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2007–2008 WINTER MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

San Diego Convention Center, San Diego, CA
January 8–9, 2008

The 2007–08 Winter Meeting of Association of Symbolic Logic was held on Jan. 8–9, 2008 at the San Diego Convention Center, in conjunction with the annual Joint Mathematical Meetings. The members of the Program Committee were Christian Rosendal, Thomas Scanlon, and Jeffrey Remmel (Chair). The program consisted of six invited 50-minute talks and seventeen contributed talks. The ASL hosted a welcoming reception on January 8th at the Marriott Hotel. In addition, on January 6–7, 2008, prior to the ASL meeting, there was a joint AMS-ASL Special Session, **Set Theory and Banach Spaces**, organized by Christian Rosendal and Stevo B. Todorcevic.

The 50-minute invited addresses were:

Gregory L. Cherlin (Rutgers University), *Model theory and algebraic groups*.
Ilijas Farah (York University), *Applications of set theory to operator algebras*.
Pavel Hrubes (Czech Academy of Science), *Proof complexity after $NP \neq coNP$*.
Victor Marek (University of Kentucky), *Answer set programming*.
Jan Riemann (University of California, Berkeley), *Measures and their random reals*.
Simon Thomas (Rutgers University), *A descriptive view of geometric group theory*.

Abstracts of the invited talks and contributed talks given (in person or by title) by members of the Association for Symbolic Logic follow.

For the Program Committee
JEFFREY REMMEL

Abstracts of invited talks

► GREGORY L. CHERLIN, *Model theory and algebraic groups*.
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We discuss three connections between model theory and the theory of algebraic groups. Model theoretic methods have been applied to diophantine problems via a model theoretic analysis of abelian varieties in categories extending the algebraic category. In the reverse direction, the structure of algebraic groups has been applied in model theoretic contexts. On one hand, the structure of compact Lie groups has been shown to exercise great control over the structure of groups definable in suitable (o-minimal) expansions of the theory of the real field, and on the other hand it has been conjectured that the simple groups arising in Zilber's analysis of uncountably categorical theories are algebraic. (Classically, one of the

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incarnations of this line of thought is the Galois theory of differential equations.) While this algebraicity conjecture remains open in general, work in this direction involves substantial connections with the theory of finite simple groups, and has applications to the study of algebraic groups as permutation groups.

► ILIJAS FARAH, *Applications of set theory to operator algebras*.
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I will present some recent applications of set theory to the structure theory of operator algebras.

► PAVEL HRUBES, *Proof complexity after NP $\neq$ coNP*.
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I will give a few motives for the study of proof complexity. I will argue that investigation into lengths of proofs would be worthwhile even if our computability conjectures were decided. This will be illustrated by problems concerning lengths of proofs in propositional logic and predicate calculus, namely the shrinkage of proofs by definitions, the use of $=$ in predicate calculus, and others.

► VICTOR W. MAREK, *Answer Set Programming*.
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Answer Set Programming (ASP) is a computational paradigm based on so-called non-monotonic logic formalisms. Typically, the user specifies a search problem $\mathcal{S}$ as a logic program, $\mathcal{P}$, (or a disjunctive program, or a default theory—depending on the underlying solver) in such a way that there is an 'easy' one-to-one correspondence between solutions to $\mathcal{S}$ and the stable models (or answer sets, or default extensions) of the program $\mathcal{P}$.

ASP is one of several formalisms grounded in foundational work, but aimed primarily at applications in computer science, computer engineering and combinatorial optimization. Like other similar areas (satisfiability, first-order and higher-order theorem proving) it aims at using fragments of logic with well-defined syntax and semantics to solve practical problems.

Like the satisfiability in propositional logic, logic programming with stable semantics captures precisely search problems in the class NP. The other formalisms mentioned above capture Sigma-2 search problems.

Answer Set Programming is now supported by a number of successful solvers, i.e., software programs finding one or more stable models (or answer sets) of the input programs. Those systems (smodels, dlv, ASSAT, NoMoRe, aspps) find applications in combinatorial optimization, computer engineering, product configuration and other applications.

We discuss the current research issues in ASP. These include studies of constructs loosely motivated by connections with integer arithmetic such as set-constraints. We also discuss connections to intermediate logics such as Goedel–Smetanich maximal nonclassical logic.

The lecture will be self-contained. The concepts of answer set semantics, set constraints, related intermediate logic etc. will be clearly outlined and the state-of-the-art of ASP presented.

► JAN REIMANN, *Measures and their random reals*.
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The duality between measures and the sets they "charge" is a central theme in modern analysis. An effective analogue of this question is: Given a real x, does there exist a (probability) measure relative to which x is effectively random in the sense of Martin-Löf (so that x is not an atom of the measure)? And if such a measure exists, can we ensure that it has certain properties (non-atomic, of a certain minimum capacity, etc.)? While every non-computable real is random with respect to some measure, there exists a nested sequence of countable Π^1_1 sets of reals that are not n-random with respect to any continuous measure. This sequence exhibits a number of interesting properties. For instance, the proof that all of the sets are countable requires the existence of infinitely many iterates of the power set of ω, similar to Borel determinacy. Furthermore, there does not seem to be a natural notion of rank for such reals.

I will first survey the basic results on continuous randomness, before discussing more recent results on continuous 1-randomness. The techniques used in the proofs are drawn from various areas of logic and analysis, such as Turing degrees, Π^1_0 classes, determinacy, fine structure of the constructible universe, or Hausdorff measures and potential theory.

► SIMON THOMAS, *A descriptive view of geometric group theory*.
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Gromov's geometric group theory seeks to classify finitely generated groups in terms of the "large scale geometry" of their Cayley graphs. In this talk, I will discuss this program from the perspective of the theory of Borel equivalence relations and point out some intriguing connections with recent work of Louveau-Rosendal on K_σ equivalence relations.

Abstracts of contributed talks

► BROOKE ANDERSEN, *A new notion of reducibility*.
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We consider Turing reduction procedures that are total on all recursively enumerable sets. This gives a (nontransitive) notion of reducibility between truth-table and Turing. Conditions are then added to guarantee transitivity, and the transitive notion is distinct from all other known reducibilities. Finally, we will discuss our ongoing work on determining the structure of the degrees with respect to this new reducibility.

► KATALIN BIMBÓ, *Semantics for positive entailment and for entailment*.
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The logic of entailment **E** and its fragments have elegant axiomatic formulations (see [1], [5] and [2]). In this talk, I introduce a consecution calculus for $\mathbf{E}_+^{tTF}$, the positive fragment of **E** that includes truth and the Church constants. The single cut rule is admissible in this calculus. Then I define a "dual gaggle" semantics (that relies on co-theories) for both this logic and **E**. (Somewhat similar semantics were defined for certain logics in [4] and [3].) When a topology is added to the structures for these logics, one can prove the variety of the algebras of the logics (with operation preserving maps) to be the dual of the class of structures for the logics (with frame morphisms).

[1] Alan R. Anderson and Nuel D. Belnap, ***Entailment. The logic of relevance and necessity***, vol. I, Princeton University Press, Princeton, NJ, 1975.

[2] ALAN R. ANDERSON, NUEL D. BELNAP, and J. MICHAEL DUNN, ***Entailment. The logic of relevance and necessity***, vol. II, Princeton University Press, Princeton, NJ, 1992.

[3] KATALIN BIMBÓ, *Relevance logics*, ***Philosophy of logic*** (Dale Jacquette, editor), Handbook of the Philosophy of Science (D. Gabbay, P. Thagard and J. Woods, editors), vol. 5, Elsevier (North-Holland), Amsterdam, 2007, pp. 723–789.

[4] KATALIN BIMBÓ and J. MICHAEL DUNN, *Relational semantics for Kleene logic and action logic*, ***Notre Dame Journal of Formal Logic***, vol. 46 (2005), pp. 461–490.

[5] J. MICHAEL DUNN, *Relevance logic and entailment*, ***Handbook of philosophical logic*** (D. Gabbay and F. Guenthner, editors), vol. 3, D. Reidel, Dordrecht, 1st ed., 1986, pp. 117–229.

► JOHN BOURKE, *Results on off-branch numbers and other invariants*.
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We consider the problem of extending certain cardinal invariants to larger cardinals. An off-branch subset of $2^{<\kappa}$ is one that has bounded intersection with each branch of the tree; the off-branch number is the least size of a maximal almost disjoint family of off-branch sets. We discuss results concerning the off-branch number and our continuing work in the area.

► JOHANNA N. Y. FRANKLIN, *Genericity and Schnorr triviality*.
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Since the Turing degrees of Schnorr trivial reals cannot be neatly characterized, it is natural to ask which properties are compatible with Schnorr triviality. Here, we will discuss the relationship between Schnorr triviality and genericity in the *tt*- and *wtt*-degrees as well as the Turing degrees. In the Turing degrees, we find that while no 2-generic real is equivalent to a Schnorr trivial real, there is a 1-generic real that has this property. However, in the *tt*-degrees, we find that no 1-generic real is equivalent to a Schnorr trivial real.

► PETER M. GERDES, *Sets with a non-uniform self-modulus*.
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For $f, g \in \omega^\omega$ denote g pointwise dominates f by $g \succ f$. Following Slaman and Groszek [1] say f is a modulus (of computation) of $X \subset \omega$ if every $g \succ f$ can compute X and that f is a self-modulus if f is a modulus for some X with $X \equiv_T f$. A modulus f of X is uniform if there is a common reduction that allows every $g \succ f$ to compute X. If X has a modulus than X is recursively encodable so by a result of Solovay [2] X must be Δ^1_1 and by [1] X must have a Δ^1_1 uniform modulus. While it is easy to construct uniform self-moduli and degrees containing no self-modulus it isn't obvious that a self-modulus that isn't a uniform self-modulus exists. We present a construction, borrowing heavily from the machinery of forcing, of just such a non-uniform self-modulus. Generalizing further for every n we construct an f such that $g \succ f \Longrightarrow g \geq_T f$ and if $h \leq_T f^{(n)}$ then h is not a uniform modulus for f. If time permits we will also discuss the general case where n is replaced with an arbitrary recursive ordinal.

[1] MARCIA J. GROSZEK AND THEODORE A. SLAMAN, unpublished.

[2] ROBERT M. SOLOVAY, *Hyperarithmetically encodable sets*, ***Transactions of the American Mathematical Society***, vol. 239 (1978) pp. 99–122.

▶ JEFFRY L. HIRST, *Reverse mathematics and Gödel's* Dialectica *interpretation* (preliminary report).
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One indispensable characteristic of Gödel's *Dialectica* interpretation is that any formula θ is provably equivalent to its *Dialectica* form (denoted θ^D). The scheme $\theta \to \theta^D$ implies the existence of Skolem functions, and so its proof clearly requires strong comprehension axioms. The converse scheme, $\theta^D \to \theta$ also requires some comprehension strength, as shown by the following.

THEOREM 1. *Over the base system* RCA_0^ω, *the scheme* $\theta^D \to \theta$ (*for arithmetical formulas* θ) *implies* ACA_0.

This talk will include a brief introduction to Gödel's *Dialectica* interpretation and will not assume extensive knowledge of reverse mathematics.

▶ MATTHEW JACKSON, *Integration and differentiation in an elementary topos*.
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If $\langle \mathcal{F}, \preceq \rangle$ is a σ-algebra, it is well known (Breitsprecher [1]) that there is an object $\mathbb{M}$ in the topos $\mathrm{Sh}(\mathcal{F})$ which can be viewed externally as the sheaf of measures.

This sheaf is described internally, using the logical machinery of the presheaf topos $\widehat{\mathcal{F}}$, the sheaf topos $\mathrm{Sh}(\mathcal{F})$, and the Grothendieck topology J (the "countable join topology") that generates $\mathrm{Sh}(\mathcal{F})$. Since it is an internal construction, it is therefore possible to find an object of j-measures in any topos $\mathcal{E}$. In the case where $\mathcal{E}$ is the topos of presheaves on some locale $\mathcal{L}$ and j is the canonical topology, the object $\mathbb{M}$ is the sheaf of continuous valuations.

This sheaf theoretic foundation for measure theory is naturally extended to include integration. In such a setting, there is a form of the Radon-Nikodym Theorem that relates the Booleanness of certain subtoposes to the existence of derivatives.

[1] SIEGFRIED BREITSPRECHER, *On the concept of a measurable space.* (I), ***Applications of sheaves (Proceedings of the Research Symposium on Application of Sheaf Theory to Logic, Algebra and Analysis)*** (University of Durham, Durham), (Michael P. Fourman, Christopher J. Mulvey and Dana S. Scott, editors) Lecture Notes in Mathematics, vol. 753 Springer, 1977, pp.157–168.

▶ ALLEN L. MANN, *Independence-friendly cylindric set algebras*.
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Independence-friendly logic is a conservative extension of first-order logic that has the same expressive power as existential second-order logic. We algebraize independence-friendly logic in the same way that Boolean algebra is the algebra of propositional logic and cylindric algebra is the algebra of first-order logic.

We will show that every independence-friendly cylindric set algebra over a structure is the expansion of a Kleene algebra. Furthermore, the class of Kleene algebras that can be expanded to an independence-friendly cylindric set algebra generates the variety of all Kleene algebras. Hence the equational theory of the class of Kleene reducts of independence-friendly cylindric set algebras is axiomatized by the axioms of Kleene algebra.

► RUSSELL MILLER, *The degree of categoricity for an algebraic number field.*
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We study computable algebraic number fields, and consider the degree of categoricity of such a field. The main negative results are that there exist such fields which are not $\mathbf{0}'$-categorical, and that an algebraic number field can have a splitting algorithm without being computably categorical. The positive result is that if R is a splitting algorithm for a computable algebraic number field F, then every computable field $\tilde{F}$ isomorphic to F is isomorphic to it via a map f with $f' \leq_T R'$. That is, the degree of categoricity of F is low relative to R. Since the splitting algorithm is always computable from $\mathbf{0}'$, this means that every computable algebraic number field is categorical in a degree whose jump is $\leq_T \mathbf{0}''$.

► GRIGORI MINTS AND TOMOHIRO HOSHI, *Parameterization in monadic logic*.
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A logical formula $F(\mathbf{X}, \mathbf{P})$ is treated here as an equation to be satisfied by solution $\mathbf{X}_0(\mathbf{P})$ for unknown predicate symbols $\mathbf{X}$ (with parametric predicate symbols $\mathbf{P}$). We find general solution for monadic predicate logic with equality. Elimination of quantifiers provides necessary and sufficient solvability conditions of the form $\vee_{i\leq m} R_i(\mathbf{P})$ with $R_i(\mathbf{P}) = \wedge_s Q_{i,s} x P_s(x)$. Here s ranges over all possible *states* $P_1^{\sigma_1}(x) \wedge \cdots \wedge P_n^{\sigma_n}(x)$ and $Q_{i,s}$ is $\exists_{=k_s}$ or $\exists_{\geq k_s}$. The general solution consists of the components $\mathbf{X}_0^i(\mathbf{P})$, $i \leq m$ such that

$$R_i(\mathbf{P}) \rightarrow F(\mathbf{X}_i^0(\mathbf{P}), \mathbf{P}) \tag{1}$$

is valid. $\mathbf{X}_0^i(\mathbf{P})$ contains ϵ-terms for $\exists$-quantifiers in R_i. This solves a parameterization problem stated by J. McCarthy (http://www-formal.stanford.edu/jmc/parameterize.html).

The components are glued together in a standard way:

$$X_0 := (R_1 \wedge X_0^1) \vee (R_2 \wedge \neg R_1 \wedge X_0^2) \vee (R_m \wedge \wedge_{1\leq i<m} \neg R_i \wedge X_0^m)$$

X_0 is a solution: $\vee_i R_i \rightarrow F(X_0, P)$, since $R_i \rightarrow (X_0 \leftrightarrow X_0^i)$, and (1).

X_0 is a general solution: for every state s and $i \leq m$ there is a substitution σ_i for the predicates occurring in X_0 but not in $\mathbf{P}$ (arbitrary constants) that turns X_0 into X_0^i in state P_s provided R_i holds. This implies

$$\vee_i R_i \rightarrow \vee_i (\sigma_i(X_0) \leftrightarrow X_0^i)$$

since $\vee_s P_s$ is valid.

► CARL MUMMERT AND FRANK STEPHAN, *Topological aspects of poset spaces*.
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Several contemporary research programs are interested in the computable or constructive content of general topology. Our work establishes a tight link between two of those programs, in Reverse Mathematics and Domain Theory, by showing they study the same class of second-countable spaces: the T_1 spaces with the strong Choquet property. This class of spaces has robust descriptive set theoretic properties but is not yet completely understood. The talk will also describe the relationship between these spaces and the "formal spaces" studied in the program of Formal Topology.

► ERMEK S. NURKHAIDAROV, *On maximal subgroups of the automorphism group of a countable arithmetically saturated model of PA*.
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Kossak-Kotlarski-Schmerl [1] shows that the stabilizer of an unbounded element a of a countable recursively saturated model of Peano Arithmetic M is a maximal subgroup of $\mathrm{Aut}(M)$ iff the type of a is selective. We extend this result by showing that if M is a countable arithmetically saturated model of Peano Arithmetic, Ω is a very good interstice, and $a \in \Omega$, then the stabilizer of a is a maximal subgroup of $\mathrm{Aut}(M)$ iff the type of a is selective and rational.

[1] R. Kossak, H. Kotlarski, and J. H. Schmerl, *On maximal subgroups of the automorphism group of a countable recursively saturated model of PA*, ***Annals of Pure and Applied Logic***, vol. 63 (1991), pp. 125–148.

► SARA QUINN, *Turing computable embeddings into p-groups*.
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The notion of a *Turing computable embedding* is an effective analogue of the Borel embeddings used in descriptive set theory to compare classes of countable structures. This notion was explored in [1]. One line of inquiry from [1] is to give general characteristics for the classes that Turing computably embed into some mathematically interesting classes of structures (finite prime fields, finite linear orderings, vector spaces over the rationals, and a particular class of equivalence structures). I have continued this line of inquiry, and in this talk I will present new results characterizing the classes that Turing computably embed into classes of reduced Abelian p-groups of various lengths.

[1] J. F. Knight, S. Miller, and M. Vanden Boom, *Turing computable embeddings*, ***The Journal of Symbolic Logic***, vol. 72 (2007), pp. 901–918.

► DIMA SINAPOVA, *A model with a very good scale and a bad scale*.
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The relationship between the Singular Cardinal Hypothesis, Jensen's square principle, very good scales and large cardinals is important in singular cardinal arithmetic and in understanding how much the universe is "L-like". Recently two old questions were answered by a paper of Gitik and Sharon: that failure of *SCH* does not imply weak square, and the existence of a very good scale does not imply weak square.

This paper was inspired by work of Cummings and Foreman, who, elaborating on the Gitik-Sharon paper, described a model in which κ has cofinality ω, *SCH* fails at κ and there is a very good scale and a bad scale at κ. The existence of a bad scale implies the failure of weak square. The natural question is whether we can get the same result for larger cofinalities; it turns out that we can.

The main result we will present is that if in the ground model *GCH* holds, κ is a supercompact cardinal and λ is a regular cardinal less than κ, then there is a generic extension in which the cofinality of κ is λ, *SCH* fails at κ, there is a very good scale at κ, and there is a bad scale at κ. The forcing we use combines ideas from Magidor's forcing for changing cofinalities of cardinals and the forcing described in the Gitik-Sharon paper.

[1] JAMES CUMMINGS AND MATT FOREMAN, *Marginalia to a theorem of Gitik and Sharon*, talk.

[2] MOTI GITIK AND ASSAF SHARON, *On SCH and the approachability property*, ***Proceedings of the American Mathematical Society***, not yet published.

[3] THOMAS JECH, ***Set theory***, Springer Monographs in Mathematics, Springer-Verlag, 2003.

[4] MENACHEM MAGIDOR, *Changing cofinality of cardinals*, ***Fundamenta Mathematicae***, vol. 99, no. 1, pp. 61–71.

► SANDRO SKANSI, *On Russell's mathematical philosophy*: *Is also set theory a part of philosophy*?

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What Russell in 1919 called 'mathematical philosophy' is basically set theory in the modern usage of the term. A point in which they differ could be equated with the difference between ZFC and nonstandard set theory today, Russell's approach, influenced by his type theory, being the nonstandard one, much like present day NF. In the years following Russell's book, there have been some major breakthroughs in whole field of logic, let alone set theory; it is our intention to show that most of these results do not influence Russell's ideas about mathematical philosophy in any essential way. This all will serve as an extended introduction to the main question of the exposition: does set theory belong to philosophy, i.e., is mathematical philosophy philosophy proper? Our aim will be to show, relying to some extent on Russell's arguments, that it belongs just as much to philosophy as it belongs to mathematics.

► DAN E. WILLARD, *On the Second Incompleteness Theorem and empirical stability*.

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We have published a series of papers about generalizations and boundary-case exceptions for Godel's Second Incompleteness Theorem [1, 2, 3, 4, 5, 6, 7]. The goal of this research was to identify when an axiom system is suitably weak so that it can verify its own consistency. The present talk will identify a common denominator that has characterized all our boundary-case exceptions to the Second Incompleteness Theorem. Thus for a fixed superscript ξ, the tuple $(\Delta_0^\xi,\ L^\xi,\ B^\xi,\ d)$ will be called a **"Transitional"** when:

1. Δ_0^ξ denotes some well-defined set of fully-bounded formulae,
2. L^ξ denotes an arithmetic language containing the constant symbols of "0" and "1".
3. B^ξ denotes a **"Base Axiom System"** that is valid in the standard arithmetic model and which is able to prove every valid Σ_1 like sentence in the language of L^ξ, and
4. d denotes some method of deduction.

The analogs of Π_1 sentences in the language L^ξ will be called Π_1^ξ sentences. Also, the Π_1^ξ sentence $\forall v\ \psi(v)$ is called **Good(N)** whenever $\psi(a)$ is valid for every integer a bounded by 2^N. In this context, the tuple $(\Delta_0^\xi,\ L^\xi,\ B^\xi,\ d)$ will be called **Empirically Stable** whenever the following invariant holds:

> If α is a set of Good(N) Π_1^ξ sentences and Φ is a Π_1^ξ theorem of $B^\xi + \alpha$ whose proof under deduction method d has a length of L then Φ is Good$(N - L)$.

Our unification of the diverse results of [1]–[7] will state that whenever $(\Delta_0^\xi,\ L^\xi,\ B^\xi,\ d)$ is empirically stable, one can construct an axiom system $\beta \supset B^\xi$ where β is able to prove a theorem affirming the non-existence of a proof of $0 = 1$ from itself under the deduction method d. (This result is only a *boundary-case exception* to the Second Incompleteness Theorem because **MOST BUT NOT ALL** ξ fail to be empirically stable.)

[1] D. WILLARD, *Self-verifying systems, the Incompleteness Theorem and tangibility*, ***The Journal of Symbolic Logic***, vol. 66 (2001) pp. 536–596.

[2] ———, *How to extend the semantic tableaux and cut-free versions of the Second Incompleteness Theorem almost to Robinson's Arithmetic Q*, ***The Journal of Symbolic Logic***, vol. 67 (2002), pp. 465–496.

[3] ———, *An exploration of the partial respects in which an axiom system recognizing solely addition as a total function can verify its own consistency*, ***The Journal of Symbolic Logic***, vol. 70 (2005), pp. 1171–1209.

[4] ———, *On the available partial respects in which an axiomatization for real valued arithmetic can recognize its consistency*, ***The Journal of Symbolic Logic***, vol. 71 (2006), pp. 1189–1199.

[5] ———, *A generalization of the Second Incompleteness Theorem and exceptions to it*. ***Annals of Pure and Applied Logic***, vol. 141 (2006), pp. 472–496.

[6] ———, ***The axiom system $I\Sigma_0$ manages to simultaneously obey and evade the Herbrandized version of the Second Incompleteness Theorem***, Electronic Notes in Theoretical Computer Science, vol. 165, 2006, pp. 213–226.

[7] ———, *Passive induction and a solution to a Paris-Wilkie open question*, ***Annals of Pure and Applied Logic***, vol. 146 (2007), pp. 124–149.

► BRIAN WYNNE, *More on decidable theories of l-groups of continuous functions*.
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A P-point is a point in a completely regular space X such that every $f \in C(X)$ is constant on some neighborhood of the point. A P-space is a completely regular space all of whose points are P-points and an essential P-space is a completely regular space with at most one non-P-point. P-spaces were named by Gillman and Henriksen [1] and essential P-spaces by Henriksen and Osba [2].

Let PEP be the class of essential P-spaces with one non-P-point and let PEP_δ be the class of PEP-spaces whose non-P-point is a G_δ. The elementary theories of the l-groups $C(X)$ with $X \in PEP_\delta$ may often be reduced to that of an associated Boolean algebra with distinguished ideal. In [3] it is shown, via the aforementioned reduction, that various classes of $C(X)$ with $X \in PEP_\delta$ have decidable theories. Continuing this work we establish similar results for $C(X)$ with $X \in PEP - PEP_\delta$, where the associated structures are now Boolean algebras with two distinguished ideals.

[1] L. GILLMAN AND M. HENRIKSEN, *Concerning rings of continuous functions*, ***Transactions of the American Mathematical Society***, vol. 77 (1954), pp. 340–362.

[2] M. HENRIKSEN AND E. A. OSBA, *Essential P-spaces: a generalization of door spaces*, ***Commentationes Mathematicae Universitatis Carolinae***, vol. 45 (2004), no. 3, pp. 509–518.

[3] BRIAN WYNNE, *Decidable theories of non-projectable l-groups of continuous functions*, ***Annals of Pure and Applied Logic***, vol. 146 (2007), no. 1, pp. 21–39.

Abstracts of papers presented by title

► ADIB BEN JEBARA, *About space and time in quantum mechanics*.
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We apply set theory with urelements ZFU to physical space, we consider locations as urelements, elements of U. U_i is a subset of U with number of elements n. $X_i U_i$ is the infinite cartesian product and a set of paths. Let us consider the set of paths of all elementary particles-locations which number is n. If n is greater than m in CC (2 through m), countable choice for k elements sets $k = 2$ through m, the set of paths will be the void set. So, physical space would become void, the universe would collapse and a Big Crunch would happen. But the matter would have to go somewhere and indeed the Big Bang happened. So, n is indeed greater than m.

Let us start the set theory ZFU with two infinite sets of urelements U_1 and U_2. Mr. Andreas Blass pointed out that their union is U, the usual set. Let physical space at the level of elementary particles be U_1 and time be U_2. As U_2 is not linearly ordered, there is no backwards time causality. In quantum mechanics, there are waves which go backward time, see : `http://www.npl.washington.edu/AV/altvw08.html`.

But if U_2 is time and is not linearly ordered, there is no traveling backward time. So, our notion of causality is less jeopardized than with backward time causality.

Maybe using $U_1 \times U_2$ for space-time would be still better.

My idea is that Dedekind cardinals are cardinalities for space and for time. For instance, the time elapsed since 36 Big Crunches/Big Bangs ago is a Dedekind cardinal. The cardinality of the physical space of the previous universe (before the Big Bang) is a Dedekind cardinal.

The negation of the axiom of choice is really true because it can be applied in physics.

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This largely expository lecture deals with aspects of traditional solid geometry suitable for applications in logic courses. A *regular* polygon has equal sides and equal angles. A *subregular* polyhedron has congruent faces and congruent [polyhedral] angles. A subregular polyhedron whose faces are all regular polygons is *regular*. Geometers before Euclid showed that there are "essentially" only five regular polyhedra: every regular polyhedron is a tetrahedron, a hexahedron or cube, an octahedron, a dodecahedron (12 faces), or an icosahedron (20 faces). The first question is whether there are subregular polyhedra that are not regular. Another question is the classification of subregular polyhedra. For example, considering the fact that the regular tetrahedra have equilateral triangles as faces, we ask which triangles other than equilaterals are faces of subregular tetrahedra. Similarly, considering the fact that the regular hexahedra have squares as faces, we ask which quadrangles other than squares are faces of subregular hexahedra. After introductory remarks that include historical and philosophical points, we concentrate on tetrahedra. A triangle that is congruent to each of the four faces of a tetrahedron is called a *generator* of the tetrahedron. The main result proved is that every acute triangle is a generator of a subregular tetrahedron. The proof includes an algorithm—implementable with scissors and paper—that constructs from any given acute triangle a subregular tetrahedron whose faces are congruent to the given triangle. Algorithm: Given any acute triangle. Construct a similar triangle whose sides are double the sides of the given triangle. Draw the three lines connecting the three midpoints of the sides (making four triangles congruent to the given triangle—a central triangle surrounded by three peripheral triangles). Make three "hinges" along the lines connecting the midpoints. "Fold" the peripheral triangles together (into a tetrahedron).