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2007 SPRING MEETING
OF THE ASSOCIATION FOR SYMBOLIC LOGIC

The Palmer House Hilton Hotel, Chicago, Illinois
April 19–21, 2007

A Spring Meeting of the Association of Symbolic Logic was held April 19–21, 2007, in Chicago, Illinois, at The Palmer House Hilton Hotel, in conjunction with the annual meeting of the Central Division of the American Philosophical Association. The Program Committee consisted of Daniel Leivant, Larry Moss, and William Tait (Chair). The ASL hosted a reception on the afternoon of April 20th.

The program included a symposium and two special sessions of invited one-hour talks:

Symposium on Directions in Logic

Yiannis Moschovakis (University of California, Los Angeles), *Mathematical and philosophical logic from computer science*.

Rohit Parikh (City University of New York), *Sentences, propositions, and logical omniscience: what does deduction tell us*?

Richmond H. Thomason (University of Michigan), *Trends in logic: philosophy, linguistics and AI*.

Special Session A

John T. Baldwin (University of Illinois, Chicago), *Why is the weak GCH true*!

Steffen Lempp (University of Wisconsin, Madison), *Separating notions of randomness*.

Special Session B: Categorical Logic

Steve Awodey (Carnegie-Mellon University), *Recent developments in categorical logic*.

Jean-Pierre Marquis (University of Montreal), *From spaces to logic via algebra: the genesis of categorical logic*.

Colin McLarty (Case-Western Reserve), Commentator.

The program also included one session of contributed papers in which five talks were presented by logicians from the US and abroad.

Abstracts of the invited talks and contributed talks given (in person or by title) by members of the Association for Symbolic Logic follow.

For the Program Committee
WILLIAM TAIT

Abstracts of invited talks

► STEVE AWODEY, *Recent developments in categorical logic*.
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This survey talk will provide an overview of some recent developments in categorical logic,

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since the development of topos theory. Topics include algebraic set theory, predicativity and constructivity, intensionality, and modal logic.

► JOHN T. BALDWIN, *Why the weak GCH is true*!
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By the weak GCH we mean the assertion: $2^\lambda < 2^{\lambda^+}$. We will argue for the acceptance of this axiom on grounds of its coherence with our intuitions of cardinal arithmetic and its consequences for mathematical practice. The second argument is mediated by the Devlin–Shelah weak diamond. We will explain this principle, which follows easily from weak GCH, and discuss some of its consequences. These include a version of Morley's theorem for $L_{\omega_1,\omega}$ (Shelah) and implications for the study of semi-abelian varieties (Zilber).

► STEFFEN LEMPP, *Separating notions of randomness*.
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The intuitive concept of randomness allows several possible concise definitions, many of which have been shown to be inequivalent. We discuss some of these notions and present a new separation result: Not every "injectively partially computably random" (and thus in particular not every "permutation partially computably random" set) is Martin–Löf random. This is a partial result toward separating Martin–Löf randomness from Kolmogorov–Loveland randomness, one of the main open questions in this area. (See J. Miller/Nies, BSL, vol. 12, 2006, pp. 390–410, for more background on this area.)

This is joint work with Bart Kastermans.

► JEAN-PIERRE MARQUIS, *From spaces to logic via algebra : the genesis of categorical logic*.
Department of Philosophy, Université de Montréal, C.P. 6128, succ. Centre-ville, Montréal QC, H3C 3J7, Canada.
E-mail: Jean-Pierre.Marquis@umontreal.ca.

In this talk, we will take a close look at the genesis of categorical logic from 1963 until 1974. Starting from Lawvere's pioneering work in 1963 on universal algebra, we will move on to results and program he proposed in the late sixties. We will then consider the interaction between topos theory and logic from 1970 until 1974, particularly how it was developed by the Montreal school, namely by Joyal, Reyes and Makkai. We will underline some of the major philosophical assumptions and relevant results of the field as we move along.

► ROHIT PARIKH, *Sentences, propositions and logical omniscience, or what does deduction tell us*?
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In deductive reasoning, if ϕ is deduced from some set Γ, then ϕ is already implicit in Γ. But then how do we learn anything from deduction? That we do not learn anything is the (unsatisfying) answer suggested by Socrates in Plato's *Meno*. This problem is echoed in the problem of logical omniscience prominent in epistemic logic according to which an agent knows all the consequences of his/her knowledge. An absurd consequence is that someone who knows the axioms of Peano Arithmetic knows all its theorems.

Since knowledge presumes belief, the lack of closure of (actual) beliefs under deduction remains an important issue.

The post-Gettier literature has concentrated on the gap between justified true belief and knowledge, but has not concerned itself with what *belief* is. This question, or at least an analysis of sentences of the form *Jack believes that frogs have ears*, **has** been prominent in the philosophy of language. But even there, less attention has been paid to *how* we know what someone believes. This turns out to matter.

The question *How we know what someone believes has*, however, been addressed by Ramsey and de Finetti (as well as by Savage) in the context of decision theory and the foundations of subjective probability. Beliefs are revealed by the choices we make, the bets we accept and the bets we refuse. And among these choices are the choices of what to say and what to assent to.

We argue that this way of thinking about beliefs not only allows us to address the issue of logical omniscience and to offer formal models of "inconsistent" beliefs, it also allows us to say something useful about Frege's problem, and whether it is sentences or propositions that we believe.

► RICHMOND H. THOMASON, *Trends in logic: philosophy, linguistics, and AI*.
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When the ASL was founded, the membership divided naturally into mathematicians and philosophers. What was once the philosophical component has now developed into a cluster of topics addressed by different specialists: by computer scientists working in A.I. and by linguists as well as by philosophers. Broadly speaking, these logicians are interested in reasoning in nonmathematical domains—in the development of appropriate logics for such domains, and in formalization problems, some of them on an unprecedented scale.

These developments represent trends that are important for logic, and that all logicians should be aware of.

In my talk I will survey work in these areas and will try to identify the challenges this field raises both for theoretical logic and for the emerging field of applied logic. In many ways, the logical formalization of large-scale domains raises problems like those that are addressed by software engineering issues, and—just as software engineering can impact the design of programming languages, these problems have implications for the design of applicable logics.

Abstracts of contributed talks

► KATALIN BIMBÓ, *Duality theorems for ortholattices and De Morgan lattices*.
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Ortholattices and bounded De Morgan lattices may be viewed as generalizations of Boolean algebras. $a \wedge (b \vee c) \leq (a \wedge b) \vee c$ may be false in an ortholattice, so may be $a \wedge a' \leq b$ and $a \leq b \vee b'$ in a De Morgan lattice.

Two classes of topological spaces are defined, concretely, the class of orthospaces and the class of De Morgan spaces. The following are proven: (1) given $\mathfrak{A}$, that is an ortholattice (a bounded De Morgan lattice), $\mathtt{A}(\mathtt{T}(\mathfrak{A}))$ is isomorphic to $\mathfrak{A}$, (2) given $\mathfrak{T}$, that is an orthospace (a De Morgan space), $\mathtt{T}(\mathtt{A}(\mathfrak{T}))$ is homeomorphic to $\mathfrak{T}$, (3) there is a 1–1 correspondence between the maps in the categories of ortholattices and orthospaces (of bounded De Morgan lattices and De Morgan spaces). ($\mathtt{A}$ and $\mathtt{T}$ denote, in a straightforward way, the functors between an algebraic category and the dual topological category.) That is, full duality theorems are proven between the categories of ortholattices (of bounded De Morgan lattices) and orthospaces (De Morgan spaces), respectively.

The orthospaces defined here are related to (but not the same as) the Stone spaces introduced in [1]. De Morgan spaces are a (relational) subclass of the ordered Stone spaces defined in [2].

[1] ROBERT GOLDBLATT, *The Stone space of an ortholattice*, ***Bulletin of the London Mathematical Society***, vol. 7 (1975), pp. 45–48.

[2] HILARY A. PRIESTLEY, *Representation of distributive lattices by means of ordered Stone spaces*, ***Bulletin of the London Mathematical Society***, vol. 2 (1970), pp. 186–190.

► DENIS F. CUDIA, *The Boltzmann Principle and context free grammars*.
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In the following context free grammar productions 4, 5, 6, 7 are the vertical columns of the genetic code table written horizontally except for the codons of Ser, Leu, Arg, resp., which are given in 8, 9, 10, resp.:

1. $\langle \text{codon} \rangle \rightarrow \langle \mathrm{U}_2 \vee \mathrm{A}_2 \rangle | \langle \mathrm{C}_2 \vee \mathrm{G}_2 \rangle$
2. $\langle \mathrm{U}_2 \vee \mathrm{A}_2 \rangle \rightarrow \mathrm{U}_2 | \mathrm{A}_2$
3. $\langle \mathrm{C}_2 \vee \mathrm{G}_2 \rangle \rightarrow \mathrm{C}_2 | \mathrm{G}_2 | \langle \mathrm{UC}_2 \vee \mathrm{AG}_2 \rangle$
4. $\mathrm{U}_2 \rightarrow \mathrm{UUY} | \langle \mathrm{CU}_2 \vee \mathrm{UU}_2 \rangle | \mathrm{AU} \langle \mathrm{Y} \vee \mathrm{A} \rangle | \mathrm{AUG} | \mathrm{GUN}$
5. $\mathrm{C}_2 \rightarrow \mathrm{CCN} | \mathrm{ACN} | \mathrm{GCN}$
6. $\mathrm{A}_2 \rightarrow \mathrm{UAY} | \mathrm{CAY} | \mathrm{CAR} | \mathrm{AAY} | \mathrm{AAR} | \mathrm{GAY} | \mathrm{GAR}$
7. $\mathrm{G}_2 \rightarrow \mathrm{UGY} | \mathrm{UGG} | \langle \mathrm{CG}_2 \vee \mathrm{AG}_2 \rangle | \mathrm{GGN}$
8. $\langle \mathrm{UC}_2 \vee \mathrm{AG}_2 \rangle \rightarrow \mathrm{UCY} | \mathrm{UCR} | \mathrm{AGY}$
9. $\langle \mathrm{CU}_2 \vee \mathrm{UU}_2 \rangle \rightarrow \mathrm{CUY} | \mathrm{CUR} | \mathrm{UUR}$
10. $\langle \mathrm{CG}_2 \vee \mathrm{AG}_2 \rangle \rightarrow \mathrm{CGY} | \mathrm{CGR} | \mathrm{AGR}$
11. $\langle \mathrm{Y} \vee \mathrm{A} \rangle \rightarrow \mathrm{U} | \mathrm{C} | \mathrm{A}$
12. $\mathrm{N} \rightarrow \mathrm{Y} | \mathrm{R}$
13. $\mathrm{Y} \rightarrow \mathrm{U} | \mathrm{C}$
14. $\mathrm{R} \rightarrow \mathrm{A} | \mathrm{G}$

This grammar is used to examine the translation of mRNA into polypeptides.

[1] D. CUDIA, *The information in the genetic code*, ***The Journal of Symbolic Logic***, vol. 53 (1988), pp. 1291–1292.

► MEERI KESÄLÄ, *Finitary abstract elementary classes*.
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We know that first order logic fails to capture many natural classes of structures that appear in mathematics. Several generalizations of model-theoretical tools to nonelementary classes have been introduced. The context of abstract elementary classes was presented by Shelah in the 1980's. He does not study structures in any specific logic, but gives axioms for an abstract elementary substructure -relation. The original framework by Shelah is very general. However, if we want to study for example stability theory in this context, we may have to add some more specific assumptions for the class.

We have collected a set of properties for classes of structures, which enable us to develop a 'geometric' approach to stability theory, including an independence calculus, in a very general framework. We study AEC's with amalgamation, joint embedding, arbitrarily large models, countable Löwenheim–Skolem number and finite character. We call such classes finitary abstract elementary classes. The novel idea is the property of finite character. It is a minor requirement, but it has turned out to make possible various constructions from elementary model theory in this much more general context. It also enables the use of a notion of a weak type instead of the usual Galois type. This context generalizes both contexts of *homogeneous classes* and *excellent classes*.

Notions of simplicity, superstability, Lascar strong type, primary model and U-rank are inroduced for finitary classes. A categoricity transfer result is proved for simple, tame finitary classes: categoricity in any uncountable cardinal transfers upwards and to all cardinals above the Hanf number. Unlike the previous categoricity transfer results of equal generality the theorem does not assume the categoricity cardinal being a successor. This is joint work with Tapani Hyttinen and is presented in the papers [1], [2] and [3].

[1] Tapani Hyttinen and Meeri Kesälä, *Independence in finitary abstract elementary classes*, ***Annals of Pure and Applied Logic***, vol. 143 (2006), pp. 103–138.

[2] ———, *Categoricity transfer in simple finitary abstract elementary classes*, submitted in 2005.

[3] ———, *Superstability in simple finitary AEC*, submitted in 2006.

► ROHIT PARIKH, *Probabilistic conditionals are almost monotonic*.
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It is well known that while probabilistic conditionals satisfy right weakening, they do not satisfy monotonicity, or strengthening the antecedent. In other words, from $A \mid\!\sim B$ we can infer $A \mid\!\sim B \vee X$ but we cannot infer $A \;\&\; X \mid\!\sim B$. Here following Adams and Edgington, $A \mid\!\sim B$ is interpreted as meaning that $p(B \mid A)$ is high.

A standard counter-example to monotonicity is where A indicates that the given creature is a bird, B that it flies, and X indicates that it is a penguin. Other examples will readily come to mind.

What we show is that if we are reasoning over a finite space W, and A, B are subsets of W such that if $A \mid\!\sim B$, i.e., the probability $p(B \mid A)$ is high, (assuming uniform distribution for simplicity), say over .95, then $p(B \mid A \;\&\; X)$ is also high, i.e., it is at least .94 for better than 99% of X's. This explains why knowing that birds fly we can comfortably conclude that female birds fly or that blue birds fly. Penguins are UNUSUAL!

Naturally, having such specific numbers as .95, .94 and .99 implies that we have actual numbers for W, A, B, in mind, but the proof will show that all we need is that the actual numbers be sufficiently large.

► DOLPH ULRICH, *Concise axiomatizations of implicational R-mingle*.
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$\mathbf{RM}_{\rightarrow}$, the implicational fragment of the system **R-mingle** of [1], was axiomatized concisely in [2]: with substitution and detachment as rules, $\mathbf{B}' = CCpqCCqrCpr$, $\mathbf{C}^* = CpCCpqq$, and either $\mathbf{RM1} = CCCpCCCqprqrr$ or $\mathbf{RM2} = CCCCCpqrCqprr$ together form a three-axiom base for $\mathbf{RM}_{\rightarrow}$ in which the total number of symbols occurring is 31. Subsequently, [4] asked (as open question **OQ29.RM**) if there exists an equivalent two-base, and (as open question **OQ30.RM**) if there exists a single axiom for $\mathbf{RM}_{\rightarrow}$.

In [3], the present author answered **OQ29.RM** affirmatively by providing a two-base for $\mathbf{RM}_{\rightarrow}$ containing a total of 32 symbols, and answered **OQ30.RM** by constructing a single axiom for that system of length 85. Shorter such bases have since been found. The useful formula $CCCCpqqrCCrsCps$ (which gives both $\mathbf{B}'$ and $\mathbf{C} = CCpCqrCqCpr$), combined with either **RM1** or **RM2**, is a two-base of length 28 for $\mathbf{RM}_{\rightarrow}$, while the 37-symbol $CCCCCCCpqrCqpCsrCsrCCttCuCvwCCxuCvCxw$ is a single axiom for it. The author conjectures that these 28-symbol two-bases are shortest possible bases for $\mathbf{RM}_{\rightarrow}$, but that a single axiom of length 29 exists.

[1] J. M. Dunn, *Algebraic completeness results for R-mingle and its extensions*, ***The Journal of Symbolic Logic***, vol. 35 (1970), pp. 1–13.

[2] Z. Ernst, B. Fitelson, K. Harris, and L. Wos, *A concise axiomatization of $RM_{\rightarrow}$*, ***Bulletin of the Section of Logic***, vol. 30 (2001), pp. 191–194.

[3] D. Ulrich, *On the existence of a single axiom for implicational R-mingle*, ***The Bulletin of Symbolic Logic***, vol. 11 (2005), p. 459.

[4] L. Wos and G. Pieper, ***Automated reasoning and the discovery of missing and elegant proofs***, Rinton Press, Paramus, 2003.

Abstracts presented by title

► ADIB BEN JEBARA, *The Big Crunch will happen after an infinite time*.
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The collapse of the universe will happen after an infinite time. For that, a singularity has to happen and I think no space (the void set) must have a part in it. We have seen that time can be infinite in "An idea about time in cosmology" in ASL Annual Meeting 2004. We have also seen that space at the level of elementary particles can be described with only a particular case of the axiom of choice holding, countable choice for a family of sets of n elements $n = 2$ through m, in "About space and time of elementary particles" in ASL Annual Meeting 2005 and in "About space being not a continuum" in Logic Colloquium 2005.

No space (the void set) cannot be reached by a finite number of times but by infinite Cartesian products of sets of locations.

The void set will be supplemented with particles and it will not be the void set any more but without there being space. Then, it will be again the singularity of the Big Bang which creates space.

To explain more, in my view, space is the set U of urelements of a Fraenkel Mostowski model and the Big Crunch will occur because of what happens with such a space. The connection between that specific set theory and physical space is my thesis, at the level of elementary particles. The conclusion of the Big Crunch happening holds because bodies are made of elementary particles. The actual infinite means that the Big Crunch will really happen.

We have to notice that time can be countable or of the cardinality of the set of urelements U or a continuum, like space can be of the cardinality of U or a continuum, in different cases.

► JOHN CORCORAN, *Second-Order Logic in Context*.
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This philosophical, historical, expository article might be subtitled: Second-Order Logics, First-Order Logics, and Basic Logics. It builds on my *Second-order Logic* in ***Logic, meaning, and computation: Essays in memory of Alonzo Church***, (C. A. Anderson, and M. Zeleny, editors), Kluwer, Dordrecht, 1998. Employing second-order propositions and second-order reasoning in a natural way, it illustrates the fact that second-order logic is actually a familiar part of our traditional intuitive logical framework—not an artificial formalism created by specialists for technical purposes. Some of the main relationships between second-order logic and first-order logic are explained by introducing *basic logic*, a kind of zero-order logic, which is more rudimentary than first-order and which is transcended by first-order in the same way that first-order is transcended by second-order. Even though it lacks variables and quantification, basic logic is a genuine underlying logic; it is not a mere logic-schema like propositional logic. Nevertheless, it serves many of the pedagogical purposes the latter now serves. The heuristic effectiveness and the historical importance of second-order logics are reviewed in the context of the contemporary debate over the legitimacy of second-order logic.

Rejection of second-order logic is viewed as radical: a paradigm shift repudiating a core part of our scientific tradition. But its rejection is also viewed as reactionary: analogous to the repudiation of symbolic logic by supporters of Aristotelian logic. But even if "genuine" logic comes to be regarded as excluding second-order reasoning, which seems less likely today than fifty years ago, its effectiveness as a heuristic instrument will remain and its essential importance for understanding the history of logic and mathematics will not be diminished. Second-order logic might some day join Aristotle's logic in the dustbin of failed logics, but—like Aristotle's logic—its usefulness and importance should never be forgotten.

► LEONA F. FASS, *Commonsense reasoning and common sense*.
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Some researchers within Logic in Computer Science and AI aim to endow machines with commonsense reasoning capabilities demonstrative of "human-level intelligence". But developing logic-based reasoning techniques; successfully implementing methods on today's machines; and having results represent humans' real-world dynamic environments are very different achievements [4]. Several benchmark problems [3] established by commonsense reasoning researchers have meticulously-crafted, excellent theoretical solutions (e.g., [5, 2]). Still, counterexamples from our own experience indicate results perhaps are partial. It's our position that there may not be a commonality of "commonsense knowledge" representing The Human Experience and available to processes or machines. Neither humans nor machines can demonstrate complete common sense.

Still, humans solve life's reasoning problems and machines solve problems important to humans, each finding solutions not representative of all commonsense knowledge, but satisfactory. We discuss cases where machines with logical-decision-making abilities (e.g., [1]) obtain results that may be finalized by knowledgeable, common-sensible human experts. We show machines and humans may combine their respective logical and commonsense reasoning capabilities, collaborating to solve many problems quite well. But not all problems can be solved; that's just common sense.

[1] A. BARTH AND J. C. MITCHELL, *Managing digital rights using linear logic*, ***Proceedings, 21st IEEE Symposium on Logic in Computer Science***, Seattle WA, August, 2006, pp. 127–136.

[2] J. CAMPBELL AND V. LIFSCHITZ, *Reinforcing a claim in commonsense reasoning*, ***Logical formalizations of commonsense reasoning***, AAAI Press, SS-03-05, 2003, pp. 51–56.

[3] *Common sense problem page*, www-formal.stanford.edu/leora/commonsense/

[4] L. F. FASS, *Theory, practice and reality*, preliminary paper, for participation in ***8th International Symposium on Logical Formalizations of Commonsense Reasoning***, held in Honor of John McCarthy's 80th Birthday at Stanford University, March 2007.

[5] L. MORGENSTERN, *Mid-sized axiomatizations of commonsense problems: A case study in egg cracking*, ***Studia Logica***, vol. 67 (2001), no. 3, pp. 333–384.

► XUNWEI ZHOU, *Knowledge constituent and knowledge in mutually-inversistic predicate calculus*.
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In mutually-inversistic logic, knowledge constituents include term such as 0, 1, 2 (term constant), x, y, z (term variable); function such as $+$, $-$, $*$, (function constant), f, g, h (function variable); predicate such as int, rat (predicate constant), p, q, r (predicate variable); fact compounder such as $\neg$, $\wedge$, $\vee$; empirical or mathematical connective such as $\leqslant^{-1}$ (mutually inverse implication), empirical or mathematical compounder such as $\neg$, $\wedge$, $\vee$; logical connective such as $\leqslant^{-1}$. Term, predicate, empirical or mathematical connective, logical connective are the main constituents. Function, fact compounder, empirical or mathematical compounder are the auxiliary constituents. Knowledge is divided into term, fact proposition, empirical or mathematical connection proposition, logical connection proposition. An auxiliary constituent connecting knowledge forms the same grade of knowledge. A main constituent (except term) connecting knowledge forms knowledge one grade higher. Knowledge is formed as follows: a function connecting terms still forms as term such as $1+1$, $x * y$; a predicate connecting terms forms a fact proposition such as int(3), rat(x), P (abbreviation of $p(x)$); a fact compounder connecting fact propositions still form a fact proposition such as parent$(x, y) \wedge$ ancestor(y, z), $P \wedge Q$; an empirical or mathematical connective connecting fact propositions forms an empirical or mathematical connection proposition such as $\mathrm{int}(x) \leqslant^{-1} \mathrm{rat}(x)$, $P \leqslant^{-1} Q$; an empirical or mathematical compounder connecting empirical or mathematical connection propositions still forms an empirical or mathematical connection proposition such as $\{\mathrm{int}(x) \leqslant^{-1} \mathrm{rat}(x)\} \wedge \{\mathrm{rat}(x) \leqslant^{-1} \mathrm{real}(x)\}$, $\{P \leqslant^{-1} Q\} \wedge \{Q \leqslant^{-1} R\}$; a logical connective connecting empirical or mathematical connection propositions forms a logical connection proposition such as $\{\mathrm{int}(x) \leqslant^{-1} \mathrm{rat}(x)\} \leqslant^{-1} \{\mathrm{rat}(x) \leqslant^{-1} \neg\mathrm{int}(x)\}$. $\{P \leqslant^{-1} Q\} \leqslant^{-1} \{\neg Q \leqslant^{-1} \neg P\}$.

► JEFFERY ZUCKER, *In search of a Church–Turing Thesis for computations on continuous data*.
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The CT (Church–Turing) Thesis establishes the scope and limits of computations by deterministic algorithms on discrete data.

In modelling the real world, we often use continuous data such as real numbers, infinite bitstreams and analog signals [4]. What are the scope and limits of computations on such data?

Over the past two decades we have developed a comprehensive theory of abstract models of computation on many-sorted algebras, with a corresponding CT thesis, involving either a high-level programming language or recursion schemes [1]. More recently, we have considered concrete computation models, which, unlike abstract models, are implementation-dependent. Such models differ significantly from the abstract models in the case of continuous data.

A correspondence between abstract and concrete computation models has been obtained [2, 3] by strengthening the abstract model in two directions: (1) adding a new atomic statement, the "choice operator", and (2) replacing exact by approximable computation.

However, a good formulation of a CT thesis for concrete computability on continuous data remains an open problem.

This is joint work with John Tucker (Swansea).

[1] J. V. Tucker and J. I. Zucker, *Computable functions and semicomputable sets on many-sorted algebras*, ***Handbook of Logic in Computer Science***, (S. Abramsky, D. Gabbai and T. Maibaum, editors), vol. 5, Oxford University Press, 2000, pp. 317–523.

[2] ———, *Abstract versus concrete computation on metric partial algebras*, ***ACM Transactions on Computational Logic***, vol. 5 (2004), pp. 611–668.

[3] ———, *Computable total functions, algebraic specifications and dynamical systems*, ***Journal of Logical and Algebraic Programming***, vol. 62 (2005), pp. 71–108.

[4] ———, *Computability of analog networks*, ***Theoretical Computer Science***, vol. 371 (2007), pp. 115–146.