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2008 SPRING MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

The Sheraton Hotel
Pasadena, California
March 20–21, 2008

A Spring Meeting of the Association of Symbolic Logic was held March 20–21, 2008, in Pasadena, California, at the Sheraton, in conjunction with the annual meeting of the Pacific Division of the American Philosophical Association. The Program Committee consisted of Michael Glanzberg and Gila Sher (Chair). The ASL hosted a reception on the evening of March 20th.

The program included a Symposium on **Quantifiers in Logic and Language** and an Invited Speakers Session on **Foundational and Historical Perspectives on Logic and Mathematics**.

The following one-hour symposium talks were presented:

Chris Barker (New York University), *Reasoning about scope-taking in a substructural logic*.
Edward L. Keenan (UCLA), *Non-classical quantification in natural language*.
Dag Westerståhl (Göteborg), *Quantifiers, freezing, compositionality, and possessives*.

The following one-hour invited talks were presented:

Geoffrey Hellman (University of Minnesota), *What can we expect of a "foundational framework for mathematics"*?
Jamie Tappenden (University of Michigan), *'Function', 'thought', and 'fruitfulness' in nineteenth century logic and mathematics*.
Edward Zalta (Stanford University), *Reflections on logical foundations for mathematics*.

The program also included a contributed papers session chaired by Gila Sher, in which two talks were presented.

Abstracts of the invited talks and contributed talks given (in person or by title) by members of the Association for Symbolic Logic follow.

For the Program Committee
GILA SHER

Abstracts of invited papers for the Symposium on Quantifiers in Logic and Language

► CHRIS BARKER, *Reasoning about scope-taking in a substructural logic*.
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What is the logic of scope-taking in natural language? From the point of view of resource-sensitive logics such as Lambek grammars, we can rephrase the question like this: what structural resources characterize natural language scope-taking? Associativity gives a first approximation. It is not sufficient, however, since it allows for quantifiers in peripheral but not in medial position (e.g., it allows the quantifier *everyone* to take sentential scope in *John saw everyone* but not in *John saw everyone yesterday*). I will argue that the crucial concept for scope-taking is a continuation. A (delimited) continuation is a (portion of) an expression's semantic context. If so, then reasoning about scope-taking requires structural postulates that explicitly articulate structures into values and their contexts. I will show how this can be viewed as a kind of hypothetical reasoning that operates entirely on the left-hand side of the turnstile. Applications discussed include quantification, bound anaphora, and the compositional analysis of scope-taking adjectives such as *same* and *different*, as in *John and Bill read the same book*.

► EDWARD L. KEENAN, *Non-classical quantification in natural language*.
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We present results for three classes of natural language quantifiers: Prop$\langle 1,1\rangle$, *proportionality quantifiers* of type $\langle 1,1\rangle$, quantifiers of type $\langle\langle 1,1\rangle,1\rangle$, and quantifiers of type $\langle 2\rangle$.

A. **Card**$\langle 1,1\rangle$	some, no, hardly any, about/exactly/less than ten, just finitely many
Co-Card$\langle 1,1\rangle$	all, nearly all, all but (at most) ten, all but finitely many
Prop$\langle 1,1\rangle$	most, seven out of ten, not one ... in ten, (about) a third of, forty percent of

Card$\langle 1,1\rangle$ and Co-Card$\langle 1,1\rangle$ are shown to be isomorphic to Type $\langle 1\rangle$, the set of generalized quantifiers (over the same domain). Prop$\langle 1,1\rangle$ is not, its members are essentially two place. Elements of Prop$\langle 1,1\rangle$ support two entailment patterns which Card and Co-Card do not:

(1) $Q(A)(B) \equiv Q(A)(\neg B)$

Exactly half the students passed	$\equiv$ Exactly half the students didn't pass
Between 1/3 and 2/3 of Americans are overweight	$\equiv$ ... are not overweight

We prove two "Mid-Point" Theorems characterizing the Q for which (1) holds.

(2) a. More than n/m As are Bs
At least $1 - n/m$ As are Cs
Ergo: Some A is both a B and a C

b. For Q conservative & right increasing,
QAB and $Q^{dual}AC$
Ergo: $\mathrm{Some}(A)(B \cap C)$

(2b) is Westerstahl's Generalization. Most English quantifiers are conservative, meaning $QAB = Q(A)(A \cap B)$, all A,B. Q is right increasing iff $QAB \Rightarrow QAC$ whenever $B \subseteq C$. (2b) favors proportionality quantifiers as they are closed under duals, whereas Card and Co-card are maximally not: Given a domain E, $\{F^{dual} \mid F \in \mathrm{Card}_E\} = \mathrm{Co\text{-}Card}_E$ and conversely.

B. Cardinal Qs of type $\langle\langle 1,1\rangle,1\rangle$ are illustrated in (3a), their proportional counterparts in (3b). (3c) shows that the analogue of (1) holds for some of the proportional quantifiers of this type

(3) a. <u>More</u> students <u>than</u> teachers signed the petition
b. <u>Proportionally more</u> students <u>than</u> teachers signed the petition
c. <u>The same percentage of</u> students <u>as</u> teachers signed / didn't sign

C. Some quantifiers of type $\langle 2\rangle$, properties of binary relations, are given in (4). That in (4a) is the composition of two type $\langle 1\rangle$ functions, those in (4b,c,d,e) are provably not.

(4) a. No politician kissed every baby ((No Politician)∘(Every Baby))(Kiss)
b. Different people like different things
c. John danced with Mary but no one else danced with anyone else
d. John and Bill support rival political parties
e. The director wanted to know which agents infiltrated which organizations

► DAG WESTERSTÅHL, *Quantifiers, freezing, compositionality, and possessives*.
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Noun phrases in general denote type $\langle 1\rangle$ quantifiers (sets of sets), often, but not always, obtained from type $\langle 1,1\rangle$ quantifiers (binary relations between sets) by freezing the first argument: the restriction. We provide some answers to two questions: (a) Which type $\langle 1\rangle$ quantifiers can be expressed as frozen type $\langle 1,1\rangle$ quantifiers? (b) For those that can be so expressed, when can one recover the type $\langle 1,1\rangle$ quantifier and/or the restriction? These questions are relevant to certain semantic issues, notably the semantics of possessives. In particular, the largely negative answer to (b) creates a problem for the compositionality of possessive constructions.

Abstracts of papers for the Invited Session on Foundational and Historical Perspectives on Logic and Mathematics

► GEOFFREY HELLMAN, *What can we expect of a foundational framework for mathematics*?
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We consider two demands on foundational frameworks for mathematics not usually considered: (1) accommodating a plurality of systems, including apparently conflicting ones such as classical analysis and smooth infinitesimal analysis; (2) accommodating indefinite extendability of "universes" for mathematical discourse. Implications with regard to set theory, topos theory and structuralism will be discussed.

► JAMIE TAPPENDEN, *Function, thought, and fruitfulness in nineteenth century logic and mathematics*.
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Among the features Frege emphasizes as significant for his logic are: that his logic represents pure thought, that it takes the mathematical concept of function as basic, and that it is capable of representing "the truly fruitful concepts" that are important for mathematics and science. I'll spell out how these themes were highly charged ones in the mathematical environment Frege worked in. The concept of function as basic, the idea that mathematics should be done with "thoughts rather than computations" and the idea that the proper choice of basic concepts would reveal itself in facilitated problem-solving were characteristic methodological positions of Riemann and his students, especially Dedekind. I'll explain what mathematical innovations were connected with this, and how Frege fits in to the story.

► EDWARD N. ZALTA, *Reflections on logical foundations for mathematics*.
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In this talk, I draw out some consequences, and examine some assumptions, of the version of neologicism defended recently in the paper (with Bernard Linsky) "What Is Neologicism?"

(The Bulletin of Symbolic Logic, 12/1, 2006, 60–99). I give a positive argument as to why object theory, formulated in weak third-order logic with abstraction instead of comprehension over objects, constitutes a new form of logicism. I argue that the resulting system achieves the epistemological goals of the original logicist program.

Abstracts of contributed talks

► JESSE ALAMA, *A formal proof of Euler's polyhedron formula.*
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Euler's polyhedron formula asserts for a polyhedron p that $V - E + F = 2$, where V, E, and F are, respectively, the numbers of vertices, edges, and faces of p. Imre Lakatos's history [1] of this theorem illustrates a dialectical process, which he calls "the method of proofs and refutations", by which definitions of polyhedra and the statements and proofs of Euler's polyhedron formula evolved. The process is dialectical because although proofs are advanced, counterexamples show either that the proofs are mistaken or that the theorem is too narrowly restricted; theorems, proofs, and definitions are revised and the critical process recommences.

The apparently turbulent history of Euler's polyhedron formula presents an interesting challenge for computer-checked formalization: to give a formal proof of the theorem and to verify the proof's validity. The author has recently completed such a proof in MIZAR [2], which is a proof-checking system based on classical first-order logic with dependent types and Tarski-Grothendieck set theory. The informal proof on which the formal counterpart is based is a linear algebraic argument due to Poincaré and actually appears in [1]. We will survey the formal proof and discuss a more philosophical aim of the project: to try to disarm Lakatos's antipathy toward formal proofs by showing how formalization mirrors or exemplifies the method of proofs and refutations in some important respects.

[1] IMRE LAKATOS, ***Proofs and refutations: The logic of mathematical discovery***, Cambridge University Press, 1976.

[2] MIZAR, available at http://www.mizar.org.

► EREZ SHOCHAT, *Automorphisms of countable short recursively saturated models of arithmetic.*
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A model of Peano Arithmetic is short recursively saturated if it realizes all its bounded finitely realized recursive types. It can be proven that short recursively saturated models of arithmetic are exactly the elementary initial segments of recursively saturated models of arithmetic. In this talk, I will survey results on short recursively saturated models of PA and their automorphisms. I will then prove that countable short recursively saturated elementary initial segments of a countable recursively saturated model of arithmetic have automorphisms which cannot be extended to automorphisms of their recursively saturated extension.

Abstracts of papers presented by title

► JOHN CORCORAN, *An information-theoretic approach to first-order logic.*
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Before the ascendancy of set theory, logicians such as Boole and Frege presupposed an information-containment conception of implication. For them, the cogency and soundness of deduction was based on information-containment insights. [Logical] implication, expressed by 'implies', was sometimes treated as a homogeneous two-place relation as in "Every conjunction *implies* each of its conjuncts". In the information-containment sense, given any two propositions, the first *implies* the second if the first contains all of the information of the second. The first *is logically equivalent to* the second if the first contains all of the information of the second and no more. [*Implicative*] *conjunctive paradigms* are implications of a given proposition by a given conjunction having the given proposition as a conjunct, e.g., "Zero is even" is implied by "Zero is even and one is odd". In a sense, conjunctive paradigms are typical not only of information-theoretic implication but also of standard deduction. One sentence [deductively] *yields* a second if the second is deducible from the first. The deductive yielding relation parallels information-containment implication. The (elementary) *Conjunctive Paradigm Proposition* is that one sentence yields a second iff the first is interdeducible with a conjunction having the second as a conjunct. This result generalizes to sets of sentences. The *General Conjunctive Paradigm Theorem* is to the effect that *every* first-order deduction relation can be understood as a generalized conjunctive paradigm: *every sentence deducible from a given set of sentences is interdeducible with a conjunction each conjunct of which is deducible from one sentence in the set*. Thus, every consequence of a given set of propositions is "virtually" a conjunction of consequences of individual members of the set. This lecture sharpens discussions in this BULLETIN, vol. 10 (2004), pp. 276–277 and vol. 13 (2007), p. 405.

► DOLPH ULRICH, *A new three-base and a single axiom for pure denumerable Łukasiewicz implication.*
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The implicational fragment of Łukasiewicz's infinite-valued sentential calculus $\mathbf{L}_{\aleph_0}$ is axiomatized in [1] using **Syl** $= CCpqCCqrCpr$, **Simp** $= CpCqp$, **In(version)** $= CCCpqqCCqpp$, and **Lin(earity)** $= CCCpqCqpCqp$ with rules substitution and detachment.

The latter two axioms can be compressed into the single formula **InLin** $= CCCpqqCCCpqCqpp$. Where Dx.y is the most general result of detaching an instance of formula y from an instance of formula x, **InLin** is **DDSyl.InDSyl.Lin**. In turn, with 1 as **Syl**, 2 as **Simp**, and 3 as **InLin**, we have D1.3 = 4. $CCCCCpqCqpprCCCpqqr$, D1.2 = 5. $CCCpqrCqr$, and D4.5 = **In**. Continuing, D1.5 = 6. $CCCpqrCCCspqr$, DD1.4.5 = 7. $CCCCCpqCqpprCqr$, and D7D6.7 = **Lin**.

The 37-symbol **Ax** $= CCCpCqpCCCCCCrssCCCrsCsrrCtuCvuwCCvtw$ is an equivalend *single* axiom, but here the going is considerably heavier.

That **Ax** is a theorem of implicational $\mathbf{L}_{\aleph_0}$ follows from the well-known facts (see, e.g., [2]) that in this system each wff α is provably equivalent to $C\mathbf{T}\alpha$ for any theorem **T**, and that provable equivalents are intersubstitutable in all theorems. Replacing Ctu with C**InLin**Ctu in **DSyl.Syl** $= CCCCtuCvuwCCvtw$ now gives $CCCC$**InLin**$CtuCvuwCCvtw$, and replacing CCC**InLin**$CtuCvuw$ therein with *its* equivalent, C***CpCqp***CCC**InLin**$CtuCvuw$, then delivers **Ax**.

Finally, taking 1 to be **Ax**, we have D1.1 = 2. $CCpqCrCsr$ and D2.1 = 3. **Simp**. Thereafter, D1.2 = 4. $CCpqCrCCCCCsttCCCstCtssCquCpu$, whereupon **DDD4.4.1.D1.4** = **Syl** while **DDD1.D3.1.1.1** = **InLin**.

[1] R. K. MEYER, *Pure denumerable Łukasiewicz implication*, ***The Journal of Symbolic Logic***, vol. 31 (1966), pp. 575–580.

[2] A. ROSE and J. B. ROSSER, *Fragments of many-valued statement calculi*, ***Transactions of the American Mathematical Society***, vol. 87 (1958), pp. 1–53.

► XUNWEI ZHOU, *The second level predicate calculus of mutually-inversistic logic is quantifier-free*.
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In mutually-inversistic logic, P, Q, R are fact variables and are the abbreviations of predicate variables acting on individual variables. For example, P is the abbreviation of $p(x)$, $p(x, y)$, etc. where p is a predicate variable, x and y are individual variables. $\neg$, $\wedge$, $\vee$ are compounders. $\leq^{-1}$ (mutually inverse implication) is a connective. $\wedge$ in $P \wedge P$ is a fact compounder, because it composes two facts, forming a fact. $\leq^{-1}$ in $P \wedge R \leq^{-1} Q \wedge R$ is an empirical or mathematical connective, because it connects two facts, forming an empirical or mathematical connection proposition. $\wedge$ in $\{P \leq^{-1} Q\} \wedge \{Q \leq^{-1} R\}$ is an empirical or mathematical compounder, because it composes two empirical or mathematical connection proposition, forming an empirical or mathematical connection proposition. $\leq^{-1}$ in $\{P \leq^{-1} Q\} \leq^{-1} \{\neg Q \leq^{-1} \neg P\}$ is a logical connective, because it connects two empirical or mathematical connection proposition, forming a logical connection proposition.

In the second level predicate calculus of mutually-inversistic logic, quantifier is not introduced. The boundness of a fact variable in a logical connection proposition is determined by its occurrences relative to the compounders and connectives. P and Q in the logical connection proposition $\{P \leq^{-1} Q\} \leq^{-1} \{\neg Q \leq^{-1} \neg P\}$ are relevantly bound, because they occur on both sides of the logical connective $\leq^{-1}$. Q in $\{P \leq^{-1} Q\} \wedge \{Q \leq^{-1} R\} \leq^{-1} \{P \leq^{-1} R\}$ is transitively bound, because it occurs on one side of the logical connective $\leq^{-1}$ but on both sides of the empirical or mathematical compounder $\wedge$. R in $\{P \leq^{-1} Q\} \leq^{-1} \{P \wedge R \leq^{-1} Q \wedge R\}$ is additionally bound, because it occurs on one side of the logical connective $\leq^{-1}$ but on both sides of the empirical or mathematical connective $\leq^{-1}$. The first and second occurrences of P in $\{P \wedge P \leq^{-1} Q\} \leq^{-1} \{P \leq^{-1} Q\}$ is juxtaposedly bound, because it occurs on one side of the empirical or mathematical connective $\leq^{-1}$ but on both sides of the fact compounder $\wedge$.

If a fact variable occurs only once in a logical connection proposition, then it is free. Because it cannot be one of the above-mentioned bound variables. If a fact variable occurs exactly twice in a logical connection proposition, then it is purely bound. Because it is just one of the above-mentioned bound variables. If a fact variable occurs three times or more in a logical connection proposition, then it is combined boundness. Because it combines the above-mentioned boundness. For example, P in $\{P \wedge P \leq^{-1} Q\} \leq^{-1} \{P \leq^{-1} Q\}$ is a combined relevant and juxtaposed boundness, because the first and second occurrences of it is juxtaposed boundness, the first and the third occurrences, the second and the third occurrences are relevant boundness.