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2007 EUROPEAN SUMMER MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC LOGIC COLLOQUIUM '07

Wrocław, Poland
July 14–July 19, 2007

Logic Colloquium 2007, the European Summer Meeting of the Association for Symbolic Logic, took place in Wrocław, Poland, on July 14–19, 2007. Major funding was provided by the Association for Symbolic Logic, the U.S. National Science Foundation, and the University of Wrocław.

The members of the Program Committee were: A. Andretta, F. Delon, U. Kohlenbach, S. Lempp (Chair), P. Maddy, J. Marcinkowski, L. Newelski, A. Pitts, P. Pudlák, S. Solecki, F. Stephan, and G. Sundholm. The success of the meeting owes a great debt to the efforts of the Organizing Committee: T. Jurdzinksi, E. Kieronski, P. Kowalski, J. Marcinkowski (Chair), and B. Rusiecka.

The first day of the Logic Colloquium was joint with the Annual IEEE Symposium on Logic in Computer Science (LICS), which immediately preceded the Logic Colloquium. This day featured two joint LC/LICS one-hour invited talks as well as four joint LC/LICS half-hour invited talks.

The Logic Colloquium program included three invited tutorials, eleven invited plenary lectures, twenty-one invited lectures in five special sessions (on logic and analysis, model theory, philosophical and applied logic at the JPL, proof complexity and nonclassical logics, and set theory), and over sixty contributed talks. Logic Colloquium 2007 included 176 participants from twenty-nine countries spread over six continents, including fifty-four students.

The following one-hour invited joint LC/LICS lectures were presented:

Martin Hyland (Cambridge), *Combinatorics of proofs*.
Colin Stirling (Edinburgh), *Higher-order matching, games and automata*.

The following half-hour invited joint LC/LICS lectures were presented:

Cristiano Calcagno (Imperial College), *Can logic tame systems programs*?
Martín Escardó (Birmingham), *Infinite sets that admit exhaustive search*.
Rosalie Iemhoff (Utrecht), *Skolemization in constructive theories*.
Alex Simpson (Edinburgh), *Non-well-founded proofs*.

The following invited tutorials were presented:

Steve Jackson (North Texas), *Cardinal Arithmetic in* $L(R)$.
Bakhadyr Khoussainov (Auckland), *Automatic structures*.
Ya'acov Peterzil (Haifa), *The infinitesimal subgroup of a definably compact group*.

The following one-hour invited plenary lectures were presented:

Albert Atserias (Barcelona), *Structured finite model theory*.
Matthias Baaz (Vienna), *Towards a proof theory of analogical reasoning*.
Vasco Brattka (Cape Town), *Computable analysis and effective descriptive Set Theory*.
Zoé Chatzidakis (Paris 7), *Model theory of difference fields, and some applications*.

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Gabriel Debs (Institute Math. de Jussieu & Le Havre), *Coding compact spaces of Borel functions*.
Fernando Ferreira (Lisbon), *On a new functional interpretation*.
Andrzej Grzegorczyk (Warsaw), *Philosophical content of formal achievements*.
Bjørn Kjos-Hanssen (Hawaii), *Brownian motion and Kolmogorov complexity*.
Piotr Kowalski (Wrocław), *Definability in differential fields*.
Paul B. Larson (Miami University), *Large cardinals and forcing-absoluteness*.
Donald A. Martin (UCLA), *Sets and the concept of set*.

Abstracts of invited and contributed talks given in person or by title by members of the Association follow.

For the Program Committee
STEFFEN LEMPP

Abstracts of invited joint ASL–LICS hour lectures

► MARTIN HYLAND, *Combinatorics of proofs*.
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Ideally interpretations of proofs should exhibit some essential combinatorial features in an interesting and appealing way. As a case study, one can consider the notion of innocent strategy which is the basis for a game semantical interpretation of proofs and programmes. Some combinatorial content of this notion is sketched in the joint LICS paper accompanying this talk, whose abstract reads as follows.

We show how to construct the category of games and innocent strategies from a more primitive category of games. On that category we define a comonad and monad with the former distributing over the latter. Innocent strategies are the maps in the induced two-sided Kleisli category. Thus the problematic composition of innocent strategies reflects the use of the distributive law. The composition of simple strategies, and the combinatorics of pointers used to give the comonad and monad are themselves described in categorical terms. The notions of view and of legal play arise naturally in the explanation of the distributivity. The category-theoretic perspective provides a clear discipline for the necessary combinatorics.

There are other instances of a kind of categorical combinatorics of proofs, but in this talk I shall restrict myself to the one instance.

► COLIN STIRLING, *Higher-order matching, games and automata*.
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We describe a particular case where methods such as model-checking as used in verification are transferred to simply typed lambda calculus. Higher-order matching is the problem given $t = u$ where t, u are terms of simply typed lambda-calculus and u is closed, is there a substitution S such that tS and u have the same normal form with respect to beta eta-equality: can t be pattern matched to u? In the talk we consider the question: can we characterize the set of all solution terms to a matching problem? We provide an automata-theoretic account that is relative to resource: given a matching problem and a finite set of variables and constants, the (possibly infinite) set of terms that are built from those components and that solve the problem is regular. The characterization uses standard bottom-up tree automata. However, the technical proof uses a game-theoretic characterization of matching.

Abstracts of invited joint ASL–LICS thirty-minute lectures

► CRISTIANO CALCAGNO, *Can logic tame systems programs*?
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We report on our experience on designing and implementing tools for automatic reasoning about safety of systems programs using separation logic. We highlight some of the fundamental obstacles that need to be overcome, such as the complexity of data structures and scalability of the methods, on the path to realistic systems programs.

► MARTIN ESCARDÓ, *Infinite sets that admit exhaustive search*.
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Perhaps surprisingly, there are infinite sets that admit mechanical exhaustive search in finite time. We investigate three related questions: (1) What kinds of infinite sets admit exhaustive search? (2) How do we systematically build such sets? (3) How fast can exhaustive search over infinite sets be performed?

We give answers to them in the realm of Kleene–Kreisel higher-type computation: (1) involves the topological notion of compactness, (2) amounts to the usual closure properties of compact sets, including the Tychonoff theorem, (3) provides some fast algorithms and a conjecture.

These two talks include my contributed LICS paper, but go beyond in two respects: a general introduction to the role of topology in computation is given, and a few new results are included, such as an Arzela–Ascoli type characterization of exhaustible sets.

► ROSALIE IEMHOFF, *Skolemization in constructive theories*.
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It has long been known that Skolemization is sound but not complete for intuitionistic logic. We will show that by slightly extending the expressive power of the logic one can define a translation that removes strong quantifiers from predicate formulas and that is related but not equal to Skolemization. Since the extended logic is constructive, the translation can be considered as an alternative to Skolemization for constructive settings. The result easily implies an analogue of Herbrand's theorem. We will apply the method to various constructive theories and compare it to other Skolemization methods and related translations like the Dialectica Interpretation.

► ALEX SIMPSON, *Non-well-founded proofs*.
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I will discuss various situations, arising in computer science, mathematics and logic, in which one is naturally led to consider associated proof systems involving interesting forms of non-well-founded proof.

Abstracts of invited tutorial talks

► STEVE JACKSON, *Cardinal Arithmetic in $L(R)$*.
Department of Mathematics, University of North Texas, P.O. Box 311430, Denton, Texas, 76203-1430, USA.
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In this series of talks we will survey the cardinal structure of the model $L(R)$ assuming the axiom of determinacy. We describe the close relationship between the cardinal structure and partition properties of the odd projective ordinals. We will present some recent simplifications to the presentation of this theory, as well as a result connecting the cardinal structure of $L(R)$ to that of the background universe V. We will attempt to make the talks as self contained as possible.

► BAKHADYR KHOUSSAINOV, *Automatic structures*.
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We study automatic structures. These are infinite structures that have automata presentations in a precise sense. By automata we mean any of the following: finite automata, tree automata, Büchi automata and Rabin automata.

Automatic structures possess a number of interesting algorithmic, algebraic and model-theoretic properties. For example, the first order theory of every automatic structure s decidable; automatic structures are closed under the first order interpretations; also, there are characterizations theorems for automatic well-founded partially ordered sets, Boolean algebras, trees, and finitely generated groups. Most of these theorems have algorithmic implications. For instance, the isomorphism problem for automatic Boolean algebras is decidable.

The first lecture covers basic definitions and presents many examples. We explain the decidability theorem that describes extensions of the FO logic in which each automatic structure has a decidable theory. The second lecture surveys techniques for proving whether or not a given structure can be presented by automata. We also talk about logical characterizations of automatic structures. The last lecture concentrates on complexities of automatic structures in terms of well-known concepts of logic and model theory such as heights of well-founded relations, Scott ranks of structures, and Cantor–Bendixson ranks of trees.

Most of the results are joint with Liu, Minnes, Nies, Nerode, Rubin, Semukhin, and Stephan.

► YA′ACOV PETERZIL, *The infinitesimal subgroup of a definably compact group*.
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Consider the compact Linear group $G = SO(3, \boldsymbol{R})$. When G is viewed in any nonstandard real closed field, the set G^{00} of all matrices in G which are infinitesimally close to the identity forms a normal subgroup. Endow the quotient G/G^{00} with a "logic topology", whose closed sets are those whose preimages in G are type-definable. It is easy to see that G/G^{00}, with this logic topology, is isomorphic to $SO(3, \boldsymbol{R})$, with the Euclidean topology.

Several years ago, A. Pillay conjectured that a similar phenomenon should be true for every "definably compact" group in an arbitrary o-minimal structure, even if the group itself was not defined over the real numbers. Roughly speaking, Pillay conjectured that every definably compact group G in a sufficiently saturated o-minimal structure has a canonical type-definable normal subgroup G^{00} such that the group G/G^{00}, when endowed with the logic topology as above, is isomorphic to a compact real Lie group. Moreover, the real dimension of this Lie group equals the o-minimal dimension of G.

My goal in these talks is to show, with the help of examples, how the interaction between different notions, such as o-minimality, Lie groups, compactness, measure theory, and Shelah's Independence property, yields a solution to the conjecture.

For background on o-minimality, see van den Dries's book [2] below. For Pillay's conjecture, see [6]. For key-steps in the solution to the conjecture, see [1, 5, 3, 4].

[1] A. Berarducci, M. Otero, Y. Peterzil, and A. Pillay, *A descending chain condition for groups in -minimal structures*, ***Annals of Pure and Applied Logic***, vol. 134 (2005), pp. 303–313.

[2] L. v. d. Dries, ***Tame topology and o-minimal structures***, Cambridge University Press, New York, 1998.

[3] M. Edmundo and M. Otero, *Definably compact abelian groups*, ***Journal of Mathematical Logic***, vol. 4 (2004), pp. 163–180.

[4] E. Hrushovski, Y. Peterzil and A. Pillay, *Groups, measure and the NIP*, ***Journal of the American Mathematical Society***.

[5] Y. Peterzil and A. Pillay, *Generic sets in definably compact groups*, ***Fundamenta Mathematicae***, vol. 193 (2007), pp. 153–170.

[6] A. Pillay, *Type-definability, compact Lie groups, and -minimality*, ***Journal of Mathematical Logic***, vol. 4 (2004), pp. 147–162.

Abstracts of invited plenary talks

► ALBERT ATSERIAS, *Structured finite model theory*.
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Many of the classical results of model theory, most notably some of the direct consequences of the Compactness Theorem, fail badly in restriction to finite structures. A classical example is the Łoś-Tarski preservation-under-substructures Theorem, which is known to fail in the finite. Motivated by certain application-areas in computer science and combinatorics, it has been argued that it might be profitable to consider some further restrictions beyond finiteness. What classical theorems of model theory hold on classes of finite trees? or finite planar graphs? or finite structures of bounded treewidth? or, more generally, minor-closed classes of finite graphs? The goal of our talk is to give an overview of the recent results in this area. Interestingly, many of these results have at its heart an application of Gaifman's Locality Theorem, which seems to play in this area a role comparable to the one played by the Compactness Theorem in classical model theory.

► MATTHIAS BAAZ, *Towards a proof theory of analogical reasoning*.
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In this lecture we compare three types of analogies based on generalizations and their instantiations:

1. Generalization w.r.t. to invariant parts of proofs (e.g., graphs of rule applications etc.).
2. Generalization w.r.t. to an underlying meaning. (Here proofs and calculations are considered as trees of formal expressions. We analyze the well-known calculation demonstrating that the 5th Fermat number is compound.)
3. Generalization w.r.t. to the premises of a proof. (This type of analogies is especially important for symbolic juridical reasoning.)

► VASCO BRATTKA, *Computable analysis and effective descriptive set theory*.
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Computable analysis can be considered as an extension of computability theory to infinite objects such as real numbers, continuous functions and closed subsets that are studied in analysis. In a similar way as discrete degrees of non-computability can be classified in the arithmetical hierarchy, the corresponding hierarchy for such classifications in analysis is the Borel hierarchy. Thus, it is natural to expect a fruitful interaction between effective descriptive set theory and computable analysis. In this talk we present an extension of the Representation Theorem of computable analysis to effectively Borel measurable maps. This theorem allows to introduce a conservative extension of the notion of effective Borel measurability to admissibly represented topological spaces as they are used in computable analysis. A resulting notion of reducibility for functions is described and techniques of completeness proofs from computability theory can be imported into this branch of effective descriptive set theory. We apply these techniques to characterise the degree of non-computability of topological operations such as the closure, the interior, the boundary and the derived set operation with respect to hyperspace representations for computable metric spaces.

[1] VASCO BRATTKA, *Effective Borel measurability and reducibility of functions*, ***Mathematical Logic Quarterly***, vol. 51 (2005), no. 1 pp. 19–44.

[2] VASCO BRATTKA AND GUIDO GHERARDI, *Borel complexity of topological operations on computable metric spaces*, ***Computation and Logic in the Real World***, (S. B. Cooper, B. Löwe, and A. Sorbi, editors), Lecture Notes in Computer Science, vol. 4497, Springer, Berlin, 2007.

► ZOÉ CHATZIDAKIS, *Model theory of difference fields, and some applications*.
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A difference field is a field with a distinguished automorphism. I will first give a survey of the model theory of the existentially closed difference fields, and then of some application(s) to diophantine geometry. The main tool for applications is the dichotomy theorem, and I will spend some time explaining what are its consequences in the particular context of difference fields, and how they can be used.

► GABRIEL DEBS, *Coding compact spaces of Borel functions*.
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Let $B(X)$ denote the space of all Borel functions on some Polish space X, equipped with the pointwise convergence topology (i.e., the topology induced by the product topology on $\mathbf{R}^X$). When X is uncountable the space $B(X)$ is clearly non metrizable. But it follows from the early work of Rosenthal and Bourgain–Fremlin–Talagrand that the **compact** subspaces of $B(X)$ possess many metric-like properties: For example most of the standard topological notions admit sequential formulations. Notice that such a compact space K needs not to be metrizable even if it is separable. However in this latter case, one can equip K with some natural "descriptive structure" that we shall describe in more detail in this talk. One consequence of the main result that we shall present here is that all these structures are actually "equivalent" in some sense. This will also provide a positive answer to a question asked by Argyros, Dodos and Kanellopoulos concerning the notion of **analytic** Rosenthal compacta that they introduced recently. Though many of the statements we shall consider are totally classical, the proofs make a crucial use of Effective Descriptive Set Theory, namely of some fundamental results of Moschovakis on inductive definability.

► FERNANDO FERREIRA, *On a new functional interpretation*.
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Gödel's functional "Dialectica" interpretation of 1958 was published with the explicit aim of being a contribution to Hilbert's program. As things go, it introduced a new technical tool in Proof Theory: one that presents a trade-off between quantifier complexity and finite-type computable functionals. Gödel's particular interpretation reduces HA (Heyting arithmetic) to a quantifier-free calculus T of finite-type functionals, thereby reducing the consistency of the former to the consistency of the latter. It is seldom observed explicitly that Gödel's interpretation can also be used to prove certain conservation results. For instance, the adjunction of certain principles (axiom of choice AC, independence of premises IP, Markov's principle MP) does not change the provably Π_2^0-sentences of HA.

We describe a new (2005) functional interpretation of HA. This functional interpretation (due to the author and Paulo Oliva) uses the same functionals as Gödel's but changes the assignment of formulas. Contrary to Gödel's assignment, our assignment does not preserve set-theoretic truth. The new interpretation yields new conservation results. Some false set-theoretic principles like the refutation of extensionality or Brouwer's FAN theorem, as well as some semi-intuitionistic principles like LLPO (lesser limited principle of omniscience) or weak König's lemma (WKL), can be adjoined to HA without changing the provably Π_2^0-sentences. More memorably, within intuitionistic logic, the above principles are not able to prove further terminating computations.

As an illustration, we show that the provably total functions of the classically inconsistent second-order **intuitionistic** theory $\mathrm{WKL}_0 + \mathrm{LLPO} + \mathrm{IP} + \mathrm{MP} + \mathrm{AC}^{\mathbf{N}} + \mathrm{FAN}$ are the primitive recursive functions, where $\mathrm{AC}^{\mathbf{N}}$ is the countable axiom of choice. By the negative translation of Gödel–Gentzen, we get, as an easy corollary, Harvey Friedman's well-known conservation result of (classical) WKL_0 over RCA_0.

Finally, we would like to draw attention to an ubiquitous principle in mathematical logic: the bounded collection scheme. It appears in bounded theories of arithmetic, in admissible set theory (as Δ_0-collection), and in analysis (if we view Brouwer's FAN principle as a form of collection) and proof mining (Kohlenbach's uniform boundedness principles). We believe that the new interpretation provides a fresh and integrated look at bounded collection.

► ANDRZEJ GRZEGORCZYK, *Philosophical content of formal achievements*.
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Two reflexions will be proposed:

The first general remark: A Perception of Philosophical Contents depends on the **Taste** of the Scholar. In Science about reality, the most important is **Truth**. This means a good **description** of real objects (events).

In formal Science, the value is an **Invention** of Consistent Construction and something which may be called: **joke** and/or **beauty**.

The second remark: In formal science, we can make the following similar **distinction** in the **taste** of scholars:

- **Maths**: Mathematicians were always proud that they can **calculate** functions. They **calculate**, e.g., **by recursion**: $f(0) = a$, $f(n+1) = F(f, n) \dots$
  Calculation is the fundamental joke (or beauty) of the construction.
- **Logic**: Logicians are more philosophers, and were always proud that they can **define** relations. They **define** using logical **connectives** and **quantifiers**.
  Definition is the fundamental joke (or beauty). It gives us a vision (of the inside).

Some arguments, an example, and a propaganda for these distinctions will be given.

► BJØRN KJOS-HANSSEN, *Brownian motion and Kolmogorov complexity*.
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Probabilistic potential theory contains results of the following form: A randomly chosen small set of reals is almost surely disjoint from a given small set of reals. Here the size of a set can be for example its Hausdorff dimension.

Using computability theory and Kolmogorov complexity, we obtain some results of this kind, for certain randomly chosen sets associated with Brownian motion. We then investigate to what extent these results can be reproduced using only potential theory.

► PIOTR KOWALSKI, *Definability in differential fields*.
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A differential field is a field equipped with a derivation. In the positive characteristic case one often needs to "improve" the notion of derivation by introducing Hasse–Schmidt derivations. I will briefly discuss the first-order theory of (differentially closed) differential fields. Then, I will focus on two special kinds of definable sets there. Sets of the first kind played a crucial role in the Hrushovski's proof of the Mordell–Lang conjecture [2]. Sets of the second kind appear in the statement of a theorem of Ax [1] and its generalizations.

[1] James Ax, *On Schanuel's conjectures*, ***Annals of Mathematics***, vol. 93 (1971), no. 2, pp. 252–268.
[2] Ehud Hrushovski, *Mordell–Lang conjecture for function fields*, ***Journal of the American Mathematical Society***, vol. 9 (1996), pp. 667–690.

► PAUL B. LARSON, *Large cardinals and forcing-absoluteness*.
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Absoluteness theorems of Levy and Shoenfield state that the truth values of Σ_1 and Σ^1_2 sentences cannot be changed by set forcing. In the presence of large cardinals, forcing-absoluteness extends to much larger classes of statements. Jumping to the end of a long story, we will talk about some of the strongest forcing absoluteness results, including extensions of Woodin's Σ^2_1 absoluteness theorem.

► DONALD A. MARTIN, *Sets and the concept of set*.
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Those who doubt that the subject matter of mathematics is a realm of abstract objects often do so because of a general disbelief in the existence of abstract objects. I will propose a very different basis for doubts about the status of mathematical objects. Many people think that various statements of set theory, e.g., the Continuum Hypothesis, are neither true nor false. I am not one of one of these people, but I do think we cannot at present be certain that the CH has a truth-value. I will argue that if it is not certain that the CH has a truth-value then it is also uncertain whether there is any system of objects that satisfies our concept of a universe of sets. It is common to say that the CH has no truth-value and to blame this on the existence of many systems of objects that satisfy the set concept. Part of my job will be to discredit this position. Another part will be to give an account of what set theoretic truth and falsity could amount to if there is no universe of sets.

Abstracts of invited talks in the Special Session on Logic and Analysis

► PHILIPP GERHARDY, *Local stability of ergodic averages*.
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'Proof mining' is the subfield of mathematical logic that is concerned with the extraction of additional information from proofs—even ineffective proofs!—in mathematics and computer science. The main focus is on developing general (and feasible) methods to unwind the computational content of proofs and to apply these methods to real existing proofs. We present an application of proof mining in the field of ergodic theory. The Mean Ergodic Theorem states that a for a nonexpansive operator T on a Hilbert space H and for any f in H, the sequence of ergodic averages $A_n f := 1/(n+1) \sum_{i \leq n} T^i f$ converges to a limit. While a full rate of convergence is not possible, we present explicit rates for the classically equivalent statement that for any number-theoretic function K, the averages $A_m f$ are stable within ε in some interval $[n, K(n)]$. These computable bounds have been obtained by applying methods of proof mining to a standard textbook proof of the Mean Ergodic Theorem.

► PETER HERTLING, *Computability and non-computability results for the topological entropy of shift spaces*.
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The topological entropy, a numerical quantity assigned to a continuous function from a compact space to itself, is invariant under topological conjugacy and serves as a tool for classifying dynamical systems. Therefore, computing the topological entropy is an important problem in dynamical systems theory. We discuss several recent positive and negative results concerning the computability of the topological entropy, mostly of shift dynamical systems.

► JULIEN MELLERAY, *Geometry of the Urysohn space: a model-theoretic approach*.
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The Urysohn space was built by Pavel Urysohn in 1924. Over the past 15 years or so, there has been growing interest in this space and its geometry. In the talk I'll try to explain how one can use model-theoretic methods (via the framework of model theory of metric structures) to tackle some problems involving the Urysohn space and its geometry. I'll discuss in particular the homogeneity properties of the space, and questions about conjugacy of isometries.

► ANDREAS WEIERMANN, *Analytic Combinatorics of the transfinite*.
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Roughly speaking, Analytic Combinatorics (AC) is the art of counting using Cauchy's integral formula. Typically (following Flajolet et al.) AC is used for investigations on the average case analysis of algorithms. In this talk we will show how to apply AC to systems of ordinal notations (for relatively small ordinals). We explain surprising connections to the theory of partitions as studied, for example, by Hardy and Ramanujan.

The resulting information is used for proving logical limit laws (joint work with Alan R. Woods) and for classifying phase transition thresholds for Gödel incompleteness.

Abstracts of invited talks in the Special Session on Model Theory

► VERA DJORDJEVIC, *Independence in structures and finite satisfiability*.
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An outline of ideas and methods concerning proving the finite submodel property by probabilistic means will be given. It appears like some notion of "sufficient independence" between definable relations is needed for a probabilistic argument to work out. We consider a notion, the n-embedding of types property, which has some relationship with the n-amalgamation property considered elsewhere. The main result about the finite submodel property applies to countably categorical simple structures with trivial forking, finite SU-rank and with the n-embedding of types property for every n.

▶ AMADOR MARTIN-PIZARRO, *Some thoughts on bad objects*.
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The Algebraicity Conjecture states that a simple 2^{ω}-categorical group can be seen as an algebraic group over an algebraically closed field. This long-standing open conjecture belongs to a wider conjecture, called "Principe du Nirvana", whose various specific instances have been refuted in the last decades. A specific programme in order to characterize such simple groups was described, which originally imposed the non-existence of certain objects called bad fields (describing an undesired configuration characterizing the Borel subgroups). We will discuss the relevance of bad fields and foster diverse links with other areas of algebraic geometry.

▶ ZIV SHAMI, *Countable imaginary simple unidimensional theories*.
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We show that these theories are supersimple, provided that forking is witnessed by pseudo-low formulas (a formula is pseudo-low if forking by it is a type-definable relation). In particular, any low or 1-based countable imaginary simple unidimensional theory is supersimple.

▶ MARCUS TRESSL, *Super real closed rings*.
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A super real closed ring is a commutative ring A with unit together with functions $F_A\colon A^n \to A$ for all n in N and every continuous function $F\colon \mathbf{R}^n \to \mathbf{R}$, satisfying the composition rules $(F \circ (\dots, G, \dots))_A = F_A \circ (\dots, G_A, \dots)$. For example, every ring $C(X)$ of continuous real-valued functions is a super real closed ring, where $F_{C(X)}$ is composition with F; also super real fields in the sense of Dales–Woodin at prime z-ideals are naturally equipped with a super real closed ring structure.

I will overview the algebraic properties of super real closed rings, give an application to o-minimal structures expanding the real field and discuss the relation to C^{∞}–rings (in the sense of Moerdijk–Reyes).

Abstracts of invited talks in the Special Session on Philosophical and Applied Logic at the Journal of Philosophical Logic

▶ G. ALDO ANTONELLI, *From philosophical logic to computer science—and back*.
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Over the past 25 years or so, computer scientists have looked at philosophical logic in search of tools for the formal modeling of a number of interactive computational processes, whereas philosophical logician have drawn new inspiration from a set of problems specific to theoretical computer science and artificial intelligence. This talk provides a (perforce idiosyncratic) survey of the interactions between the two fields, pointing out interdisciplinary connections and fruitful cross-fertilizations.

► HORACIO ARLÒ COSTA, *Philosophical logic meets formal epistemology*.
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Formal epistemology is a branch of epistemology that uses formal methods to articulate and solve traditional epistemic problems. The talk surveys some recent interactions between philosophical logic and formal epistemology. We focus on epistemic problems related to the representation of attitudes and their change, both for single and multiple agents. The consideration of these problems has motivated, for example, new work in areas like belief revision, epistemic and dynamic logic or formalisms to represent uncertainty. More expressive and sophisticated formalisms have been developed (first and second order extensions of well known formalisms as well as new multi-modal and multi-agent approaches). Finally the consideration of some paradoxes concerning the relation of probability and qualitative belief, as well as problems related to judgment aggregation, have motivated researchers to rethink the very foundations of the Kripkean semantic program in modal logic. We survey some of these new developments as well as the philosophical problems that motivated them. The talk was not given; it was presented by title.

► GREG RESTALL, *Proof theory and meaning: three case studies*.
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Many of us are attracted to the idea that inference rules play a role in determining meanings. In this talk I will illustrate options for infererentialist approaches to meaning by looking at three case case studies discussed in the recent literature: (1) classical logic, (2) modalities, and (3) inductive and coinductive types.

► ALBERT VISSER, *Interpretations in philosophical logic*.
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In my talk, I will discuss the use of interpretations as a tool for comparing theories. The talk consists of two parts. In the first half I will discuss some general issues.

- I will sketch the various notions of interpretation and the various notions of sameness of theories definable in terms of interpretations.
- I will compare comparison of theories based on interpretations with other notions of comparison, like conservativity.
- I will comment on the question: what does it tell us when we have an interpretation of one theory in another?

In the second half, I will illustrate a use of interpretations to study the Predicative Frege Hierarchy obtained by iterating Predicative Comprehension plus Frege Function. (See for a description John Burgess' book Fixing Frege.) Roughly, the result is that, modulo mutual interpretability, we get Robinson's Arithmetic Q plus all iterated consistency statements for Q, so $Q+\mathrm{con}(Q)+\mathrm{con}(Q+\mathrm{con}(Q))+\cdots$. The second level of this hierarchy, i.e., $Q+\mathrm{con}(Q)$, is mutually interpretable with Elementary Arithmetic. It is open whether there is a natural hierarchy of faster and faster functions corresponding to the iterated consistencies.

Abstracts of invited talks in the Special Session on Proof Complexity and Nonclassical Logics

► EMIL JEŘÁBEK, *Proof systems for modal logics*.
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We discuss some issues in proof complexity of calculi for propositional modal and intermediate logics. We consider the usual Frege ("Hilbert style") proof systems, as well as their variants (extended Frege, substitution Frege) inspired by the classical case. We are interested in polynomial simulations, lower bounds, feasible disjunction properties, feasible partial conservativity results, and similar questions.

► GEORGE METCALFE, *Substructural fuzzy logics*.
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In this talk I will discuss a family of substructural logics defined by algebras based on the real unit interval [0,1]; well known examples being Łukasiewicz logic and Gödel–Dummett logic. Gentzen systems can often be obtained for these logics simply by extending sequent calculi for substructural logics like Linear Logic to the level of hypersequents. Moreover, the elimination of a special "density rule" from proofs in such calculi can be used to show completeness results for both Gentzen and Hilbert systems for the logics. Indeed, in certain cases, conditions that guarantee cut-elimination for the calculi also guarantee density-elimination, providing a syntactic characterization of the logics from this family.

► ALASDAIR URQUHART, *Complexity problems for substructural logics*.
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Substructural logics such as relevance logics and linear logics are the most complex nonclassical propositional calculuses investigated to date. They include undecidable systems, as well as logics that are decidable, but provably intractable. This talk surveys some of the work in the area, and also lists some open questions.

Abstracts of invited talks in the Special Session on Set Theory

► MÁRTON ELEKES, *Partitioning κ-fold covers into κ many subcovers*.
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Motivated by a question of A. Hajnal we investigate the following set of problems. Let X be a set, κ a cardinal number, and $\boldsymbol{H}$ a family that covers each x in X at least κ times. Under what assumptions can we decompose $\boldsymbol{H}$ into κ many subcovers? Equivalently, under what assumptions can we colour $\boldsymbol{H}$ by κ many colours so that for each x in X and each colour c there exists H in $\boldsymbol{H}$ of colour c containing x?

The assumptions we make can be, e.g., that $\boldsymbol{H}$ consists of open, closed, compact, convex sets, or polytopes in $\boldsymbol{R}^n$, or intervals in a linearly ordered set, or we can make various restrictions on the cardinality of X, $\boldsymbol{H}$, or elements of $\boldsymbol{H}$.

Besides numerous positive and negative results, many questions turn out to be independent of the axioms of set theory.

The speaker's research was partially supported by OTKA grants no. 49786, 61600, F43620 and the Öveges Project of NKTH and KPI.

This is a joint work with T. Mátrai and L. Soukup.

► GUNTER FUCHS, *Maximality principles for closed forcings*.
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I am going to talk about the modifications of the maximality principles, which were originally introduced by Stavi and Väänänen, and later (independently) by Hamkins, that result from restricting them to subclasses of $< \kappa$-closed forcings. The subclasses I consider are the entire class of $< \kappa$-closed forcings, the $< \kappa$-directed closed forcings and the collapses to κ. So for example, the maximality principle for $< \kappa$-closed forcings says that any statement that can be forced to be true by a $< \kappa$-closed poset in such a way that it remains true in any further generic extension by $< \kappa$-closed forcing, is already true. The talk will center around the following aspects: Consistency of the principles, the compatibility with large cardinal properties of κ, the relationships between the different versions of the principle, outright consequences, the possibilities of combining them in the sense that they hold at many regular κ at the same time, and the limitations of the extent to which they may be combined.

► VICTORIA GITMAN, *Scott's Problem for proper Scott sets*.
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Given a model of Peano Arithmetic (PA), define its "standard system" to be the collection of subsets of the natural numbers that arise as intersections of the definable sets of the model with its standard part. In 1962, Scott observed that standard systems satisfy certain basic computable theoretic and set theoretic properties. A collection of subsets of the natural numbers having these properties came to be known as a "Scott set". As a partial converse, Scott showed that countable Scott sets are exactly the *countable* standard systems and in 1982, Knight and Nadel extended his result to standard systems of size ω_1. The question of whether Scott sets are exactly the standard systems of models of PA came to be known as "Scott's Problem". I will introduce the history of Scott's Problem and talk about my results for standard systems of size ω_2, which were obtained using forcing constructions with models of PA together with the Proper Forcing Axiom. In particular, I will define the notion of proper Scott set and show, under PFA, that every proper arithmetically closed Scott is the standard system of a model of PA. I will also focus on the set theoretic questions (and answers) involving the existence of proper Scott sets.

► LIONEL NGUYEN VAN THÉ, *The Urysohn sphere is oscillation stable*.
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In 1994, Odell and Schlumprecht built a uniformly continuous map from the unit sphere of the Hilbert space into the unit interval and which does not stabilize on any isometric copy of the sphere. This result allowed to show that the Hilbert space has a property known as 'distortion'. The purpose of the present talk is to show that this situation is different when working with another remarkable metric space sharing many common features with the Hilbert sphere: the Urysohn sphere.

This is joint work with Jordi López Abad (Université Paris 7) and Norbert Sauer (University of Calgary).

► ASGER TÖRNQUIST, *Classifying measure preserving actions up to conjugacy and orbit equivalence*.
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Extending a well-known theorem by Foreman and Weiss we show: If H is an infinite subgroup of a discrete countable group G, then the measure preserving actions of H that can be extended to G cannot be classified by countable structures. As a consequence, the measure preserving actions of a countable group with the relative property (T) cannot be classified up to orbit equivalence by countable structures.

► MATTEO VIALE, *The constructible universe for the anti-foundation axiom system ZFA*.
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Aczel [1], and independently Forti and Honsel [2], introduced a strengthening of the axiom of foundation asserting the existence of a unique transitive collapse for every binary relation which is a set. The first order axiomatization of set theory with foundation replaced by this axiom is currently named in the literature ZFA. Forti and Honsel [3] also show that if M and N are transitive models of ZFA such that $M \cap WF = N \cap WF$, then $M = N$ (where WF is the definable class of well-founded sets). We exploit this idea to show that there are natural Gödel operations which are absolute for the theory ZFA such that the smallest transitive class which is closed under these Gödel operation is the "constructible" universe of the theory ZFA. The results that we present appeared in [4].

[1] Peter Aczel, ***Non well-founded set theories***, CSLI lecture notes 14, CSLI publications, 1988, Stanford.

[2] Marco Forti and Furio Honsel, *Set Theory with free construction principles*, ***Annali Scuola Normale Superiore di Pisa, Classe di Scienze***, vol. 10 (1983), pp. 493–522.

[3] ———, *Axiom of Choice and free construction principles I*, ***Bulletin de la Société Mathématique de Belgique***, vol. 36(B) (1984), pp. 69–79.

[4] Matteo Viale, *The cumulative hierarchy and the constructible universe of ZFA*, ***Mathematical Logic Quarterly***, vol. 50 (2004), no. 1, pp. 99–103.

Abstracts of contributed talks

► SERIKZHAN A. BADAEV, SERGEY S. GONCHAROV, AND ANDREA SORBI, *Completion of numberings*.
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Complete numberings play an important role in computability theory: they have an effective fixed point property, and their degrees are not splittable, while the completion operator gives us a regular way to construct a complete numbering α_a for any numbering $\alpha\colon \omega \longrightarrow \mathcal{A}$ and any special element $a \in \mathcal{A}$. We continue the study of the completion operator, started in [1].

As observed by Ershov, [2], $\alpha \leq \alpha_a$, and $\alpha_a \leq \beta$ for every numbering $\beta \geq \alpha$ which is complete with respect to a. In particular, $\alpha_a \equiv \alpha$ if and only if α is complete with respect to a.

We prove that no minimal numbering can be complete and, for every Friedberg numbering α, the interval $(\deg(\alpha), \deg(\alpha_a))$ consists of incomplete numberings (relative to any element of $\mathcal{A}$).

We show that, for some numbering α of a two-element set $\mathcal{A}$, the segment $[\deg(\alpha), \deg(\alpha_a)]$ is isomorphic to the upper semilattice of c.e. $\boldsymbol{m}$-degrees. This is a partial solution to an open problem posed in [1].

[1] S. Badaev, S. Goncharov, and A. Sorbi, *Completeness and universality of arithmetical numberings*, ***Computability and models***, (S. B. Cooper and S. S. Goncharov, editors), Kluwer / Plenum Publishers, pp. 11–44, New York, 2003.

[2] Yu. L. Ershov, ***Theory of numberings***, Nauka, Moscow, 1977.

► ILNUR BATYRSHIN, *The algebraic structure of quasi-degrees.*
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Tennenbaum (see [5]) introduced the notion of quasi-reducibility for sets: $A \leq_Q B$ if there is a computable function f such that $x \in A \Leftrightarrow W_{f(x)} \subseteq B$. Q-reducibility is a natural generalization of many-one reducibility (m-reducibility) and is closely connected with enumeration reducibility. Besides, on c.e. sets Q-reducibility implies T-reducibility: if a c.e. set $W \leq_Q A$ then $W \leq_T A$.

Quasi-reducibility plays [4] the main part in the Marchenkov's solution to the Post's problem in his spirit. He showed that no $\eta-$hyperhypersimple set is Q-complete and since Q-reducibility coincides with T-reducibility on semirecursive c.e. sets, each semirecursive noncomputable η-hyperhypersimple set is a solution to the Post's problem.

It also turned out that quasi-reducibility is closely connected to the word problems of groups: Dobritsa proved that for every set of natural numbers X there is a word problem with the same Quasi-degree as that of X, and Belegradek [2] showed that for any computably presented groups G and H, if G is a subgroup of every algebraically closed group of which H is a subgroup, then G's word problem must be quasi-reducible to that of H.

The study of the algebraic structure of Quasi-degrees of c.e. sets was started by the paper of Downey, LaForte and Nies [3], who established the density of the c.e. Q-degrees. Arslanov and Omanadze [1] started the study of the algebraic structure of Quasi-degrees of n-c.e. sets. In my report I will present some new results in this direction.

[1] M. M. Arslanov and R. Sh. Omanadze, *Q-degrees of n-c.e. sets*, to appear.

[2] O. Belegradek, *On algebraically closed groups*, ***Algebra i Logika***, vol. 13, no.3 (1974), pp. 813–816.

[3] R. R. Downey, G. G. LaForte and A. Nies *Computably enumerable sets and Quasi-reducibility*, ***Annals of Pure and Applied Logic***, vol. 95 (1998), pp. 1–35.

[4] S. S. Marchenkov, *A class of incomplete sets*, ***Matematicheskie Zametki***, vol. 20 (1976), pp. 473–478; ***Mathematical Notes*** vol. 20 (1976), pp. 823–825, (English translation).

[5] H. Rogers, Jr. ***Theory of recursive functions and effective computability***, McGraw-Hill, 1967.

► ACHIM BLUMENSATH, *Simple monadic theories.*
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We give an overview over recent developments concerning the model theory of monadic second-order logic. On the one hand, there are tree-like structures whose monadic theory is simple enough to develop a structure theory. On the other hand, there are structures with definable pairing functions where monadic second-order logic is as expressive as full second-order logic. According to a conjecture of Seese [2] these two cases form a dichotomy: either a structure is 'tree-like' or it has a definable pairing function. For graphs (or structures with relations of arity at most 2) a variant of this conjecture has recently been proved by Courcelle and Oum [1]. In this talk we will present first partial results concerning the general case. We consider structures without pairing function and we prove that the partition width of such structures is bounded by $2^{2^{\aleph_0}}$.

[1] B. Courcelle and S.-I. Oum, *Vertex-minors, monadic second-order logic, and a conjecture by Seese*, ***Journal Combinatorial Theory B***, vol. 97 (2007), pp. 91–126.

[2] D. Seese, *The structure of the models of decidable monadic theories of graphs*, ***Annals of Pure and Applied Logic***, vol. 53 (1991), pp. 169–195.

► ANDREW BROOKE-TAYLOR, *Definable well-ordering, the GCH, and large cardinals.*
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It is known that one may force a definable well-order of the universe to exist, for example by McAloon's technique of coding into the continuum function, or using Jensen coding to make the universe constructible from a real. However, it would be nice to be able to force to obtain a definably well-orderable universe where the GCH holds and very large cardinals exist, a situation that cannot be achieved by either of those methods. We show how one can achieve it, using a class length forcing iteration which may be constructed so as to preserve a proper class of such large cardinals as n-superstrong, n-huge, or κ^+-supercompact κ.

► FRANQUI CÁRDENAS, *A forcing extension of a simplified* $(\omega_2, 1)$ *morass with no simplified* $(\omega_2, 1)$ *morass with linear limits using an unfoldable cardinal.*
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Lee Stanley [3] has observed that if κ is supercompact cardinal then there is a forcing extension where there is a $(\omega_2, 1)$-simplified morass but there is no $(\omega_2, 1)$-simplified morass with linear limits. Using the lottery preparation, a technic developed by Joel Hamkins in [1], we just need a strongly unfoldable cardinal for the same observation.

Let κ an unfoldable cardinal, the forcing $\mathbb{P}$ which adds a $(\kappa, 1)$-simplified morass is κ-closed and it is also κ^+-c.c., although we can't extend the unfoldable embeddings in the ground model using directly this forcing (since it is too big to be in any M transitive model of ZFC^- of size κ), it is possible to do it thanks to a properness condition [2].

[1] Joel David Hamkins, *The lottery preparation*, ***Annals of Pure and Applied Logic***, vol. 101 (2000), no. 2, pp. 103–146.

[2] Thomas Johnstone, *Strongly unfoldable cardinals made indestructible*, ***The Journal of Symbolic Logic***, submitted.

[3] Dan Velleman, *Simplified Morasses with linear limits*, ***The Journal of Symbolic Logic***, vol. 49 (1984), no. 4, pp. 1001–1021.

► PETER CHOLAK, ROD DOWNEY, AND NOAM GREENBERG, *Strong jump-traceability I: the computably enumerable case.*
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This talk will discuss a project which determines the relation between Turing degrees which are *super jump traceable* and those which are K*-trivial*. By work of Nies and others the K-trivials are very robust class of degrees; for example, see [2].

We say that a function $h\colon \omega \to \omega \setminus \{0\}$ is an *order* (Schnorr) if h is computable, nondecreasing and $\lim_s h(s) = \infty$. We say that a function $f : \omega \to \omega$ is *computably traceable* with respect to the order h if there is a computable sequence $\langle F_x \rangle_{x<\omega}$ of finite sets such that for all x, $|F_x| \leq h(x)$ and $f(x) \in F_x$. We will say that a degree $\boldsymbol{a}$ is computably traceable iff there is some order h such that every f of degree $\boldsymbol{a}$ or less can be computably traced with respect to h. Finally, we will say that $\boldsymbol{a}$ is *strongly* computably traceable iff it is computably traceable with respect to any order. Here the idea is that the real is *computationally feeble*, in the sense that we have very good approximations to computations using A as an oracle. Perhaps one would expect that such reals would be highly non-random.

We have shown:

Theorem 1 ([1]). *Every c.e. strongly jump-traceable set is* K*-trivial.*

Thus for the first time, we have an example of a combinatorial property that at least *implies* K-triviality. The proof of this result relies on a new combinatorial technique using a kind of amplification of the traceability along the lines of the decanter or golden run method. It is beyond known technology; we believe that it could have other applications within computability theory and randomness.

On the other hand we also prove the following.

THEOREM 2 ([1]). *There is a K-trivial c.e. set that is* not *strongly jump-traceable. Indeed it is not jump traceable with a bound of size roughly* $\log \log n$.

This is the first example of a class defined by cost functions which we know does not coincide with the K-trivials in the proof technique is novel, since it is the first time a cost function has been used which still allows for the defeat of one involving Kolmogorov complexity.

[1] PETER CHOLAK, ROD DOWNEY, AND NOAM GREENBERG, *Strong jump-traceability and K-triviality*.

[2] ANDRÉ NIES, *Lowness properties and randomness*, ***Advances in Mathematics***, vol. 197 (2005), no. 1, pp. 274–305.

► PETR CINTULA AND GEORGE METCALFE, *Structural completeness for fuzzy logics*.
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A consequence relation is structurally complete iff all of its proper extensions have new theorems. That is, if a schematic rule is admissible (preserves the set of theorems), then it is derivable in any formal system axiomatizing the consequence relation. Classical logic has this property, but for many families of non-classical logics such as Intermediate, Modal, and Substructural logics, it is relatively rare, often existing only for certain fragments of the systems in question.

In this work we investigate structural completeness for the class of (t-norm based) fuzzy logics. These include Gödel–Dummett logic, known to be structurally complete, and Łukasiewicz logic, where the positive fragment is known to have the property, but not the full logic. We show that all fragments of the third fundamental fuzzy logic, Product logic, are structurally complete, a result that extends to related logics such as Cancellative Hoop Logic and Abelian Logic. We also give very general methods to show that a wide range of logics (fuzzy or otherwise) are not structurally complete. By combining these approaches we are able to answer the question of structural completeness for most (if not quite all) prominent fuzzy logics.

► BENJAMIN CLAVERIE, *Changing u_2 by stationary set preserving forcing*.
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We show that if I is a precipitous ideal on ω_1 and $\theta > \omega_1$ is regular, then there is a stationary set preserving forcing which adds a countable structure (M, J) which iterates to (H_θ, I) in ω_1 steps. Therefore, if there is a precipitous ideal on ω_1 and the universe is closed under sharps, then there is a stationary set preserving forcing which increases u_2. If BMM holds and there is a precipitous ideal on ω_1, then $u_2 = \omega_2$.

► IOANNA MATILDE DIMITRIOU AND PETER KOEPKE, *Equiconsistency of choiceless higher Chang conjectures with one Erdős cardinal*.
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We are interested in the connection between cardinals defined by certain combinatorial properties and cardinals satisfying certain model theoretic relations. In particular we will look at Erdős cardinals and generalised Chang conjectures.

Even though with the axiom of choice (AC) several versions of the Chang conjecture have consistency strengths in the realm of strong cardinals, higher or are even inconsistent, without AC we show that some versions of the Chang conjecture are equiconsistent with just one Erdős cardinal.

By a Chang conjecture we mean that for some cardinals κ, κ', λ and λ', the statement $\langle\kappa, \lambda\rangle \to \langle\kappa', \lambda'\rangle$ holds, i.e., for any structure with domain A of size κ with a unary predicate $B \subseteq A$ of size λ in a countable language, there is an elementary substructure $\langle A', B', \dots\rangle \prec \langle A, B, \dots\rangle$ with $|A'| = \kappa'$ and $|B'| = \lambda'$. On the other hand, a cardinal κ is λ-Erdős if it is the least such that $\kappa \to (\lambda)^{<\omega}$ holds, i.e., for every partition f of the finite subsets of κ into two colours, there is a homogeneous set for f of size λ.

We show that for λ a regular cardinal, the existence of a λ-Erdős cardinal κ is equiconsistent with $\langle\lambda^+, \lambda\rangle \to \langle\lambda, \nu\rangle$ for any infinite $\nu < \lambda$. For the forward direction we construct a symmetric model where κ is λ^+ and still λ-Erdős. In that model, $\langle\lambda^+, \lambda\rangle \to \langle\lambda, \nu\rangle$ holds for all ν with $\lambda > \nu \geq \omega$. For the converse we use Jensen's indiscernibles lemma to show that if one of these versions of the Chang conjecture holds, then $(\lambda^+)^V$ is λ-Erdős in K, the Dodd–Jensen core model.

► NATASHA DOBRINEN AND SY-DAVID FRIEDMAN, *On the consistency strength of the tree property at the double successor of a measurable cardinal.*
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Given any regular cardinal κ and a weak compact λ above κ, Mitchell showed that λ can be collapsed to κ^+ in such a way that in the forcing extension, κ^+ has the tree property. His methods used a mixed-support iteration of various Cohen forcings. We show that iterated generalized Sacks forcing can achieve the same goal. We then use these iterations to obtain a model where the tree property holds at the double successor of a measurable cardinal, using much weaker hypotheses than were previously known to suffice.

► ANDREY N. FROLOV, Δ^0_n*-categorical linear orderings.*
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I'll talk about Δ^0_n-categorical linear orderings. In particular, I will show that there is a strictly Δ^0_{n+1}-categorical linear ordering. A linear ordering is strictly Δ^0_{n+1}-categorical, if it is Δ^0_{n+1}-categorical and it is not Δ^0_n-categorical. Also I'll talk about Δ^0_n-stable linear orderings.

► NICOLAS GUZY, *Towards a notion of p-adic closure for a ring.*
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In [1] L. Bélair generalizes results in [4] by giving an axiomatization, denoted by $ALpC_{e,f}$, of local p-adically closed rings of p-rank d, i.e., henselian local rings with residue field which are p-adically closed field of p-rank d ($d = ef$ with e and f fixed). The results extend the ones obtained by E. Robinson about the p-adic spectrum. Then he completes the theory $ALpC_{e,f}$ in a scheme of axioms $AIpC_{e,f}$ whose models are henselian valuation rings whose residue fields are p-adically closed fields of p-rank d and value groups are divisible.

Particular examples of models of $AIpC_{e,f}$ are provided and studied in Section 3 of [1]: if C is an affine curve on $\mathbb{Q}_p$ then L. Bélair shows that for any prime non maximal ideal $\mathfrak{p}$ of the ring of continuous definable functions $C \to \mathbb{Q}_p$, denoted $\mathcal{C}(C)$, we have $\mathcal{C}(C)/_{\mathfrak{p}}$ is a model of $AIpC_{e,f}$.

To this effect, the powerful tool used in this paper is the existence of an homeomorphism between the p-adic spectrum of $\mathbb{Q}_p[V]$ (V is an affine variety over $\mathbb{Q}_p$) and the ring spectrum of $\mathcal{C}(V)$.

In [2], L. Bélair generalizes his previous approach to the ring of continuous definable functions on V over $\mathbb{Q}_p$, denoted by $\mathcal{C}(V)$, for any affine p-adic variety V. He studies, in particular, first-order properties of the quotient rings $\mathcal{C}(V)/\mathfrak{p}$ by a prime ideal $\mathfrak{p}$ of $\mathcal{C}(V)$.

This talk is devoted to give an account of the work in [3]. In this work, we introduce a new class of rings called p-adically closed rings. For this purpose, we build the p-adic closure of a given ring A in a similar way as in [5] for the real closure of a ring. Namely, the topological properties of the p-adic spectrum of A plays a central role in this construction.

Then it allows us to define the notion of a p-adically closed ring and to generalize the results from [1] and [2] to the p-adically closed rings.

[1] L. Bélair, *Anneaux p-adiquement clos et anneaux de fonctions d'efinissables*, ***The Journal of Symbolic Logic***, vol. 56 (1991), pp. 539–553.

[2] ———, *Anneaux de fonctions p-adiques*, ***The Journal of Symbolic Logic***, vol. 60 (1995), no. 2, pp. 484–497.

[3] N. Guzy, *Towards a notion of p-adic closure for a ring*, preprint, 2007.

[4] E. Robinson, *The geometric theory of p-adic fields*, ***Journal of Pure and Applied Algebra***, vol. 40 (1986), pp. 281–296.

[5] N. Schwartz, *The basic theory of real closed spaces*, ***Memoirs of the American Mathematical Society***, vol. 397 (1989).

► PETR HÁJEK, *Complexity of fuzzy predicate logics with witnessed semantics*.
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Investigated are continuous t-norm based fuzzy predicate logics with standard semantics (the set of truth degrees is the closed real unit interval), as introduced in my Metamathematics of fuzzy logic (Kluwer 1998); in particular, Łukasiewicz, Gödel and product logic. Semantics is Tarski-style; over a crisp non-empty domain predicates are interpreted by fuzzy ([0, 1]-valued) relations, the value of a universally quantified formula being the infimum of the values of instances and analogously for existential quantification and supremum. A tautology is a formula having the value 1 in each interpretation; a formula is satisfiable if it has the value 1 in some interpretation. The arithmetical complexity of the set of tautologies and the set of satisfiable formulas is known to be: for Łukasiewicz Π_2-complete, Π_1-complete; for Gödel Σ_1-complete, Π_1-complete; for product logic non-arithmetical, non arithmetical. An interpretation is witnessed if the truth value of each universally quantified formula is the minimum of truth values of instances (the infimum is taken) and the truth value of each existentially quantified formula is the maximum of values of instances. A witnessed tautology is a formula having the value 1 in each witnessed interpretation; similarly for witnessed satisfiability. Now the complexity of the set of witnessed tautologies/witnessed satisfiables: for Łukasiewicz Π_2-complete, Π_1-complete; for Gödel Σ_1-complete, Π_1-complete (for both logics the same complexity as without the assumption of witnessedness); for product logic the set of witnessed tautologies is Π_2-hard (nothing more is known) whereas the set of witnessedly satisfiable formulas is Π_1-complete. For Göodel and product the set of witnessedly satisfiable formulas is equal to the set of formulas in the classical Boolean logic. Much more is known; a survey will be given.

[1] P. Hájek, ***Metamathematics of fuzzy logic***, Kluwer, 1998.

[2] ———, *On witnessed models in fuzzy logic*, ***Mathematical Logic Quaterly***, vol. 53 (2007), pp. 66–77.

[3] ———, *On witnessed models in fuzzy logic II*, submitted.

► JAMES HUNTER, *Higher-order reverse topology*.
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Reverse Mathematics studies equivalences between logical axioms and mathematical theorems. Traditional Reverse Mathematics is limited to subsystems of second-order arithmetic, and thus can consider only those mathematical theorems expressible in the language of second-order arithmetic.

In a recent paper, Kohlenbach showed that Reverse Mathematics extends nicely to higher-order theories, and examined certain higher-order statements in mathematical analysis. The same techniques can be applied to statements about topological spaces of size continuum, allowing for an examination of statements more general than those studied by traditional Reverse Mathematics.

► KOICHIRO IKEDA, *A remark on a characterization of non-forking in generic structures*.
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Let L be a countable relational language and $\mathbf{K}$ a class of finite L-structures with non-negative predimension. Then a countable L-structure M is said to be $\mathbf{K}$-generic, if (i) any finite $A \subset M$ belongs to $\mathbf{K}$; (ii) for any finite $A \leq B \in \mathbf{K}$ with $A \leq M$ there is a copy B' of B over A with $B' \leq M$; (iii) there are finite $A_0 \leq A_1 \leq A_2 \leq \cdots \leq M$ with $M = \bigcup_i A_i$. Let M be a saturated generic structure and $\mathcal{M}$ a big model of $\mathrm{Th}(M)$. In [1], [2] and [3], non-forking in $\mathcal{M}$ has been characterized as follows: For any $A \leq B, C \leq \mathcal{M}$ with $A = B \cap C$ *algebraically closed*, $\mathrm{tp}(B/C)$ does not fork over A if and only if B, C are free over A and $BC \leq \mathcal{M}$. It is seen that one cannot remove the condition that A is algebraically closed in the above statement. Our aim is to generalize the statement as follows: For any $A \leq B, C \leq \mathcal{M}$ with $A = B \cap C$, $\mathrm{tp}(B/C)$ does not fork over A if and only if B, C are free over $\mathrm{acl}(A)$ and $BC \cup \mathrm{acl}(A) \leq \mathcal{M}$. As a corollary, it can be proved that there is no saturated generic structure that is superstable but not ω-stable.

[1] J. T. Baldwin and N. Shi, *Stable generic structures*, ***Annals of Pure and Applied Logic***, vol. 79 (1996) pp. 1–35.

[2] V. Verbovskiy and I. Yoneda, *CM-triviality and relational structures*, ***Annals of Pure and Applied Logic***, vol. 122 (2003), no. 1–3, pp. 175–194.

[3] F. O. Wagner, ***Relational structures and dimensions, automorphisms of first-order structures***, The Clarendon Press, Oxford University Press, New York, 1994, pp. 153–180

► CHRIS IMPENS AND SAM SANDERS, *What if computers could count to infinity*?
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This paper describes work in progress in weak systems of arithmetic and applications. We define a nonstandard version of the theory of polynomial time functions in which the usual fundamental principles of nonstandard mathematics, like saturation and transfer, are integrated. Our main result is that, over the finite numbers, the nonstandard polynomial time functions contain the recursive functions. Given the big difference in computational complexity between the standard polynomial time functions and the recursive ones, the result is surprising.

Hypercomputation with computers effectively executing infinitely many calculations in a finite time has already been considered in quantum mechanics and elsewhere. Our nonstandard theory of polynomial time functions is a very natural abstraction of this computational concept, and allows unexpected insights in the feasibility and power of such a hypercomputer.

The authors wish to thank professor Ulrich Kohlenbach (Technische Universität Darmstadt) and professor Andreas Weiermann (University of Ghent) for their valuable advice.

► BART KASTERMANS AND STEFFEN LEMPP, *Separating notions of randomness*.
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The intuitive concept of randomness allows several possible concise definitions, many of which have been shown to be inequivalent. We discuss some of these notions and present a new separation result: Not every "permutation partially computably random" set is Martin-Löf random. This is a partial result toward separating Martin-Löf randomness from Kolmogorov–Loveland randomness, one of the main open questions in this area. (See J. Miller/Nies, BSL, vol. 12 (2006), pp. 390–410, for more background on this area.)

► THOMAS F. KENT AND ANDREA SORBI, *Non-splitting enumeration degrees*.
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There are not many properties that hold for all proper ideals of Σ^0_2-enumeration degrees. For example, it is known that there are ideals of Σ^0_2-enumeration degrees which contain only properly Σ^0_2 degrees except $\mathbf{0}_e$ (and hence no low-degree), while there are also non-trivial ideals consisting of only low-degrees. The authors have shown that any proper ideal contains a non-splitting degree, and thus the non-splitting degrees are downwards dense in the Σ^0_2-enumeration degrees. In proving this result, they also introduce a new form of permitting, which allows them to build below an arbitrary Σ^0_2-set.

► SATORU KURODA, *Bounded arithmetic and LOGCFL*.
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The class LOGCFL is defined to be the class of sets LOGSPACE-reducible to some context-free language. This class is equivalent to the class SAC^1 of sets which are computed by some semi-unbounded fan-in, logarithmic depth, polynomial size circuit families.

Using the latter characterization, we define a two-sort bounded arithmetic $V\text{–}Q^{SAC}(\Sigma^B_0)$ [2] which has a bit comprehension axiom for formulae stating that there is a tree certificate of SAC^1 circuit on a given input string. Then we prove that a function is provably total in $V\text{–}Q^{SAC}(\sum^B_0)$ if and only if it is polynomially bounded and bitwise computable in LOGCFL.

The main feature of the proof is that $V\text{–}Q^{SAC}(\Sigma^B_0)$ can prove the closure of SAC^1 under complementation. This closure property of SAC^1 was proved by Borodin et al. [1] using the inductive counting argument and our proof is a formalization of it within the system.

[1] A. Borodin, S. A. Cook, P. W. Dymond, W. L. Ruzzo and M. Tompa, *Two applications of inductive counting for complementation problems*, ***SIAM Journal on Computing***, vol. 18 (1989), no. 3, pp. 559–578.

[2] S. Kuroda, *Generalized quantifier and a bounded arithmetic for LOGCFL*, to appear in ***Archive for Mathematical Logic***, (2007).

► ROBERT S. LUBARSKY AND FRED RICHMAN, *Signed-bit representations of real numbers and the constructive Stone–Yosida theorem*.
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The signed-bit representation of reals, as developed classically, is like binary, only in addition to 0 and 1 you can also use -1. This representation lends itself especially well to the constructive (intuitionistic) theory of the reals. (For background on constructive analysis, see [1] or [2].) We develop the signed-bit equivalents of three common notions of real numbers: Dedekind cuts, Cauchy sequences, and Cauchy sequences with moduli of convergence.

This theory is then applied to representations of Riesz spaces. A Riesz space is a lattice ordered vector space (here taken to be over the rationals), and a representation of such is a homomorphism into the reals. The canonical example of a Riesz space is a space of real-valued functions, and a representation is evaluation at a point in the domain. Constructively, in order to find a point in the domain, one must often make an additional assumption, such as Dependent Choice. This is proven for example in [3], where the authors ask whether DC is necessary for this result. In this talk, the existence of (appropriate) Riesz space representations is recast in terms of signed-bit representations, and a possible way to show that to be independent from the regular axioms of set theory sans DC, using a topological model (as in [4], [5], and [6]), is then sketched.

[1] Errett Bishop, ***Foundations of constructive analysis***, McGraw-Hill, 1967.

[2] Douglas Bridges and Luminita Vita, ***Techniques of constructive analysis***, Springer, 2006.

[3] Thierry Coquand and Bas Spitters, *Formal topology and constructive mathematics: the Gelfand and Stone–Yosida representation theorems*, ***Journal of Universal Computer Science***, vol. 11 (2005), pp. 1932–1944.

[4] M. P. Fourman and J. M. E. Hyland, *Sheaf models for analysis*, ***Applications of sheaves***, (M. P. Fourman, C. J. Mulvey, and D. S. Scott, editors), Lecture Notes in Mathematics, vol. 753, Springer-Verlag, 1979, pp. 280–301.

[5] R. J. Grayson, *Heyting-valued models for intuitionistic set theory*, ***Applications of Sheaves***, (M. P. Fourman, C. J. Mulvey, and D. S. Scott, editors), Lecture Notes in Mathematics, vol. 753, Springer-Verlag, 1979, pp. 402–414.

[6] Robert Lubarsky, *On the Cauchy completeness of the constructive Cauchy reals*, accepted for publication, available at `www.math.fau.edu/Lubarsky`.

► KERKKO LUOSTO, *Unary quantifiers and built-in successor*.
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Unary quantifiers binding a single formula were introduced by Andrzej Mostowski 50 years ago. The notion of generalized quantifiers was subsequently enhanced by Lindström 1966, but unary quantifiers remain an interesting special case. The definability theory of unary quantifiers is well-understood and leads to some interesting combinatorics involving Ramsey theory (see, e.g., Nešetřil and Väänänen 1996 or Luosto 2000). However, the impact of built-in relations in the structures has not yet been thoroughly explored. There are some indications that the definability theory in ordered structures is difficult (Luosto 2004). Here, we discuss the case of built-in successor relation and show that except for the change in quantifier ranks, built-in successor is not strong enough to enhance the expressive power of unary quantifiers.

► SATOSHI MATSUOKA, *MLL proof nets as error-correcting codes*.
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The study of the multiplicative fragment of Linear Logic without multiplicative constants (for short MLL) is successful from both semantical and syntactical point of view. In the semantical point of view there are good semantical models including coherent spaces. In the syntactical point of view the theory of MLL proof nets has obtained a firm status without doubt. On the other hand IMLL, an intuitionistic version of MLL is also studied. IMLL can be seen as a subsystem of MLL. IMLL is easier to be studied more deeply than MLL, because we can use intuitions inspired from the conventional lambda-calculus theory as well as graph-theoretical intuitions from the MLL proof nets theory.

In order to study MLL more deeply, how should we do? One approach is to interpret MLL intuitionistically by using Gödel's double negation interpretation. However in such an approach multiplicative constants must be introduced. Definitely introducing multiplicative constants makes things complicated. Another approach we propose in this paper is to adopt *coding theoretic* framework.

Coding theory is very useful for real world applications. A notable example is digital television. Basically, coding theory is to study a way of detecting and/or correcting data that may be true or false. In this paper we propose a novel approach for analyzing proof nets of MLL by coding theory. We define families of proof structures and introduce a metric space for each family. In each family,

1. an MLL proof net is a real code
2. a proof structure that is not an MLL proof net is a false code.

In this talk we describe a summary about results we have obtained so far, using examples.

► HEIKE MILDENBERGER, *There may be infinitely many near coherence classes under* $\mathfrak{u} < \mathfrak{d}$.
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We show that in the models from [2], where the ultrafilter characteristic is strictly less than the dominating number, there are infinitely many near-coherence classes of ultrafilters. Thus we answer Banakh's and Blass' Question 30 of [1] negatively. By an unpublished result of Canjar, there are at least two classes in these models. On the way we prove some general facts on ultrafilters under $\mathfrak{u} < \mathfrak{d}$. This talk is mainly a report on [3].

[1] Taras Banakh and Andreas Blass, *The number of near-coherence classes of ultrafilters is either finite or* 2^c, ***Set theory***, (Joan Bagaria and Stevo Todorcevic, editors), Trends in Mathematics, pp. 257–273. Birkhäuser, 2006.

[2] Andreas Blass and Saharon Shelah, *Ultrafilters with small generating sets*, ***Israel Journal of Mathematics***, vol. 65 (1989), pp. 259–271, [BsSh:257].

[3] Heike Mildenberger, *There may be infinitely many near-coherence classes under* $\mathfrak{u} < \mathfrak{d}$, ***The Journal of Symbolic Logic***, to appear.

► CYRUS NOURANI, *Positive realizability morphisms and Tarski models.*
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Definition 1. Let Σ (x 1 ... x n) be a set of formulas of L. Say that a positive theory T in L positively locally realizes Σ iff there is a formula φ x 1 ... x n) in L s.t.

(1) φ is consistent with T

(2) for all $\sigma \varepsilon \Sigma$, T $\models \varphi$ or T $\cup \sigma$ is not consistent.

Theorem 1. *Let* L_1, L_2 *be two positive languages. Let* $L = L_1 \cap L_2$. *Suppose* T *is a complete theory in* L *and* T 1 $\subset$ T, T 2 $\subset$ T *are consistent in* L_1, L_2, *respectively. Suppose there is model* M *definable from a positive diagram in the language* $L_1 \cup L_2$ *such that there are models* M 1 *and* M 2 *for* T 1 *and* T 2 *where* M *can be homomorphically embedded in* M 1 *and* M 2.

(1) T 1 $\cup$ T 2 *is consistent.*

(*ii*) *There is model* N *for* T 1 $\cup$ T 2 *definable from a positive diagram that homomorphically extend that of* M 1 *and* M 2.

THEOREM 2. *Let* L_1, L_2 *be two positive languages. Let* $\mathrm{L} = \mathrm{L}_1 \cap \mathrm{L}_2$. *Suppose* T *is a complete theory in* L *and* $\mathrm{T}\,1 \subset \mathrm{T}$, $\mathrm{T}\,2 \subset \mathrm{T}$ *are consistent in* L_1, L_2 *respectively. Then*
(1) $\mathrm{T}\,1 \cup \mathrm{T}\,2$ *has a model* M, *that is positive end extension on Models* M 1 *and* M 2 *for* T 1, *and* T 2, *respectively*;
(*ii*) M *is definable from a positive diagram in the language* $\mathrm{L}_1 \cup \mathrm{L}_2$.

Positive forcing (author 1981) had defined T^* to be T augmented with induction schemas on the Generic diagram functions

THEOREM 3. *Considering a Tarskian presentation* P *for a theory* T *that has a positive local realization, with* T^* *we can assert the following. Every formula on the presentation* P *is completable in* T^*.

► MICHAEL O'CONNOR, *The order-theoretic structure of free Heyting algebras*.
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A new result on the structure of Lindenbaum algebras of intuitionistic propositional logic in finitely many variables is discussed.

► ROBERT OWEN, *Outer models, class forcing, and weakly self-defining classes*.
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M. C. Stanley described [1] a notion of class forcing that allows $0^\sharp$ to be realized from a ground model of $V = L$ assuming such models exist in the universe. We consider the Definability Lemma for such forcings and use this as a springboard to discuss set-theoretic complexity classes lying between Σ_n/Π_n and Δ_{n+1}, and their self-definability.

[1] M. C. STANLEY, *Outer models and genericity*, ***The Journal of Symbolic Logic***, vol. 68 (2003), no. 2, pp. 389–418.

► ALEXANDRA REVENKO, *Automatic linear orders*.
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A structure $\mathfrak{A} = (A, R_1, \dots, R_n)$ is automatic if its domain A and all its relations R_i are finite automaton recognisable (automatons for relations working synchronously on tuples of finite words). The structure has an automatic presentation if it is isomorphic to some automatic structure.

It is known that there exists an algorithm that given a relation which is first order definable (with parameters) in automatic structure with an additional quantifier $\exists^\infty$ constructs an automaton recognising this relation. Hence the first order theory with this additional quantifier of an automatic structure is decidable [1, 5].

Our investigations concern the recursive isomorphism problem for two automatic presentations of linear order [3, 4].

Delhomme achieved the next result:

THEOREM 1 ([2]). *An ordinal* α *is automatically presentable if and only if* $\alpha < \omega^\omega$.

This fact was generalized by S. Rubin and B. Khoussainov [5]. If we factor a linear order by the equivalence relation "there is a finite number of elements between x and y" then FC-rank of the linear order is the least number of such factorisations after which we get a dense order or **1**-order (order with 1 element) from the given order.

THEOREM 2 ([5]). *The FC-rank of every sutomatic linear order is finite.*

Thus the next results were achieved:

THEOREM 3. *Every two automatic presentations of ordinal* $\alpha < \omega^\omega$ *are recursive isomorphic.*

The linear order is scattered if it does not contain a nontrivial dense subordering

THEOREM 4. *Every two automatic presentations of automatic scattered linear order with FC-rank* 2 *are recursive isomorphic.*

THEOREM 5. *Every automatic linear order is definable in appropriate automatic linear order with FC-rank* 1.

In addition examples of non-periodic automatic linear order were provided. The linear order is periodic if it is $\sum_{i\in\omega^*} \mathcal{A}_i + \mathcal{B} + \sum_{i\in\omega} \mathcal{C}_i$, where $\mathcal{A}_i = \mathcal{A}_j = \mathcal{A}$ for all $i \in \omega^*$, $\mathcal{C}_i = \mathcal{C}_j = \mathcal{C}$ for all $i \in \omega$ and $\mathcal{A}, \mathcal{B}, \mathcal{C}$—some linear orders

[1] A. BLUMENSATH, *Automatic structures*, Diploma thesis, RWTH Aachen, 1999.

[2] C. DELHOMME, *Non-automaticity of ω^ω*, manuscript, 2001.

[3] YU. L. ERSHOV AND S. S. GONCHAROV, *Constructive models*, Siberian School of Algebra and Logic, 1999.

[4] H. ROGERS, JR., ***Theory of recursive functions and effective computability***, McGraw-Hill, New York, 1967.

[5] S. RUBIN, *Automatic structures*, a thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, The University of Auckland, 2004.

► GEMMA ROBLES, JOSÉ M. MÉNDEZ AND FRANCISCO SALTO, *Relevance logics and intuitionistic negation*.

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Since the beginning of the relevance enterprise, relevance logicians have been interested in exploring the frontiers of relevance logics. This explains the considerable attention that has been paid to the paradoxical R-Mingle or the motivation behind Meyer and Routley's Classical Relevant Logic. We have investigated the result of adding the K rule to relevance logics in the context of a constructive negation and in the presence of a non-constructive negation in [4] and [1], respectively. In the present contribution, we study the effect of adding a constructive intuitionistic-type negation including the EFQ ('E falso quodlibet') axioms $A \to (\neg A \to B)$, $\neg A \to (A \to B)$ and the ECQ ('E contradictione quodlibet') axiom $(A \wedge \neg A) \to B$ to positive relevance logics up to Relevance Logic R+ plus the mingle axiom $A \to (A \to A)$ and show that the K rule $\vdash A \Rightarrow \vdash (B \to A)$ and so, the K axiom $A \to (B \to A)$ are not derivable. Thus, a spectrum of constructive logics that, though having the standard paradoxes of consistency, lack the key paradoxes of relevance are defined.

We employ the semantics defined in [3]. Part of the results presented appear in [2].

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[1] J. M. MÉNDEZ AND G. ROBLES, *Relevance logics, paradoxes of consistency and the K rule II. A non-constructive negation*, ***Logic and Logical Philosophy***, vol. 15 (2006), pp. 175–191.

[2] ———, *Relevance logics and intuitionistic negation*, ***Journal of Applied Non-Classical Logics***, accepted.

[3] G. ROBLES, J. M. MÉNDEZ, AND F. SALTO, *Minimal negation in the ternary relational semantics*, ***Reports on Mathematical Logic***, vol. 39 (2005), pp. 47–65.

[4] ———, *Relevance logics, paradoxes of consistency and the K rule*, ***Logique et Analyse***, forthcoming, June issue vol. 50 (2007), no. 198. (An abstract of this paper was read at the Logic Colloquium 2006, Nijmegen, Holland, 27 July–2 August 2006).

► LUCA MOTTO ROS, *General reducibilities for sets of reals*.
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Intuitively, a set of reals A is *simpler* than—or as complex as—a set of reals B if the problem of verifying membership in A can be reduced to the problem of verifying membership in B. This observation has led W. Wadge to introduce the notion of *continuous reducibility*, where A is reducible to (i.e., simpler than) B if there is some continuous function $f : \mathbb{R} \to \mathbb{R}$ such that $f^{-1}(B) = A$. The study of this relation (on the Baire space) has led to the development of a very rich and fascinating theory, now known as Wadge theory. We will give a brief history of some of the most important results in this area and present a very general approach to the study of various notions of reducibility on the Baire space.

► DENIS I. SAVELIEV, *On cardinals in set theory without choice and regularity*.
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We show that a considerable part of standard results on cardinals can be recovered (or almost recovered) in set theory without Choice and Regularity. In the point of well-known "negative" consistency results (of Gitik and others), such "positive" provable results are rather unexpected. Further, we show that Regularity can influence on cardinals. Quite often relationships of cardinalities of well-founded sets to other cardinalities look like relationships of alephs to cardinalities of non-well-orderable sets.

► PHILIPP SCHLICHT, *Thin projective equivalence relations and inner models*.
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Greg Hjorth proved in 1993 under the assumption that all reals have sharps, that for an inner model M, M has representatives of all equivalence classes of all thin Π^1_2 equivalence relations if and only if M is Σ^1_3 correct in V and computes ω_1 correctly. I will present a similar description of the inner models that have representatives of all equivalence classes of all thin Π^1_{2n} equivalence relations, assuming PD. These are exactly the inner models that are $\Sigma^1_{(2n+1)}$-correct in V and compute the canonical tree from a $\Pi^1_{(2n-1)}$-scale correctly.

This is joint work with Greg Hjorth and Ralf Schindler.

► M. SHEREMET, F. WOLTER, AND M. ZAKHARYASCHEV, *A modal logic of metric spaces*.
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In an attempt to extend Tarski's programme of algebraising topology to metric spaces, we introduce three binary operators on metric spaces (I, d): for $A, B \subseteq I$,

$$\begin{aligned} A \leftleftarrows B &= \{u \in I \mid d(u, A) < d(u, B)\}, \\ A \leftharpoonup B &= \{u \in I \mid \forall b \in B\, \exists a \in A\, (d(u, a) < d(u, b))\}, \\ A \equiv B &= \{u \in I \mid \exists a \in A\, \forall b \in B\, (d(u, a) \le d(u, b))\}, \end{aligned}$$

where $d(u, A) = \inf\{d(u, a) \mid a \in A\}$ if $A \neq \emptyset$, and $d(u, \emptyset) = +\infty$.

Denote by $\mathcal{L}$ the logic obtained by adding the operators $\leftleftarrows$, $\leftharpoonup\!\!\!=$, $\equiv\!\!\!\!\leftarrow$ to classical propositional logic and interpreting it over metric spaces (propositional variables are interpreted as their arbitrary subsets).

It is easy to see that $\mathcal{L}$ is more expressive than Tarski's $S4$; for example, $A \leftleftarrows \neg A$ is the interior of A. We show the following:

(1) $\mathcal{L}$ is as expressive over metric spaces as the logic $\mathcal{L}'$ with the operators $\exists x$ (there exists $x > 0$), $\exists^{<x}$ (in the open x-neighbourhood), and $\exists^{\leq x}$ (in the closed x-neighbourhood), where formulas starting with $\exists^{<x}$ or $\exists^{\leq x}$ are only allowed in a Boolean combination immediately after $\exists x$. For example, $A \leftleftarrows B$ is equivalent to $\exists x(\exists^{<x} A \sqcap \neg\exists^{<x} B)$.
(2) There is a natural Hilbert-style finite axiomatisation of $\mathcal{L}$.
(3) The decision problem for $\mathcal{L}$ is ExpTime-complete.
(4) Satisfiability of $\mathcal{L}$-formulas is undecidable over $\mathbb{R}$ (which is proved by reduction of the solvability problem for Diophantine equations).

► ELLIOTT SPOORS, *Provably recursive functions in extensions of a predicative arithmetic*.
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We provide a short summary of recent research into extensions of the weak first order theory of arithmetic $EA(I;O)$ developed by S. Wainer and previous research students, e.g., [2]. $EA(I;O)$ replaces the full induction scheme of PA by what can be seen as a predicative induction rule, incorporating the idea of variable separation from Bellantoni & Cook [1]. The principle effect is that the provably recursive functions are now the Kalmar elementary functions with the bounding functions in the ordinal analysis coming from the slow growing hierarchy.

In this talk we will present a conservative extension of $EA(I;O)$ which provides a more natural approach to composition of provably recursive functions. Furthermore, this theory can be easily extended into a hierarchy to capture higher levels of the Grzegorczyk hierarchy. The difficultly in proving so comes about in establishing upper bounds. We look in detail at the first level above our base theory where we can employ a similar finitary analysis as used on $EA(I;O)$ in [3]. In doing so we raise a number of interesting issues such as composition of the slow growing G_α functions.

[1] S. Bellantoni and S. Cook, *A new recursion theoretic characterization of the polytime functions*, ***Computational Complexity***, vol. 2 (1992), pp. 97–110.

[2] N. Cagman, G. E. Ostrin and S. S. Wainer, *Proof theoretic complexity of low subrecursive classes*, ***Foundations of Secure Computation***, (F. L. Bauer and R. Steinbrueggen, editors), IOS Press Amsterdam, 2000, pp. 249–286.

[3] G. E. Ostrin and S. S. Wainer, *Elementary arithmetic*, ***Annals of Pure and Applied Logic***, vol. 133 (2005), pp. 275–292.

► JACK D. STECHER, HARRIE DE SWART, AND KIRA PRONIN, *Intuitionism and repeated games*.
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In non-cooperative game theory, a stage game is one that is played repeatedly; a standard example is an iterated prisoners' dilemma. A strategy can be part of an equilibrium of an infinitely repeated stage game, yet not part of any equilibrium if the number of stages is finite. Subjects in laboratory experiments typically play finitely repeated stage games, at least during an initial segment, as if they were playing an infinitely repeated stage game. By doing so, the subjects often receive higher payoffs than they would receive from playing equilibrium strategies.

We provide a theoretical explanation for this observed behavior by modeling the players of a game as reasoning intuitionistically. In particular, we take the position that the players of a game may not necessarily know what game they will play or what opponents they will face at stages in a sufficiently remote future. In this setting, any strategy a subject chooses in an infinite stage game is also one that the subject would be willing to choose in any stage game that had a sufficiently similar initial segment; this is essentially Brouwer's continuity principle. We show that the classically problematic behavior is intuitionistically quite sensible.

► ALEXEY STUKACHEV, *On Ershov semilattices of degrees of* Σ*-definability of structures*.
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The notion of Σ-definability of a structure in an admissible set, introduced by Yu. L. Ershov [1], is an effectivization of the model-theoretical notion of interpretability of structures, and, at the same time, a generalization of the notion of constructivizability of a structure on natural numbers. For structures $\mathfrak{M}$ and $\mathfrak{N}$, let $\mathfrak{M} \leqslant_\Sigma \mathfrak{N}$ means that $\mathfrak{M}$ is Σ-definable in $\mathbb{HF}(\mathfrak{N})$, the least admissible set over $\mathfrak{N}$. Preordering $\leqslant_\Sigma$, considered for structures of cardinality $\leqslant \alpha$, generates the upper semilattice $\mathcal{S}_\Sigma(\alpha)$. Σ-degrees of some uncountable structures (fields of real, p-adic and complex numbers, dense linear orders, etc.) were studied in [1, 2, 3].

We show that the semilattices of Turing and enumeration degrees are embeddable in a natural way into the semilattices of Σ-degrees, by means of Σ-degrees of structures having a degree (resp., e-degree). The notion of a structure having a degree, known in computable model theory, gives only a partial measure of complexity, since there are a lot of structures which do not have a degree. Σ-degrees, as well as degrees of presentability with respect to different effective reducibilities [4], are natural measures of complexity which are total, i.e., defined for any structure.

In this talk we consider some recent results on some local and global properties of Ershov semilattices [5].

[1] Yu. L. Ershov, ***Definability and computability***, Plenum, NY, 1996.

[2] A. I. Stukachev, *Uniformization property in hereditary finite superstructures*, ***Siberian Advances in Mathematics***, vol. 7 (1997), pp. 123–132.

[3] ———, Σ*-definability in hereditary finite superstructures and pairs of models*, ***Algebra and Logic***, vol. 46 (2004), pp. 258–270.

[4] ———, *On mass problems of presentability*, ***TAMC2006***, (J.-Y. Cai, S. B. Cooper, and A. Li, editors), Lecture Notes in Computer Science, vol. 3959, Springer, 2006, pp. 774–784.

[5] ———, *On degrees of presentability of structures, I.II*, ***Algebra and Logic***, to appear.

► DARIUSZ SUROWIK, *A new axiomatization of* IK_t *system*.
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There are several systems of intuitionistic tense logic [1], [2], [3]. In the talk we would like to consider one of them given in [1] and called of IK_t.. This is a minimal system of intuitionistic tense logic (no conditions are imposed upon earlier-later relation).

We will propose a new axiomatization for this system. Our set of axioms is smaller than the set of axioms proposed by Ewald, then the new axiomatization is simpler. We will prove our axiomatization of IK_t is equivalent to the Ewald's axiomatization.

[1] W. B. Ewald, *Intuitionistic tense and modal logic*, ***The Journal of Symbolic Logic***, vol. 51 (1986), no. 1.

[2] D. Surowik, *Some remarks about intuitionistic tense logic*, ***On Leibniz's Philosophical Legacy***, 1997.

[3] K. Trzęsicki, *Intuitionism and Indeterminism (tense logical considerations)*, ***Jan Woleński, Philosophical Logic in Poland***, Kluwer Academic Publishers, 1994.

► LUIS ADRIAN URTUBEY, *An analysis of some basic philosophical thesis about non-deductive inference in the framework of CHC models*.
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The so called CHC Models, where CHC stands for conjectures, hypothesis and consequences, have been introduced in Trillas et al. [2000] and further developed in Ying and Wang [2002] and Qiu [2007]. A CHC model is a quite general algebraic model for conjectures, hypothesis and consequences. In this model, all statements or propositions are represented as elements of an orthocomplemented lattice. Operators defined in these lattices formalize the intuitive notions of conjecture, consequence and hypothesis in an algebraic setting. Consequently, CHC models provide with a framework in which, these notions appear as mathematical objects and in which several concepts concerning issues related with conjectures, hypothesis and consequences can be properly addressed.

It is nowadays widely recognized that Charles Peirce is one of the noteworthy precursors of the logical study of non-deductive inference. Peirce coined some new terminology to designate these alternative forms of reasoning, notably the current characterization of abduction is owed to him, and he also studied some relationships among these inference patterns. This contributed talk aims at formulating some scattered philosophical thesis concerning non-deductive inference in the setting of CHC models. More specifically, the purpose of this talk is to consider, in the algebraic framework of CHC models, the representation of some facts about abduction advanced by Peirce and lately addressed by other specialists, in order to contribute to the clarification of their conceptual analysis.

[1] D. Qiu, *A note on Trillas CHC models*, ***Artificial Intelligence***, 2007, in press.

[2] E. Trillas, S. Cubillo, and E. Castineira, *On conjectures in orthocomplemented lattices*, ***Artificial Intelligence***, vol. 117 (2000), pp. 255–275.

[3] M. Ying and H. Wang, *Lattice-theoretic models of conjectures, hypothesis and consequences*, ***Artificial Intelligence***, vol. 139 (2002), pp. 253–267.

► HEINRICH WANSING AND YAROSLAV SHRAMKO, *Harmonious many-valued propositional logics and the logic of computer networks*.
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In many-valued logic, sometimes a distinction is made not only between designated and undesignated (not designated) truth values, but between designated, undesignated, and antidesignated truth values. But even if the set of truth values is, in fact, tripartitioned, usually only a single semantic consequence relation is defined that preserves the possession of a designated value from the premises to the conclusions of an inference. We shall argue that if the set of anti-designated values does not constitute the complement of the set of designated values, it is natural to define *two* entailment relations, a positive one that preserves possessing a designated value from the premises to the conclusions of an inference, and a negative one that preserves possessing an antidesignated value from the conclusions to the premises. Once this distinction has been drawn, it is quite natural to reflect it in the logical object language and to contemplate many-valued logics whose language is split into a positive and a matching negative logical vocabulary. If the positive and the negative entailment relations do not coincide, the interpretations of matching pairs of connectives are distinct, and nevertheless the positive entailment relation restricted to the positive vocabulary is isomorphic to the negative entailment relation restricted to the negative vocabulary, then such a many-valued logic is called *harmonious*. We shall present examples of harmonious finitely-valued logics. These examples are not *ad hoc*, but emerge naturally in the context of generalizing Nuel Belnap's ideas on how a single computer should think to how interconnected computers should reason.

► JAMES WORTHINGTON, *Automatic proof generation in Kleene algebra with tests*.
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Kleene algebra (KA) is the algebra of regular events. Familiar examples of Kleene algebras include regular sets, relation algebras, and trace algebras. A Kleene algebra with tests (KAT) is a Kleene algebra with an embedded Boolean subalgebra. The addition tests allows one to encode while programs as KAT terms, thus the equational theory of KAT allows one to reason about (propositional) program equivalence. More complicated statements about programs can be expressed in the Hoare theory of KAT, which suffices to encode Propositional Hoare Logic.

We prove the following results. First, there is a PSPACE transducer which takes equations of Kleene Algebra as input and outputs proofs of them in an algebraic proof system based on a form of bisimulation. Second, we give a feasible reduction from the equational theory of KAT to the equational theory of KA. Combined with the fact that the Hoare theory of KAT reduces efficiently to the equational theory of KAT, this yields an algorithm capable of generating proofs of a large class of statements about programs.

Our results have applications to areas such as Proof-Carrying Code, where it is necessary that a formal proof be produced. There are two traditional approaches to the equational theory of KA: interactive protocols for generating proofs and the decision procedure of Stockmeyer and Meyer to determine finite automata equivalence. Each has its own drawbacks: interactive protocols are by definition not automatic, and the output of a decision procedure is just one bit, and therefore not efficiently verifiable. Our method, which is completely automatic and outputs a proof, combines the best features of each.

► URSZULA WYBRANIEC-SKARDOWSKA AND ZBIGNIEW BONIKOWSKI, *On vague set $\sqrt{s}$ and vague logic*.
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The subject matter of the consideration touches the problem of vagueness from the logical, set-theoretical and the computer science perspective. The paper proposes a new, formal-logical approach to the problem.

The starting point is the concept of agent's unit information about any object o of discovered reality with respect to a relation R defined on U. It is the image $\vec{R}(o)$ of the object o wrt R. If ${}_{\rho}\sqrt{\vec{R}(o)}$ is unknown to the agent, then the unit information for him can be given as the equation:

$${}_{\rho}\sqrt{\vec{R}(o)}, \qquad (e)$$

where X is an unknown quantity. Its scope is the at least a two-element family $\mathbf{V}_o$ of sets (relations) that are possible solutions of (e) from the point of view of the agent. Then we say that the agent's unit information about o wrt R is *vague* and the family $\mathbf{V}_o$ is a *vague set*.

A language representation of (e) is the vague sentence:

$$a \text{ is } V \text{ or } V(a), \qquad (re)$$

where a is the singular term of o and V is the name-predicate (resp. vague-predicate) corresponding to X; the denotation of V is the vague set $\mathbf{V}_o$—the family of all denotations (extensions) of sharp terms representing V from the agent's point of view. The upper and lower approximations (limits) of $\mathbf{V}_o$ are algebraic boundaries in $P(U)$. Some operations on vague sets and their algebraic properties are also presented. Some important conditions about the membership relation for vague sets, in connection to Zadeh's fuzzy sets (1965), Pawlak's rough sets (1982) and Blizard's multisets (1989) are established as well. A view on the problem of logic of vague sentences (*vague logic*) based on vague sets is also discussed. The considerations intend to take into account a 'conservative', classical approach to reasoning based on vague premises.

► IKUO YONEDA, *CM-triviality and geometric elimination of imaginaries*.
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CM-triviality is a geometric notion of forking introduced by Hrushovski in [3]. The original definition is given in terms of eq-structures. Usually the CM-triviality of a generic relational structure is shown as follows; first show the weak elimination of imaginaries of the structure, and then working in the real sort we complete the proof.

In my talk, I present a much more direct way: I define the notion of *CM-triviality in the real sort which implies* both the geometric elimination of imaginaries and the CM-triviality in the original sense.

I also give a sufficient condition for the geometric elimination of imaginaries in simple theories and (pregeometric) surgical theories.

[1] David M. Evans, $\aleph_0$*-categorical structures with a predimension*, ***Annals of Pure and Applied Logic***, vol. 116 (2002), pp. 157–186.

[2] Jerry Gagelman, *Stability in geometric theories*, ***Annals of Pure and Applied Logic***, vol. 132 (2005), pp. 313–326.

[3] Ehud Hrushovski, *A new strongly minimal sets*, ***Annals of Pure and Applied Logic***, vol. 88 (1993), pp. 147–166.

[4] Ikuo Yoneda, *Forking and some eliminations of imaginaries*, submitted, October 2006.

► RYAN YOUNG, *'Asymmetric' systems of natural deduction*.
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The construction of natural deduction systems normally requires the basic symmetry that both elimination and introduction rules are given for each connective within the system. There are many good reasons for this requirement, however it is not a necessary requirement for the construction of a logic. The project of this paper is to explore the consequences of relaxing this restriction in a particular direction, namely removing $\vee$-*Introduction*.

The motivation for this investigation is the observation that uniquely among the standard classical inference rules, $\vee$-*Introduction* seems out of place in ordinary human argumentation. One does not normally argue from a singular premise, say "It will rain tomorrow", to the conclusion that "It will rain tomorrow or my watch will break tomorrow". Unless there is some known connection between it raining, and the state of my watch, such an argument would be considered wrong in ordinary discourse.

Given that Natural Deduction most closely resembles human reasoning, and human reasoning normally proceeds without the $\vee$-*Introduction*, it should be worth investigating Natural Deduction systems without the $\vee$-*Introduction*, it should be worth investigating Natural Deduction systems without the $\vee$-*Introduction* rule. In this paper, one such system will be the focus: Intuitionistic Natural Deduction without $\vee$-*Introduction*. This system suffers form the limitations that are to be expected when one removes the basic symmetry of normal Natural Deduction systems. It is, for example, not complete in any normal sense, and there is no obvious model for it. Nevertheless, it has a number of unusual properties that are worth exploring. For example, one can prove consistency directly from the inference rules; it collapses to Classical Logic upon the addition of the Law of the Excluded Middle as an axiom; and it is 'negatively explosive'. That is, from a contradiction the negation of every formula is provable, but not every formula. The paper concludes with a brief discussion of the semantic interpretation of this system, and the significance of the formal results

Abstracts of papers presented by title

► ADIB BEN JEBARA, *An interpretation about space and time in quantum mechanics*.
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There was a repeated experiment where at first, two protons are joined and of opposite spins. Then, the second is taken far away, and it is acted upon the first to modify its spin. The second proton will change its spin to keep it the opposite of the spin of the first. For further details: http://mist.npl.washington.edu/npl/int_rep/tiqm/TI_24.html#2.4.1

Now, if you will assume with me that we can apply the set theory ZFU to physical space, U (urelements) being physical space, you will see that we get an interpretation of the experiment.

Indeed, as it is not possible to define a distance in U, the second proton will not be any more far away from the first.

Also, if we consider time to be U, we cannot say that the protons were separated a long time ago and that there should be no more influence.

Such two hypothesis about space and time were already made in "About time and time of elementary particles" in ASL Annual Meeting 2005. It is not yet clear: are space and time alternatively U? Is U space-time?

Continuums are still approximations.

Mr. Andreas Blass pointed out that, in the experiment, the first proton is acted upon for measurement, and he also pointed out that only the most used distances are not defined in U. He is skeptical about the assumptions.

As the hypothesis apply to cosmology, the unity of physics would be increased in such a direction.

There was another repeated experiment with a photon, expected to go one way, going both two quite separated ways. Here, again, if we assume something else about space, the two ways would be not that much separated.

► ERICA CALARDO, *Admissible rules for the multimodal logic of knowledge and time* LTK_1.
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Multi-modal propositional logics with operators representing both time and knowledge are particularly effective to describe the interaction of agents throughout the flow of time [2, 1, 3, 4, 5, 7]. These systems are usually generated by adding to an existing propositional logic two sets of modalities: one to model the flow of time and one to describe agents' knowledge. The interaction of such modalities gives a precise account of the dynamic development of agents' knowledge. However, despite the power of multi-modal propositional logics, multi-modal languages can only express formulae which are static in a way: the statements only fix a fact, and cannot handle a changing environment, although this is required to model human reasoning, computation and multi-agent environments. Sometimes it might be more useful to discover what follows given some premises, rather than knowing logical truths. For this purpose, inference rules, or logical consecutions, are a core instrument.

Our research aims at investigating a multi-modal propositional logic, LTK (Linear Time and Knowledge), which combines tense and knowledge modalities. This logic is semantically defined as the set of all LTK-valid formulae, where LTK-frames are multi-modal Kripke-frames combining a linear and discrete representation of the flow of time with special S5-like modalities, defined at each time cluster and representing agents' knowledge.

So far we have proved that: (i) LTK has the finite model property [2]; (ii) LTK has a finite axiomatisation [3]; (iii) LTK_1, a weaker version of LTK with only one agent operating in the system, is decidable with respect to its admissible inference rules [1]. Our latest results is to show that LTK_1 has a finite basis for admissible inference rules, i.e., all those rules under which the logic itself is closed.

[1] Erica Calardo, *Admissible inference rules in the linear logic of knowledge and time* LTK, ***Logic Journal of the IGPL***, vol. 14 (2006), no. 1, pp. 15–34.

[2] Erica Calardo and Vladimir V. Rybakov, *Combining time and knowledge, semantic approach*, ***Bulletin of the Section of Logic***, vol. 34 (2005), no. 1, pp. 13–21.

[3] ———, *An axiomatisation for the multi-modal logic of knowledge and linear time LTK*, ***Logic Journal of the IGPL***, to appear.

[4] Ronald Fagin, Joseph Y. Halpern, Yoram Moses, and Moshe Y. Vardi, ***Reasoning about knowledge***, MIT Press, 1995.

[5] Dov Gabbay, Agi Kurucz, Frank Wolter, and Michael Zakharyaschev, ***Many-dimensional modal logics: theory and applications***, Studies in Logic and the Foundations of Mathematics, vol. 148, Elsevier, North-Holland, New York–Amsterdam, 2003.

[6] Robert Goldblatt, *Logics of time and computation*, ***Center for the Study of Language and Information***, 1987.

[7] Joseph Y. Halpern, Ron van der Meyden, and Moshe Y. Vardi, *Complete axiomatization for reasoning about knowledge and time*, ***SIAM Journal on Computing***, vol. 33 (2004), no. 3, pp. 674–703.

► JOHN CORCORAN, *Aristotle's many-sorted logic*.
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As noted in 1962 by Timothy Smiley, if Aristotle's logic is faithfully translated into modern symbolic logic, the fit is exact. If categorical sentences are translated into many-sorted logic *MSL* according to Smiley's method or the two other methods presented here, an argument with arbitrarily many premises is valid according to Aristotle's system if and only if its translation is valid according to modern standard many-sorted logic. As William Parry observed in 1973, this result can be proved using my 1972 proof of the completeness of Aristotle's syllogistic.

MSL Using Sortal Variables

The ranges of the Sortal variables are all non-empty. In the examples, ess ranges over spheres, pee over polygons.

Every sphere is a polygon.	$\forall s \exists p\ s = p$
Some sphere is a polygon.	$\exists s \exists p\ s = p$
No sphere is a polygon.	$\forall s \forall p\ s \neq p$
Some sphere isn't a polygon.	$\exists s \forall p\ s \neq p$

MSL Using Range-Indicators with General (Non-Sortal) Variables

Each initial variable occurrence follows an occurrence of a quantified range-indicator or "common noun" that determines the range of the variable in each of its occurrences in the quantifier's scope. To each range-indicator, a non-empty set is assigned as its "extension". In the example, the extension of ess is the spheres, pee the polygons.

For every sphere x, there exists a polygon y such that $x = y$.	$\forall \mathrm{S}x \exists \mathrm{P}y\ x = y$
For some sphere x, there exists a polygon y such that $x = y$.	$\exists \mathrm{S}x \exists \mathrm{P}y\ x = y$
For every sphere x, for every polygon y, x isn't p.	$\forall \mathrm{S}x \forall \mathrm{P}y\ s \neq y$
There exists a sphere x such that, for every polygon y, x isn't y.	$\exists \mathrm{S}x \forall \mathrm{P}y\ x \neq y$

Many-sorted logic with range-indicators and non-sortal variables was pioneered by Anil Gupta in his 1980 book. Also see my *Logical form of quantifier phrases: quantifier-sortal-variable* this BULLETIN, vol. 5 (1999) pp. 418–419.

► NIKOLAOS GALATOS, *Axiomatizations for intersections of substructural logics*.
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Substructural logics include classical, intuitionistic, linear, relevant and many-valued logics, and their study provides a framework for the comparative study of various logics. Substructural logics are defined as axiomatic extensions of the sequent calculus FL; the latter is essentially obtained from Gentzen's system LJ for intuitionistic propositional logic, by omitting the structural rules of exchange, contraction and weakening. Residuated lattices form algebraic semantics for substructural logics and algebraic methods have been applied successfully to the logical investigation. Substructural logics ordered under inclusion, form a lattice. Given axiomatizations for two substructural logics, their join in the lattice (the least logic containing both of them) is axiomatized by the union of the axiomatizations. On the other hand, the meet of the two logics (the greatest logic containing both of them) is their intersection, but the intersection of their axiomatizations does not axiomatize the meet; for example, it can be empty. Using algebraic methods, we provide axiomatizations for intersections of substructural logics that also work in case the language lacks the disjunction connective. In particular, we prove that the intersection of two finitely axiomatized subcommutative substructural logics has a finite axiomatization, as well. En route, we investigate the finitely subdirectly irreducible algebras of residuated (semi)lattices (if join is not included in the language) and show that if they form an elementary class, the finitely axiomatized extensions of the corresponding substructural logic are closed under intersections.

This is joint work with J. G. Raftery and J. S. Olson.

[1] N. GALATOS, *Equational bases for joins of residuated lattice varieties*, ***Studia Logica***, vol. 76 (2004), no. 2, pp. 227–240.

[2] N. GALATOS, J. S. OLSON, AND J. G. RAFTERY, *Irreducible residuated semilattices and finitely based varieties*, manuscript.

► V. V. REMATSKI, *An explicit basis for admissible rules of modal logics extending S4.1*.
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Let a modal logic $\lambda \supseteq S4.1$ has (i) Finite Model Property; (ii) a weak co-cover property (WCCP for short); (iii) the disjunction property be (cf. [2]).

We introduce the following sequence of inference rules:

$$\mathcal{R}_n := \frac{\Box(A_{n,1} \wedge \neg(A_n \wedge B)) \vee \Box z}{\Box\neg A_n \vee \Box z};$$

where $A_n := \bigwedge_{1 \le i \le n} \diamond p_i$; $\quad A_{n,1} := \Box[\bigwedge_{1 \le i \le n} p_i \to \neg\Box q]$; $\quad B := q \vee \neg\Box q$; $n > 1, n \in N$

THEOREM 0.1. *Let modal logic $\lambda \supseteq S4.1$ have: (i) FMP; (ii) WCCP; (iii) disjunction property. Then rules $\mathcal{R}_n$, $n \in N$, form a basis for λ-admissible rules.*

CONCLUSION 0.2. *Rules $\mathcal{R}_n$, $n \in N$, form a basis for all admissible rules of logics S4.1, Grz.*

THEOREM 0.3. *Let modal logic $\lambda \supseteq Grz$ have: (i) FMP; (ii) $\mathcal{L}$ WCCP (iii) disjunction property; (iv) the width of λ is equal to finite $\mathcal{L}$. Then rules $\mathcal{R}_n$, $1 < n \leq \mathcal{L}$, form basis for all λ-admissible rules.*

THEOREM 0.4. Let logic $\lambda \supseteq$ *Int* satisfies to conditions (i)–(iii) of theorem 0.1 or (i)–(iv) of theorem 0.3 Then rules $\{\mathcal{R}_n^i\colon T(\mathcal{R}_n^i) = \mathcal{R}_n, n > 1, n \in N\}$ form a basis for λ-admissible rules. If the width of λ is equal finite to $\mathcal{L}$ then a basis for λ-admissible rules is $\{\mathcal{R}_n^i\colon T(\mathcal{R}_n^i) = \mathcal{R}_n, 1 < n \leq \mathcal{L}\}$.

[1] V. V. RYBAKOV, ***Admissibility of logical inference rules***, Studies in Logic and the Foundations of Mathematics, vol. 136, Elsevier Science Publishers, New York–Amsterdam, 1997, 617 pp.

[2] ———, *Construction of an explicit basis for rules admissible in modal system S4*, ***Mathematical Logic Quarterly***, vol. 47 (2001), no. 4, pp. 441–451.

► VLADIMIR RYBAKOV, *Discrete linear temporal logic with current time point clusters*.

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Diverse variations of linear temporal logics, as special bi-modal logics, were studied very profoundly. Though, even nowadays there are not investigated pathways. Only recently [1] it has been shown that all finitely axiomatizable linear tense logics are decidable and coNP-complete. We study temporal logic $\mathcal{TL}(N_C, \Box_w^{+-})$ based on linear time with current time point clusters (bi-modal clusters situated in place of natural numbers imitating time flow). Its language uses, together with standard modalities $\diamond^+$ (possible in future) and $\diamond^-$ (possible in past), special temporal operations, $-\Box_w^+$ (weakly necessary in future) and $\Box_w^-$ (weakly necessary in past). Continuing [1], we first show that the logic $\mathcal{TL}(N_C, \Box_w^{+-})$ itself is decidable. We propose an deciding algorithm based on reduction of formulas to rules and converting rules in special reduced normal form, and (then) on checking validity of such rules in models of single exponential size in reduced forms. Then we show how to reduce in $\mathcal{TL}(N_C, \Box_w^{+-})$ the admissibility problem of inference rules to the decidability of $\mathcal{TL}(N_C, \Box_w^{+-})$ itself. So, we prove that the admissibility problem for $\mathcal{TL}(N_C, \Box_w^{+-})$ is also decidable. This fact (based on checking validity of inference rules and presence of a definable universal modality) extends previous results [2, 3].

[1] T. LITAK AND F. WOLTER, *All finitely axiomatizable tense logics of linear time flows are coNP-complete*, ***Studia Logica***, vol. 81 (2005), no. 2, pp. 153–165.

[2] V. V. RYBAKOV, *Logical consecutions in discrete linear temporal logic*, ***The Journal of Symbolic Logic***, vol. 70 (2005), no. 4, pp. 1137–1149.

[3] ———, *Linear temporal logic with until and before on integer numbers, deciding algorithms*, Lecture Notes in Computer Science, vol. 3967, Springer, 2006, pp. 322–334.

► JACEK WALDMAJER, *On logical theory of structures*.

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In the paper certain criteria of adequacy of representation of knowledge are presented. The criteria refer to situations in which some objects are cognized by means of other objects. The formulation of the criteria of adequacy of representation of knowledge is made within two formal-logical, axiomatic theories: *theory of cognitive tuples* ***T*** *and theory of structures* ***TS*** built over the former.

The primitive notions of ***T*** are the concepts: a *cognitive tuple* and *holding* (*occurrence*) *of a cognitive tuple*. Introduction of the notions requires adapting the following two postulates: (1) certain objects are available for cognition, (2) certain objects that are available for cognition make it possible to cognize other objects.

A *cognitive tuple* is, at the same time, intuitively understood as a tuple of objects by means of which we want to cognize something. If objects of these tuples are connected with one another, so that information (knowledge) of the cognized objects is transferred by their means, then we can say about the cognitive tuples that they *hold* (*occur*), and their description is called *adequate representation of knowledge*.

Since cognition of a given object often occurs by means of more than one cognitive tuple, representation of knowledge usually refers to a certain set of cognitive tuples. This leads to taking into account, in theoretical considerations, a notion of *structure* $_{p}\sqrt{-}$ indispensable in scientific research [1]. This concept is introduced in the theory ***TS***. ***TS*** describes these properties of the structures which point out some criteria of adequacy of representation of knowledge (cf. [2]). An important notion of this theory is the one of *adequate structure*. Such a structure is characterized by that all of its cognitive tuples are holding (occurring) tuples, and the knowledge, which is transferred by them is *adequate*.

The theory ***TS*** has its interpretation in *ZF* set theory and as such is consistent.

[1] Nicolas Bourbaki, *The architecture of mathematics*, ***American Mathematical Monthly***, (1950), no. 57, pp. 221–231.

[2] Jacek Waldmajer, ***On structures and their adequacy, Directions in universal logic***, Polimetrica, International Scientific Publisher, (to appear).

► XUNWEI ZHOU, *The first level predicate calculus of mutually-inversistic logic is quantifier-free*.

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The first level predicate calculus of mutually-inversistic logic is quantifier-free. A proposition in the form of $\text{parent}(x, y) \wedge \text{ancestor}(y, z) \leqslant^{-1} \text{ancestor}(x, z)$ is called a first-order single empirical or mathematical connection proposition, where x, y, z are term variables (corresponding to individual variables in classical logic), $\wedge$ is a fact compounder, $\leqslant^{-1}$ (mutually inverse implication) is an empirical or mathematical connective. The boundness of a term variable in a first-order single empirical or mathematical connection proposition is determined by its occurrences relative to empirical or mathematical connective, fact compounder, predicate, and function. A term variable is relevantly bound if it occurs on both sides of the empirical or mathematical connective, such as the x in $\text{man}(x) \leqslant^{-1} \text{mortal}(x)$. A term variable is transitively bound if it occurs on one side of the empirical or mathematical connective but on both sides of a fact compounder, such as the y in $\text{parent}(x, y) \wedge \text{ancestor}(y, z) \leqslant^{-1} \text{ancestor}(x, z)$. A term variable is additionally bound if it occurs on one side of the empirical or mathematical connective but on both sides of a binary predicate, such as the z in the elimination law of addition $x + z = y + z \leqslant^{-1} x = y$. A term variable is juxtaposed bound if it occurs on one side of a predicate but on both sides of a binary function such as the z in $x * x + y * y + z * z \leqslant 1 \leqslant^{-1} x * x + y * y \leqslant 1$.

If a term variable occurs only once in a first-order single empirical or mathematical connection proposition, then it is free, Because it cannot be one of the above-mentioned bound variables. If a term variable occurs in a first-order single empirical or mathematical connection proposition just twice, then it is purely bound. Because it is just one of the above-mentioned boundness. For example, in $\text{parent}(x, y) \wedge \text{ancestor}(y, z) \leqslant^{-1} \text{ancestor}(x, z)$, x and z are purely relevantly bound, y is purely transitively bound. If a term variable occurs in a first-order single empirical or mathematical connection proposition three times or more, then it is combined boundness. Because it combines two or more of the above-mentioned boundness. For example, x in $x < y \wedge x < z \leqslant^{-1} x < y + z$ is combined relevant and transitive boundness. Because the first and second occurrences of x are transitively bound, the first and third occurrences of x are relevantly bound, the second and the third occurrences are relevantly bound.