical analysis*, ***Soviet Mathematics Doklady***, vol. 23 (1980), no. 1, pp. 46–50.

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We show that natural generalizations of Turing machines can capture the arithmetical hierarchy, the hyper-arithmetical sets, and the analytical hierarchy. All one needs is to consider a broader semantic interpretation of Chandra–Kozen–Stockmeyer's 1980 alternating Turing machines (where non-determinism can be used both existentially and universally).

Such machines are traditionally defined in terms of local branching conditions, and consequently are no more powerful than deterministic machines. We generalize the definition of acceptance by alternating machines by referring to more global conditions on existential and universal branching. We show that such machines accept precisely the inductive (i.e., Π^1_1) sets. Alternating deciders, which either accept or reject each input, accept the hyper-elementary (Δ^1_1) sets. When a uniform bound k is imposed on alternations along every execution path, we obtain level k of the arithmetical hierarchy, in perfect analogy with the characterization of the polynomial-time hierarchy in terms of time-bounded alternating Turing machines. Finally, further equipping machines with negation states (endowed with a strong semantics), yields precisely the Π^1_{k+1} sets when such states occur up to k times along every execution path.

We further show how these characterizations shed light on classical results of Higher Recursion Theory.

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A predicate extension $\mathbf{SQHT}^{=}$ of the Smetanich's logic of here-and-there had been introduced by V. Lifschitz, D. Pearce, and A. Valverde (A characterization of strong equivalence for logic programs with variables, Proceedings of International Conference on Logic Programming and Nonmonotonic Reasoning (LPNMR), 2007) to characterize the notion of strong equivalence of logic programs with variables and equality with respect to stable models. The semantics for this logic is determined by intuitionistic Kripke models $M = \langle W, R, D\rangle$ where $W = \{h, t\}$, R is reflexive and hRt, and D is a non-empty constant individual domain. Equality is decidable. Derivable objects of our formulation are *hyperequents* $\Gamma \Rightarrow \Delta * \Pi \Rightarrow \Phi$ to be read: $\Gamma \Rightarrow \Delta$ here or $\Pi \rightarrow \Phi$ there.

Translation into formulas: $\&\Gamma \rightarrow \vee\Delta \vee (\&\Pi \rightarrow \neg\neg \vee \Phi)$.

The system has familiar axioms, classical Gentzen rules for all connectives in $\Pi \Rightarrow \Phi$ and for all connectives except $\Rightarrow\rightarrow, \Rightarrow \neg$ in $\Gamma \Rightarrow \Delta$. Special rules for $\Rightarrow_h\rightarrow, \Rightarrow_h \neg$, equality (in addition to familiar rules) and h-t-transfer:

$$\frac{A, \Gamma \Rightarrow \Delta, B * \Pi \Rightarrow \Phi \quad \neg B, \Gamma \Rightarrow \Delta, \neg A * \Pi \Rightarrow \Phi}{\Gamma \Rightarrow \Delta, A \rightarrow B * \Pi \Rightarrow \Phi}$$

$$\frac{\Gamma \Rightarrow \Delta, *A, \Pi \Rightarrow \Phi}{\Gamma \Rightarrow \Delta, \neg A * \Pi \Rightarrow \Phi} \quad \frac{t = s, \Gamma \Rightarrow \Delta * t = s, \Pi \Rightarrow \Phi}{\Gamma \Rightarrow \Delta * t = s, \Pi \Rightarrow \Phi} \quad \frac{A, \Gamma \Rightarrow \Delta * A, \Pi \Rightarrow \Phi}{A, \Gamma \Rightarrow \Delta * \Pi \Rightarrow \Phi}$$

THEOREM 1. *The rules are sound and complete.*

Existence is definable: $\exists x\, Px \Leftrightarrow \forall y(\forall x(Px \to Py) \to Py)$.

Let **WEM** be an extension of the intuitionistic logic with $\neg p \vee \neg\neg p$, decidable equality, the formula of constant domains and the equivalences for moving $\neg$ through $\forall$, $\exists$.

THEOREM 2. **SQHT** $=$ *is conservative over* **WEM** *for the formulas without embedded positive occurrences of implication.*

► CYRUS F. NOURANI, *Positive realizability on Horn filters*.
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During 2005–2007 the author explored positive and Horn fragment categories based on his 1981 on positive forcing on Kiesler fragments. Positive forcing had defined T^* on a theory T to be T augmented with induction schemas on the generic diagram functions, for a model M fo T. Let P be a poset, F a family of sets, G subset of P. The author defined positive local realizability on 2005–2007 publications (ASL).

THEOREM 1 (1981). *The positive forcing T^* is a F-generic filter.*

LEMMA. *T^* is a principal proper filter.*

PROPOSITION. *Let I be the set T^*. Let $\varphi(x_1 \dots x_n)$ be a Horn formula and let $\Re i\ i\,\varepsilon\, I$ be models for language L, let $a_1 \dots a_n\, \varepsilon\, \Pi_i\, \varepsilon I\, A_i$. The $\Re I$ are fragment Horn models.*

If $\{i\,\varepsilon\, I : \Re i \models \varphi[(a_1(i) \dots a_n(i)]\}$ then the direct product $\Pi_D \Re I \models \varphi[a_1 D \dots a_n D]$, where D is the generic filter on T^*.

Completabililty theorem at T^* on universal Horn sentences might be provable without $CH: 2^\omega = \omega^+$.

Applying positive local realizability from 2, 3:

THEOREM 2. *Let $(P, \leq)$ be a positive Horn Poset and $p\,\varepsilon\, P$. If D is a countable family of dense subsets of P then $T^*(P)$ is D-generic filter F in P such that $p\,\varepsilon\, F$ and every $p\,\varepsilon\, F$ has a positive local realization.*

The above theorem is a Horn densithy counterpart ot the Rasiowa–Sikorski lemma.

[1] C. F. NOURANI, *Functional generic filter*, ***ASL, Montreal, May 2006***.
[2] ———, *Functorial models and positive realizability*, ***ASL, Florida, March 2007***.
[3] ———, *Positive omitting types and fragment consistency*, ***Gödel Society conference, April 24–25, Brasil***.

► EUGENIO OMODEO AND ALBERTO POLICRITI, *The decidability of the Bernays–Schönfinkel–Ramsey class for set theory*.
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As is well-known, the Bernays–Schönfinkel–Ramsey class of all prenex $\exists^*\forall^*$-sentences which are provable in first-order predicate calculus is decidable. This paper shows that an analogous result holds when the only available predicate symbols are $\in$ and $=$, no constants or function symbols are available, and one moves inside a (rather generic) set theory whose axioms yield the well-foundedness of membership and the existence of infinite sets.

► EUGENE N. PAVLOVSKY, *Complexity of prime computable models*.
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In [1] by S. S. Goncharov anv J. F. Knight there was suggested an approach to solve computable characterisation problem for arbitrary classes of models. Due to this approach it is interesting to find computable characterisation for such a natural classes as prime models, for example. This work devoted to giving precise estimations for index sets of such classes in arithmetical and hyperarithmetical hierarchy.

It is known, that for any computable language σ there exists an universal computable enumeration of computable models of σ (in the case of infinite language we take enumeration with every finite submodels in finite sublanguage). Let K be a class of computable models, closed under computable isomorphisms.

Definition 1. *Index set* $I(K)$ of class K is a set of indices i, such that model with index i of the universal enumeration lies in K.

Definition 2. Model $\mathcal{M}$ is *prime* in $Th(\mathcal{M})$, if for every model $\mathcal{N} \models Th(\mathcal{M})$ there exists an elementary embedding $f : \mathcal{M} \to \mathcal{N}$.

Let us fix a Turing degree d.

Definition 3. Model $\mathcal{M}$ is called *d-decidable*, if its full diagram $FD(\mathcal{M})$ is d-computable.

Recall the Ershov hierarchy of differences of Σ_n sets. Denote $\Sigma_n^{-1,d}$ a set, which is a difference of two $\Sigma_n^{0,d}$ sets, relative to oracle d. Also recall hyperarithmetic hierarchy.

Theorem 4. *For every language σ, containing a predicate symbol of arity $\geqslant 2$ or a functional symbol of arity $\geqslant 2$, is is true that for every arithmetical Turing degree d index set of d-decidable prime computable models of σ is m-complete $\Sigma_3^{-1,d}$ set.*

Theorem 5. *For every language σ, containing infinite number of predicate symbols of arity $\geqslant 2$ or functional symbols of arity $\geqslant 2$, is is true that index set of all prime computable models of σ is a $\Pi^0_{\omega+2}$-complete set.*

[1] S. S. Goncharov and J. Knight, *Computable structure and non-structure theorems*, ***Algebra and Logic***, vol. 41 (2002), no. 6, pp. 351–373.

► ALEXEY G. VLADIMIROV, *Kripke models for intuitionistic set theory*.
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Let *ZFI2C* be usual first order intuitionistic Zermelo-Fraenkel set theory with scheme *Collection* in two-sorted language. Axioms and rules of this system are: all usual axioms and rules of intuitionistic predicate logic *HPC*, all usual axioms of intuitionistic arithmetic (*HA*) for variables of sort 0, and all usual axioms and schemes of Zermelo-Fraenkel system for variables of sort 1. Kripke models are used intensively for investigations of intuitionistic arithmetic and analysis ([1]),[3]). For set theory Kripke models were used, for example, in [2] to prove that *ZFI2C* does not have the full existence property, and that classes of provably recursive functions in *ZFI2C* and in *ZFI2R* (with Replacement without Collection) are different, so, *ZFI2C* is stronger than *ZFI2R*. It is well-known that Smorinsky operation over Kripke models for Heyting arithmetic *HA* permit to get a lot of results about *HA* (see [1], [3]). But for Kripke models for set theory it is not clear, how to define an analog of this operation, because in the case of *HA* it is very important that for each *HA*-model Kripke there a is common part of all domains, but in case of *ZFI2C*-models Kripke it is not clear, what would be a such common part (see [1]) for discussion of Smorinsky operation for *HA*-models).

We consider Kripke models for this theory in some generalized sense in the style of [1]. This let us work without using Smorinsky operation for Kripke models for set theory.

Using this semantic we prove the following statement for $ZFI2C$ and for $ZFI2C + DCS$ the following statements:

1. DP without parameters implies EP without parameters (some variant of well-known result of H. Friedman; DP for $ZFI2C$ was proved in [4]):
2. Admissibility of Markov's rule without parameters;
3. Underivability of Weak Markov principle;
4. Underivability of the principle P;
5. Admissibility of the rule P without parameters;
6. Underivability of the Least Number Principle.

All these results can be generalized to every set theory $T = ZFI2C + \Gamma$ such that if K is a Kripke model for T then K' is a Kripke model for T. For example, this is true for $ZFI2C + DCS$.

[1] A. G. Dragalin, ***Mathematical intuitionism. Introduction to the proof theory***, Translations of Mathematical Monographs, vol. 67, 1988.

[2] H. Friedman and A. Scedrov, *The lack of definable witheness and provably recursivve functions in intuitionistic set theories*, ***Advances of Mathmatics***, vol. 57 (1973), pp. 1–13.

[3] A. S. Troelstra, ***Metamathematical investigations of intuitionistic arithmetic and analysis***, Lecture Notes in Mathematics, vol. 344, 1973.

[4] A. G. Vladimirov, *Effectivity properties for intuitionistic set theory with scheme Collection*, to appear.

► XUNWEI ZHOU, *Rule-based inference is suitable for human while hypothetical inference is suitable for computer*.
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Take "if triangle ABC is congruent with triangle A′B′C′, then side AB equals side A′B′" as an example. Rule-based inference is described by

$$\frac{\Delta \mathrm{ABC} \cong \Delta \mathrm{A'B'C'}}{\therefore\ \mathrm{AB} = \mathrm{A'B'}}$$

where the general law $\Delta\mathrm{ABC} \cong \Delta\mathrm{A'B'C'} \rightarrow \mathrm{AB} = \mathrm{A'B'}$ is used as an inference rule, inferring the unknown fact $\mathrm{AB} = \mathrm{A'B'}$ from the known fact $\Delta\mathrm{ABC} \cong \Delta\mathrm{A'B'C'}$. Hypothetical inference is described by

$$\frac{\begin{array}{l}\Delta \mathrm{ABC} \cong \Delta \mathrm{A'B'C'} \rightarrow \mathrm{AB} = \mathrm{A'B'} \\ \Delta \mathrm{ABC} \cong \Delta \mathrm{A'B'C'}\end{array}}{\mathrm{AB} = \mathrm{A'B'}}$$

where the general law $\Delta\mathrm{ABC} \cong \Delta\mathrm{A'B'C'} \rightarrow \mathrm{AB} = \mathrm{A'B'}$ is used as a geometrical theorem, taken as the major premise, $\Delta\mathrm{ABC} \cong \Delta\mathrm{A'B'C'}$ is taken as the minor premise, inferring the conclusion $\mathrm{AB} = \mathrm{A'B'}$.

In rule-based geometry system, there are many inference rules, a computer cannot choose from them automatically, but a human can. Therefore, rule-based inference is not suitable for computer but is suitable for human. Geometrical hypothetical inference system transforms all these inference rules into geometrical theorems, taking hypothetical inference as the sole inference rule. Because there is only one inference rule, every time a computer wants to make inference, the computer uses it. As to the many geometrical theorems and facts, the computer searches them top-down, from left to right, depth-first plus backtracking, like the way Prolog does. In this way, the inference can be made automatic. So, hypothetical inference is suitable for computer. But because there are too many repetitive work, it is not suitable for human.

[1] XUNWEI ZHOU, *Rule-based inference v. hypothetical inference*, ***2008 Association for Symbolic Logic Annual Meeting***, Irvine, California, USA, March 27–30, 2008.