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REVIEWS

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References such as 49***5*** or 280***1*** are to entries so numbered in *A bibliography of symbolic logic* (the JOURNAL, vol. 1, pp. 121–218).

GREGORY CHERLIN AND EHUD HRUSHOVSKI. ***Finite structures with few types.*** Annals of Mathematics Studies. Princeton University Press, 2003, vi + 196 pp.

The work in this monograph is a continuation and perhaps perfection of ideas and methods from a line of investigation started by B. Zilber in his work on uncountably categorical theories and their models [B. Zilber, ***Uncountably categorical theories***, Translations of Mathematical Monographs, vol. 117, American Mathematical Society, 1993] and A. Lachlan, in his work on stable homogeneous structures; for an overview, see [A. H. Lachlan in ***Algebraic model theory***, pp. 145–159, edited by B. T. Hart, A. H. Lachlan, M. A. Valeriote, Kluwer Academic Publishers, 1997]. These two studies were merged in the work on $\aleph_0$-categorical $\aleph_0$-stable structures [G. Cherlin, L. Harrington and A. H. Lachlan, ***Annals of Pure and Applied Logic***, vol. 28 (1985), pp. 103–135], and subsequently the ideas were taken further in the study of smoothly approximated structures, in particular the primitive ones, in [W. M. Kantor, M. W. Liebeck, H. D. Macpherson, ***Proceedings of the London Mathematical Society*** vol. 59 (1989), no. 3, pp. 439–463]. Briefly, the two main themes involved have been that of understanding the structure of infinite "minimal" definable sets (where I here take "minimal" to mean, for some $n < \omega$, the set cannot be split definably into n disjoint infinite parts) and of "coordinatization", which, roughly speaking, tells us how the infinite minimal definable sets are organized within the whole structure.

Finite Structures with Few Types studies in great detail a class of infinite $\aleph_0$-categorical structures called *smoothly approximable*, which includes all previously mentioned structures.

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A structure M is smoothly approximable (by finite structures) if it is $\aleph_0$-categorical and every finite subset of M is included in a finite substructure $N \subseteq M$ such that

- for any k and any relation $R \subseteq M^k$ which is definable in M (without parameters) the relation $R \cap N^k$ is definable in N (without parameters), and
- for every pair of tuples $\bar{a}, \bar{b}$ from N of the same length, $\bar{a}$ and $\bar{b}$ realize the same type with respect to M if and only if they realize the same type with respect to N.

An increasing chain of such finite N's the union of which is M can be seen as a sequence of approximations of M. The theory developed for smoothly approximable structures can also be viewed as a study of such finite approximating structures, or via results developed, as an investigation of finite structures in a given language which have a fixed finite bound on the number of 4-types; in fact a complete classification of such structures is given.

Part of the work shows that smooth approximability of M is equivalent with other interesting properties, of which I will mention two. The first is that the complete theory of M has a model M^* in a nonstandard universe the size of which is an infinite nonstandard integer and the number of n-types (for standard n) of M^* is at most c^{n^2} for some finite c.

Another equivalent property, that of being Lie coordinatizable, amounts to a description of the fine structure of smoothly approximable structures. Roughly, a structure M is *Lie coordinatized* if:

(1) M's infinite minimal sets are finite unions of projective or affine versions of *linear geometries* where the latter may be a pure set (with no other structure), a vector space over a finite field possibly expanded by an inner product or by a quadratic form, a bipolar space (two vector spaces with a nondegenerate bilinear pairing) or a 'quadratic geometry' (the most complicated case).

(2) M's elements as well as its minimal sets can be associated with the nodes of a tree with finite height in such a way that
  (a) the root of the tree is in the algebraic closure (in the model theoretic sense) of $\emptyset$,
  (b) if two elements are on the same branch then the one closest to the root is in the algebraic closure of the other, and
  (c) for any $a \in M$, there is b on the same branch as a and closer to the root such that: either a belongs to the algebraic closure of b or there is a b-definable minimal set to which a belongs.

So, somewhat simplified, the relationship between elements $a_1, \dots, a_n$ in a Lie coordinatized structure depends on where they are placed on the tree and how elements $b_1, \dots, b_m$ $(m \leq n)$, where each b_i is either identical to one of $a_1, \dots, a_n$ or on the same branch as one of $a_1, \dots, a_n$ and closer to the root, are related within a common minimal set to which they belong, if such set exists.

A structure is *Lie coordinatizable* if it is bi-interpretable with a structure which is Lie coordinatized.

The theory of smoothly approximable structures generalizes many results concerning $\aleph_0$-categorical $\aleph_0$-stable structures, but a smoothly approximable structure need not be stable. Nevertheless notions of rank and independence can be defined and a theory is developed for these notions. As independence in this setting has certain properties, some of which are known within stability/simplicity theory as symmetry, transitivity, monotonicity and extension, it follows that every smoothly approximable structure is simple. In addition, the rank of any type is finite when working in a smoothly approximable structure and the rank coincides with the so-called SU-rank from simplicity theory. The analysis of independence shows that smoothly approximable structures are "well-behaved" from a stability/simplicity theoretic point of view since they satisfy some "nice" properties which exclude a too difficult behaviour of the independence relation. These nice properties include 1-basedness, called modularity in the book, and the independence property over algebraically closed sets, called type-amalgamation property in the book. The model theoretic analysis also shows that the class of structures which have a smoothly approximable expansion can be characterized as the class of structures which satisfies nine model theoretic properties, a few of which have just been mentioned. Many of the results presented depend on weaker assumptions than smooth approximability, such as $\aleph_0$-categoricity and finite rank.

As shown in [Kantor, Liebeck, Macpherson, *op. cit.*], every smoothly approximable structure M has the following property, called *pseudofiniteness*: If φ is a sentence which is true in M then φ is true in a finite substructure of M. It follows that the complete theory of a smoothly approximable structure is not finitely axiomatizable. By developing further the method of envelopes and techniques of G. Ahlbrandt and M. Ziegler (used for strongly minimal totally categorical theories) it is shown that the complete theory of any smoothly approximable structure is *quasifinitely axiomatizable*, meaning that the theory is axiomatized by a finite number of axiom schemes. This extends earlier results about pseudofiniteness and quasifinite axiomatizability for $\aleph_0$-categorical $\aleph_0$-stable theories.

In the last chapter, which builds on methods used for quasifinite axiomatizability, decidability problems are considered. For instance, given a language with a finite signature containing only relation symbols and a sentence in this language, it is decidable whether the sentence has a stable homogeneous model. Also, in a language with finite signature, one can decide whether a sentence has a finite model with a given number of 4-types.

For some results the classification of finite simple groups is used and a discussion of this is given at the end of the book.

The book is technically difficult, proofs are often quite compressed and it expects that the reader is familiar with various notions from model theory and algebra; so the book demands plenty of the reader. An error (the statement of Lemma 2.4.8) is corrected in [G. Cherlin, M. Djordjević, E. Hrushovski, ***The Journal of Symbolic Logic***, vol. 70 (2005), pp. 1359–1364].

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GRAHAM PRIEST. ***Towards non-being: the logic and metaphysics of intentionality.*** Oxford University Press, 2005, xi + 190 pp.

Priest's recent book is a homage to his deceased friend and collaborator Richard Sylvan (formerly Routley). Priest defends, with Sylvan, an ontological position derivative of Meinong's they call *noneism*. Noneism, basically, is the view that there are non-existent objects. This view is motivated by semantic considerations about what Priest calls "intentional" verbs, which are also often called "intensional" verbs. Priest puts to work many clever formal tools, the book contains several interesting historical excursion, a short story, many technical appendixes, and lots of good philosophy. Parts of the book overlap with Priest's earlier publications, but much of it is new. It is a notable achievement.

The book is divided into two parts. The first part presents Priest's proposal for a semantics of intentional verbs, the technical framework on which it is based, a discussion of identity and substitutivity, and his solution to one of the main problems that Sylvan thought his account of non-existent objects never fully provided: the characterization problem. The characterization problem is the problem to say which non-existent objects there are. On the one hand one might hold there are only a few, maybe just Santa and the Easter Bunny. On the other hand one might hold that there as many as conceivably could be: for any set of properties there is an object which has just those properties. Priest defends a version of the latter view, with Sylvan. But a naive version of this leads to triviality. Take the property of *being self-identical and p*. If anything has this property then it follows that *p*. The characterization problem is to say which sets of properties characterize an object, i.e., for which sets of properties is there an object (existing or not) which has exactly those properties. Traditionally, there have been two strategies to solve this problem. One is to allow only combinations of certain kinds of properties, the other is to claim that the non-existent objects don't really have the properties that characterize them, they merely "encode" them. Priest proposes a third solution. According to him any combination of properties is allowed, and the objects have the properties that characterize them in the usual sense. However, they might not actually have them. They instead might have them merely in other 'worlds', which includes impossible and inconsistent worlds. To defend this position, and thus to propose a solution to a problem that Sylvan felt was still open, is one of the main goals of the book.

The second part of the book is the more philosophical part. It discusses various philosophical objections to noneism, and applies the theory to philosophical issues about fiction and mathematics. It also includes an intriguing discussion of what Priest takes to be the best objection to noneism, which is related to the paradoxes of denotation. (His solution is based on the idea of allowing terms to denote more than one object.)

Overall, the book is a significant advancement over earlier attempts to solve the same problems, in part because of the more sophisticated technical tools. Priest's treatments of open and inconsistent worlds, various "non-standard" logical systems, and the way they are brought together are without question progress turned to ink. But there are also a few aspects of the book where it falls short of full glory. I will briefly mention two. The first is the account of noneism and Priest's claim that his semantics for intentional verbs supports it. Noneism is the view that there are non-existent objects, and it is contrasted with Meinongianism in that it claims that non-existent objects have no ontological status at all, whereas Meinong held that non-existent objects had some secondary status, that of subsisting. But whether noneism so characterized is really a substantial view, in particular one related to ontology is not so clear. In a sense, everyone agrees with it. We all believe that Santa doesn't exist, i.e., that Santa is non-existent, and thus there is an innocent sense in which Santa is a non-existent object. But then, who would deny that there are non-existent objects, with Santa being one of them? Ordinary talk seems to allow for an apparently innocent way of saying this truly. But noneism, as a philosophical position, probably puts some more substantial weight on this talk about what doesn't exist. After all, Priest holds that his substantial and intricate semantics of intentional verbs supports noneism. But there isn't too much discussion in the book how that semantics supports this metaphysics. Sure enough, Priest talks about lots of things which don't exist in giving the truth conditions of sentences that talk about things that don't exist. But does that alone support a particular metaphysical view of the non-existing? And could it support noneism over Meinongeanism? I wish he would have said more here.

A second aspect of Priest's book that makes it less than maximally glorious is this: When discussing the semantics of propositional attitude verbs Priest writes that "contemporary discussion of how to treat intentional verbs with non-propositional objects is all but non-existent" (p. 6). And thus Priest does not discuss or cite any such treatments. But this is a real oversight on his part. There is in fact a quite active and sophisticated discussion of the semantics of intentional verbs in the more linguistic literature. This literature deals with the same question as Priest: how to give a semantics for intentional verbs. There are two main traditions in this semantic literature, and Priest, in effect, sides with one of them. The first goes back to Montague and treats the arguments of transitive intentional/intensional verbs as higher type objects. The second, which is congenial to Priest's semantics for certain tricky cases, treats intentional verbs to ultimately express propositional attitudes. Both of these approaches have their advantages and their problems, and which one is the better candidate for the semantics of natural language is the subject of quite a bit of debate in the literature on natural language semantics. For example, the sentence

(1) Fred is looking for a wife.

has at least three readings (someone to marry / any wife / a particular wife). How such sentences are to be understood semantically is not just restricted to specifying what the truth conditions are in each case, but also to explain why one and the same sentence has these three different readings, i.e., how the readings arise from this sentence. The linguistic literature is very much concerned with this question, and I think Priest could have profited from considering it.

To understand our talk about non-existing things, and what metaphysical position it requires, we need to look at least at philosophy, logic, and natural language semantics. Priest neglects contemporary natural language semantics, contemporary natural language semantics neglects the technical tools Priest and others have developed. One day soon everyone will sit at the same table, and then we will see what approach to the semantics of talk about what doesn't exist is best, and what follows for metaphysics from this. We are not there yet, but Priest has gotten us closer.

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MOHAMMAD ARDESHIR. ***Manteghé Riazi* (*Mathematical Logic*).** Hermes Publications, Tehran, 2005, 262 pp.

Ardeshir's book, written in Farsi (Persian), is based on the author's lecture notes, developed over several years, for a one-semester course in mathematical logic for undergraduate mathematics majors at Sharif University of Technology, one of the leading institutes of higher learning in Iran. At first glance, this compact textbook seems to provide a standard treatment of topics one would expect to encounter in a rigorous introductory course. But a closer look reveals the author's distinct personal imprint, manifested through the style of exposition, as well as the selection and omission of certain topics.

The text begins with a rather philosophical introduction that traces the origins, aims, and scope of the discipline of mathematical logic. Chapter one then proceeds to thoroughly develop the syntax and semantics of propositional logic, all the way to soundness and completeness theorems pertaining to *three* different flavors of proof systems (Hilbert style deduction, natural deduction, and sequent calculus). This chapter also contains a section devoted to an algebraic treatment of propositional logic via the framework of Boolean algebras, maximal ideals, and ultrafilters. Chapter two then focuses on the syntax and semantics of first order logic, extending each of the three aforementioned proof systems to the context of first order logic, with the aim of revisiting soundness and completeness within the framework of first order logic. Chapter three presents the rudiments of model theory, including many applications of the compactness theorem, upward and downward Löwenheim–Skolem theorems, elementary submodels, nonstandard models of arithmetic and analysis, model completeness, and elimination of quantifiers. The final chapter focus on second order logic and discusses the strengths and weaknesses of second order logic, its connection to set theory, and Henkin's semantics for second order logic. Each of the four chapters includes a section devoted to exercises and a section containing historical, philosophical, and sometimes philological commentary. The text also includes an appendix devoted to a brief description of the works and biographies of 12 founding figures of mathematical logic, as well as a pair of glossaries (in English–Farsi, and Farsi–English).

Ardeshir's text is a welcome addition to the small but influential family of mathematical logic texts in Farsi. The exposition is clear and rigorous, and the exercises range from routine confidence-building ones all the way up to those that are formidable and quite challenging. The philosophical and historical interludes can sometimes be obscure and even controversial, but they are always thought provoking. In the reviewer's opinion, the most glaring omission in the choice of topics in the text is the absence of any material on computability theory and incompleteness theorems. Of course there is only so much one can accomplish during a mere semester, and there is always the option of reaching for supplementary materials and texts, but I hope that the author would consider enriching the future editions of this text with a treatment of these two important topics.

I wish to conclude this review with a brief discussion of Farsi logic textbooks in order to provide a background for the work reviewed here. The first modern logic textbook in Farsi was published in Iran more than half a century ago in 1955. It was authored by G. H. Mosaheb, a gifted and admirably tireless mathematician, educator, and man of letters. Thanks to the industry of Alonzo Church, the book was reviewed two years later in ***The Journal of Symbolic Logic*** by none other than L. A. Zadeh, the founder of fuzzy logic; see JSL, vol. 22, no. 4 (Dec. 1957), pp. 354–355. The next series of notable mathematical logic texts to arrive on the Iranian scene were the Farsi translations of Enderton's ***A Mathematical Introduction to Logic***, and Hamilton's ***Logic for Mathematicians***, which appeared three decades later in the mid-1980's and helped to introduce the subject to a new generation of students. In the early 1990's Zia Movahed (an expert in philosophical logic, who is also well-known in Iran as a literary scholar and poet) authored a respectable introductory textbook on modern logic, and more recently (2002) he published a substantial textbook on modal logic, both of which will hopefully be reviewed by BSL in the near future.

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Truth and games: Essays in honour of Gabriel Sandu. edited by Tuomo Aho and Ahti-Veikko Pietarinen, Acta Philosophica Fennica, vol. 78. Societas Philosophica Fennica, Helsinki, 2006, vi + 322 pp.—therein:

JAAKKO HINTIKKA AND BESIM KARAKADILAR. *How to prove the consistency of arithmetic*. Pp. 1–15.

SAMSON ABRAMSKY. *Socially responsive, environmentally friendly logic*. Pp. 17–45.

J.C. BRADFIELD. *Independence: logics and concurrency*. Pp. 47–70.

JOUKO VÄÄNÄNEN. *A remark on nondeterminacy in IF logic*. Pp. 71–77.

TAPANI HYTTININEN. *Remark on spectrums of formulas with Henkin quantifiers*. Pp. 79–90.

SHAHID RAHMAN AND CÉDRIC DÉGREMONT. *The beetle in the box: exploring IF-dialogues*. Pp. 91–122.

JOHAN VAN BENTHEM. *Logical construction games*. Pp. 123–137.

VEIKKO RANTALA. *Learning and epistemic logic*. Pp. 139–157.

JAN VON PLATO. *Normal form and existence property for derivations in Heyting arithmetic*. Pp. 159–163.

This collection of eighteen essays honours the fiftieth birthday of Gabriel Sandu, Professor of Theoretical Philosophy at the University of Helsinki. A great deal of Sandu's research has been in the areas of "independence-friendly" (IF) logic and game-theoretical semantics. Most of the essays are based on these themes, and as a result, the collection has more unity than most contemporary *festschriften*. Papers that are mostly devoted to history, philosophy or linguistics are not reviewed here.

IF logic was introduced by Hintikka and Sandu in a paper of 1989, and is closely related to partially ordered quantification, invented by Leon Henkin (JSL XXX 96). IF logic extends normal first-order logic by adding quantifiers such as $\exists y/\forall x$; this is interpreted semantically as representing a choice of y that is independent of an earlier choice of x. If we give a game-theoretical semantics for this logic, then the appropriate games are games of partial information, just as first-order logic corresponds to games of full information. Since there are games of partial information that are not determined, the law of excluded middle fails in IF logic. It is more expressive than first-order logic, however, since it can express directly "There are infinitely many F's"; consequently, like Henkin's branching quantifier logic, it is not axiomatizable, and in fact, there are faithful translations in both directions between IF logic and the Σ^1_1 fragment of second order logic. Readers interested in the philosophical motivation for IF logic can profitably consult Hintikka's 1996 monograph *The Principles of Mathematics Revisited*, as well as the very detailed review of this book by Harold Hodes (JSL VXIII 1615). Those who would like to know more of the technical details can read the excellent exposition by Jouko Väänänen in ***Dependence Logic: A New Approach to Independence Friendly Logic***, Cambridge University Press 2007.

Jaakko Hintikka and Besim Karakadilar sketch a proof that arithmetic based on IF logic can prove its own consistency, in a sense. By a diagonal construction, they produce a sentence D stating its own disprovability in that system. Since $\neg D$ can be proved in arithmetic based on extended IF logic, they conclude that this system can prove its own consistency, and go on to argue that this constitutes a solution to Hilbert's consistency problem for arithmetic.

The semantics of IF logic is naturally expressed in terms of a two player game. Abramsky's article is devoted to developing a logic for multi-player games. He develops an algebraic formalism in which connectives subscripted by players represent (for example) parallel and sequential composition of games. The semantics of this logic is given in terms of multiple concurrent strategies, formalized as closure operations on the concrete domains of Kahn and Plotkin

Bradfield's article is devoted to a related theme, the application of IF logic to modal logics for concurrent and distributed systems. In the logic of distributed systems, we can define modal operators for each agent, in which the corresponding accessibility relations model the actions available to that agent. Bradfield gives a syntax and semantics for the modal counterparts of the Henkin quantifier, and also for a modal version of IF logic.

Jouko Väänänen observes that many important examples of infinite structures can be characterized, in a negative sense, by IF logic. For example, if $\mathcal{N}$ is the standard model of arithmetic, then there is a sentence ϕ of IF logic that is true in $\mathcal{M}$ if and only if $\mathcal{M}$ is *not* isomorphic to $\mathcal{N}$. He observes the somewhat odd consequence that follows: the same sentence ϕ that characterizes $\mathcal{N}$ is necessarily nondetermined in exactly the models isomorphic to $\mathcal{N}$.

Tapani Hytinnen addresses the spectrum problem for logics with Henkin quantifiers (a sentence ϕ's spectrum is the set of natural numbers n such that ϕ has a model of size n). His main result is a characterization of the spectrum of sentences of the form $\exists\vec{y}\forall x\forall z\exists y/\forall z\exists v/\forall x\phi$, where ϕ is a quantifier-free sentence in the empty vocabulary. He shows that the spectrum of such sentences is essentially trivial (for sufficiently large n, it is $\emptyset$, $\mathbb{N}$, $2\mathbb{N}$, or $2\mathbb{N}+1$). More complicated sets are definable if we allow more Henkin quantifiers or a larger vocabulary.

There is a close connection between game theoretical semantics and the dialogical approach to logic of Paul Lorenzen and other logicians, such as Walter Felscher. Rahman and Dégremont work out some of the details by providing tableau formulations of some fragments of IF logic. They propose these formulations as a possible implementation of Hintikka's interpretation of Wittgenstein's private language argument (hence the title). The "beetle in a box" corresponds in their version to certain private moves made by players in their games.

Johan van Benthem also addresses the relation between semantic tableaux and games. His article, written in a relaxed and inviting style, and based on his lectures at the University of Amsterdam, shows how to interpret Beth-style tableaux in game-theoretical terms, and goes on to propound game-style approaches to modal and intuitionistic logic, as well as the contraction-free fragment of first-order predicate logic.

Veikko Rantala is concerned with the notorious problem of omniscience in epistemic logic (the apparently counter-intuitive assumption that an agent knows all logical consequences of known propositions). His solution is to introduce a relativized version of an epistemic modality, written as $K(\alpha)\phi$, to be interpreted as "The agent knows that ϕ after the learning process α." The learning processes are modeled as non-normal (partial) worlds, and the semantic interpretation is given in terms of accessibility relations $R(\alpha)$ relativized to learning processes. Rantala proves completeness for his version of epistemic logic.

Finally, Jan von Plato's short note provides a proof of the existence property of Heyting arithmetic by a direct transformation of formal derivations in a natural deduction calculus.

The papers in this volume generally assume some background on the part of the reader, and so would not be well suited for an introduction to the area of IF logic and game semantics—that purpose is better served by the books by Hintikka and Väänänen. However, they give a good impression of the diversity of work in this part of logic, and can be recommended on that basis.

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