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REVIEWS

The Association for Symbolic Logic publishes analytical reviews of selected books and articles in the field of symbolic logic. The reviews were published in ***The Journal of Symbolic Logic*** from the founding of the JOURNAL in 1936 until the end of 1999. The Association moved the reviews to this BULLETIN, beginning in 2000.

 Authors and publishers are requested to send, for review, copies of books to *ASL, Box 742, Vassar College, 124 Raymond Avenue, Poughkeepsie, NY 12604, USA.*

In a review, a reference "JSL XLIII 148," for example, refers either to the publication reviewed on page 148 of volume 43 of the JOURNAL, or to the review itself (which contains full bibliographical information for the reviewed publication). Analogously, a reference "BSL VII 376" refers to the review beginning on page 376 in volume 7 of this BULLETIN, or to the publication there reviewed. "JSL LV 347" refers to one of the reviews or one of the publications reviewed or listed on page 347 of volume 55 of the JOURNAL, with reliance on the context to show which one is meant. The reference "JSL LIII 318(3)" is to the third item on page 318 of volume 53 of the JOURNAL, that is, to van Heijenoort's *Frege and vagueness*, and "JSL LX 684(8)" refers to the eighth item on page 684 of volume 60 of the JOURNAL, that is, to Tarski's *Truth and proof*.

References such as 49***5*** or 280***1*** are to entries so numbered in *A bibliography of symbolic logic* (the JOURNAL, vol. 1, pp. 121–218).

GERALDINE BRADY. ***From Peirce to Skolem. A neglected chapter in the history of logic.*** Elsevier, Amsterdam, 2000, xi + 468 pp.

Perhaps the most important thing a potential reader of this book should know is that it is *not* a source book on the model of Jean van Heijenoort's 1967 ***From Frege to Gödel: A Source Book in Mathematical Logic, 1879–1931***. We still need a careful collection of source articles and book selections covering the Boole–Peirce–Schröder–Löwenheim–Skolem chain of logical investigations; a suitable title might be something like ***From Boole and Peirce to Skolem: A Source Book in Symbolic and Mathematical Logic, 1847–1923***. The book being reviewed makes the need all the more evident.

Even though the book is not a source book but a history—as the second part of its title implies, it does provide some useful sources: original translations of seven of Ernst Schröder's "Lectures" (pp. 207–427) and selections from Norbert Wiener's dissertation (pp. 429–444).

This book could have an immediate impact on current debates in history of logic. Among other things, it could help clarify two "priority" issues. The first concerns the origin of *symbolic logic*, the distinctively modern form of logic *per se*, where logic *per se* is a field of study with a more or less continuous development since its founding by Aristotle (Tarski, ***Introduction to logic and to the methodology of the deductive sciences***, trans. O. Helmer, ed. J. Tarski, Oxford UP, New York, 1994, p. 18). In ***Prior Analytics***, Aristotle addressed the two central problems of logic *per se*: (1) how to show that a given conclusion follows from given premises that *do* formally imply it and (2) how to show that a given conclusion does not follow from

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given premises that *do not* formally imply it. In a way, Aristotle was moving toward seeking two decisive tests or criteria: one for determining if the conclusion follows and one for determining if the conclusion does not follow. In particular, the first issue is whether any person or persons deserve to be regarded as having initiated symbolic logic and if so, who and how.

The second priority issue, entirely separate from the first, concerns the branch of mathematics whose theorems are metatheorems about logics (or parts of logics or applications of logics) and which is known today (somewhat misleadingly) as *mathematical logic*—even though it might with more justice be called *mathematics of logic*. This includes two principal subfields: proof theory, more properly called derivation theory, which investigates purely syntactic entities (that might but need not express proofs) and model theory, more properly interpretation theory, which studies set-theoretic "interpretations" of formalized languages. In short, the second issue is whether any person or persons deserve to be regarded as having founded mathematical logic and if so, who and how. At stake is identification of the origins of proof theory and model theory.

One would think that logic *per se* would have had to have reached a high level of maturity before its content could be subjected to distinctively mathematical investigation. Surprisingly, Aristotle and some nearly contemporaneous Stoic logicians are thought to have proved some theorems about their own respective logics: qualified scholars such as Timothy Smiley and Benson Mates have argued that there were completeness proofs in antiquity. But it is also clear that even if such proofs existed, they were not taken to be paradigmatic: they did not give rise to a field of study that was (1) dedicated to proving theorems about logics and (2) was capable of attracting mathematically talented people to apply their talents to formulating and proving such theorems.

In regard to the first issue, currently the two main candidates seem to be George Boole (1815–1864) and Gottlob Frege (1848–1925). This book focuses on a chain of post-Boole developments largely untouched by any Fregean influence—beginning with C. S. Peirce (1839–1914) and ending with Thoralf Skolem (1887–1963). The book mentions but does not describe what Peirce took from Boole. Boole was committed to the view that all propositions were equations between classes and that the exclusive role of the concepts expressed by 'and', 'or', 'if' and the like was to compound classes not propositions (G. Boole, ***Laws of Thought***, 1854, Cambridge: Macmillan. Reprinted with introduction by J. Corcoran, 2003, Buffalo: Prometheus Books, p. 401). This view inspired the search for analogies between logic and analysis including solving equations—something totally unprecedented in logic before Boole (J. Corcoran, *Aristotle's* ***Prior Analytics*** *and Boole's* ***Laws of Thought***, ***History and Philosophy of Logic*** 24 (2003), 261–288). It also "justified" expressions such as *algebra of logic*, *algebra of relations*, and *algebraic logic* which permeated the literature of this development. Moreover, it helped to effectively block discovery of propositional logic as we know it from the Fregean tradition or even from ancient Stoic sources. As this book suggests, it might have been Skolem who joined the Boolean algebraic-equational viewpoint to the "predication" viewpoint taken by Aristotle, Frege, Peano, and many others.

Unlike other revolutionary logical innovators including Frege, Boole's greatness as a logician was recognized almost immediately, and by Peirce, the greatest native-born American logician of his time. In 1865, hardly a decade after Boole's 1854 ***Laws of Thought*** and not even a year after Boole's tragic death, Boole's logic was the subject of Peirce's Harvard University lecture "Boole's Calculus of Logic". Peirce opened his lecture with these prophetic words:

> Perhaps the most extraordinary view of logic which has ever been developed with success is that of the late Professor Boole ... [whose] book ***Laws of Thought*** is destined to mark a great epoch in logic; for it contains a conception which in point of fruitfulness will rival that of Aristotle's ***Organon***" (Peirce 1865 *Boole's Calculus of Logic*, in ***Writings of Charles S. Peirce: A Chronological Edition,*** Vol. I, 1982, Bloomington: Indiana UP, pp. 223–4.)

Thus, if the importance of the Peirce-to-Skolem development can be established, this book will strengthen the case for Boole's candidacy and further weaken that for Frege's. If this book is at all successful, we can expect that it will be welcomed by Boole's supporters (which included Tarski, ***Introduction to logic and to the methodology of the deductive sciences***, trans. O. Helmer, ed. J. Tarski, Oxford UP, New York, 1994, p. 18, and includes this reviewer, *Aristotle's* ***Prior Analytics*** *and Boole's* ***Laws of Thought***, ***History and Philosophy of Logic*** 24, 2003, pp. 261ff) but that it will be viewed negatively by Frege's supporters.

In regard to the second issue, current discussion has not progressed far enough to identify "main candidates" but it is clear that Frege is not in the running: his philosophy might even have excluded the kind of metalogical deliberations involved in articulating and proving metatheorems. Moreover, Boole, whose "metaproofs" are hardly models of cogency, might turn out to be at best a weak candidate, but perhaps a winner by default. Intellectual history is replete with paradigms whose founders turn out to have been inept at what they founded. Accordingly, it must be said that Boole's claims of metatheorems—including, e.g., his "law of development" and his "completeness" assertions—have influenced the course of mathematical logic, even though many of his supporters prefer to ignore his embarrassing "application of Maclaurin's theorem" to the expansion of an "elective" function "in ascending powers of x" (which is not overlooked by a detractor such as Dummett JSL XXIV 203–4). This book treats early mathematical accomplishments in logic which are not known to historians as well as they deserve to be. In this respect, the book contributes to discussion of the second issue.

As the author says, this book, published in 2000, began in 1990 as a master's thesis at the University of Chicago supervised by William Tait, a distinguished philosopher of logic, and William Howard, a distinguished mathematical logician. The thesis developed into an "earlier version" of the book, which was accepted in 1997 as a doctoral dissertation in History of Science at the University of Oslo; the supervisor was Anil Nerode, a distinguished mathematician known for achievements in the theory of formal languages. The preface mentions several important figures that helped in various ways; some read part or all of the manuscript: Volker Peckhaus, Saunders Mac Lane (Nerode's "doctor-father"), Dagfinn Føllesdal, Burton Dreben, William Lawvere, Michael Dummett, and others.

Given that this is the author's first book, it is surprising to find in chapter after chapter sentences that read more like magisterial pronouncements—the casual assertions of established scholars confident of being generously interpreted—than like the carefully crafted statements of a junior scholar trying to earn respect. A few examples are necessary. On page 14, we read "Boole was not interested in the axiomatic method." This surprising assertion is unsupported and unaccompanied by explanation. The following is from pages 21 and 22.

> When Peirce tried to use Boole's propositional logic with a smattering of syllogistic to represent geometric reasoning, he realized that Aristotle + Boole, or what we call monadic predicate logic, was not expressive enough to represent the reasoning of geometry, which is almost entirely in terms of the two binary relations incidence and congruence. As soon as these binary relations are introduced much if not all reasoning in Euclid can [*sc*. be] approached.

The reader is to take these points on the authority of the author, who gives no evidence that Boole had a "propositional logic" or that Peirce ever said a word about expressive strength. In addition, no explanation is given of why most geometrical reasoning has no need for three-place and four-place geometrical relations such as "betweenness" (a is between b and c) and "proportionality" (a is to b as c is to d). Moreover, on page 24 in a discussion of the kinds of questions Peirce treated, the book says the following.

> First, there is his [*sc*. Peirce's] use of formal algebraic computations to deduce complex logical theorems from simple ones. Second, there is his codification of

> rules about relations to reveal the mathematical structure of formal logic itself. This was a precursor of metamathematics and proof theory.

We are not told how Peirce thought that deduction could be achieved by means of computation. There is no explanation of what "the mathematical structure of formal logic" is or what Peirce revealed to the world about it. And there is no hint as to what the author takes metamathematics and proof theory to be, how the two are distinguished, what Peirce did that qualifies him to be a precursor, or whether other historians have discussed these points.

Unfortunately, the book is marred by careless scholarship. Page 430 repeats 12 lines of a quotation from page 24 of Russell's ***Principles of Mathematics*** that have already been quoted in footnote 5 on page 7. Moreover, on page 430 the author has misquoted Russell by interpolating five words, 'of the Algebra of Relatives', that are not on Russell's page 24, not on the author's page 7, and not marked *scilicet* inside editorial brackets. There are numerous assertions that cry out for clarification, and, of the clear statements, many are unsupported. There are many typographical and other errors. The index omits several technical terms and it omits several occurrences of several it lists. This deficiency is somewhat mooted by the fact that the publisher has provided readers with a fully searchable e-version of the book on `Amazon.com`. The reviewer—a mathematically trained logician who has been publishing in history of logic, philosophy of logic, philosophy of mathematics, and mathematical logic for over forty years—found this book very difficult to read: the author often seemed unaware of how difficult the material is and what needs to be said to convey it to a reader. The definitive history of this philosophically and mathematically rich series of logical investigations remains to be written, but it will build on this work. Nevertheless, the reviewer found the book fascinating and rewarding.

This review benefited from the assistance of Pierre Adler, Roger Maddux, Daniel Merrill, and others.

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Graham Priest. ***An introduction to non-classical logic: From If to Is.*** Second Edition. Cambridge University Press, Cambridge, United Kingdom, 2008, xxxii + 613 pp.

As stated in the title, this is a second edition of the book with the title ***An introduction to non-classical logic***; for a review of the first edition see this journal (BSL XII 294). The subtitle ***From If to Is*** indicates that whereas the first edition concerned only propositional logics the present version deals also with predicate calculi, i.e., calculi with quantification. Part I (pages 1–260) is an extended and revised version of the first edition with 11 chapters bearing the same names as in first edition but most of them containing some new material. There are two important addenda. First in Chapter 11 section 11.7a on t-norm based fuzzy logic. The whole Chapter 11 of first edition (*Fuzzy logics*) was criticized as not being up-to-date and not referring to recent development of formal systems of (mathematical, symbolic) fuzzy propositional and predicate calculi. This is improved in the present edition. The second addendum is a new chapter 11a on many-valued modal logics.

Part II—*Quantification and identity* (pages 261–583) has Chapters 12 to 25; we list their names and add some comments. All chapters end with sections on *History, Further Readings and Problems*. Chapter 12—*Classical first-order logic* contains a section 12.6 on some philosophical problems. Chapter 13—*Free logics*: there is a unary predicate **E** of existence; models may contain both existing and non-existing objects. In positive free logics non-existing objects may satisfy positive atomic formulas; in negative free logics not. In neutral free logics atomic formulas referring to non-existing objects are neither true nor false. Chapters 14–18 deal with modal logics, with constant domains and variable domains and with necessary identity and contingent identity. Chapter 18 studies non-normal modal logics, where some possible worlds may be non-normal: then formulas of the form $\Box A$ and $\Diamond A$ are assigned truth values not depending on the truth values of A (in a non-normal world logical truths may fail). Chapter 19—(quantified) conditional logic, having an additional binary connective $>$ with semantics studied in Part I. Chapter 20—(quantified) intuitionist logic, Chapter 21—(quantified) many-valued logics. Chapter 22—*First Degree Entailment*. In a model, each predicate and each tuple of its arguments is assigned not one truth value but no truth value, one truth value (0 or 1) or two truth values (both 0 and 1). Alternatively one may use just four different truth values. This extends to connectives as in Part I, and in a natural way to quantifiers. Chapter 23 named *Logic with gaps, gluts and worlds* presents four particular logics serving as a bridge to the relevant logic of chapter 24, named *Relevant logics*. The term relevant logic seems to occur first on p. 172 (part I) where it is said that a propositional logic is relevant if whenever $A \to B$ is logically valid then A and B have a propositional parameter in common. In the present chapter quantification and identity is studied in mainstream relevant logics. The final chapter 25 is devoted to quantified fuzzy logic; first Łukasiewicz continuum-valued predicate logic is studied, philosophical comments on vagueness, Sorites paradox and fuzzy identity follow; finally, quantification and identity in t-norm based fuzzy logic is presented. Small warning: fuzzy logic presented here is mathematical fuzzy logic or fuzzy logic in the narrow sense, just a kind of many-valued logic with a comparative notion of truth, to be distinguished from Lotfi Zadeh's fuzzy logic in the broad sense, which is just theory and applications of fuzzy sets.

I have found only very few small mistakes but I do not list them; the author announces in the preface the web address `www.cambridge.org/priest` where errata will be found. The book contains an extremely large amount of information on surprisingly many non-classical logical systems, their deductive and semantical properties and mutual relations. It is a very valuable source for many logicians interested in non-classical logics of some kind.

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