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REVIEWS

The Association for Symbolic Logic publishes analytical reviews of selected books and articles in the field of symbolic logic. The reviews were published in ***The Journal of Symbolic Logic*** from the founding of the JOURNAL in 1936 until the end of 1999. The Association moved the reviews to this BULLETIN, beginning in 2000.

In a review, a reference "JSL XLIII 148," for example, refers either to the publication reviewed on page 148 of volume 43 of the JOURNAL, or to the review itself (which contains full bibliographical information for the reviewed publication). Analogously, a reference "BSL VII 376" refers to the review beginning on page 376 in volume 7 of this BULLETIN, or to the publication there reviewed. "JSL LV 347" refers to one of the reviews or one of the publications reviewed or listed on page 347 of volume 55 of the JOURNAL, with reliance on the context to show which one is meant. The reference "JSL LIII 318(3)" is to the third item on page 318 of volume 53 of the JOURNAL, that is, to van Heijenoort's *Frege and vagueness*, and "JSL LX 684(8)" refers to the eighth item on page 684 of volume 60 of the JOURNAL, that is, to Tarski's *Truth and proof*.

References such as 49***5*** or 280***1*** are to entries so numbered in *A bibliography of symbolic logic* (the JOURNAL, vol. 1, pp. 121–218).

TORKEL FRANZÉN. ***Inexhaustibility: a non-exhaustive treatment.*** Lecture Notes in Logic, vol. 16. Association for Symbolic Logic, A K Peters, Ltd., Wellesley, Massachusetts, 2004, xi + 251 pp.

TORKEL FRANZÉN.. *Transfinite progressions: a second look at completeness*. ***The Bulletin of Symbolic Logic***, vol. 10, no. 3 (2004), pp. 367–389.

Both the book and the paper concern a phenomenon that Torkel Franzén calls *inexhaustibility of mathematical knowledge*. He describes it as a certain positive aspect of Gödel's incompleteness theorems (***Inexhaustibility***, p. 10): "we have a way of extending any system of axioms for mathematics that we recognize as correct to a logically stronger system that we will also recognize as correct, namely by adding the statement that the old system is consistent. ... Thus, our mathematical knowledge would appear to be inexhaustible in the sense that it cannot be pinned down in any one formal axiomatic theory."

If one takes the concept of (mathematical) knowledge seriously, many aspects of the inexhaustibility phenomenon appear rather puzzling. For example, one can ponder what is it specifically that makes our knowledge inexhaustible, why is it impossible to pin down exactly which axioms we recognize as correct? On the other hand, one can ask what would happen if one simply adds a consistency assertion as a new axiom to a given system and repeats this process ad infinitum. Would the limits of our knowledge be reached by such a process? In the book under review, the author explores many of such questions and scrutinizes the various concepts and ideas involved. Thus, it is a book on epistemological and foundational *interpretations* of mathematical results around Gödel's incompleteness theorems, rather than simply an exposition of these results.

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In the main focus of the book are the so-called transfinite progressions of theories obtained by repeatedly adding to some initial theory its consistency assertion (or some stronger reflection principle). Alan Turing in 1939 was the first to introduce and systematically study this subject. Later, it was set on a fully rigorous basis and further developed in a classical paper by Solomon Feferman, *Transfinite recursive progressions of axiomatic theories*, JSL, vol. 21, no. 3 (1962), pp. 259–316. However, due to rather heavy technicalities, the details of these results—and one can also say, their meaning—is not so widely known.

Torkel Franzén explains a philosophy behind the study of such progressions following rather closely, as far as I can judge, the original views of Kreisel and Feferman. In a sense, the book can be considered as an extended commentary on Feferman's papers. However, the author makes explicit many intuitions and claims that were only implicit or just briefly touched upon there. Also, he often puts his own emphasis on various particular aspects. Among such strongly emphasized points one can mention, for example, the fact that Gödel's theorem does not really show a 'weakness' of any formal theory. Rather, it shows the inability of a formal system even to *consistently postulate* its own consistency—which is a peculiar feature of the consistency assertion.

Another such point is the role of non-standard definitions of sets of axioms of formal systems that appear in the proofs of Turing and Feferman completeness results for transfinite progressions. Franzén rightly argues that because of the use of non-standard definitions these completeness results have little epistemological significance. In contrast, the notion of autonomous progression appears to be more suitable for the analysis of acquisition of mathematical knowledge.

With great skill, Torkel Franzén presents some technical material in a form accessible to a philosophically rather than mathematically minded reader. He starts at a very basic level, beginning with a discussion of the main concepts of mathematical logic such as truth, provability and computability. He discusses the relation between formal and actual (informal) mathematical proofs and illustrates it by sketching a formalization of some results on prime numbers in PA.

The first twelve chapters of the book are an exposition of Gödel's incompleteness theorems and of various aspects thereof. The material is mostly standard, but is presented in an rather original way. I would mention here a nice informal treatment of truth-definitions for various languages, which is lacking in most textbooks of a comparable level. For example, the author shows the bi-interpretability of a second order arithmetic ACA and a natural extension of PA by a truth-predicate. He also presents some more recent and less standard results on local reflection principles, e.g., his own proof of the so-called Goryachev theorem.

Chapters 13–15 deal more directly with transfinite progressions. Feferman's completeness theorem is presented in a certain weaker but technically easier form than the original result (completeness w.r.t. Π^0_2-sentences). This deficiency is compensated in the accompanying survey article where the reader can find a full streamlined and readable proof of Feferman's completeness result.

Introducing explicitly the notion of knowledge in the discussion of the meaning of incompleteness theorems seems an inevitable step. In fact, the words 'mathematical' or 'arithmetical knowledge' appear quite often in the non-technical parts of the book. However, Franzén does not try to explicate in any detail the intuitive notion of knowledge he is referring to. To continue the line of thought taken in this book, it could be fruitful to develop an appropriate notion of knowledge on a more formal level.

This thought-provoking book and the accompanying paper can be recommended to anyone interested in deeper understanding of the incompleteness phenomenon.

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Rudolf Taschner. ***The Continuum.*** Friedrich Vieweg & Sohn Verlag, Wiesbaden, Germany, 2005, 136 pp.

Rudolf Taschner's idiosyncratic book is simultaneously charming and annoying. Entitled ***The Continuum***, it is an introduction to constructive analysis. The topics are the real numbers (a definition thereof and their basic properties), metric spaces (with such properties as completeness, compactness, locatedness, and separability), and continuous functions (including pointwise and uniform continuity, and sequences of such functions).

The next fact a potential reader would likely want to know is who the intended audience is. The preface indicates that these are students, advanced undergraduate and graduate. There are two problems with this, though. For one, there are no exercises. Even worse, there is virtually no discussion of constructivism. That is, in the introductory chapter there are just over two pages on this topic (pp. 13–15). There the essence of the matter is described as follows: "The salient point of "Brouwer's new intuitionism" can be concentrated into one sentence: He replaced Dedekind cuts, i.e., vertical *lines* in the Farey table [rationals], by vertical *stripes*" (p. 13). And that, we are told, is Brouwer's intuitionism. Excluded middle is not mentioned even once, nor are its consequences that constructivists routinely consider, such as Markov's Principle (If a decidable property of the natural numbers is not always false then it's somewhere true.). There is no discussion about the meaning of existence proofs. The limited principle of omniscience (LPO: Every binary sequence is all 0s or has a 1.) is the single exception, being mentioned once (p. 70), in the only Brouwerian counter-example in the text (If all subsets of a metric space are located, then LPO holds.), and even then LPO is not referred to by name. A student could learn constructivism from this book neither by explicit instruction nor by seeing it in action. (Of course, if the constructivism is supposed to be secondary, then many of the proofs and notions here will seem unnecessarily complicated.)

Moving on to the substance of the book, that is where the charm begins. The first topic is the Farey fractions, which is a way of generating the rationals by how a third grader would like to add fractions: the Farey mediant of $p\backslash n$ and $q\backslash m$ is $p + q\backslash n + m$. This construction is then found in the pentagram, which is connected with the Fibonacci numbers, continued fractions, and certain square roots, culminating in Dedekind cuts. This mathematics is quite interesting, rich in content, easily accessible, and not so well known, and so, while not any of the mainstream approaches to the foundations of analysis, it is well worth one's time.

A similar *fin de siècle* elegance is also provided by an air of old-fashioned, Old World stateliness. This starts with the title, which is a self-conscious reference to Hermann Weyl's "Das Kontinuum" from 1918, almost a century ago. The photographs of Weyl and Brouwer at the beginning (pp. vi–vii), in which even Hermann Hesse makes an appearance, put one in a historical frame of mind. The brief history of the subject seems to end with Weyl in 1927. Two sentences are dedicated to the later work of Errett Bishop, Douglas Bridges, and Bishop's "American school of constructivism"; this more recent tradition is then dismissed with an approving citation of an imaginary mathematician's rejection of "hard labor and fewer results," so that the author can return to Brouwer's intuitionism (p. 15). And so we remain in interregnum Europe.

A different kind of charm is afforded by the introduction to Dedekind cuts. While referring to Dedekind's published work on the subject, mention is made of the publisher of one of his booklets in the body of the text (p. 9), in contrast to normal modes of attribution (footnotes and endnotes) and unique even in this book. The reason for this unusual decision is, presumably, that the publisher so referred to just so happens to be the publisher of the current work. This is of course completely unprofessional, but rather than being unpleasant, comes across as charming in an amateurish kind of way. Also contributing to this sense of amateurness are the diagrams, two of which have appeared already by this point (pp. 4 & 7). The quality of their printing is really lousy. Better output can be produced with TEX, for instance. Other inexplicable layout decisions similarly distract when reading. For instance, proofs are introduced with the boldface word **Proof**. The corresponding action is not taken, though, when introducing the statements of theorems. Sure, named theorems do have their names in bold beforehand, and the statements of all theorems are italicized. But what happens when coming across an italicized assertion unflagged as a theorem or a lemma or something is that one immediately wonders why this particular statement is being emphasized, and then does a double-take when one realizes what's going on. Ignoring this nearly universal and quite sensible standard is jarringly amateurish.

The same sense that the author doesn't quite know how it's done comes across in the use of language. By way of praise, most of the time the language is idiomatic and does not distract from the content; since the author and both of his language assistants seem to be non-native English speakers, that's quite an accomplishment. Every once in a while, though, there is something strange that you just can't help stumbling over. The identity function is called "the identical function" (p. 96) (and more than once—see p. 100 and the index too—so it's not a typo). In the section entitled "topological concepts" (p. 78), the cover of a set (of points of a metric space) is defined to be what's usually called the closure of a set, and has nothing to do with open covers. He sometimes talks about detaching a real number (p. 81), and I still don't know what that means.

Often mathematicians abuse language and notation to make it easier on the reader, and not uncommonly one finds a phrase or symbol introduced with the apology "by abuse of notation." In this case, the author chose to give this apology in French—"*par abus de langage*"—even though it can be perfectly well expressed in English, as is commonly done. I assume this was done that way by reference, even if unintended, to Bourbaki, who first introduced the phrase, naturally enough in the language in which they were writing. Doing so fits in well with the historical atmosphere of the book, even if it anachronistically puts us in the postwar period. But the intended reader, a beginning English-speaking student, will know neither French nor the phrase in English to enable them to figure the translation out, leaving them to guess it has something to do with languishing on a bus. Furthermore, instead of indicating that the phrase is foreign with italics, as is typical, it is enclosed in quotes, and sometimes set off with dashes too for good measure, as in: "The assignment that appoints to each argument x one and the same value y_0 is of course a function that can be defined on every set. It is called a *constant* function which—"par abus de langage"—can be identified with its only value y_0" (p. 96).

Occasionally the phrasing is very cumbersome. Consider for instance the statement of the following theorem (p. 99):

THEOREM 1. *Suppose that the function* $f : U \to T$ *and the point* $\xi \in U$ *fulfill the following condition*: *For each real number* $\varepsilon > 0$ *it is possible to construct a real number* $\delta > 0$ *such that for all* $x \in U$ *the inequality* $\|x - \xi\| < \delta$ *implies the inequality* $\|f(x) - f(\xi)\| < \varepsilon$. *Then* f *is continuous at the point* ξ.

While this looks to be the exact definition of continuity at a point, and so seems to be a definition and not a theorem, the content of this statement as a theorem is actually in order, as the definition of continuity in the text is non-standard. (Continuity was defined on the cover—that is, closure—of a set, so that a function f can be continuous at a point where it's not even defined, apparently so that f can be extended to these points and—"par abus de langage"—the same name used for the extended function. Why this benefit outweighs the disadvantage of using a notion of continuity unique in the literature was not explained.) My purpose in repeating this statement is to contrast it to the much lighter:

THEOREM 2. *For* $f : U \to T$ *and* $\xi \in U$, *suppose that for all* $\varepsilon > 0$ *there is a* $\delta > 0$ *such that* $\|x - \xi\| < \delta$ *implies* $\|f(x) - f(\xi)\| < \varepsilon$. *Then* f *is continuous at* ξ.

The only thing the extra verbiage adds to the double-length original version is weight.

Returning finally to the substance of the book, after the introduction the real numbers are defined. They are defined, essentially, as numbers that are given by our standard, base ten, decimal representation. No explanation for this choice is given. As a mathematician, I find binary, or signed binary (whereby -1 is also allowed as a digit—much more sensible constructively), or arbitrary Cauchy sequences of rationals, natural, and base ten not. A reason for this unnatural choice should have been given. As for the rest of the chapter, it is quite in order that the mathematics is not advanced, given the intended audience. The downside is that the reader is presented with page after page of tedious verification of statements that look like trivialities, and little else. The theorems are the basic properties of $>$ and $\geq$ (transitivity, irreflexivity in the former case, and such like), absolute value, the triangle inequality (in three versions no less), equality and apartness, convergence, and so on. Perhaps the most frustrating part of this is that the differences with the classical theory, just the thing that might intrigue a reader, are nowhere brought out.

I could go into comparable detail about the remaining two chapters, but the upshot will be the same. With the essence of the constructivism hidden, the text reads like unnecessarily difficult proofs of things you would find in any standard, classical text, often just left to the reader there, except for those notions, inherently constructive, that, in this context, just make no sense at all.

If one wanted an introduction to constructive analysis, there are any of a number of other texts that stand up well against the current one. Some of the best known are Michael Beeson's ***Foundations of Constructive Mathematics*** (Springer, Berlin-Heidelberg-New York, 1985), Errett Bishop's ***Foundations of Constructive Analysis*** (McGraw-Hill, New York, 1967), Errett Bishop's and Douglas Bridges's ***Constructive Analysis*** (Springer, BerlinHeidelbergNew York, 1985), Douglas Bridges's and Fred Richman's ***Varieties of Constructive Mathemataics*** (Cambridge University Press, Cambridge, 1987), and Anne Troelstra's and Dirk van Dalen's ***Constructivism in Mathematics***, vol. 1 (North Holland, Amsterdam-New York-Oxford-Tokyo, 1988). Perhaps the most apt comparison can be made with Douglas Bridges's and Luminiţa Vîţa's ***Techniques of Constructive Analysis*** (Springer, Berlin-Heidelberg-New York, 2006), being dedicated solely to the same subject and having appeared just about a year after the text under review. Authored by arguably the current leading constructive analyst and a student of his, it does in 47 pages what ***The Continuum*** does in 128, plus a lot more: exercises, a section on constructive logic, a fuller history, more notions defined and principles identified. To say nothing of the remaining 153 pages. Admittedly parts of this book would be rough going for the audience in question of introductory students. But I'd rather have my students struggle with advanced, inherently difficult material than with tedium.

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NATHAN SALMON. ***Metaphysics, mathematics, and meaning.*** Collected papers, vol. 1. Oxford University Press, Oxford, 2005, xiv + 419 pp.

Volume 1 of Nathan Salmon's collected papers contains nineteen papers written from 1984 to 2005, two of which are previously unpublished. The chief topics are the metaphysics of modality and semantics. Many of the papers are required reading for those with either interest.

The previously unpublished *Modal logic Kalish-and-Montague style* presents natural deduction systems for $S5$, $S4$, B, and T. Chapter 6 reprints Salmon's substantially negative review of David Lewis's ***The plurality of worlds***. *The limits of human mathematics* discusses the philosophical significance of Gödel's Incompleteness Theorems.

Tense and intension presents a four-layered semantic theory (akin to David Kaplan's three-layered account) with the fourth layer designed to accommodate the double temporal relativity that occurs in contexts such as in 'Sometimes the present President is a Republican' and thus allow propositions to be taken as eternal. *Pronouns as variables* defends Peter Geach's claim that anaphoric pronouns (excepting pronouns of laziness) are bound variables.

The three papers *Existence*, *Nonexistence*, and *Mythical objects* are closely related. The first argues against Quine's slogan "to be is to be the value of a variable." Salmon argues *contra* Kant, that existence is a property and that we need to allow variables to range over non-actual and even impossible objects in order for the semantics of natural language to work out. *Nonexistence* takes up the problem of nonreferring proper names. Denials that the referent of such names exist are argued to be ambiguous, with a reading on which they are true, and one on which they lack truth value. An account of discourse about fictional entities is developed and 'accidental fiction' is introduced to cover cases of inadvertent failure of reference. In both of the first two papers an account of propositions is developed on which sentences that contain names referring to impossible objects express possible propositions that do not themselves exist, yet some of them are true. *Mythical objects* exploits the notion of accidental fiction to address Peter Geach's problem of how to make sense of Hob and Nob having thoughts about the same witch, given that there are no witches.

Impossible worlds, *The logic of what might have been*, and *This side of paradox* all discuss an argument of Salmon's (derived from earlier work of Hugh Chandler and Roderick Chisholm) for the conclusion that the S4 axiom $\Box\phi \rightarrow \Box\Box\phi$ has false instances. The argument proceeds as follows: let t be a table, made from some particular hunk of matter h_1. On non-essentialist views regarding material composition there will be other hunks of matter sufficiently similar to h_1 such that t could have been constructed out of them. But not any hunk will do. So, let h_3 be a hunk that overlaps with h_1 but is just slightly too different from h_1 for it to be possible that t was constructed from h_3. It is thus necessarily true that t was not made from h_3. Now, let h_2 be a hunk of matter intermediate between h_1 and h_3. It is, by hypothesis, possible that t had been constructed from h_2. But had it, it would have been possible that the table had been formed from h_3. So, while it is necessary that the table was not formed from h_3, it is not necessarily necessary that it was not.

Salmon draws a number of morals from this argument. First, in *Impossible worlds*, he concludes there are possible worlds that are impossible relative to the actual world. Second, in *This side of paradox* (itself devoted to a discussion of a related argument of Timothy Williamson's) Salmon rejects the principle (W) that the differences between two artifacts constructed in two possible worlds from sufficiently similar hunks of matter (such as h_1 and h_2) are small enough that they stand in the relationship of trans-world identity. He prefers principle (A$'$) to the effect that given an artifact fashioned from one hunk of matter, there could not have been a distinct artifact fashioned from a sufficiently overlapping hunk. The reason Salmon denies (W) is clear—if we accept it and the transitivity of identity, we would have to say that the possible table formed from h_3 is trans-world identical to the actual table t formed from h_1 as t could have been formed from h_2, and any table formed from h_2 could, it its turn, have been formed from h_3. On the other hand, if we deny that the possible table formed from h_2 stands in a relation of trans-world identity to t, it is no longer clear that the existence of a table manufactured from h_2 in some possible world shows that t could have been formed from h_2. Furthermore, it is unclear to me how the denial that the possible table fashioned from h_2 is distinct from t differs from the claim that t and that possible table are trans-world identical. Third, *The logic of what might have been* deploys the anti-$S4$ argument coupled with a conception of possible worlds as maximal scenarios to argue that some worlds are absolutely impossible worlds (rather than merely impossible relative to the actual world). An example such world is one in which Nathan Salmon is a Visa credit card. Worlds reflecting violations of logical consistency are another class of such impossible worlds.

The paper defends these views by insisting on a distinction between a way for things *to be* and a way things *might have been*. Thus, the impossible world with a Visa card that is also a well-know philosopher is counted as merely a way for things to be. While the distinction seems fruitful—it is necessarily true that there is no largest prime, but it seems coherent to say that there is a way that things would have been were there, even though that way is not possible—it is open to question whether 'Salmon is a Visa card' is sufficiently coherent so as to count as a specification of a way for things to be at all.

The short *The fact that* $x = y$ provides a proof that if x and y are possible individuals in two different possible worlds, then if $x = y$ there is no qualitative fact about x and y that makes this so. The argument is simple: there is no qualitative fact about x in virtue of which $x = x$. Thus, if $x = y$, then by the indiscernability of identicals, there is no such fact about y in virtue of which $x = y$, either. *Identity facts* deploys a similar argument for the conclusion that there are no objects that are indeterminately identical. The core idea (Salmon discusses several variants) is easily stated. Let $\langle x, y\rangle$ be a pair of objects that are indeterminately identical. It is determinate that the members of the pair $\langle x, x\rangle$ are identical. But then the pairs $\langle x, x\rangle$ and $\langle x, y\rangle$ are distinct. Thus, $x \neq y$ and there is a determinate identity fact after all. This proof is a *reductio* and that allows the indeterminate identity theorist to complain that the appeal to bivalence is illicit. Much of the paper consists of Salmon's discussion of such a response by Terence Parsons.

The previously unpublished *Personal identity: What's the problem?* starts from a case of brain transplantation from a Woody Allen joke. Salmon exploits this and related cases to make a number of distinctions in modal metaphysics such as ones between different grades of Haecceitism. Ultimately, he argues for a brain essentialism solution to the problem of personal identity.

Whole, parts, and numbers considers the problem of how many oranges there are on a table which had three oranges until one was cut in half and eaten. Considerations are adduced against all of the answers 2, 2.5, and 3. Salmon himself tentatively endorses 2.5, seeing it as a property of the plurality of oranges and thus suggests that we might adopt plural quantification to accommodate this. A further consequence that Salmon claims to find is that numerical quantifiers such as '2' are then nonextensional. This is supposed to be so since 'There are exactly 2 whole oranges' is true, yet 'There are exactly 2 oranges' is false, as there are exactly 2.5. This, however, only counts against the extensionality of '2' if we take, as Salmon does, the phrases 'whole orange' and 'orange' to be extensionally equivalent. It seems more natural to me to take the difference in truth value between the two sentences not as a sign that the quantifier is nonextensional, but as a sign that 'orange' and 'whole orange' are not extensionally equivalent.

'On Designating' takes up the interpretive challenges posed by the 'Gray's *Elegy*' passage of Russell's *On denoting*. At nearly 50 pages it is a formidable paper. Much of it consists of a close reading of the passage, together with a 'translation' aimed at resolving the notorious ambiguity in Russell's use of quotation marks. Salmon rejects the interpretive positions that Russell in that passage was attacking Frege or his own earlier view in the ***The principles of mathematics***. Instead, he suggests that the specific target of the passage was the view that a definite description has a semantic content that determines what the description designates. He further maintains that the ultimate aim of the attack was to show that such descriptions are not singular terms. Thus, Salmon sees the argument of the passage as having a much broader scope than many other interpreters would accord to it.

A problem in the Frege–Church theory of sense and denotation argues that Church's solution to the Paradox of Analysis whereby an analysis is informative because the analysandum and analysans, while referring to the same things, differ in customary sense is in conflict with two other claims: (1) the Fregean claim that terms that occur in propositional attitude ascriptions refer to their usual sense, and (2) the claim from Church's Translation Argument that expressions like 'brother' and 'male sibling' may be translated into the same expression of another language that has only one phrase for the concept while preserving sense. Salmon proposses to resolve the conflict by abandoning Church's solution to the paradox, noting that doing so appears to undermine the original impetus behind Frege's distinction between *Sinn* and *Bedeutung*. *The very possibility of language* also discusses Church's Translation Argument, arguing that the moral of that argument shows that Dummett's account of language collapses into the absurd position that it is impossible for us to know a language.

The volume contain a helpful bibliography of Salmon's writings from 1979–2005. Unfortunately, there is no bibliography of the works cited; the details are included only in footnotes. There is a good deal of cross-referencing amongst the included papers; regrettably, these references refer to other papers that appear in the collection by their original pagination. Furthermore, cross-references to footnotes internal to an individual paper seem not always to have been updated when changes to the text affected the numbering of the notes. The text is well bound, attractively if densely type-set, and relatively free from typographical errors. Happily, those that exist are easily rectified.

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IAN CHISWELL AND WILFRID HODGES. ***Mathematical logic.*** Oxford Texts in Logic, vol. 3. Oxford University Press, Oxford, England, 2007, 250 pp.

This is an introductory textbook to mathematical logic on the undergraduate level. As such, it is comparable to many other books, such as the well-known ones by Enderton and Mendelson. Not surprisingly, it is at the same time unique in several ways, and so might be the text of choice, depending on the instructor's and institution's preferences and abilities.

Regarding the content, the goal of Chiswell–Hodges is to prove the Completeness Theorem for first-order logic. This is done by proving it for ever larger fragments: first for propositional logic, then for the quantifier-free predicate calculus, and finally for the full first-order theory. This can be contrasted with Enderton in two ways. For one, after the chapter in Enderton culminating in completeness, there is another chapter almost as long which highlights the various incompleteness and undecidability theorems, the climax of that text. (In Chiswell–Hodges, those latter theorems are mentioned only in the four-page postlude.) Another contrast is that Enderton develops the propositional calculus without proving completeness, and then immediately moves to full quantifier logic. Similarly, Mendelson has a full chapter on incompleteness, followed by one on set theory and another on computability. Apparently the goal of that work is to be a well-rounded introduction to all of logic and foundations. Even though it's intended for a two-semester course, the author recommends that a one-semester version go through incompleteness. The lead-up to completeness is structured similarly to Enderton's, with the sentential calculus introduced first and completeness proved only for the full first-order language, only Mendelson introduces equality as a dedicated symbol only after the proof of completeness (whereas Enderton does it before). So the goals of the book at hand are more modest than those of these other two.

Personally, I find it unfortunate that incompleteness gets such short shrift in Chiswell–Hodges. Incompleteness is exactly what's most powerful and interesting behind introductory logic (when studied for its own sake: this does not apply if the goal is applications, such as in CS or linguistics). In fact, the completeness theorem becomes significant only in the context of the search for a decision procedure for provability or such like. While a thorough proof of any of the incompleteness theorems might well be unnecessary, or even inadvisable, depending on the audience involved, some discussion as well as some indication of the proof is necessary in order to show something of the excitement and depth that logic can offer.

The development in Chiswell–Hodges is too brief, and calling its chapter a postlude only segregates it even more and sends the wrong message.

Another noticeable difference among the texts under consideration is the choice of a deductive system. Enderton uses a Hilbertian system, whereby a deduction is a linear sequence of formulas, each of which is an axiom or follows from the preceding ones by modus ponens—and that only for the quantified part, since the sentential axioms are taken to be all tautologies. Mendelson is similar, only there the list of sentential axioms is quite modest. Chiswell–Hodges, however, uses a combination of the sequent calculus and natural deduction for both the propositional and predicate parts. This leads naturally enough to another difference, the practice given to translations between natural (including mathematical) language and the formal system. All of the textbooks here spend a section on examples and exercises on such translations between statements in first-order logic and those in mathematics or natural English. Chiswell–Hodges in addition dedicates a whole chapter to motivating the rules of natural deduction via examples of the use of the sentential connectives in elementary mathematical contexts. Those of us who have noticed that our students have difficulty connecting the formal with the informal languages might well hope that the extra energy allotted to this practice might produce better results.

Besides the formalism of the object language, there is also a choice of formalism necessary in order to carry out proofs about the language(s) studied. This is necessary to give precise answers to questions such as what a formula is, what a proof is, what a theorem is. Enderton accomplishes this, at least in principle, by a section on induction and recursion (on general sets, not just natural numbers). On this basis, he can refer to the least set generated by or closed under certain rules, and he does in fact make such references, but only in passing, omitting the details, and mercifully so, since my students would otherwise not just lose the forest for the trees, but also miss the trees too. Mendelson handles this informally, by using induction and making only a passing reference to it if at all. For instance, sentential formulas are defined to be sentential letters, and combinations of formulas via certain connectives, and nothing else; this is then explained in more detail only in a footnote. Also, some proofs say they are by induction (on the natural numbers) with no explanation as to what induction is, presuming this knowledge of the reader; others don't even bother to make the induction explicit. What Chiswell–Hodges has to fill this role are trees. A propositional formula is defined to be a labeled tree, with variables on the leaves and connectives on the internal nodes. A natural deduction proof is a tree. A propositional formula, with a truth assignment of its variables, is evaluated via its tree. Substitution is explained through trees. First-order terms are certain trees. And so on. In order to support all the weight placed on trees, half of an introductory section is devoted to just the graph theory behind them, including precise definitions of trees, heights, and paths. I find this to be too much. Getting my students competent with one formalism is challenge enough; having them dwell on the formalism about a formalism is overkill. It's good to have something along those lines there, for those able to deal with it; the problem here is that it's so deeply embedded in the book it becomes hard to avoid. (To be fair, I have not taught from Chiswell–Hodges, so I am not speaking from direct experience here, just general experience. Perhaps there are redeeming virtues to this approach that I have missed.)

This book has other distinctive aspects that are quite interesting. What I like most were the two short chapters, called interludes, about the psychology behind elementary truth-table analysis in application. The first, the Wason selection task, has people consider the statement "if a card has a vowel on one side, it has an even number on the other side." The subject is then shown four cards each with a letter on one side and a number on the other, with respectively a vowel, a consonant, an even number, and an odd number showing. They are then asked what card(s) would have to be turned over in order to evaluate the truth of the statement. The authors ran this experiment in their logic class after having studied propositional logic.

They report the results, which include 0% getting the correct answer, and in the following discussion mention that their results are typical.

The second interlude is the Linda problem. Linda is described (in more detail than this) as being socially concerned and left-leaning. Then eight statements (a)–(h) about Linda are listed, and the subject is asked to rank them in order of probability. Six are red herrings. The two of interest are: (f) Linda is a bank teller, and (h) Linda is a bank teller and is active in the feminist movement. Apparently the majority of participants rank (h) as more likely than (f), presumably because it contains some information more consistent with the overall image of Linda, even though (h) implies (f) whereas (f) does not imply (h).

(The authors report that these two phenomena and variants have been intensely studied by psychologists. In fact, one of the researchers behind the latter test, Kahneman, was awarded the Nobel Memorial Prize in Economic Sciences in 2002 for this work, done with Tversky, who had by then passed away.)

Another charm of the book are the many photographs of (or, if that is not possible, graphics related to) important figures in logic, with a snippet about their contribution to the field. In fact, the first chapter is just two pages devoted to five mathematicians from Euclid to Gentzen addressing the question as to what mathematics is (as well as a one-page pronunciation guide). Along the way we also meet de Morgan, Łukasiewicz, Hilbert, Peirce, Post, Hintikka, Bolzano, and others.

When deciding whether to use this for your course, beside the considerations already mentioned, another along which this fares favorably is price. At \$59, it is a chunk cheaper than Mendelson (\$85) and Enderton (\$102), even if the former is paperback and the latter two hardcover. With the authors' teaching experience behind them, their offering is well worth considering for your introductory, undergraduate logic course.

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JOHANN HEINRICH LAMBERT. ***Philosophische Schriften. Vol. VIII: Kleinere philosophische Abhandlungen und Rezensionen.*** edited by A. Emmel and A. Spree, Georg Olms Verlag, Hildesheim, Zurich and New York 2007, Part 1: xlii + pp. 1–474; Part 2: pp. 475–763.

Hans Werner Arndt was responsible for a meritorious edition of the philosophical works of Johann Heinrich Lambert (1728–1777), one of the last universalists in philosophy and science. Arndt's edition was intended to cover 10 volumes of which volumes I–IV, VI–VII, and IX appeared between 1965 and 1969. When Arndt died in 2004 Lothar Kreimendahl (University of Mannheim) with his team took over the editorial work. Volume V, also edited by Armin Emmel and Axel Spree, presents Lambert's ***Kosmologische Briefe über die Einrichtung des Weltbaues*** (Cosmological letters on the constitution of the world). It appeared in 2006. The volume under review, volume VIII of the series, contains ***Kleinere philosophische Abhandlungen und Rezensionen*** (Smaller philosophical tracts and reviews). It is organized in two separate partial volumes, the first giving the tracts, the second the reviews.

The new volume is of particular interest for the history of logic because it comprises important contributions to the theory of logical calculi.

In 1764 Lambert published his main contribution to logic and epistemology, his ***Neues Organon, oder Gedanken über die Erforschung und Bezeichnung des Wahren*** (New organon, or thoughts about the investigation and designation of the true). In March 1764 he announced this book in ***Göttingische Anzeigen von gelehrten Sachen*** (pp. 33–35), "reporting some [of Lambert's] efforts to improve logic by applying mathematical expressions". Lambert presents his line diagrams, a geometric representation of categorial syllogistics which allows to draw a syllogistic inference simply by drawing the linear representations of the two premisses. He also suggests a non-standard syllogistics with restricted quantifiers, using a symbolism like "$C\frac{1}{2}$ is B" standing for "I know only with half certainty that C is B".

The central piece is the famous "Sammlung der Schriften, welche den logischen Calcul Herrn Prof. Ploucquet's betreffen, mit neuen Zusätzen" (Collection of writings concerning Professor Ploucquet's logical calculus, with new additions) (pp. 37–316), first published by August Friedrich Bök in 1766. It contains the documents of an argument about the qualities of Lambert's line calculus in comparison with the geometrical calculus of Gottfried Ploucquet, professor of philosophy in Tübingen, who used rectangulars on the basis of an identity theory of judgements. Further contributions by Heinrich Wilhelm Clemm and Georg Jonathan Holland are included in this collection.

A short tract "De universaliori calculi idea disquisitio una cum adnexo specimine" (pp. 327–359) first published in ***Nova acta eruditorum*** (1765) explicitly extends the Leibnizian idea of a universal calculus. Lambert suggests a calculus of properties and qualities using algebraic symbols for logical operations and relations. It is Lambert's only publication on calculi of intensions. His "Sechs Versuche einer Zeichenkunst in der Vernunftlehre" (Six attempts of an art of characters in a theory of reason) appeared posthumously (see vol. VI of the ***Philosophische Schriften***).

Considerations on a characteristic art and its application to different fields are taken up in other papers like "In algebram philosophicam cl. Richeri breves adnotationes" (pp. 361–372) or "Observations sur quelques dimensions du monde intellectuel" (pp. 373–390) with an application to esthetics.

Partial volume 2 is devoted to Lambert's reviews published in the ***Allgemeine Deutsche Bibliothek*** between 1768 and 1777, among them several comments on works on logic. An index of the books reviewed makes access easier.

This edition provides facsimile reprints of the texts. A comprehensive introduction gives useful information on the background and summaries of the material edited. This important edition of Lambert's philosophical works will be finished in 2008 with volume X, containing further philosophical writings, drafts and reviews with new material from the *Nachlass*. The edition provides a reliable textual basis for investigating the philosophical ideas of this major figure in European enlightenment who wrote in Leibnizian spirit.

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Kosta Došen and Zoran Petrić. ***Proof-net categories.*** Polymetrica International Scientific Publisher, Monza, Italy, 2007, viii + 147 pp.

Before discussing the content of the book, let me write some words about the publishing method. There is an electronic edition of the book which is freely accessible. The "Polymetrica License B" explicitly grants anyone the right to distribute the contents of the work, provided that authors and publisher are recognised and mentioned. Furthermore, the printed version is available at very low cost compared to similar books by other scientific publishers. In view of the fact that scientific publishing is in the hands of a few big companies who unscrupulously exploit the scientific community, we should be grateful for the publishing choice that Došen and Petrić have made.

The book is a monograph on the intersection between category theory and proof theory, more precisely, it is about the well-known correspondence between Michael Barr's *star-autonomous categories* and Jean-Yves Girard's *proof nets*: two proofs in *unit-free multiplicative linear logic* represent the same morphism in the freely generated star-autonomous category if and only if they are presented by the same proof net. There is a well-known problem with this correspondence: while the two *units* 1 and $\perp$ of linear logic are a central part of a star-autonomous category, the proof nets are *unit-free*. Thus, we only have a relation between proof nets and "unit-free star-autonomous categories"; suprisingly, we find here an embarrassing gap in the literature of linear logic. This correspondence between proof nets and "unit-free star-autonomous categories" has never been worked out in detail. In fact, there is not even a widely accepted definition of what a "unit-free star-autonomous category" actually is. The book by Došen and Petrić is a serious attempt to fill this gap. They give a definition of a "unit-free star-autonomous category", and because of the relation to proof nets, they call these categories *proof net categories*. (I find this choice of name misleading since proof nets do also exist for other logics.) The main result of the book is the coherence theorem for these categories. Roughly speaking, coherence says that two maps in the category are identical if they have the same "coherence graph". This also applies to star-autonomous categories with units, as long as the involved objects (i.e., formulas) use the units only in a sufficiently trivial way.

Although the subject is mathematically not very difficult, the book is not easy to read. One of the reasons is the non-standard notation. For example, the standard symbols for the operations in multiplicative linear logic (accepted by category theorists *and* proof theorists) are $\otimes$, $\parr$, and $\multimap$. But the authors use the symbols $\wedge$, $\vee$, and $\rightarrow$ from classical logic, without giving a warning that these operations behave very differently from what the reader might expect. Consequently, the authors have to write $f : A \vdash B$ instead of $f : A \rightarrow B$ for denoting a morphism from A to B. The names of the basic morphisms, which are part of the definitions, follow an unusual pattern. For example the associativity morphism is often denoted by $a_{A,B,C}$ or $\mathsf{assoc}_{A,B,C}$ or $\alpha_{A,B,C} : A \otimes (B \otimes C) \rightarrow (A \otimes B) \otimes C$. Došen and Petrić write $\hat{b}^{\rightarrow}_{A,B,C}$. The authors never write the types of a morphism, let alone a diagram when writing an equation. To give a simple example, they write (page 30):

$$\hat{b}^{\rightarrow}_{\neg p \vee p, B_1, B_2} \circ \hat{\Sigma}_{p, B_1 \wedge B_2} = \hat{\Sigma}_{p, B_1} \wedge 1_{B_2}$$

which might take the reader a couple of minutes to decipher, whereas the diagram

$$\begin{tikzcd} B_1 \otimes B_2 \arrow[r, "\Sigma_{p,(B_1 \otimes B_2)}"] \arrow[dr, "\Sigma_{p,B_1} \otimes 1_{B_2}"'] & (p \parr p^{\perp}) \otimes (B_1 \otimes B_2) \arrow[d, "\mathsf{assoc}"] \\ & ((p \parr p^{\perp}) \otimes B_1) \otimes B_2 \end{tikzcd}$$

conveys the point of the equation within seconds.

Let me emphasize that, taken one-by-one, all these points of criticism are negligible and, in fact, should not be mentioned in such a review. The problem is that all of them together make the book essentially unreadable.

Furthermore, the book does not exhibit the possible beauty and simplicity of algebraic arguments. In fact, the authors succesfully avoid using the most important algebraic property of star-autonomous categories, namely the natural bijection between the morphisms of type $A \otimes B \rightarrow C$ and those of type $A \rightarrow B^{\perp} \parr C$ (or, equivalently, $A \rightarrow B \multimap C$), also know as *currying*. The consequence of not using this property is that the definitions contain even more equations and the proofs become more cumbersome. For example, the proof of the equivalence between the categories **PN** and $\mathbf{PN}^{\neg}$ (in one the objects are the formulas in negation normal form, and in the other negation is allowed everywhere in formulas) takes 6 pages, although it should be easy.

On the other hand, often proofs are left to the reader, saying that they are "easily proved by induction on the complexity of the arrow term" (e.g., Lemmas 2D and 3D, pp. 25–26), or that they "rely on lengthy, but also rather straightforward, derivations" (p. 69).

Usually, the heart of a "coherence proof" is some sort of cut elimination. It is a pity that the authors fail to clearly exhibit this relation between proof theory and category theory. They bury their cut elimination in the p-q-r Lemma (p. 26) and refer the reader to another of their books for the proof. The situation becomes even more confusing, when Došen and Petrić give a "Gentzenization" of a star-autonomous category and prove a cut elimination theorem. First, it is not clear how their "Gentzen terms" relate to Gentzen's sequent calculus, and second, for proving cut elimination they rely on a coherence result. Nonetheless, the whole construction takes 12 pages (pp. 76–88), although in the ordinary (more space consuming) sequent notation the cut elimination for multiplicative linear logic takes less than 3 pages and does not need a coherence result. Finally, in the proof of the coherence theorem for star-autonomous categories, Došen and Petrić rely on the connectedness condition of proof nets (p. 90) without introducing proof nets and without any justification why this can be done.

Whenever we write a mathematical text, we face the problem of deciding what should be explained in detail and what can be left to the reader. I feel that Došen and Petrić often fail to make the right decision in that respect. Of course, there is no place in the book where I can object when Došen and Petrić write that something is "easily proved". I must agree that certain things are obvious provided one has sufficient maturity in the area. But I cannot see the difference between the "easy things" and the things that they choose to elaborate in detail.

Even though this is a very subjective view, I am afraid that any researcher or student who is reading the work in order to understand the deep relation between proof nets and star-autonomous categories will be disappointed when reaching the end of the book.

In the last chapter the authors finally establish the relation between their work and proof nets via the following well-known correspondence: a proof net is the coherence graph of a morphism together with the formula trees, or, conversely, the coherence graph is obtained from the proof net by forgetting the structure of its conclusion. But there is a problem: The proof net categories of Došen and Petrić do *not* correspond to proof nets of unit-free multiplicative linear logic, simply because there are proofs of formulas like $(A \parr A^{\perp}) \otimes (B \parr B^{\perp})$ which would correspond to a map $1 \to (A \parr A^{\perp}) \otimes (B \parr B^{\perp})$, but the 1 is not there. They can only speak about maps $A \to A \otimes (B \parr B^{\perp})$ which can be written without units although they are obtained via the composition $A \to A \otimes 1 \to A \otimes (B \parr B^{\perp})$. This means that, independent from the other criticism in this review, Došen and Petrić cannot claim to have solved the problem of defining the notion of "unit-free star-autonomous categories". In fact, there are alternative approaches for giving a definition for "unit-free star-autonomous categories" that successfully address the problem of capturing *all* proof nets by additional algebraic constructions. These are cited by Došen and Petrić on page 4 in the book, but discarded as being "not equational" (whatever that means). Thus, the name "proof net categories" is misleading in an additional way.

In the remainder of the last chapter, the authors make some general claims about proof nets, which most people familiar with the field might find amusing, but novices in the area might find them confusing. For example, on p. 123 the authors indicate that the main purpose of proof nets is to check whether there is a proof of a certain formula. This is not the case. The authors speak of the "theoremhood problem" and the "graph-theoremhood problem", but they seem unaware of the fact that the first one is the provability problem in multiplicative linear logic and is NP-complete, and the second one is equivalent to checking the correctness of a proof net, which can be done in linear time.

As a matter of fact, the main purpose of proof nets is to provide a "bureaucracy-free" presentation of proofs. That is, they abstract away from the irrelevant order in which the inference rules are applied for constructing the proof. The lack of this information is not a disadvantage of proof nets, as Došen and Petrić indicate on pp. 128–129, but it is their *key feature*. One can speak about proof nets as concrete proof objects without having to resort to equivalence classes of proof terms, as Došen and Petrić prefer to do. It seems that Došen and Petrić have written this book to reintroduce the bureaucracy that proof nets have successfully removed.

Finally, the sentence "The notion of proof net is not decidable in an elementary way" (p. 131) is plain wrong. A proof net consists of a sequence of formulas, equipped with the axiom links (i.e., the coherence graph). The question whether such an object is correct, (i.e., represents an actual proof) can be decided in linear time.

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Current topics in logic and analytic philosophy. edited by Concha Martínez, José L. Falguera and José M. Sagüillo, Colloquium on Logic and Analytic Philosophy at Santiago de Compostela, 2001–2005. Universidade de Santiago de Compostela, 2007, 288 pp. — therein:

JOHN CORCORAN. *Notes on the founding of logics and metalogic: Aristotle, Boole, and Tarski*. Pp. 143–176.

IGNACIO JANÉ. *On the general adequacy of Tarski's definition of logical consequence*. Pp. 177–201.

CONCHA MARTÍNEZ-VIDAL. *John Corcoran in Spanish: His contribution to the improvement of logical knowledge at the University of Santiago de Compostela*. Pp. 203–220.

ANGEL NEPOMUCENO-FERNÁNDEZ. *Information and logic*. Pp. 221–232.

STEPHEN READ. *Farewell to opacity*. Pp. 233–246.

JOSÉ M. SAGÜILLO. *Corcoran the philosopher*. Pp. 247–269.

STEWART SHAPIRO. *Corcoran the mathematician*. Pp. 271–288.

This volume is a collection of fourteen papers presented at the Colloquium on Logic and Analytic Philosophy at Santiago de Compostela between 2001 and 2005. In the first part of the collection, seven papers (not reviewed here) are devoted to topics in analytic philosophy, including philosophy of language, philosophy of logic, and philosophy of science. The second part consists of seven papers presented at "The Corcoran Symposium," a two-day conference held in connection with the conferral of the degree Doctor Honoris Causa on Professor John Corcoran in October 2003.

Corcoran's article is a conspectus of the history of logic, with Aristotle, Boole and Tarski as representative of the earliest, transitional and most recent periods. Aristotle's contribution is described as consisting of the systematization of methods of proving validity in a limited domain of propositions expressed in canonical notation, as well as the method of countermodels for establishing invalidity. Boole presented the first system of mathematical logic, and was the founder of logic as formal ontology by including the study of tautologies and metalogical laws such as excluded middle. Tarski in his fundamental papers of the 1930s on truth, consequence and definability identified and addressed the need for systematic metatheory of logical theories.

Ignacio Jané is concerned with the "common concept of consequence" discussed by Tarski in his well known paper of 1936 on the concept of logical consequence (JSL II 83). Jané provided detailed arguments in an earlier article (BSL XII 1–42) showing that the "common concept" that Tarski had in mind was the conception current in the development of mathematical axiomatics. In the present article, he addresses the question of whether Tarski's definition is adequate in more general settings, outside formal axiomatics. The overall upshot is negative, due to the fact that a given informal argument can be held to be logically valid or invalid, depending on the formalization chosen.

The article by Concha Martínez-Vidal discusses five papers by Corcoran that have been translated into Spanish, and their influence on logical work at the U.S.C. These include articles on the interpretation of Aristotle's logic and argumentation theory. She concludes with a survey of Corcoran's general views on logic: mathematical logic as a technique of modelling mathematical discourse, and the contrast between the atemporal, objective notion of logical consequence and the timebound, personal activity of producing concrete proofs.

Angel Nepomuceno-Fernández's contribution takes up the information theoretic approach to logic developed by Corcoran in three papers published in the 1990s. Given a collection of propositions, a set $\mathfrak{I}$ is taken to represent the entire information of the domain of investigation; each proposition α is then assigned an informative content $I(\alpha) \subseteq \mathfrak{I}$. The properties of the function I are given as a set of axioms, perhaps best explained by an example. If we take $\mathfrak{I}$ to be the set of all interpretations for a given language, and $I(\alpha)$ the set of interpretations that do not satisfy α, then this definition satisfies all of Corcoran's axioms (though it should not be taken as definitive of his axiomatic conception). Nepomuceno-Fernández proposes an information-based approach to defining relevant consequence and deductive-nomological explanation in philosophy of science.

Stephen Read's article is a vigorous defence of the law of indiscernability of identicals, and of the view that there are no opaque contexts. Quine's views to the contrary are countered by arguing that quotation contexts should not be taken as counter-examples to the view, and that such contexts are radically different from modal and intensional contexts.

José M. Sagüillo touches on several of the themes in Corcoran's work mentioned above, as well as others, such as his advocacy of second-order logic, his influential work on the interpretation of Aristotle's syllogistic as a natural deduction system, and his views on logical paradoxes.

Finally, Stewart Shapiro's contribution surveys Corcoran's more technical contributions to logic, including natural deduction formulations of modal logic, work on variable binding term operators, string theory and investigations of categoricity in second order languages.

The editors have succeeded in assembling an interesting collection of diverse papers reflecting the work of a philosopher-logician who has been influential both through his writing and teaching.

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Leopold Löwenheim (1878–1957). ***History and Philosophy of Logic***, Special Issue, edited by C. Thiel, vol. 28, no. 4 (November 2007), pp. 289–336.

Leopold Löwenheim, remembered today above all for the Löwenheim-Skolem theorem, often referred to in the philosophy of mathematics as "Löwenheim-Skolem paradox", died on 5 May 1957 in Berlin-Charlottenburg. On the occasion of the 50th anniversary of his death, Christian Thiel edited a special issue of the journal ***History and Philosophy of Logic***. This edition brings together the editor's "A Short Introduction to Löwenheim's Life and Work and to a Hitherto Unknown Paper" (pp. 289–302) and this "hitherto unknown paper", Löwenheim's "Funktionalgleichungen im Gebietekalkül und Umformungsmöglichkeiten im Relativkalkül", written around 1935, prepared for publication, but victim of historical circumstances.

Thiel's introduction gives the dates of Löwenheim's life with some hints on his family. Löwenheim studied mathematics and natural sciences at the Friedrich-Wilhelms University at Berlin and the Technische Hochschule at Charlottenburg. He never made a Ph.D., became teacher in Berlin. In 1934 he was forced to retire, but survived the Third Reich not least due to the support of anthroposophic friends.

In the second section of the introduction Löwenheim's work in the algebra of logic is sketched. Thiel shows that Löwenheim obviously underestimated the main results of his pioneering paper "Über Möglichkeiten im Relativkalkül" of 1915. Löwenheim considers first-order equations that are valid in every finite domain but not valid universally (i.e., *Fluchtgleichungen*, fleeing equations). He proves that for every equation of this type there are values of its predicate letters such that the equation is not valid in any denumerably infinite domain: If F is finitely, but not universally valid, then F is not $\aleph_0$-valid, which is classically equivalent to the formulation that if F is satisfiable then it is finitely satisfiable or $\aleph_0$-satisfiable. Thiel writes: "Apparently it does not occur to him in this context that the theorem implies the existence of denumerable models also of first-order axiom systems intended to characterize ('implicitly define') a non-denumerable domain like that of the real numbers" (p. 295). Löwenheim's further results are mentioned: the decidability of the monadic fragment of first-order logic with equality, furthermore the theorem that every relative or first-order equation is equivalent to a binary one, and other results in the Boolean Algebra and the Algebra of Logic, such as his contributions to the elimination and solution problems.

In the third section the reader is introduced to Löwenheim's paper published here for the first time. The introduction closes with a complete bibliography of the works of Leopold Löwenheim (42 items) with a list of mimeographed and unpublished typescripts (8 items). Bibliographies of Löwenheim's father Detmold Louis Löwenheim (7 items) and of his mother Elise Löwenheim-Röhn (3 items) are added.

The paper "Funktionalgleichungen im Gebietekalkül und Umformungsmöglichkeiten im Relativkalkül" was intended for publication in the Polish journal ***Fundamenta Mathematicae***. The typesetting was completed, but the publication became impossible after the German occupation of Poland. The galley proofs survived and are the basis for the edition. The first part continues research from the time before the First World War on the solution problem. Löwenheim gives several methods for finding reproductive solutions of Boolean functional equations from given particular solutions. He also simplifies some of Ernst Schröder's calculation methods. The second part continues the research of his famous 1915 paper discussing the limits of elimination.

The paper begins with an interesting motivation for Löwenheim's habit to keep the algebraic Peirce-Schröder notation instead of switching to the Peano-Russell signs as most fellow logicians of his time. Löwenheim regrets that the natural harmony and beauty of mathematics has been given up without good reasons. "With Russell's signs I would not have discovered much of what I found, and in any case I would not have my big propensity of calculi. Inappropriate signs paralyze creativity" (p. 305).

The special issue is illustrated with five photographs, an official portrait photograph of 1936, Löwenheim's birthplace in Krefeld, a portrait of his father, and Löwenheim among the staff members of his school in Berlin-Lichtenberg after the Second World War, from which an enlargement is added.

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