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2007 ANNUAL MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

University of Florida
Gainesville, Florida
March 10–13, 2007

The Annual Meeting of the Association was held on the campus of the University of Florida, Gainesville, March 10–13, 2007. The program included the Eighteenth Annual Gödel Lecture, eleven one-hour invited talks and four special sessions. There were also sessions of contributed talks. The members of the Program Committee were Matthias Aschenbrenner, Mirna Džamonja (Chair), Neil Immerman, Jean Larson, Joseph Miller, and Jamie Tappenden. The members of the Local Organizing Committee were Douglas Cenzer, Gérard Emch, William Mitchell, Greg Ray, Meera Sitharam and Jindrich Zapletal (Chair). There were 79 registered participants at the meeting, including 30 graduate students. Generous financial support was provided by the University of Florida, Department of Mathematics. The conference complemented the activities of the NSF-funded Special Year in Logic at the University, and included a satellite workshop on *Singular Cardinal Combinatorics and Inner Models*, organized by Matthew Foreman, William Mitchell, and John Steel, on March 5–9, 2007.

The ASL hosted a conference reception and an additional reception was offered by the Department of Philosophy and the Society for Exact Philosophy in honor of the thirty-fifth anniversary of the ***Journal of Philosophical Logic***.

The Eighteenth Gödel Lecturer was Ehud Hrushovski (Hebrew University of Jerusalem). Unforeseen circumstances prevented Hrushovski from attending and in his place Thomas Scanlon (University of California-Berkeley) gave an hour-long lecture on *Algebraic Model Theory*. The other hour-long invited addresses were:

Michael Benedikt (Oxford University (UK)), *Decision problems for regular tree languages and the model theory of trees*.

Andreas Blass (University of Michigan-Ann Arbor (USA)), *Ultrafilters, cardinal characteristics, and forcing*.

Natasha Dobrinen (University of Vienna (Austria)), *Co-stationarity of the ground model*.

Noam Greenberg (University of Wellington (New Zealand)), *Lowness properties of computably enumerable degrees*.

Denis R. Hirschfeldt (University of Chicago (USA)), *Randomness, triviality and computablity*.

Salma Kuhlmann (University of Saskatchewan (Canada)), *Integer parts and complements to valuation rings of ordered fields*.

Hannes Leitgeb (University of Bristol (UK)), *Formal and informal provability*.

William J. Mitchell (University of Florida-Gainesville (USA)), *Perspectives on the inner model program*.

Thomas Wilke (University of Kiel (Germany)), *Temporal logics from a computer science perspective*.

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Richard Zach (Univesrity of Calgary (Canada)), *The decision problem and the development of metalogic*.

There were four special sessions (with the organizers given in parentheses): Algebraic Model Theory (M. Aschenbrenner), Computability Theory (N. Greenberg), Connections of Set Theory with Boolean Algebras and Topology (M. Džamonja and J. Larson), and Logic Research for Undergraduates (W. White). These included a total of 36 speakers, each of whom gave a 25-minute talk.

Abstracts of the invited talks and the contributed talks (given in person or by title) by members of the Association for Symbolic Logic follow.

For the Program Committee
MIRNA DŽAMONJA

Abstract of the 18th Annual Gödel Lecture

► EHUD HRUSHOVSKI, *Model theory and numbers of points*.
Department of Mathematics, Hebrew University at Jerusalem, 91904 Jerusalem, Israel.
E-mail: ehud@math.huji.ac.il.

In Euclidean plane geometry, two figures are shown to be "equal" by cutting them into matching congruent figures. This contrasts with the viewpoint consolidated in the 19th century, where a number, the "area", belonging to an external number system constructed independently of the geometry, is assigned to each figure separately.

Grothendieck, having developed a formidable analytic machinery directed largely at the problem of counting points over finite fields of algebraic varieties, introduced a program of "motives", intended to show that the same can be achieved by purely internal means. The most elementary manifestation of this is the Grothendieck group of varieties. A variety, for our purposes, can be thought of as the definable set (in an algebraically closed field) corresponding to the equation $F = 0$, where F is a rational polynomial in n variables. Two varieties are defined to be "equal" if they can be compared by cut-and-paste operations, as in the Pythagorean procedure. This implies they have the same number of points when specialized to almost any finite field; and the class in the Grothendieck group can be viewed as a model theoretic replacement to the numbers obtained by counting points. Coarser equivalence relations preserving this property are also considered; but they are no less internal and definable, and preserve or even display the hidden symmetries of the geometry.

Another kind of counting is p-adic volumes, or the closely related number of points modulo powers of a fixed prime. Kontsevich invented an analogue of p-adic integration, called motivic integration, where again numbers are replaced by algebraic varieties (or classes in a Grothendieck ring, broadly defined.) The properties of this kind of integration were brought out using the model theory of Henselian fields by Denef and Loeser.

The Grothendieck group point of view generalizes to any first-order theory, and is very nicely compatible with the model theoretic viewpoint. The Grothendieck group of pseudo-finite fields was connected, again by Denef and Loeser, with one of the next steps in Grothendieck's program (Chow motives). Further relations to the model theory of difference fields are beginning to be visible.

The Grothendieck group of definable sets over valued fields (in Robinson's model completion) reduces to those of the residue field and value group, in both a point-counting and measure-theoretic sense. This is an expression of the Ax-Kochen-Ershov principle. It turns out to generalize the Kontsevich "motivic integration", but in a way that also brings into play the Grothendieck group of the value group.

There are connections to classical model theoretic notions (orthogonality, analyzability), and many open questions, both in the general setting and regarding specific theories.

This talk was presented by title.

Abstracts of invited plenary talks

► MICHAEL BENEDIKT, *Decision problems for regular tree languages and the model theory of trees*.
Oxford University Computing Laboratory, Wolfson Building, Parks Road, Oxford, OX1 3QD, UK.
E-mail: michael.benedikt@comlab.ox.ac.uk.
Bell Laboratories, 2701 Lucent Ln., Lisle, IL 60532, USA.
E-mail: benedikt@research.bell-labs.com.

Surprisingly, the model theory of first-order logic when restricted to many "well-behaved" classes of finite models—such as finite trees—is not at all well-understood. As an example consider the problem of "characterizing first-order definability on trees"—determining whether a regular tree language (equivalently, tree automaton) is defined by some first-order logic sentence. First-order here means either first-order over the vocabulary including only node labels and the child relation, or in the vocabulary including also the descendant relation. The decidability of first-order definability in the latter sense remains open after two decades; in this talk I outline our proof that first-order definability in the first sense is decidable. In the case of strings rather than trees, the analogous problem is of deciding whether a regular string language is "locally threshold testable". This was shown decidable by Beauquier and Pin, who characterized first-order definability via identities satisfied in the monoid associated with a regular language. Our characterization and decision procedure for first-order logic on trees uses an approach that combines the algebraic approach with locality arguments. We show how the characterization of definability gives a new "semigroup-free" proof of Beauquier and Pin's theorem, and how it can be useful in resolving other questions in the finite model theory of trees, in particular recovering consequences of the interpolation theorem for trees.

This is joint work with Luc Segoufin.

► ANDREAS BLASS, *Ultrafilters, cardinal characteristics, and forcing*.
Mathematics Department, University of Michigan, Ann Arbor, MI 48109-1043, USA.
E-mail: ablass@umich.edu.

I plan to describe some recent results concerning the number of near-coherence classes of nonprincipal ultrafilters on a countable set. Some of these results are connected with a strong inequality between certain cardinal characteristics of the continuum and with unusual forcing constructions.

► NATASHA DOBRINEN, *Co-stationarity of the ground model*.
Kurt Gödel Research Center for Mathematical Logic, Universität Wien, Währinger Strasse 25, 1090 Wien, Austria.
E-mail: dobrinen@logic.univie.ac.at.

Most of the work presented is joint with Sy-David Friedman. Given $V \subseteq W$ models of ZFC with the same ordinals and $\kappa < \lambda$ cardinals in W with κ regular, let $(\mathcal{P}_\kappa(\lambda))^W$ denote the collection of subsets of λ of size less than κ in W. We say that the ground model is *co-stationary* in $(\mathcal{P}_\kappa(\lambda))^W$ if $(\mathcal{P}_\kappa(\lambda))^W \setminus V$ is stationary in $\mathcal{P}_\kappa(\lambda)$.

Abraham showed in [1] that if $\mathbb{P}$ is a c.c.c. partial ordering which adds a new subset of ω, then the ground model is co-stationary in $(\mathcal{P}_{\aleph_1}(\aleph_2))^{V^{\mathbb{P}}}$. Gitik improved this in [5] by showing the following: Suppose $V \subseteq W$ are models of ZFC with the same ordinals, κ is a regular cardinal in W, $\lambda \geq (\kappa^+)^W$, and there is a subset of ω in $W \setminus V$. Then the ground model is co-stationary in $(\mathcal{P}_\kappa(\lambda))^W$. Hence, a new real makes the ground model *globally co-stationary*.

We consider problems of generalizing Abraham's and Gitik's Theorems to pairs of models in which there is no new real in the larger model.

1. Co-stationarity of the ground model in $\mathcal{P}_{\kappa^+}(\lambda)$ for fixed κ, λ, given a new subset of κ.
2. Global co-stationarity of the ground model, given a new subset of κ.
3. Co-stationarity of the ground model in $\mathcal{P}_{\kappa^+}(\lambda)$ for fixed κ, λ, given a new sequence of length κ.
4. Global co-stationarity of the ground model, given a new sequence of length κ.

We present equiconsistencies from [4], and upper bounds on consistency strengths from [4] and [2] for various values of κ. An upper bound from [3] on the internal consistency strength of global co-stationarity from a new subset of $\aleph_1$ will be presented, as well as many open problems.

[1] Uri Abraham and Saharon Shelah, *Forcing closed and unbounded sets*, ***The Journal of Symbolic Logic***, vol. 48 (1983), no. 3, pp. 643–657.

[2] Natasha Dobrinen, *Global co-stationarity of the ground model and new ω-sequences*, Submitted.

[3] Natasha Dobrinen and Sy-David Friedman, *Internal consistency and global co-stationarity of the ground model*, Submitted.

[4] ———, *Co-stationarity of the ground model*, ***The Journal of Symbolic Logic***, vol. 71 (2006), no. 3, pp. 1029–1043.

[5] Moti Gitik *Nonsplitting subsets of $\mathcal{P}_\kappa(\kappa^+)$*, ***The Journal of Symbolic Logic***, vol. 50 (1985), no. 4, pp. 881–894.

► NOAM GREENBERG, *Lowness properties of computably enumerable degrees*.
Victoria University of Wellington, School of Mathematics, Statistics and Computer Sience, 436 Cotton Bldg., Kelburn, Wellington, 6005 New Zealand.
E-mail: `Noam.Greenberg@mcs.vuw.ac.nz`.

The low degrees are close to being computable, as demonstrated for example by Robinson's low guessing trick. Nevertheless, recent investigations have discovered a rich structure within these nether regions of the c.e. degrees, starting from the smallest known class of low degrees and ending with a transfinite hierarchy covering the low_2 degrees. These classes are related to the Turing jump operator and traceability, multiple permitting arguments and lattice embeddings, algorithmic randomness and measure; for some we get natural definability results.

► DENIS R. HIRSCHFELDT, *Randomness, triviality, and computability*.
Department of Mathematics, University of Chicago, 5734 S. University Ave., Chicago, IL 60637, USA.
E-mail: `drh@math.uchicago.edu`.

I will survey some results and open problems in the theory of K-trivial sequences, particularly focusing on their computability theoretic connections with 1-random sequences.

► SALMA KUHLMANN, *Integer parts and complements to valuation rings of ordered fields*.
Department of Mathematics and Statistics, University of Saskatchewan, McLean Hall, 106 Wiggins Road, Saskatoon, SK S7N 5E6, Canada.
E-mail: `skuhlman@math.usask.ca`.

An integer part (IP for short) Z of an ordered field K is a discretely ordered subring, with 1 as the least positive element, and such that for every element x of K, there is an element z of Z such that x is between z and z + 1.

Mourgues and Ressayre establish the existence of an IP for any real closed field K by showing that there is an order preserving embedding of K into an appropriate field of generalized power series, such that the image of the embedding is a truncation closed subfield.

An IP of K obtained in this way (i.e., from a truncation closed embedding) is called a truncation integer part of K.

IPs appear naturally in model theoretic arithmetic, algebra and analysis; e.g., Shepherdson showed that IPs of real closed fields are precisely the models of a fragment of Peano Arithmetic called Open Induction, whereas truncation IPs played a crucial role in Ressayre's investigations of the model theory of the real exponential field.

In this talk, we analyze IPs from a valuation theoretic viewpoint and summarize their main special features. We investigate their connection to special (additive) complements of valuation rings of ordered fields. This approach reveals new interesting valuation theoretic properties of valued fields; depending on whether such special complements exist. We discuss these properties and their implications, thereby giving an intrinsic valuation theoretic interpretation of truncation closed embeddings in fields of power series.

► HANNES LEITGEB, *Formal and informal provability*.
Departments of Philosophy and Mathematics, University of Bristol, 9 Woodland Road, Bristol BS8 1TB, UK.
E-mail: Hannes.Leitgeb@bristol.ac.uk.

We will deal with informal provability, i.e., provability as it is understood in mathematical practice (rather than in proof theory). In particular, we will argue that

1. informal provability differs conceptually from formal provability in a recursively axiomatized system,
2. the notion of informal provability has semantic and intuitive components that the notion of formal provability lacks (and it is possible to make sense of Gödel's view on these kinds of components),
3. the logic of informal provability differs from the logic of formal provability.

► WILLIAM J. MITCHELL, *Perspectives on the inner model program*.
Department of Mathematics, University of Florida, PO Box 118105, Gainesville, Florida 32611-8105, USA.
E-mail: mitchell@math.ufl.edu.

I will discuss the development and current state of inner models in set theory, with an emphasis on the relation of this program to singular cardinal combinatorics. The talk will assume some knowledge of set theory, but will be aimed at a general audience.

► THOMAS SCANLON, *Algebraic model theory*.
Department of Mathematics, Evans Hall #3840, U.C.B., Berkeley, CA 94720-3840, USA.
E-mail: scanlon@math.berkeley.edu.

In set theory, we define the operations on cardinal numbers in terms of categorical operations on sets. Generalizing this construction to other categories (especially to categories of definable sets), via the devices of Grothendieck (semi-)rings, we obtain a way to "count" and to perform combinatorics in other contexts. We specialized the discussion to the recent model theoretic work on motivic integration by Denef, Cluckers and Loeser, and especially its refinement due to Hrushovski and Kazhdan.

► THOMAS WILKE, *Temporal logics from a computer science perspective*.
Department of Computer Science, Christian-Albrechts-Universität zu Kiel, 24098 Kiel, Germany.
E-mail: wilke@ti.informatik.uni-kiel.de.

Logics such as linear-time temporal logic and related ones are used extensively in computer science, especially in the areas of verification and specification. The following questions have been studied:

Expressiveness: Given two such logics, can they express the same properties?
Succinctness: Given two such logics with the same expressive power, can certain properties be expressed in one logic with much smaller formulas than in the other?
Hierarchies: Given such a logic, are there natural hierarchies with respect to expressiveness (such as nesting depth in a temporal operator or quantifier alternation or number of variables used)?
Classification: Given such a logic and a property, in which fragment of the logic can the property be expressed?

Techniques that have been used to tackle these questions include: formal language theory, combinatorics on words, finite automata theory, finite semigroup theory, and Ehrenfeucht–Fraïssé games.

In this talk I will illustrate the above questions by concrete examples, describe the major results that have been obtained, explain how the techniques mentioned above are used, and what the open problems are.

As a particular example, I will focus on FO^2, first-order logic with two variables interpreted on finite linear orderings. This logic is interesting because there are several logics that are equivalent to this logic with respect to their expressive power. For instance, it is equivalent to the fragment of linear-time temporal logic where "sometime in the future" is the only temporal operator.

► RICHARD ZACH, *The decision problem and the development of metalogic*.
Department of Philosophy, University of Calgary, Calgary, Alberta T2N 1N4, Canada.
E-mail: rzach@ucalgary.ca.

In parallel with their work on proof theory in the 1920s and early 1930s, Hilbert and his collaborators and students—in particular, Ackermann, Behmann, Bernays, and Schönfinkel—did substantial work towards a positive solution for the decision problem. This begins with an unpublished talk by Behmann in 1921 in which the term "Entscheidungsproblem" first appears, and continues until the early 1930s with a number of published as well as unpublished contributions. Approaches to the decision problem evolved significantly during this time, from a purely algebraic approach in the style of Schröderian algebra of logic to relatively modern proofs which establish the finite controllability of certain prefix classes of formulas. This evolution goes hand-in-hand with an evolution of attendant concepts, in particular, semantic concepts such as satisfiability. An analysis of this work sheds light on the development of the semantics of first-order logic in the 1920s, on changing views as to what constitutes a "decision procedure," and on the connection between the decision problem and the consistency problem.

Abstracts of invited talks in the Special Session on Algebraic Model Theory

► SALIH AZGIN, *Model theory of valued difference fields*.
Department of Mathematics, University of Illinois at Urbana-Champaign, 1409 W. Green Street (MC-382), Urbana, IL 61801-2975, USA.
E-mail: azgin@math.uiuc.edu.

A classical result by Ax-Kochen and Ershov provides a very good understanding of the model theory of finitely ramified henselian valued fields. "Model Theory of Frobenius on Witt Vectors" (preprint, Belair–Macintyre–Scanlon) pursues a similar goal in the context of valued difference fields, i.e., valued fields with a distinguished automorphism.

Our goal is to prove an analogue of the main result of the BMS paper in equal characteristic $p > 0$. We shall start by considering valued difference fields in equal characteristic zero, where the interaction between the valuation and the distinguished automorphism is different than that of the BMS context. Even though this seems irrelevant to the question at hand, it provides good insight to the model theory of valued fields and valued difference fields in equal characteristic $p > 0$.

► ALF DOLICH, *Model theory around dense pairs of o-minimal structures*.
Department of Mathematics, Statistics and Computer Science, University of Illinois at Chicago, 851 S. Morgan St. (M/C 249), Chicago, IL 60607-7045, USA.
E-mail: adolich@math.uic.edu.

In [1] van den Dries studied dense pairs of o-minimal structures. Namely he considered for a fixed o-minimal theory T models $\mathfrak{M} \preceq \mathfrak{N}$ with M dense in N and studied the resulting theory of the pair. In this talk we investigate various model theoretic properties of such pairs and then consider these properties in a more general context of densely ordered structures. We also consider how results of van den Dries for dense pairs of o-minimal structures may be generalized to dense pairs of other topological structures.

[1] Lou van den Dries, *Dense pairs of o-minimal structures*, ***Fundamenta Mathematicae***, vol. 157 (1998), pp. 61–78.

► CLIFTON EALY, *Superrosy groups*.
Department of Mathematics, University of Illinois at Urbana-Champaign, 1409 W. Green Street (MC-382), Urbana, IL 61801-2975, USA.
E-mail: clif@math.uiuc.edu.

Superrosy groups are a wide class of groups characterized by a well-behaved notion of dimension, and include superstable groups, supersimple groups, and groups definable in o-minimal theories. I will provide an overview of recent work in the area (joint with Krzyzstof Krupinski and Anand Pillay) focusing in particular on superrosy groups which are dependent (a class which includes superstable groups and groups definable in o-minimal theories).

► PANTELIS ELEFTHERIOU, *Compact domination for groups definable in linear o-minimal structures*.
Department of Mathematics, University of Notre Dame, 255 Hurley Hall, Notre Dame, IN 46556-4618, USA.
E-mail: pelefthe@nd.edu.

Let $\mathcal{M}$ be a saturated ordered vector space over an ordered division ring, and G a definably compact group definable in $\mathcal{M}$. Then G is compactly dominated by $(G/G^{00}, \mathbf{m}, \pi)$, where $\mathbf{m}$ is the Haar measure on G/G^{00} and π is the canonical homomorphism from G onto G/G^{00}. Intuitively, this means that π is a kind of intrinsic 'standard part map' ([1]).

[1] E. Hrushovski, Y. Peterzil, and A. Pillay, *Groups, measures, and the NIP*, To appear in the ***Journal of the American Mathematical Society***.

► JOHN GOODRICK, *The Schröder–Bernstein problem for weakly minimal groups*.
Department of Mathematics, University of California at Berkeley, Evans Hall, Berkeley, CA 94720-3840, USA.
E-mail: goodrick@math.berkeley.edu.

A theory has the *Schröder–Bernstein property* if any two of its elementarily bi-embeddable models are isomorphic. We study this problem in the context of stable, weakly minimal groups (that is, stable groups of U-rank 1). Our main conjecture is that if G is a weakly minimal group with a countable theory then the following two conditions are equivalent:

1. $\mathrm{Th}(G)$ does not the Schröder–Bernstein property;
2. There is some model H of $\mathrm{Th}(G)$, an automorphism f of H, and an element a of the universal domain such that the set $\{a, f(a), f^2(a), \dots\}$ is algebraically independent over H.

In this talk, we will discuss the proofs of some special cases of this conjecture, such as the direction $1 \Rightarrow 2$, and the case where all the definable structure on G comes from the group operation.

► AYHAN GUNAYDIN, *The real field with two small multiplicative subgroups*.
Department of Mathematics, University of Illinois at Urbana-Champaign, 1409 W. Green Street (MC-382), Urbana, IL 61801-2975, USA.
E-mail: gunaydin@math.uiuc.edu.

Paper [1] studies the field $\mathbb{R}$ of real numbers with a subgroup G of $\mathbb{R}^{>0}$ satisfying a certain smallness condition, called the Mann property. In this talk, I consider $\mathbb{R}$ with two subgroups A, G of $\mathbb{R}^{>0}$ satisfying the Mann property with $G < A$. Roughly, I classify structures (K, A, G) up to elementary equivalence, where K is a real closed field and A, G are subgroups of $K^{>0}$ satisfying the Mann property with $G < A$, for example $(\mathbb{R}, 2^{\mathbb{Z}} \cdot 3^{\mathbb{Z}}, 2^{\mathbb{Z}})$, where $\mathbb{Z}$ denotes the integers.

[1] L. van den Dries and A. Gunaydin, *The fields of real and complex numbers with a small multiplicative group*, ***Proceedings of the London Mathematical Society***, vol. 93 (2006), pp. 43–81.

► JONATHAN KIRBY, *Blurred complex exponentiation*.
Department of Mathematics, Statistics and Computer Science, University of Illinois at Chicago, 851 S. Morgan St. (M/C 249), Chicago, IL 60607-7045, USA.
E-mail: kirby@math.uic.edu.

In the early 1990s, Hrushovski produced his "new strongly minimal sets" by a new amalgamation construction. Zilber started searching for "natural" examples of this construction, and conjectured that the complex exponential field is such an example. Zilber's full conjecture is stronger than the very difficult Schanuel conjecture of transcendental number theory, so is considered out of reach. However, I will describe how to "blur" the number theoretic details and prove the resulting form of the conjecture.

► JANA MAŘÍKOVÁ, *Definable sets generated by standard part sets*.
Department of Mathematics, University of Illinois at Urbana-Champaign, 1409 W. Green Street (MC-382), Urbana, IL 61801-2975, USA.
E-mail: marikova@math.uiuc.edu.

Let R be a sufficiently saturated o-minimal expansion of a real closed field, set $\mathcal{O} := \{r \in R : |r| < M \text{ for some } M \in \mathbb{N}\}$, and let $\mathrm{st}\colon \mathcal{O} \to \mathbb{R}$ be the standard part map. It induces for every n a corresponding standard part map $\mathrm{st}\colon \mathcal{O}^n \to \mathbb{R}^n$, and for $X \subseteq R^n$ we put $\mathrm{st}(X) := \mathrm{st}(X \cap \mathcal{O}^n)$. Let $\mathbb{R}_{\mathrm{ind}}$ be the field $\mathbb{R}$ expanded by all sets of the form $\mathrm{st}(X)$, where $X \subseteq R^n$ is definable in R, $n = 1, 2, \ldots$. Hrushovski, Peterzil and Pillay observe in their paper *Groups, measures, and the NIP* that the structure $\mathbb{R}_{\mathrm{ind}}$ is o-minimal, as a consequence of a theorem by Baisalov and Poizat. Without using this theorem we obtain the following stronger version of this fact: the subsets of $\mathbb{R}^n$ that are definable in $\mathbb{R}_{\mathrm{ind}}$ are exactly the finite unions of sets of the form $\mathrm{st}(X) \setminus \mathrm{st}(Y)$, where $X, Y \subseteq R^n$ are definable in R.

► ALICE MEDVEDEV, *Difference fields in analysis and in model theory*.
Department of Mathematics, University of California at Berkeley, Evans Hall, Berkeley, CA 94720-3840, USA.
E-mail: alice@math.berkeley.edu.

Difference fields, i.e., fields with a distinguished automorphism, are an abstract setting for studying functional equations like $f(x+1) = (f(x))^2$ or $f(3x)+4f(x)+1 = 0$. The model theory of generic difference fields has been studied extensively by Chatzidakis, Hrushovski, Peterzil, and others.

Seidenberg showed that every countable differential field can be embedded into a field of germs of meromorphic functions, with the usual differential operator. We aim toward proving a similar result for difference fields. The automorphism could be taken to be the additive shift $f(x) \mapsto f(x+1)$ or a multiplicative shift $f(x) \mapsto f(qx)$ for a fixed q, and it is necessary to go beyond meromorphic functions and include some logarithmic expressions.

We will discuss some partial results and obstructions to several approaches.

► ALF ONSHUUS, *On theories interpretable in o-minimal structures*.
Departamento de Matemáticas, Universidad de los Andes, A.A. 4976, Bogotá, Colombia.
E-mail: onshuus@uniandes.edu.co.

In this talk we will analyze the behavior of stability and instability in super rosy theories with NIP, concentrating on theories which are interpretable in o-minimal structures.

We will first survey some results from Shelah which guarantee that stable sets in theories with NIP are easily defined and have nice properties. We will then use this definition to prove the following:

- We will define o-minimal-by-finite types and prove that in any theory interpretable in an o-minimal structure the union of stable and o-minimal-by-finite types is dense in the Stone topology.
- Any unstable theory interpretable in an o-minimal structure interprets an o-minimal order.
- For any super-rosy dependent group G there is a stable normal subgroup H such that any infinite definable subset of the quotient G/H is unstable.

► KATHRYN VOZORIS, *The complex field with a predicate for the integers*.
Department of Mathematics, Statistics and Computer Science, University of Illinois at Chicago, 851 S. Morgan St. (M/C 249), Chicago, IL 60607-7045, USA.
E-mail: vozoris@math.uic.edu.

There are several intriguing, open questions related to the complex field with exponentiation. The integers are definable in this structure, making the complex field with a predicate for the integers an interesting object to consider. Although the complex field with a predicate for the integers is unstable, it is possible to give a reasonable characterization of its theory and definable sets. I will discuss model theoretic properties of the theory, and some results on definability.

Abstracts of invited talks in the Special Session on Computability Theory

► GEORGE BARMPALIAS, *Randomness, lowness and degrees*.
School of Mathematics, University of Leeds, Leeds, LS2 9JT, UK.
E-mail: barmpalias@yahoo.com.uk.

Say that $A \leq_{LR} B$ if every B-random is A-random. This reducibility was introduced by Nies and has turned out to be a very interesting relativisation of the notion of 'low for random' (although a straightforward relativisation gives a different reducibility). We study the structure of LR degrees globally and locally (i.e., restricted to computably enumerable degrees) and their relationship with the Turing degrees. Among other results we show that every generalized superhigh degree bounds $2^{\aleph_0}$ degrees and we give sample results which demonstrate how various techniques from the theory of c.e. degrees can be used to prove results about the c.e. LR degrees. Following is a sample of our results.

THEOREM 1. *If H is (generalized) superhigh then in the LR degrees the degree of H bounds $2^{\aleph_0}$ degrees.*

THEOREM 2. *If W is an incomplete c.e. set, i.e., $\emptyset' \not\leq_T W$, then (uniformly in W) there is a c.e. set B such that $B \leq_{LR} W$ and $B \not\leq_T W$.*

THEOREM 3. *If A is c.e. and not low for random then there are c.e. B, C such that*

- $B \cap C = \emptyset$
- $B \cup C = A$
- $B \not\leq_{LR} C$ *and* $C \not\leq_{LR} B$.

Most of this work is joint with Andy Lewis and Mariya Soskova.

► STEPHEN BINNS, *Density results for the Muchnik lattice*.
Department of Mathematics, The University of Connecticut, 196 Auditorium Road, University of Connecticut, U-3009, Storrs, CT 06269-3009, USA.
E-mail: `binns@math.uconn.edu`.

The outstanding problem in the study of the Muchnik lattice of Π^0_1 classes is the problem of density. That is, given two Π^0_1 classes $P >_w Q$, does there exist a third class R such that $P >_w R >_w Q$? Significant partial results have been obtained but the general theorem has not yet been proven. Some approaches to the problem are described.

► JOHN CHISHOLM, *Strongly Δ^0_2 categorical structures*.
Department of Mathematics, Western Illinois University, Macomb, IL 61455, USA.
E-mail: `ja-chisholm@wiu.edu`.

A computable structure A is *strongly Δ^0_2-categorical* iff there is a sequence $\{f_i : i \in \omega\}$ of uniformly Δ^0_2 bijections on ω such that, for every computable B isomorphic to A, some f_i is an isomorphism from B to A.

The *index set* for a computable structure A is the set $\{e : B_e \approx A\}$, where $\{B_e : e \in \omega\}$ effectively enumerates all partial computable structures of the language of A.

A is said to have a *Scott family* iff the orbit (under automorphisms) of any tuple $\bar{a}$ in A is definable by an existential formula.

THEOREM 1. *Let A be 2-decidable. Then A is strongly Δ^0_2-categorical iff there is some finite tuple $\bar{p}$ in A such that (1) $(A, \bar{p})$ has a Scott family, and (2) the index set for $(A, \bar{p})$ is a Σ^0_3 set.*

► E. B. FOKINA, S. S. GONCHAROV, V. S. HARIZANOV, J. F. KNIGHT, AND S. MILLER, *Intrinsically Σ^0_α relations and Δ^0_α categoricity*.
Department of Mathematics, Notre Dame University, 255 Hurley Hall, Notre Dame, IN 46556-5683, USA.
E-mail: `knight.1@nd.edu`.

A relation R on a computable structure $\mathcal{A}$ is *intrinsically* Σ^0_α if for all isomorphisms F from $\mathcal{A}$ onto a computable copy $\mathcal{B}$, $F(R)$ is Σ^0_α. It is *relatively intrinsically* Σ^0_α if for all isomorphisms F from $\mathcal{A}$ onto a copy $\mathcal{B}$ (not necessarily computable), $F(R)$ is Σ^0_α relative to $\mathcal{B}$. Clearly, if R is relatively intrinsically Σ^0_α on $\mathcal{A}$, then it is intrinsically Σ^0_α. Manasse showed that the converse fails for $\alpha = 1$. In [2], it was shown that the converse fails for all computable successor ordinals α. We show that the converse also fails for computable limit ordinals. Soskov [4] showed that if R is "intrinsically hyperarithmetical", then it is "relatively intrinsically hyperarithmetical"—the definitions follow the same pattern as above.

A computable structure $\mathcal{A}$ is Δ^0_α *categorical* if for all computable copies $\mathcal{B}$ there is a Δ^0_α isomorphism from $\mathcal{A}$ onto $\mathcal{B}$. It is *relatively* Δ^0_α *categorical* if for all copies $\mathcal{B}$ (not just computable copies), there is an isomorphism which is Δ^0_α relative to $\mathcal{B}$. Clearly, if $\mathcal{A}$ is relatively Δ^0_α categorical, then it is Δ^0_α categorical. Goncharov [1] showed that the converse fails for $\alpha = 1$. In [2], it is shown that the converse fails for all computable successor ordinals α. We show that the converse also fails for computable limit ordinals.

[1] S. S. GONCHAROV, *The quantity of non-autoequivalent constructivizations*, ***Algebra and Logic***, vol. 16 (1977), pp. 257–282 (Russian), pp. 169–185 (English translation).

[2] S. S. GONCHAROV, V. HARIZANOV, J. KNIGHT, C. MCCOY, R. MILLER, AND R. SOLOMON, *Enumerations in computable structure theory*, ***Annals of Pure and Applied Logic***, vol. 136 (2005), pp. 219–246.

[3] M. MANASSE, *Techniques and counterexamples in almost categorical recursive model theory*, Ph.D. Thesis, University of Wisconsin, Madison, 1982.

[4] I. N. SOSKOV, *Intrinsically Δ^1_1 relations*, ***Mathematical Logic Quarterly***, vol. 42 (1996), pp. 469–480.

► DENIS R. HIRSCHFELDT, CARL G. JOCKUSCH, JR., BJØRN KJOS-HANSSEN, STEFFEN LEMPP, AND THEODORE A. SLAMAN, *Ramsey's Theorem for pairs and the DNR axiom*.
Department of Mathematics, University of Illinois, 1409 W. Green Street, Urbana, IL 61801, USA.
E-mail: jockusch@math.uiuc.edu.

Perhaps the most interesting and challenging of the remaining open questions on the reverse mathematics of Ramsey's Theorem is whether Ramsey's Theorem for pairs (denoted RT^2_2) implies Weak König's Lemma in the standard base theory RCA_0. We prove a modified version of this implication in RCA_0 by replacing both the hypothesis and the conclusion by weaker statements. Specifically, we show in [1] that stable Ramsey's theorem for pairs (SRT^2_2) implies DNR in RCA_0. (A coloring of pairs of natural numbers is *stable* if for each a the color of $\{a, b\}$ has a limit as b approaches infinity. SRT^2_2 is the restriction of Ramsey's Theorem to stable 2-colorings of pairs. DNR is the assertion that for each set X of natural numbers there is a function f which is diagonally noncomputable relative to X, i.e., $(\forall e)[f(e) \neq \{e\}^X(e)]$. Weak König's Lemma is equivalent to asserting that for all X there is a $\{0, 1\}$-valued function f which is diagonally noncomputable relative to X.) This result was used by Hirschfeldt and Shore [2] to show that the chain-antichain principle CAC does not imply SRT^2_2 in RCA_0. (CAC asserts that each partial ordering of the natural numbers has an infinite chain or antichain.) We also discuss the question of whether SRT^2_2 implies RT^2_2 in RCA_0.

[1] DENIS R. HIRSCHFELDT, CARL G. JOCKUSCH, JR., BJØRN KJOS-HANSSEN, STEFFEN LEMPP, AND THEODORE A. SLAMAN, *The strength of some combinatorial principles related to Ramsey's Theorem for pairs*, to appear in ***Proceedings of the Program on Computational Prospects of Infinity***, IMS Proceedings, (C. T. Chong, Qi Feng, Theodore Slaman, Hugh Woodin, and Yue Yang, editors), World Scientific, 2007.

[2] DENIS R. HIRSCHFELDT AND RICHARD A. SHORE, *Combinatorial principles weaker than Ramsey's Theorem for pairs*, ***The Journal of Symbolic Logic***, vol. 72 (2007), pp. 171–206.

► JEFFRY L. HIRST, *Ramsey's Theorem on trees (Preliminary report)*.
Department of Mathematical Sciences, Appalachian State University, Boone, NC 28608, USA.
E-mail: jlh@math.appstate.edu.

Consider the following extension of Ramsey's Theorem: Given a finite coloring of the linearly ordered n-tuples of elements in a full binary tree, there is a subtree which is order isomorphic to the full binary tree such that all the linearly ordered n-tuples of the subtree have the same color. We will consider results of reverse mathematics and computability theory related to this version of Ramsey's Theorem.

This is joint work with Jennifer Chubb and Tim McNicholl.

► BJØRN KJOS-HANSSEN, *Computing Brownian slow points*.
Department of Mathematics, Cornell University, Ithaca, NY 14853, USA.
E-mail: bjoern@math.cornell.edu.

We consider a fixed algorithmically random path of Brownian motion f associated with Chaitin's halting probability Ω. Depending on the amount of oscillation in the graph near a point $(x, f(x))$, probabilists say that x is a *fast*, *slow*, or *ordinary* point. From work of Fouché, Nerode and the speaker we know that the set of all points that are able to compute non-ordinary points has measure zero. However, both fast points and slow points exist, and we show that not all fast points compute slow points.

► ANDY LEWIS, *A fixed point free minimal degree*.
Dipartimento di Scienze Matematiche ed Informatiche Roberto Magari, University of Siena, Pian dei Mantellini 44, 53100 Siena, Italy.
E-mail: aemlewis@aemlewis.co.uk.

In 1993 Kumabe provided a positive solution to a question of Sacks by constructing a fixed point free minimal degree. The technical brilliance of the solution is remarkable. Unfortunately, however, the complexity of the construction and the dense style of presentation render Kumabe's paper almost impossible to read. Other researchers in the field have been unable to verify the validity of the proof and the paper has remained unpublished. Recently, while working on the old question of Yates as to whether or not every minimal degree has a strong minimal cover, I was interested to observe connections with Kumabe's result. It became necessary to decipher the construction in order to be able to extend the techniques involved. Having done so, I have now undertaken the project of producing a readable version of the paper. In my talk I shall explain the basic idea behind the proof.

► JOSEPH MILETI, *The strength of the Rainbow Ramsey Theorem*.
Department of Mathematics, University of Chicago, 5734 S. University Ave., Chicago, IL 60637, USA.
E-mail: mileti@math.uchicago.edu.

The Rainbow Ramsey Theorem is essentially an "anti-Ramsey" theorem which roughly states that certain colorings always have a large heterogeneous section. Surprisingly, this version follows easily from Ramsey's Theorem, even in the sense of reverse mathematics. We answer the question of the converse implication for pairs, showing that the Rainbow Ramsey Theorem for pairs is in fact strictly weaker and sits in a very interesting position in the reverse mathematical hierarchy below Ramsey's Theorem for pairs. The separation involves techniques from the theory of randomness, and also provides as a corollary a new proof of an old theorem of Martin on the measure of the set of hyperimmune degrees.

This is joint work with Barbara Csima.

► CARL MUMMERT, *Reverse mathematics and one-point compactifications*.
Department of Mathematics, University of Michigan, 530 Church St., Ann Arbor, MI 48109, USA.
E-mail: mummertc@umich.edu.

Hirst [1] studied the reverse mathematics of one-point compactifications of countable locally compact metric spaces. Methods developed for the reverse mathematics of metrizability allow us to expand this study to potentially uncountable spaces. This talk will describe results on the reverse mathematics of one-point compactifications of locally compact complete separable metric spaces.

[1] JEFFRY L. HIRST, *Embeddings of countable closed sets and reverse mathematics*, ***Archive for Mathematical Logic***, vol. 32 (1993), no. 6, pp. 443–449.

► STEPHEN G. SIMPSON, *Bounded limit recursiveness*.
Department of Mathematics, Pennsylvania State University, McAllister Building, University Park, State College, PA 16802, USA.
E-mail: simpson@math.psu.edu.

Let X be a Turing oracle. A function $f(n)$ is said to be boundedly limit recursive in X if it is the limit of an X-recursive sequence of X-recursive functions $f(n, s)$ such that the number of times $f(n, s)$ changes is bounded by a recursive function of n. Let us say that X is BLR-low if every function which is boundedly limit recursive in X is boundedly limit recursive in 0. This is a lowness property in the sense of Nies. These notions were introduced by Joshua A. Cole and the speaker in a recently submitted paper on mass problems and hyperarithmeticity. The purpose of this talk is to compare BLR-lowness to similar properties which have been considered in the recursion-theoretic literature. Among the properties discussed are: K-triviality, superlowness, jump-traceability, weak jump-traceability, total omega-recursive enumerability, array recursiveness, array jump-recursiveness, and strong jump-traceability.

► GUOHUA WU, *Quotient structure R/NCup and a diamond theorem*.
Division of Mathematical Sciences, School of Physical and Mathematical Sciences, Nanyang Technological University, Singapore 639798.
E-mail: guohua@ntu.edu.sg.

A computably enumerable (c.e.) degree **a** is cuppable if there is an incomplete c.e. degree **b** such that $\mathbf{a} \cup \mathbf{b} = \mathbf{0}'$. A c.e. degree is noncuppable, if it is not cuppable. Ambos-Spies, Lachlan and Soare proved the following cupping continuity property: if **a** cups an incomplete c.e. degree **b** to $\mathbf{0}'$, then **a** also cups a c.e. degree **c** below **b** to $\mathbf{0}'$.

In this talk, we show that there are two computably enumerable degrees **a**, **b** such that $\mathbf{a} \cup \mathbf{b} = \mathbf{0}'$, and that no incomplete c.e. degree can cup both **a** and **b** to $\mathbf{0}'$.

As a corollary, we have that the diamond lattice can be embedded into the quotient structure R/NCup, the c.e. degrees modulo the noncuppable degrees, preserving 0 and 1.

This is a joint result with Angsheng Li and Yue Yang.

Abstracts of invited talks in the Special Session on Connections of Set Theory with Boolean Algebras and Topology

► MOHAMED BEKKALI AND LATIFA FAOUZI, *Constructions and applications of Boolean Algebras*.
Department of Mathematics, Faculty of Sciences and Technology, Sidi Mohamed Ben Abdellah University, Fez 30 000, Morocco.
E-mail: bekka@menara.ma.
E-mail: latifaouzi@menara.ma.

Set Theory opened new horizons in studying Boolean algebras. We survey the use of some combinatorial techniques, axioms and cardinal functions to construct various types of Boolean Algebras. On the other hand, we give the latest constructions of Boolean algebras over partially ordered sets as well as new applications in topology.

[1] I. Juhasz, ***Cardinal functions in topology, Ten years later***, Mathematisch Centrum, 1980.

[2] J. D. Monk and R. Bonnet (editors), ***Handbook on Boolean algebras***, (3 volumes), North Holland, Amsterdam, 1989.

[3] J. Donald Monk, ***Cardinal Invariants on Boolean algebras***, Progress in Mathematics, vol. 142, Birkhäuser Verlag, Basel-Boston-Berlin, 1996.

► JENNIFER BROWN, *Topologies on pseudotrees*.
Department of Mathematics, Kenyon College, Gambier, OH 43022-9623, USA.
E-mail: brownja@kenyon.edu.

If T is a tree, the interval topology τ_I on T is the one generated by the base consisting of sets of type $(s, t]$, together with singletons $\{t\}$ where t is a minimal element of T. The fine wedge topology τ_F on T is the one whose sub-base consists of the up-sets $T \uparrow t = \{s \in T : s \geq t\}$ and their complements, for t in T. (The fine wedge topology is also the one whose base is the Boolean algebra Treealg(T).) Much is known about these topologies in the case where T is a tree; see, for example, [1]. We investigate how properties of (T, τ_I) and (T, τ_F) can vary if T is only required to be a *pseudotree*, and we give internal characterizations of several topological properties of these spaces. (A pseudotree is a partial order $(T, \leq)$ such that for all t in T, the set $T \downarrow t$ of elements below t is a linear order. This is a more general notion than that of a tree, where the sets $T \downarrow t$ are required to be well-ordered.)

[1] P. J. Nyikos, *Various topologies on trees*, ***Proceedings of the Tennessee Topology Conference***, (P. R. Misra and M. Rajagopalan, editors), World Scientific Publishing Company, 1997, pp. 167–198.

► ALAN DOW, *Tie points in βN*.
Department of Mathematics, UNC¸Charlotte, Charlotte, NC 28223, USA.
E-mail: adow@uncc.edu.

This talk is motivated by the study of rigidity properties of $\beta\mathbb{N}\backslash\mathbb{N}$ or $\mathcal{P}(\mathbb{N})/fin$ in the absence of CH. We say that a point is a *tie point* of $\mathbb{N}^*$ if (picture a bow-tie) there are closed sets A, B covering $\mathbb{N}^*$ and x is the unique common adherent (or limit) point of each. CH implies that every point of $\mathbb{N}^*$ is a tie point and that each of the closed sets A, B witnessing this are homeomorphic copies of $\mathbb{N}^*$. If A, B witness that x is a tie point, then x is a P-point of at least one of A or B. Our interest will be in the case that x is actually a P-point of $\mathbb{N}^*$ and will be motivated by the observation that $\mathbb{N}^*$ is homeomorphic to the quotient space of the free union $A \otimes B$ obtained by identifying the two copies of the point x, denoted $A \otimes B/\{x, x\}$. Again CH implies that if P-points x, y are tie points of $\mathbb{N}^*$, witnessed by A, B and A', B' respectively, then $(A \otimes B')/\{x, y\}$ is a copy of $\mathbb{N}^*$. One may note that if $\mathbb{N}^*$ is actually homeomorphic to $A \otimes A/\{x, x\}$, then there is a (nontrivial) autohomeomorphism of $\mathbb{N}^*$ with a unique fixed point. Our main result is to show that it is consistent to have different types of tie points, x, y, in the sense that $C \otimes D/\{x, y\}$ is not homeomorphic to $\mathbb{N}^*$ for any $C \in \{A, B\}$ and $D \in \{A', B'\}$. We modify the technique from a paper by Brendle and Shelah [1].

This is joint work with Saharon Shelah.

[1] Jörg Brendle and Saharon Shelah, *Ultrafilters on ω—their ideals and their cardinal characteristics*, ***Transactions of the American Mathematical Society***, vol. 351 (1999), no. 7, pp. 2643–2674.

► TODD EISWORTH, *Proper forcing and the Continuum Hypothesis*.
Department of Mathematics, Ohio University, Athens, OH 45701, USA.
E-mail: eisworth@math.ohiou.edu.

We will give a thumbnail sketch of ongoing research concerning the strength of the Continuum Hypothesis as it concerns constructions in general topology. Most of the results discussed are already fairly well-known among set-theoretic topologists; our intent in this talk is to dwell on the issues in pure set theory that have arisen in this investigation.

This work was presented by title.

► SU GAO, *Borel structures on ordinals*.
Department of Mathematics, PO Box 311430, University of North Texas, Denton, Texas 76203-1430, USA.
E-mail: sgao@unt.edu.

I will talk about a complete classification of ordinals up to Borel isomorphism in ZFC. The problem is motivated by an earlier classification of the order topologies on ordinals. But it turns out that the complete invariants for Borel isomorphism types are very different. Specifically, consider the class of ordinals consisting of all cardinals and ordinals of the form $\kappa \cdot \lambda$, where κ is regular and $\lambda \leq \kappa$ is a cardinal. Two ordinals $\alpha < \beta$ are Borel isomorphic iff there is no ordinal γ in the above class such that $\alpha < \gamma \leq \beta$. We also consider the same classification problem in the context of AD and for product spaces with ordinal factors. I will talk about some partial results for these variations.

This is joint work with Jackson and Kieftenbeld.

► STEFAN GESCHKE, *The cellularity of the phase space of a minimal dynamical system*.
Department of Mathematics, Boise State University, 1910 University Drive, Boise, ID 83725-1555, USA.
E-mail: geschke@math.boisestate.edu.

Generalizing a theorem of Balcar and Błaszczyk [1], it was shown in [2] that whenever (G, X) is a minimal dynamical system and G is $\aleph_0$-bounded, then the Boolean algebra $\mathrm{ro}(X)$ of regular open subsets of X is the completion of a free Boolean algebra. In particular, X is of countable cellularity. This result is clearly related to an older result of Uspenskiĭ [4], who showed that if an $\aleph_0$-bounded group acts continuously and transitively on a compact space X, then X is Dugundji and hence of countable cellularity.

Using elementary substructures of initial segments of the set-theoretic universe, we show that if G is κ-bounded and (G, X) is a minimal dynamical system, then the cellularity of X is at most κ [3]. It might be interesting to investigate the relations between other cardinal invariants of the group and the phase space of a minimal dynamical system.

[1] B. Balcar and A. Błaszczyk, *On minimal dynamical systems on Boolean algebras*, ***Commentationes Mathematicae Universitatis Carolinae***, vol. 31 (1990), pp. 7–11.

[2] S. Geschke, *A note on minimal dynamical systems*, ***Acta Universitatis Carolinae. Mathematica et Physica***, vol. 45 (2004), no. 2, pp. 35–43.

[3] ———, *An upper bound for the cellularity of the phase space of a minimal dynamical system*, ***Topology Proceedings***, to appear.

[4] V. V. Uspenskiĭ, *Why compact groups are dyadic*, ***General topology and its relations to modern analysis and algebra VI, Proceedings of the 6th Symposium, Prague/Czech. 1986***, Research and Exposition in Mathematics, vol. 16, pp. 601–610, 1988.

► MICHAEL HRUSAK, *Borel ideals and their quotients*.
Instituto de matematicas, Campus Morelia, Universidad Nacional Autónoma de México, Apartado Postal 61-3 (Xangari), C.P. 58089, Morelia, Michoacán, Mexico.
E-mail: `michael@matmor.unam.mx`.

We will study combinatorial properties of Borel ideals on the set of integers and the corresponding quotient Boolean algebras, mostly expressed in terms of cardinal invariants.

This work was presented by title.

► JUDITH ROITMAN, *Paracompact box products*.
Mathematics Department, University of Kansas, Lawrence, KS 66044, USA.
E-mail: `roitman@math.ku.edu`.

The question of whether the box product of countably many compact first countable spaces is really paracompact remains open. This talk will discuss the combinatorics of this problem and its relationship to cardinal functions of the continuum.

► DIEGO ROJAS-REBOLLEDO, *Cofinalities of F_σ ideals on ω*.
IMATE, UNAM: A.P. 61-3 (Xangari), Morelia, Michoacán, C.P. 58089, México.
E-mail: `rriego@gmail.com`.

We will talk about cofinalites of F_σ ideals on ω. Mainly on fragmented F_σ ideals. For a fragmented F_σ ideal $\mathfrak{I}$ there is a family of weakly bounded sets $X_i \subset [\omega]^{<\omega}$ of $\mathfrak{I}$ and a partition of ω into finite sets F_n such that

$$\mathfrak{I} = \{A \subset \omega \colon (\exists i \in \omega)(\forall n \in \omega)(A \cap F_n \in X_i)\}.$$

► ANDRZEJ ROSŁANOWSKI AND SAHARON SHELAH, *Lords of the iteration*.
Department of Mathematics, University of Nebraska at Omaha, Omaha, NE 68182, USA.
E-mail: `roslanow@member.ams.org`.
URL Address: `http://www.unomaha.edu/logic`.
Institute of Mathematics, The Hebrew University of Jerusalem, Jerusalem, 91904, Israel.
E-mail: `shelah@math.huji.ac.il`.
URL Address: `http://shelah.logic.at`.

Since 1980 it has been known that the theory of proper forcing does not admit naive generalization to the context of larger cardinals and iterations with larger supports (see [6, Appendix 3.6(2)]). Some steps towards developing the theory of forcing iterated with uncountable supports were done in Shelah [7], but the properties introduced there were aimed at situations when we do not want to add new subsets of λ (corresponding to the case of *no new reals in CS iterations of proper forcing notions*). Later Rosłanowski and Shelah [5] introduced an iterable property implying that λ^+ is not collapsed (and even more). This property works nicely for λ-support iterations (where $\lambda = \lambda^{<\lambda}$ is essentially arbitrary) and forcings adding new subsets of λ, however it does not cover many natural examples of forcing notions. Rosłanowski and Shelah [4, 3] concentrated on the case of inaccessible λ and gave somewhat better iteration theorems and new examples of forcing notions for this case.

In the current talk we will present a survey of recent results concerning not collapsing λ^+ in λ-support iterations. We will highlight the main points of the articles [1, 2], giving results applicable to both the case of inaccessible λ as well as those working for successor cardinals.

[1] ANDRZEJ ROSLANOWSKI AND SAHARON SHELAH, *Lords of the iteration*, `http://front.math.ucdavis.edu/math.LO/0611131`.

[2] ———, *Last forcing standing*, in preparation.

[3] ———, *Reasonably complete forcing notions*, ***Quaderni di Matematica***, accepted, `http://front.math.ucdavis.edu/math.LO/0508272`.

[4] ———, *Sheva-Sheva-Sheva: Large Creatures*, ***Israel Journal of Mathematics***, accepted, `http://front.math.ucdavis.edu/math.LO/0210205`.

[5] ———, *Iteration of λ-complete forcing notions not collapsing λ^+*, ***International Journal of Mathematics and Mathematical Sciences***, vol. 28 (2001), pp. 63–82.

[6] SAHARON SHELAH, ***Proper and improper forcing***, Perspectives in Mathematical Logic, Springer, 1998.

[7] ———, *Not collapsing cardinals $\leq \kappa$ in $(< \kappa)$–support iterations*, ***Israel Journal of Mathematics***, vol. 136 (2003), pp. 29–115.

► WILLIAM WEISS, *Cardinal sequences of compact scattered spaces*.
Department of Mathematics, University of Toronto, Toronto, ON M5S 1A1 Canada.
E-mail: `weiss@math.utoronto.ca`.

The Cantor-Bendixon process recursively eliminates isolated points from topological spaces. Compact scattered spaces are those for which this process terminates with the empty set. They are Stone spaces of superatomic Boolean algebras. The sizes of the removed sets give rise to a sequence of cardinal numbers. We characterize those sequences of cardinals of length less than the second uncountable ordinal which can actually arise in this way. The G.C.H. is assumed in the case of the longer sequences.

This is joint work with I. Juhasz and L. Soukup.

This talk was presented by title.

Abstracts of invited talks in the Special Session on Logic Research for Undergraduates

► CHRISTOPHER S. HARDIN, *My logic experience as an undergraduate*.
Department of Mathematics, Smith College, Northampton, MA 01063, USA.
E-mail: `chardin@email.smith.edu`.

I will describe my experience doing research in logic as an undergraduate, and how it led to and influenced my subsequent work in logic in graduate school and afterward. I will also talk about how I came to study logic in the first place. Although this will be my own perspective as a former student, I will emphasize the role of faculty in making the experience successful.

► JEFFRY L. HIRST, *Overcoming barriers to undergraduate research in logic*.
Dept. of Mathematical Sciences, Appalachian State University, Boone, NC 28608, USA.
E-mail: `jlh@math.appstate.edu`.

Undergraduates who otherwise might be drawn to participate in research activities are sometimes daunted by the prospect of choosing a topic, searching the literature, and typesetting their results. A handful of classroom demonstrations, introductory assignments, and web pages can eliminate these barriers.

► ROBERT S. MILNIKEL, *A sampling of capstone papers in logic*.
Department of Mathematics, Kenyon College, Gambier, Ohio 43022-9623, USA.
E-mail: milnikelr@kenyon.edu.

Every Senior Mathematics major at Kenyon writes a capstone paper outside the context of any classes. The subject is completely up to the student, but is usually an extension of material covered in a course. Over the last few years a number of students have expressed interest in writing on aspects of logic and set theory, despite Kenyon's meager offerings in those areas. Finding appropriate topics for students with slight background has been a challenge, but the results have been rewarding. I will discuss a handful of student papers in logic and theoretical computer science from the last several years.

► WALKER M. WHITE, *What is undergraduate research in logic?*.
Department of Computer Science, Cornell University, Ithaca, NY 14853, USA.
E-mail: wmwhite@cs.cornell.edu.

Research Experiences for Undergraduates (REUs) are becoming an increasingly important part of undergraduate education. Not only do they provide a bridge between undergraduate and graduate mathematics, but they also have some influence on the type of research that students go into when they reach graduate school. Therefore, if we are to recruit more students in mathematical logic, then we need to make such research opportunities available to them in their undergraduate education.

With that said, there are many different ways of achieving that goal. In this talk, we will examine several of the different ways to get undergraduate students interested in working in mathematical logic. We will examine the advantages and disadvantages of each. Finally, we will conclude with a discussion of what can be done to increase the availability of logic research experiences to undergraduate students.

Abstracts of contributed talks

► EVANGELIA ANTONAKOS, *Epistemic logic with common and justified common knowledge*.
CUNY Graduate Center, Ph.D. Program in Mathematics, 365 Fifth Avenue, New York, NY 10016, USA.
E-mail: Eva@Antonakos.net.

A multi-agent epistemic logic $\mathsf{S4}_n^{CJ}$ is defined which encompasses both traditional Common Knowledge C (cf. [4, 5]) and Justified Knowledge J [3]. Justified Common Knowledge, introduced by Artemov [3], is a more general approach to the concept of common knowledge which has been shown to have better proof-theoretic and complexity behavior. The notion of justified knowledge grew from the studies of the Logic of Proofs [2]. The discovery of the justified common knowledge J brought under consideration a whole variety of common knowledge operators for a given set of agents.

Here we introduce the logic $\mathsf{S4}_n^{CJ}$, which is formalized as a normal modal logic of n agents each represented by an $\mathsf{S4}$ modality K_i for $i = 1, 2, \ldots, n$. The two types of common knowledge are also $\mathsf{S4}$ modalities and satisfy the connection axiom for all i : $J\varphi \to K_i\varphi$ and $C\varphi \to K_i\varphi$. In addition, there is an induction axiom for C alone: $[\varphi \wedge C(\varphi \to E\varphi)] \to C\varphi$. In $\mathsf{S4}_n^{CJ}$ the modality C is the conventional common knowledge operator whereas J represents any common knowledge operator. In particular, $J\varphi \to C\varphi$ is provable, hence C is provably the weakest of common knowledge operators.

We show that $\mathsf{S4}_n^{CJ}$ is sound and complete for a class of epistemic Kripke models. As a result, we also have that $\mathsf{S4}_n^{CJ}$ has the finite model property and is decidable. $\mathsf{S4}_n^{CJ}$ is conservative over both $\mathsf{S4}_n^{C}$ and $\mathsf{S4}_n^{J}$, the analogous systems with a single common knowledge modality, C or J respectively. This then offers a convenient testbed for analyzing different approaches to the common knowledge phenomenon (cf. [1]).

[1] E. ANTONAKOS, *Justified knowledge is suffcient*, Technical Report TR-2006004, CUNY Ph.D. Program in Computer Science, 2006.

[2] S. ARTEMOV, *Explicit provability and constructive semantics*, ***The Bulletin of Symbolic Logic***, vol. 7 (2001), no. 1, pp. 1–36.

[3] ———, *Justified common Knowledge*, ***Theoretical Computer Science***, vol. 357 (2006), no. 1–3, pp. 4–22, 2006.

[4] R. FAGIN, J. HALPERN, Y. MOSES, AND M. VARDI, ***Reasoning about knowledge***, MIT Press, 1995.

[5] J.-J. CH. MEYER AND W. VAN DER HOEK, ***Epistemic logic of AI and computer science***, Cambridge Tracts in Theoretical Computer Science, vol. 41, Cambridge University Press, 1995.

► PAUL BRODHEAD, *Effectively closed sets and enumerations*.
Department of Mathematics, University of Florida, 429 Little Hall, P.O. Box 118105, Gainesville, FL 32611‸8105, USA.
E-mail: brodhead@math.ufl.edu.

An effectively closed set in 2^ω is also called Π^0_1 class and may viewed as the set of infinite paths through a computable tree. We consider structural questions concerning the upper semilattice of numberings of Π^0_1 classes. Two numberings that are in the same equivalence class under the ordering of this structure are said to be acceptably equivalent. First we show that many commonly used numberings acceptably equivalent. Furthermore a permutation exists between any two such numberings preserving the computable content. Next we consider the minimal elements of this structure, the enumerations without repetition, and note that such numberings do exist for Π^0_1 classes. We extend this study to enumerations of different families of Π^0_1 classes: string verifiable, decidable, homogeneous, small, and thin classes. For example, enumerations of string verifiable, decidable, and strongly homogeneous classes exist. In fact, decidable classes may be enumerated without repetition. However there are no computable enumerations of small classes. This work is an extension of the work presented at the Third International Conference on Computability and Complexity in Analysis.

[1] P. BRODHEAD, *Enumerations of Π^0_1 classes: Acceptability and decidable classes*, ***Proceedings: CCA 2006, Gainesville, FL***, Electronic Notes in Theoretical Computer Science, Elsevier Science, (2006), to appear.

► GREGORY MCCOLM, *Guarded quantification*.
Department of Mathematics, University of South Florida, Tampa, FL 33620, USA.
E-mail: mccolm@cas.usf.edu.
URL Address: http://www.math.usf.edu/~mccolm.

A *guarded quantification* is of the form

$$(\exists \vec{y}.G(\vec{x},\vec{y}))\theta(\vec{x},\vec{y},\vec{z}) \equiv (\exists \vec{y})[G(\vec{x},\vec{y}) \wedge \theta(\vec{x},\vec{y},\vec{z})]$$

or

$$(\forall \vec{y}.G(\vec{x},\vec{y}))\theta(\vec{x},\vec{y},\vec{z}) \equiv (\forall \vec{y})[G(\vec{x},\vec{y}) \rightarrow \theta(\vec{x},\vec{y},\vec{z})].$$

Although guards go back to Dykstra (and "bounded quantification" even further), much current research in model theory began with [1]. There is a small but growing collection of guards, mostly based on the form of the guard relation G or its relation to the predicate θ; for example, in [1] (which has clear connections with logical game theory), G was an primitive relation and $\vec{z}$ was an empty tuple. Since then various authors have looked at guards where G was symmetric and binary, where G is a conjunction of primitive relations, where it is a clique, and so on.

We present a classification of what we shall call *guard queries*, which will classify extant guards, as well as generate additional examples. We shall describe two classes of results:

- Descriptive complexity results: imposing guarded quantification (and disallowing unguarded quantification) on a logic creates a new descriptive complexity class.
- Representation theorems of guards: certain nice classes of guard queries have normal representations.

In particular, we look at guarded least fixed point logics possessing the same expressive power as unguarded least fixed point logic, and time permitting, explore the fine structure of the hierarchy of descriptive complexity classes (defined by number-of-variables measures) within these logics.

[1] H. ANDRÉKA, J. VAN BENTHEM AND I. NÉMETI, *Modal languages and bounded fragments of predicate logic*, ILLC Research Report ML 1996-03, Technical Notes Series, 1996.

► R. GREGORY TAYLOR, *Symmetric propositions and the language of first-order logic*.
Manhattan College, Riverdale, NY 10471 USA.
E-mail: gregory.taylor@manhattan.edu.
URL Address: http://home.manhattan.edu/~gregory.taylor.

Let $\mathcal{H}^{\theta}_{\mathfrak{D},\Sigma}$ be the Zermelo logic of inaccessible index θ over domain $\mathfrak{D}$ and signature Σ. Let $Q^{1,1}$ be a dyadic predicative first-order quantifier that is logical. If Φ is any sentence of language $\mathcal{L}^{Q^{1,1}}(\mathcal{H}^{\theta}_{\mathfrak{D},\Sigma})$ containing no occurrences of individual symbolic constants, then canonical expansion $[\Phi]^{+}_{\mathfrak{D}}$ over $\mathfrak{D}$ is a symmetric proposition in the sense that $\mathrm{Mod}^{\theta}_{\mathfrak{D},\Sigma}([\Phi]^{+}_{\mathfrak{D}})$ is invariant under domain permutations.

► DOLPH ULRICH, *A short axiom pair and a shortest possible single axiom for monothetic BCI*.
Department of Philosophy, Purdue University, 100 North University Street, West Lafayette, IN 47907, USA.
E-mail: dulrich@purdue.edu.

The C-pure substitution/detachment system BCI has as axioms $B = CCpqCCrpCrq$, $C = CCpCqrCqCpr$, and $I = Cpp$. Humberstone [1] has recently stimulated study of a related system, the smallest extension of BCI in which each theorem provably implies every theorem. Dubbing this system "monothetic BCI", Humberstone conjectures that it is axiomatizable by B, C, and $I^* = CCppCqq$ and reduces that conjecture to the question of whether it can be proved in BCI^* that B implies an instance of I. The author understands that there is a proof of the conjecture forthcoming in [2] but has not seen that paper and so does not know if it uses Humberstone's approach or some other (though certainly his is viable, the author's own proof of $CCCpqCCrpCrqCss$ in BCI^* running thirteen steps in length).

Considerably shorter axiomatic bases for monothetic BCI may be of some interest. Among these, the pair consisting of I^* and the remarkable theorem $CCCCpqqrCCrsCps$ of BCI, which gives, but is not given by, both B and C, provides a two-base in which the total number of symbols occurring is just 22. Better still, the BCI^*-thesis $CCCCppqrCCsCrtCqCst$, of length 19, is a single axiom for monothetic BCI and, in terms of both number of axioms and total length, is in fact a shortest possible base for it.

[1] L. HUMBERSTONE, *Variations on a theme of Curry*, ***Notre Dame Journal of Formal Logic***, vol. 47 (2006), pp. 101–131.
[2] T. KOWALSKI AND S. BUTCHART, *A note on monothetic BCI*, ***Notre Dame Journal of Formal Logic***, to appear.

Abstracts of papers presented by title

► ADIB BEN JEBARA, *Negation of the axiom of choice and Evil and Fate*.
Apt. F3, Residence Badr Manar 1 Tunis, 2092 Tunisia.
E-mail: adib.jebara@topnet.tn.

In "All things are numbers" in Logic Colloquium 2001 and in "About the strength of Evil" in ASL Winter Meeting 2004 2005, we have seen that the numbers of attributes of Evil are infinite sums and products of integers.

Let $U(1)$ be the notation for the universe of set theory with urelements (non sets) where even a particular case of the axiom of choice does not hold, U being the set of urelements. $1+1+1+\cdots+1+\cdots$ equal to aleph zero is a number of attributes of Evil, the only infinite number in $U(1)$.

So, Evil is even more restricted than when a particular case of the axiom of choice holds.

However, people are subjected to Fate as they cannot choose a sequence of willful elementary acts from $U_1 \times U_2 \times \cdots \times U_i \times \cdots$ which is the void set if U is the set of all possible willful elementary acts. So, Fate does exist.

At first, I was considering elementary acts but Mr. Andreas Blass pointed out that even Fate cannot produce a sequence of such acts, so, it makes sense to consider willful elementary acts.

$U(m)$ being the universe where $CC(2$ through $m)$ holds (the countable axiom of choice for families of sets of n elements, $n = 2$ through m), one could go from $U(1)$ to $U(m)$ and get less subjected to Fate, as $U_1 \times U_2 \times U_3 \times \cdots \times U_i \times \cdots$ could be not void in $U(m)$.

Let us notice that I consider the set of all possible willful elementary acts to be infinite.

Also, let us notice that the less the size of the sets of willful acts to be chosen from, the more likely is Fate to be escaped because a particular case of the axiom of choice holds.

► JOHN CORCORAN, *Information-theoretic properties of truth-functional connectives*.
Philosophy, University at Buffalo, Buffalo, NY 14260-4150, USA.
E-mail: corcoran@buffalo.edu.

According to the information-theoretic view, which can be regarded as a faithful intensional analogue or reinterpretation of standard extensional model-theoretic logic, one given proposition *follows from* (is a *logical consequence* of, is *implied* by) a second if the information of the first is [contained] in that of the second. Two propositions are *logically equivalent* if they have the same information. Although Jevons is the most articulate proponent of this view, clear indications of it are found in Boole, De Morgan, Frege, Peirce, and others writing in the 1800s before the model-theoretic view emerged. Vestiges occur in later logicians including Carnap and Quine. It accords closely with views of the nature of logical consequence implicit in the writings of scholars, scientists, and mathematicians not schooled in the standard model-theoretic view. To limit discussion, all information *pertinent* here is conveyed by a first-order language interpreted in the natural numbers. The information in the negation of a given proposition is exactly the pertinent information not in the given proposition: a proposition and its negation disjointly divide all pertinent information. The information of a conjunction is that of both conjuncts. The information in a disjunction is the information shared between the disjuncts. The information in a conditional is exactly the information of the consequent not in the antecedent. It follows that the conditional is implied by its consequent and also by the negation of its antecedent, and also moreover that the conditional contains exactly the information of the disjunction of the consequent with the negation of the antecedent. A tautology (respectively contradiction) is a proposition containing no (respectively all) pertinent information. This paper, which deduces many other familiar and unfamiliar results, builds on my lecture *Fertile postulate sets*, ***The Bulletin of Symbolic Logic***, vol. 10 (2004), pp. 276–277.

► MAURO DI NASSO, *A Euclidean measure of sets*.
Dipartimento di Matematica, Università di Pisa, Italy.
E-mail: dinasso@dm.unipi.it.

DEFINITION. Let $\mathbb{V}$ be a universe, and $\mathfrak{R} = (\mathfrak{R}, 0, 1, +, \cdot, <)$ a discretely ordered ring. A map $\mathfrak{s}\colon \mathbb{V} \to \mathfrak{R}$ is a *Euclidean measure* if the following hold:

- Sum Principle: $\mathfrak{s}(A \cup B) = \mathfrak{s}(A) + \mathfrak{s}(B)$ whenever $A \cap B = \emptyset$.
- Product Principle: $\mathfrak{s}(A \times B) = \mathfrak{s}(A) \cdot \mathfrak{s}(B)$.
- Unit Principle: $\mathfrak{s}(\{a\}) = 1$ for all singletons $\{a\}$.

The classical five *common notions* from Euclid's *Elements* are satisfied by any Euclidean measure. In particular—contrarily to Cantorian cardinality—the ancient principle that the *whole is greater than the part* is maintained. An Euclidean measure coincides with cardinality on all and only finite sets. We say that an Euclidean measure is *Half-Cantorian* if $\mathfrak{s}(X) = \mathfrak{s}(Y) \Rightarrow |X| = |Y|$. We say that $\mathfrak{s}$ is *Normal* when the following holds: If $\langle X_\alpha \mid \alpha < \kappa \rangle$ and $\langle Y_\alpha \mid \alpha < \kappa \rangle$ are *normal approximations* of the uncountable sets X and Y respectively, and $\mathfrak{s}(X_\alpha) \leq \mathfrak{s}(Y_\alpha)$ for all $\alpha < \kappa$, then $\mathfrak{s}(X) \leq \mathfrak{s}(Y)$. A *normal approximation* of a set X is an increasing sequence $\langle X_\alpha \mid \alpha < \kappa \rangle$ such that $|X_\alpha| < |X|$, $X_\lambda = \bigcup_{\alpha<\lambda} X_\alpha$ for limit $\lambda < \kappa$, and $X = \bigcup_{\alpha<\kappa} X_\alpha$.

THEOREM.
1. *Every universe $\mathbb{V}$ admits Half-Cantorian Euclidean measures $\mathfrak{s}\colon \mathbb{V} \to \mathfrak{N}$ where $\mathfrak{N} = {}^*\mathbb{Z}$ is a set of nonstandard integers.*
2. *Assume $\aleph_\omega = \beth_\omega$. Then there are Normal Euclidean measures $\mathfrak{s}\colon \mathbb{V} \to \mathfrak{N}$ where $\mathbb{V} = V(X)$ is the superstructure over a given set of atoms $|X| < \aleph_\omega$, and $\mathfrak{N} = {}^*\mathbb{Z}$ is a set of nonstandard integers.*

OPEN QUESTIONS. Can one prove in ZFC the existence of normal Euclidean measures? Is the existence of Euclidean measures which are both Half-Cantorian and Normal provable in ZFC? (or even consistent with ZFC?)

This is a joint work with Vieri Benci and Marco Forti.

[1] V. BENCI AND M. DI NASSO, *Numerosities of labelled sets: a new way of counting*, ***Advances in Mathematics***, vol. 173 (2003), pp. 50–67.

[2] V. BENCI, M. DI NASSO, AND M. FORTI, *An Aristotelian notion of size*, ***Annals of Pure and Applied Logic***, vol. 143 (2006), pp. 43–53.

► LEONA F. FASS, *Logical considerations in real world Web development*.
P.O. Box 2914, Carmel, CA 93921, USA.
E-mail: lff4@cornell.edu.

We describe some logic-based and real-world constraints affecting development of the Web, the Semantic Web and Web-based e-services. These include Web completeness issues related to sufficiency of information, and processes for realizing users' requirements and fulfilling their specified tasks. There are also termination issues related to searches, and decisions about whether accessed information and processes are the best that might be found. While we have a theoretician's perspective on such Web-related problems, interactions with actual Web developers have shown us what is practicable. Furthermore, observations of everyday users have shown us that real-world environments exhibit problems neither encountered nor resolved within "theory".

Acknowledging significant constraints, we describe instances we believe demonstrate relative success and others demonstrating relatively resolvable problems. Some of the real-world Web-related examples we observe and analyze include "verification" of e-Government services; utilization of "trusted" processes by overly-trusting users; and inconsistency, security and privacy problems resulting from attempts at information integration when user classes are, necessarily, disjoint. Based on our observations we identify approaches that (Semantic) Web researchers might take to resolve problems and features that might maximize success.

[1] L. F. FASS, *Ubiquity and common sense for the Web*, ***Proceedings of the Joint Conference on Information Sciences/International Symposium on Web Intelligence and Security***, Salt Lake City UT, July 2005, pp. 1433–1436.

[2] ———, *A logical look at agents' problem-solving on the Semantic Web*, Short Paper presented at ***21st IEEE Symposium on Logic in Computer Science, 4th Federated Logic Conference*** (***FLoC 2006***), Seattle WA, August 2006, abstract at http://www.easychair.org/FLoC-06/LICS-day226.html.

[3] ———, *Remembering the real world when developing the Semantic Web*, invited for participation in the ***International Workshop on Applications of Logic Programming in the Semantic Web and Semantic Web Services, FLoC 2006***, Seattle WA, August 2006.

► ELENA NOGINA, *Epistemic completeness of GLA*.
BMCC CUNY, Dept. of Mathematics, 199 Chambers Street, New York, NY 10007, USA.
E-mail: ENogina@bmcc.cuny.edu.

The logic of proofs and provability GLA [6] is an arithmetically complete system that combines the provability logic GL and Artemov's Logic of Proofs LP (cf. [2]) in their joint language. GLA has become a prototype of epistemic logics with justification [2, 3, 4]. In addition to axioms and rules of LP and GL, GLA has axioms $t : \varphi \to \Box\varphi$, $\neg t : \varphi \to \Box\neg t : \varphi$, $t : \Box\varphi \to \varphi$, and a rule $\vdash \Box\varphi \Rightarrow \vdash \varphi$.

In this paper we establish the completeness of GLA with respect to epistemic semantics. An epistemic model for GLA is a Fitting – Artemov model (cf. Def. 2 of [1]) with

- a reverse well-founded transitive frame $(W, \prec)$
- an equivalence relation $\equiv$ on W as an evidence accessibility relation such that $\prec \subseteq \equiv$
- a possible evidence function ([1, 5]) $\mathcal{E}$ satisfying the stability property:

$$\mathcal{E}(u, t) = \mathcal{E}(v, t) \text{ whenever } u \equiv v.$$

[1] S Artemov, *Justified common knowledge*, ***Theoretical Computer Science***, vol. 357 (2006), pp. 4–22.

[2] ———, *Modal logic in mathematics*, (Blackburn, van Benthem, and Wolter, editors), ***Handbook of Modal Logic***, pp. 931–973, Elsevier, 2006.

[3] S. Artemov and E. Nogina, *On epistemic logic with justification*, (R. van der Meyden, editor), Theoretical Aspects of Rationality and Knowledge, ***Proceedings of the Tenth Conference (TARK 2005), June 10–12, 2005, Singapore***, pp. 279–294, National University of Singapore, 2005.

[4] ———, *Introducing justification into epistemic logic*, ***Journal of Logic and Computation***, vol. 15 (2005), pp. 1059–1073.

[5] M. Fitting, *The logic of proofs, semantically*, ***Annals of Pure and Applied Logic***, vol. 132 (2005), pp. 1–25.

[6] E. Nogina, *On logic of proofs and provability*, ***The Bulletin of Symbolic Logic***, vol. 12 (2006), p. 356.

► CYRUS F. NOURANI, *Functorial models and positive realizability*.
USA Academia California, PO Box 278, Cardiff by The Sea, CA 92007, USA.
E-mail: cyrusfn@alum.mit.edu.

Based on the author 1996 functorial model theory, ASL SF 1996, a functor $F : L\omega_{1,K} \to$ *Set* can be defined by sets F_i, where F_i's are defining a free structure on some subfragment of $L_{\omega,1,F}$.

Theorem 1. *Applying F_i models on the fragment as an ω-chain from the elementary diagrams on the $Th(A(F_i))$ a generic model is defined by the limit.*

Definition. Let $\Sigma(x_1 \dots x_n)$ be a set of formulas of L. Say that a positive theory T in L positively locally realize Σ iff there is a formula φ $(x_1 \dots x_n)$ in L s.t.

(i) φ is consistent with T
(ii) for all $\sigma \varepsilon \Sigma$, $T \models \varphi$ or $T \cup \sigma$ is not consistent.

Theorem 2. *Let $L1, L2$ be two positive languages. Let $L = L1 \cap L2$. Suppose T is a complete theory in L and $T1 \supset T$, $T2 \supset T$ are consistent in $L1$, $L2$, respectively. Then*

(i) *$T1 \cup T2$ has a model M, that is a positive end extension on Models $M1$ and $M2$ for $T1$, and $T2$, respectively,*
(ii) *M is definable from a positive diagram in the language $L1 \cup L2$.*

Proposition 1 (positive morphic extensions). The following are equivalent:

(i) Every positive sentence holding on $\mathfrak{R}$ also holds on $\mathfrak{Y}$.
(ii) There are elementary extensions $\mathfrak{R} < \mathfrak{R}'$, $\mathfrak{Y} < \mathfrak{Y}'$ such that $\mathfrak{Y}$ is a homomorphic image of $\mathfrak{R}'$.

Let $L_{P,\omega}$ be the positive fragment category based on the positive Keisler fragment.

THEOREM 3. *Based on positive language fragments only there are iterated elementary extensions $\mathfrak{Y}_{i+1} > \mathfrak{Y}_i$ and an embedding $f_i : \mathfrak{R}_i < \mathfrak{R}i$ on positive fragments s.t. a slalom between the infiinte language pairs L_i, L_{i+1} reaches a limit model for L, a limit model M for $L_{P,\omega}$.*

► GONG QIRONG, *Orthodox mathematical logic is not a reasoning theory*.
North region of Guizhou University, Guiyang huaxi 550025, P. R. China.
E-mail: gongmuwen@163.com.

The real meaning of the material implication of the truth connectives in orthodox first-order predicate calculus system ***F*** which is the foundation of orthodox mathematical logic is that it is not the case in which the implicans is true whereas the implicate is false. The truth value of the implication proposition "**A→B**" is the truth function of the truth values of the implicans and the implicate. But the truth value of the sufficient condition proposition "if **A**, then **B**" in ordinary logical thinking process of human beings as is described by classical formal logic is not the truth function of the truth values of the implicans and the implicate. Semantically, material implication is utterly different from the "if ... then ... " which expresses the sufficient condition relation. However, in orthodox mathematical logic material implication is usually understood as "if ... then ... ", and the implication relation regarded as an abstraction of the sufficient condition relation. Hence

(1) In orthodox first-order predicate calculus, the correct reasoning pattern, which is induced by traditional formal logic and which allows new information to be acquired from the known, is changed into the implication tautology of repetitive language. However, an implication tautology can only be one of the following three types: ① the implicans is always false; ② the implicate is always true; ③ whether the implicans is true or not is based on the truth of the implicate. Nevertheless none of the three types of implication tautology is the correct reasoning from by which new information can be obtained from the known.

(2) Material implication, which is understood as "if ... then ... " but is actually different in meaning from "if ... then ... ", has given rise to innumerable implication paradoxes in ***F***. Of course, implication paradoxes are not reasoning forms either. It is very clear that material implication, which is the essence of first-order predicate calculus, is not an instrument of logical reasoning.

In the system ***F***, the universal quantifier $\forall_{\mathbf{x}}$ which ranges only over individual variables means "for every **x**". For the domain of finite individuals, to verify a proposition with a universal quantifier $\forall_{\mathbf{xA(x)}}$ amounts to verifying an endless sequence of singular propsitions one by one. That is apparently impracticable and nobody has ever done so. The system ***F*** which involves quantifiers cannot do the reasoning to acquire new information from the known. Even when it operates as a particular discrete mathematics, the involvement of quantifiers plunges ***F*** into many unavoidable troubles during calculation.

In a word, orthodox first-order predicate logic is not a correct reasoning theory.

► XUNWEI ZHOU, *Definitions of sufficient condition in mutually-inversistic logic*.
Institute of Information Technology, Beijing Union University, 97 Beisihuandong Road, Beijing, China.
E-mail: zhouxunwei@263.net.

In mutually-inversistic logic, the definition of sufficient condition is divided into decision definition and property definition. The decision definition is as follows: suppose A is true, if we can infer that B is true, then we establish that A is a sufficient condition of B. The property definition is as follows: suppose A is a sufficient condition of B, if A is true, then we can infer that B is true. The two definitions are different from each other. The decision definition is from inference to sufficient condition. The property definition is from sufficient condition to inference. The decision definition resembles the decision theorem in Euclidean geometry. The property definition resebles the property theorem in Euclidean geometry.