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2006–07 WINTER MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

New Orleans Marriott and Sheraton New Orleans
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January 7–8, 2007

A Winter Meeting of the Association of Symbolic Logic was held January 7–8, 2007 in New Orleans at the New Orleans Marriot and Sheraton New Orleans Hotels in conjunction with the annual Joint Mathematics Meetings. The Program Committee consisted of Timothy Carlson, Charles Delzell, and Marcia Groszek (Chair). The program included seven invited 50-minute talks and ten contributed talks. The ASL hosted a reception on the evening of January 7th in the Sheraton Hotel. On January 5th, prior to the ASL meeting, there was a joint AMS-ASL Special Session, **Logical Methods in Computational Mathematics**, organized by Saugata Basu and Charles Delzell. On January 6th, also prior to the ASL meeting, an AMS-ASL-MAA Panel Discussion on **Contemporary Perspectives on Hilbert's Second Problem and the Gödel Incompleteness Theorems** was held, with Harvey Friedman, David E. Marker, and Michael Rathjen as the panelists and Akihiro Kanamori moderating.

The 50-minute invited addresses were:

Matthew Foreman (University of California at Irvine), *Classifying measure preserving transformations*.

Su Gao, (University of North Texas), *Countable group actions and hyperfinite equivalence relations*.

Valentina Harizanov (George Washington University), *Back and forth through computable model theory*.

Ulrich Kohlenbach (Darmstadt University of Technology), *Recent uses of proof theory in nonlinear analysis and geodesic geometry*.

Michael Rathjen (Ohio State University), *From Hilbert's programme to ordinal analysis*.

Reed Solomon (University of Connecticut), *Almost everywhere domination*.

Carol Wood (Wesleyan University), *Reducts of omega-categorical theories*.

Abstracts of the invited talks and contributed talks given (in person or by title) by members of the Association for Symbolic Logic follow.

For the Program Committee
MARCIA GROSZEK

Abstracts of invited talks

► MATTHEW FOREMAN, *Classifying measure preserving transformations*.
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1079-8986/07/1303-0004/$2.10

Many dynamical systems carry a time-invariant probability measure. These measures can be used to describe the statistical information of the system qualitatively. For example they can be "completely random" (Bernoulli) or "completely ordered" (0-entropy). Classifying the possible ergodic probability measures that arise this way is a problem dating back to the 1950's and before, involving work of Halmos, von Neumann, Ornstein, Furstenberg, Zimmer and others.

The classification problem can be put into the context of Polish Group actions, where it turns out to have very high complexity even in very concrete cases.

► SU GAO, *Countable group actions and hyperfinite equivalence relations*.
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I will give a survey of progress made on the countable union problem for hyperfinite equivalence relations. The main focus will be the theorem (which is a special case of the countable union problem) that every countable abelian group action induces a hyperfinite equivalence relation. Behind the proof of this theorem is an invariant geometric group theory. I will present other related results in this theory and some applications of it, for instance to a Borel coloring problem.

This is joint work with Steve Jackson.

► VALENTINA S. HARIZANOV, *Back and forth through computable model theory*.
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Interaction of computability theory with model theory and other areas of mathematics has resulted in computable model theory. We use computability theoretic tools and methods to study various algorithmic phenomena on countable mathematical structures. Hence computable model theory also aims to better understand how algebraic properties of structures interact with the algorithmic ones. We will survey some earlier results and present recent developments in our investigation of the computability theoretic complexity of structures and their isomorphic copies, complexity of additional relations on structures, complexity of automorphisms, and the correspondence between computability and definability. For example, we will show how Turing degree spectra of relations relate to the degree spectra of structures via the so-called spectrally universal structures. We will present results in general model theoretic setting, as well as results about concrete well-known classes of algebraic structures.

[1] W. Calvert, V. Harizanov, J. Knight, and S. Miller, *Index sets of computable structures*, ***Algebra and Logic***, to appear.

[2] J. Chisholm, J. Chubb, V. Harizanov, D. Hirschfeldt, C. Jockusch, T. McNicholl, and S. Pingrey, Π^0_1 *classes and strong degree spectra of relations*, submitted.

[3] B. Csima, V. Harizanov, D. Hirschfeldt, and R. Soare, *Bounding homogeneous models*, ***The Journal of Symbolic Logic***, vol. 71 (2007), pp. 305–323.

[4] B. Csima, V. Harizanov, R. Miller, and A. Montalbán, *Computability of Fraïssé limits*, in preparation.

[5] M. Dabkowska, M. Dabkowski, V. Harizanov, and A. Togha, *Spaces of orders and their Turing degree spectra*, in preparation.

[6] V. Harizanov, C. Jockusch, and J. Knight, *Chains and antichains in computable partial orderings*, in preparation.

[7] V. Harizanov and R. Miller, *Spectra of structures and relations*, ***The Journal of Symbolic Logic***, vol. 71 (2007), pp. 324–348.

[8] V. Harizanov, R. Miller, and A. Morozov, *Automorphism degree spectra*, in preparation.

► ULRICH KOHLENBACH, *Recent uses of proof theory in nonlinear analysis and geodesic geometry*.
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In recent years proof-theoretic metatheorems have been established which provide general methods for the extractability of effective strongly uniform bounds from large classes of proofs in functional analysis and geodesic geometry. 'Strongly uniform' here refers to the fact that the bounds are independent from parameters in abstract classes of spaces as long as some local bounds on certain metric distances are given. In addition to this, in certain contexts the metatheorems make it possible to generalize proofs by eliminating strong existence assumptions made in the results to be proved which might not be satisfied in general and even where they are often require complicated functional analytic proofs (sometimes involving ultrapowers). In particular, we will show the soundness of a general principle for such generalizations for a large class of statements. The classes of structures covered include metric, hyperbolic (in the sense of Kirk and Reich), δ-hyperbolic (in the sense of Gromov), CAT(0), normed, uniformly convex spaces as well as $\mathbb{R}$-trees (in the sense of Tits). The extraction method uses functional interpretation based on a novel concept of majorizability relative to suitably chosen reference points in the spaces at hand. During the last couple of years this approach has led to many new results in nonlinear analysis and geodesic geometry and also appears to be promising in other areas such as ergodic theory or geometric group theory.

In this talk we will give a general survey of some of these developments.

► MICHAEL RATHJEN, *From Hilbert's programme to ordinal analysis*.
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Ordinal analysis of theories is a core area of proof theory. The origins of proof theory can be traced back to the second problem on Hilbert's famous list of problems. Proof theory was invented as the main tool for carrying out Hilbert's programme. In the main, ordinal-theoretic proof theory came into existence in 1936, springing forth from Gentzen's consistency proof of arithmetic. The intent of the talk is to elucidate the underlying notions and the rationale of ordinal-theoretic proof theory by relating the developments from Gentzen up to recent advances in ordinal analysis of strong theories.

► REED SOLOMON, *Almost everywhere domination*.
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A set $A \in 2^\omega$ is *almost everywhere* (*a.e.*) *dominating* if for almost every $X \in 2^\omega$ it is the case that for every function g which is computable from X, there is a function f computable from A such that f dominates g. Recently, Dobrinen and Simpson used this concept (as well as a uniform version called *u.a.e. dominating*) in a study of the regularity properties of the Lebesgue measure on 2^ω in reverse mathematics. This talk will be a survey of the work done over the last two years by numerous people on computability theory and reverse mathematics aspects of the notions of a.e. domination and u.a.e. domination.

► CAROL WOOD, *Reducts of omega-categorical theories*.
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In 1991 Simon Thomas proved there are only finitely many reducts of the theory of random graphs, and described all of them. This work was extended to other classes of structures by Thomas, and by his student James Bennett in the mid-1990's. We will describe their combinatorial approach to identifying all reducts, as well as Yun Lu's recent applications of these methods to bipartite graphs with various cross-types.

Abstracts of contributed talks

► KATALIN BIMBÓ, *On the failure of Craig interpolation in dynamic logics*.
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By 'Craig interpolation' we mean that if a formula $A \to B$ is provable (where A and B share some nonlogical symbol), then there is a wff C which contains a nonlogical symbol only if that occurs both in A and B, and the two wffs $A \to C$ and $C \to B$ are provable too (cf. [2, Thm. 5]). The problem if dynamic logic (see e.g., [4, Ch. 10]) has this property attracted considerable interest in the past three decades, with several positive conjectures published recently.

Urquhart proved (see [5]) the failure of amalgamation for the variety of distributive lattices with a binary additive operation with identity, as well as for some other classes of algebras. In this talk, amalgamation is proven to fail for the variety of dynamic algebras (that is defined by the Lindenbaum algebra of propositional dynamic logic). First, * is added to the $\mathfrak{R}$ algebras from [5]—using the idea that the Kleene star is inductively definable from $;$, **1'** and $+$ in a relation algebra. (A similar idea was utilized in one of the semantics given for Pratt's action logic in [1].) Then dynamic algebras are defined from relation algebras with transitive closure. The failure of amalgamation implies the failure of Craig interpolation. The result is unaffected by the most common variations on dynamic logic, for instance, when converse is added.

[1] Katalin Bimbó and J. Michael Dunn, *Relational semantics for Kleene logic and action logic*, ***Notre Dame Journal of Formal Logic***, vol. 46 (2005), pp. 461–490.

[2] William Craig, *Linear reasoning. A new form of the Herbrand-Gentzen theorem*, ***The Journal of Symbolic Logic***, vol. 22 (1957), pp. 250–268.

[3] J. Michael Dunn, *A representation of relation algebras using Routley-Meyer frames*, ***Logic, meaning and computation. Essays in memory of Alonzo Church*** (C. A. Anderson and M. Zelëny, editors), Kluwer, Dordrecht, 2001, pp. 77–108.

[4] Robert Goldblatt, ***Logics of time and computation***, 2nd ed., CSLI Lecture Notes, vol. 7, CSLI, Stanford, CA, 1992.

[5] Alasdair Urquhart, *Failure of interpolation in relevant logics*, ***Journal of Philosophical Logic***, vol. 22 (1993), pp. 449–479.

► DENNIS F. CUDIA, *The Boltzmann principle and protein primary structure*.
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The headings U, C, A, G, I (inosine) are the middle bases of the anticodons of the twenty amino acids. The integers are W(aa) = ad(aa), the anticodon degeneracy of the amino acid aa for each middle base of an anticodon for aa.

U		C	A	G	I
Lys 3	Tyr 3	Arg 13	Ile 4	Thr 5	Thr 5
Asn 3	Gln 6	Ser 3	Met 2	Ala 5	Ala 5
Glu 3	His 6	Gly 5	Val 5	Ser 5	Ser 5
Asp 3		Trp 2	Leu 13	Pro 10	Pro 10
		Cys 3	Phe 3		

UUI, CUI, GUI, IUI, UCI, UAU, AAI are anticodons for exactly two different amino acids. AUU and AUC are not anticodons for any amino acid. The remaining 116 triples are anticodons for exactly one amino acid. Using the Boltzmann Principle the configurational entropy for any polypeptide $\mathrm{p} = \mathrm{aa}_1 \cdot \cdots \cdot \mathrm{aa}_n$ is $\mathrm{S(p)} = \log_2 \mathrm{W(p)} = \Sigma \log_2 \mathrm{ad}(\mathrm{aa}_1)$.

The configurational entropy for TMV capsomere, whale myoglobin, mouse NGF, pancreatic ribonuclease, chymotrypsinogen, light chain EU, variable part heavy chain EU, constant part heavy chain EU, human proinsulin, human insulin, and p53 is given.

[1] J. D. WATSON, ***Molecular biology of the gene***, 3rd ed., Benjamin, New York, 1977.

[2] F. H. C. CRICK, *Codon-anticodon pairing: the wobble hypothesis*, ***Journal of Molecular Biology***, vol. 19 (1966), pp. 548–555.

► FRANÇOIS G. DORAIS, *Effective Souslin trees and degrees in α-recursion theory*.
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Souslin trees are a staple of the set theorist's toolkit, but their applicability to higher recursion theory has been largely unexplored. We will give an effective notion of Souslin trees which is suitable for use in α-recursion theory. These effective Souslin trees can be used to construct minimal α-degrees which provide an alternative proof of a result of Shore [1]. The same method can be used to realize various lattices as initial segments of the upper semi-lattice of α-degrees.

[1] RICHARD A. SHORE, *Minimal α-degrees*, ***Annals of Pure and Applied Logic***, vol. 4 (1972), pp. 393–414.

► STEPHEN HARNISH, *Hypersets with a universal set—two axiomatizations for Bi-AFA$^\sim$ set theories*.
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There are no universal sets in the $(\mathrm{ZFC}^- + \mathrm{AFA}^\sim)$ set theories as presented in Aczel's *Non-well-founded sets* [1] and [2]. Bi-AFA$^\sim$ set theories, however, do contain a universal set as well as inner models of $(\mathrm{ZFC}^- + \mathrm{AFA}^\sim)$. To avoid standard antinomies, these bi-AFA$^\sim$ theories restrict the sets to which separation, replacement, powerset and choice apply. They also replace the AFA$^\sim$ axiom by a bi-AFA$^\sim$ axiom which allows canonical pictures of sets with two kinds of nodes—those whose children are exactly the *elements* of the node, and those whose children are the *non-elements* of the node. Hence, the bi-AFA$^\sim$ canonical picture of the universal set has one node of the second kind and no edges. This paper presents two axiomatizations for the bi-AFA$^\sim$ set theories.

[1] P. ACZEL, ***Non-well-founded sets***, Lecture Notes, no. 14. Center for the Study of Language and Information, Stanford, 1988.

[2] T. E. FORSTER, ***Set theory with a universal set***, Oxford University Press, New York, 1995.

► ROSALIE IEMHOFF AND GEORGE METCALFE, *Proof theory for admissible rules*.
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Admissible rules of a logic are rules under which the set of theorems of the logic are closed. Algebraically, investigating admissible rules corresponds to the study of quasi-varieties generated by free algebras, while from a computational perspective, the interest lies with the fact that extending a system with admissible rules may improve proof search. In Classical logic, all admissible rules are also derivable, but for Intuitionistic logic and many other non-classical logics this is no longer the case. In recent years, one successful approach to characterizing such rules has been via bases, which may be viewed as providing axiomatizations for the admissible rules of a logic [3, 4, 5]. In this work, we introduce an alternative characterization: a uniform framework for defining Gentzen-style proof systems for admissible rules. Just as Gentzen systems for derivability manipulate sequents, so Gentzen systems for admissibility manipulate sequent rules. We define proof systems for the admissible rules of a large class of modal logics including K4, S4, and GL. We also treat intermediate logics, including Intuitionistic logic and De Morgan (Jankov) logic. In both cases we make use of results of Ghilardi [1, 2] on unification and projective approximations. With minor restrictions, proof search in these systems terminates, giving decision procedures for admissibility in the corresponding logics.

[1] S. Ghilardi, *Unification in intuitionistic logic*, ***The Journal of Symbolic Logic***, vol. 64 (1999), no. 2, pp. 859–880.

[2] ———, *Best solving modal equations*, ***Annals of Pure and Applied Logic***, vol. 102 (2000), no. 3, pp. 184–198.

[3] R. Iemhoff, *On the admissible rules of intuitionistic logic*, ***The Journal of Symbolic Logic***, vol. 66 (2001), no. 1, pp. 281–294.

[4] E. Jerabek, *Admissible rules of modal logics*, ***Journal of Logic and Computation***, vol. 15 (2005), pp. 411–431.

[5] V. Rybakov, ***Admissibility of logical inference rules***, Elsevier, 1997.

► JOE MILETI, *The strength of the Rainbow Ramsey Theorem*.
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The Rainbow Ramsey Theorem is essentially an "anti-Ramsey" theorem which roughly states that certain colorings always have a large chaotic section. Surprisingly, this version follows easily from Ramsey's Theorem, even in the sense of reverse mathematics. We answer the question of the converse implication for pairs, showing that the Rainbow Ramsey Theorem for pairs is in fact strictly weaker and sits in a very interesting position in the reverse mathematical hierarchy below Ramsey's Theorem for pairs. The separation involves techniques from the theory of randomness, and also provides as a corollary a new proof of an old theorem of Martin on the measure of the set of hyperimmune degrees.

This is joint work with Barbara Csima.

► SARA MILLER, *Turing computable embeddings into equivalence structures*.
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The notion of *Turing computable embeddings* was explored in [1], and a main tool of that paper is the "Pull-back Theorem," which states that, if a class K Turing computably embeds into a class K', then computable infinitary sentences acting as invariants for the structures of K' can be pulled back to computable infinitary sentences of the same complexity, acting as invariants for the structures of K. The Pull-back Theorem can be used to characterize the classes that Turing computably embed into mathematically interesting classes of structures, and this is done in [1] for finite prime fields, finite linear orderings, and $\mathbb{Q}$-vector spaces, as well as for a particular class of equivalence structures. In this talk I will give a new result characterizing the classes that Turing computably embed into the class of arbitrary equivalence structures.

[1] J. F. Knight, S. Miller, and M. Vanden Boom, *Turing computable embeddings*, ***The Journal of Symbolic Logic***, vol. 72 (2007), pp. 901–918.

► REHANA PATEL, *Classification of a family of countably universal H-free graphs*.
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Given a fixed graph H, a graph G is said to be *H-free* if H does not embed into G as a subgraph, induced or otherwise. In [1], Cherlin, Shelah and Shi show the existence of a universal countable H-free graph when $H = K_l \dot{+} K_3$, $l \geq 3$. (The case $l = 3$ was first proven by Komjath.) In this talk, we will locate the theories of these graphs within Shelah's SOP_n hierarchy.

[1] G. Cherlin, S. Shelah, and N. Shi, *Universal graphs with forbidden subgraphs and algebraic closure*, ***Advances in Applied Mathematics***, vol. 22 (1999), pp. 454–491.

► R. GREGORY TAYLOR, *Symmetric propositions and logical quantifiers*.
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Symmetric propositions over a fixed domain and signature are characterized following Zermelo, and a correlation of such propositions with logical quantifiers is described. Any logical quantifier is expressible as the join of a set of categorical quantifiers.

► DAN E. WILLARD, *On computer robots recognizing their own geometric self-consistency*.
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This talk will show how a very recent paper in the December 2006 issue of the JSL [2] offers a refreshing new geometric-based perspective on Hilbert's Second Problem. Let $\text{Add}(x, y, z)$ and $\text{Mult}(x, y, z)$ be predicates indicating $x + y = z$ and $x * y = z$. A system will be said to *recognize* addition and multiplication as *Total Functions* iff it contains the axioms $\forall x \, \forall y \, \exists z \; \text{Add}(x, y, z)$ and $\forall x \, \forall y \, \exists z \; \text{Mult}(x, y, z)$. Our year-2002 article [1] *roughly showed* that no consistent system containing these two axioms *about integer arithmetic* can prove a theorem verifying its own semantic tableaux consistency.

In contrast, [2] proved the same *is not true* under a computer's floating point arithmetic. Thus, let x and y be ordered pairs, (M_x, E_x) and (M_y, E_y), whose two components represent the mantissas and exponents of floating point real numbers, and L_i denote the bit-length of the mantissa M_i. Let the term *simulated real arithmetic* refer to addition and multiplication among simulated real numbers where the resulting mantissas, *after rounding is done*, have lengths equal to $1 + \text{Max}(L_x, L_y)$. Our paper [2] showed:

A. It is surprisingly possible to develop axiom systems that recognize their own semantic tableaux consistency and the simulated arithmetic operations to be total functions.
B. In contrast, the above evasion of the Second Incompleteness Theorem will *collapse entirely* if our axiom systems view as "total" the operation of "LongMultiplication"— which differs from "Simulated Multiplication" by having its *unrounded* resulting mantissa M_i have a bit-length of $L_x + L_y$.

While (A) is clearly *only a limited evasion* of Gödel's second theorem, it is philosophically fascinating because human beings, *when viewed as computerized robots*, do not know the precise number of atoms, precise geometric dimensions or precise weights and velocity of the objects that they lift and move in their geometric environments. Instead, these quantities are treated as simulated real numbers.

[1] D. Willard, *How to extend the semantic tableaux and cut-free versions of the second incompleteness theorem almost to Robinson's arithmetic Q*, ***The Journal of Symbolic Logic***, vol. 67 (2002), pp. 465–496.

[2] ———, *On the available partial respects in which an axiomatization for real valued arithmetic can recognize its consistency*, ***The Journal of Symbolic Logic*** vol. 71 (2006), pp. 1189–1199. (See also Springer LNCS, vol. 3702 (2005), pp. 292–306.)

Abstracts presented by title

► ADIB BEN JEBARA, *Physical space is infinite*.
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There could be an infinite physical space of which we know only a finite part. So far, mathematics have been applied successfully to physics, remain, among other theories, the actual infinite of set theory. We already assumed that time could be infinite in "An idea about time in cosmology" in ASL Annual Meeting 2004. We also assumed an infinite number of locations in "About space and time of elementary particles" in ASL Annual Meeting 2005.

We assumed that physical space is Dedekind infinite, which is some model to explain things such as the movements of particles. The energy-matter does not have to be infinite and because of the Big Bang, I can see no way it can be, but what is around energy-matter could be infinite. I think we went from no space in the Big Bang to an infinite physical space.

As we find further and further part of the physical space, we can assume reasonably that we will go on doing so. In such a case, only the brain could see the whole physical space and neither the eyes nor devices. Space can exist without matter, as empty space can be obtained in laboratories. Does that really fit the physical space to use a mathematical infinite universe?

A finite physical space would have a boundary with no space and no space would be surrounding space. But in "About space being not a continuum" in Logic Colloquium 2005, we have seen that no space (the void set) can be reached only in an infinite number of steps, at the level of the elementary particles. Thus, physical space is infinite at the level of the elementary particles and therefore at our level. Except for a finite part, physical space is completely empty.

► PAOLA CATTABRIGA, *Observations concerning Gödel's 1931*.
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This paper demonstrates the invalidity of Theorem VI in Gödel's monograph of 1931 [1], by showing that

$$\overline{xB_\kappa(17\text{ Gen }r)} \longrightarrow \mathrm{Bew}_\kappa[Sb(r^{17}_{Z(x)})], \tag{15}$$

$$xB_\kappa(17\text{ Gen }r) \longrightarrow \mathrm{Bew}_\kappa[\mathrm{Neg}(Sb(r^{17}_{Z(x)})], \tag{16}$$

(derived by means of definition (8.1) $Q(x,y) \equiv \overline{xB_\kappa[Sb(y^{19}_{Z(y)})]}$) are false in P. This is achieved in two steps. First, the predicate complementary to the well-known Gödel's predicate $\mathrm{Bew}(x)$ is defined by adding a new relation $\mathrm{Wid}(x)$. In accordance, new logical connections are established, Lemma (6), $\overline{\mathrm{Bew}_\kappa(y)}$ iff $\overline{xB_\kappa y}$ and $\overline{\mathrm{Wid}_\kappa(y)}$ iff $xB_\kappa y$. Second,

$$\overline{\overline{xB_k(17\text{ Gen }r)} \to \mathrm{Bew}_k[Sb(r^{17}_{Z(x)})]}, \tag{I}$$

$$\overline{xB_\kappa(17\text{ Gen }r) \to \mathrm{Bew}_k[\mathrm{Neg}(Sb(r^{17}_{Z(x)})]}, \tag{II}$$

are derived from Lemma (6) and definition (8.1). It amounts to saying that (15) and (16) are false and unacceptable for the system (P). On the account of that, the two cases "1. 17 Gen r is not κ-PROVABLE." and "2. Neg(17 Gen r) is not κ-PROVABLE." can not be drawn (unless we say that they are true since ((15) & (I)) $\supset$ 1. and ((16) & (II)) $\supset$ 2. are examples of $(p\ \&\ \sim p) \supset q$).

[1] KURT GÖDEL, *On formally undecidable proposition of Principia mathematica and related systems I.* (1931), ***Collected Works***, vol. I, Oxford University Press, New York, 1986, pp. 144–195.

► JOHN CORCORAN, *Aristotle, Boole, and Tarski*.
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This paper is a series of notes focusing on certain of the logical achievements of Aristotle, Boole and Tarski. Aristotle founded logic as organon—as "formal epistemology". He was the first to systematically attempt a theory of demonstrative proof and the first to develop criteria of validity and invalidity of premise–conclusion arguments. He was the first to treat formal deduction and the first to treat independence proofs. Boole founded logic as science—as "formal ontology". He was the first to explicitly recognize the role of tautologies in deduction and to attempt a systematic treatment of "laws of thought"—his expression which was later used in essentially the same sense by Tarski. Tarski founded metalogic—the science explicitly conducted in the metalanguage and focusing among other things on syntax and semantics of idealized languages of sciences including logic. Much of Tarski's theory of metalogic, or "formal methodology", appeared in the 1933 truth-definition monograph, the 1936 consequence-definition paper, and the 1986 logical-notion-definition paper. The notes presented here using concepts introduced or formalized by Tarski contribute toward two main goals: to compare Aristotle's system with the one Boole constructed intending to broaden and to justify Aristotle's, and to give a modern perspective to both logics. Choice of these three logicians has other advantages. In the history of logic, Aristotle is the best representative of the earliest period, Boole the best of the transitional period, and Tarski the best of the most recent period. In the philosophy of logic, all three were amazingly successful in having their ideas incorporated into mainstream logical theory. This paper continues the thinking in my 1989 lecture *Logical methodology: Aristotle and Tarski*, ***The Journal of Symbolic Logic***, vol. 57 (1992), p. 374.

► DAVID FERNÁNDEZ, *Dynamic topological completeness for* $\mathbb{R}^2$.
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Dynamic topological logic (**DTL**) combines topological and temporal modalities to express asymptotic properties of dynamic systems on topological spaces [1]. A dynamic topological model is a triple $\langle X, f, V\rangle$, where X is a topological space, $f : X \to X$ a continuous function and V a truth valuation. Valid formulas are those that are true in every model, independently of X or f. A natural question that arises is what logics are obtained on familiar spaces, such as $\mathbb{R}^n$. In [2] it was shown that any satisfiable formula of **S4C**, the fragment which uses only the topological operator and "next", can be satisfied in some $\mathbb{R}^n$ for n large enough, but the question of how the logic varies with n remained open.

S4C has the property that any satisfiable formula can be satisfied in a model based on an Aleksandrov space. In fact, there are larger fragments of **DTL** that share this property, such as $\mathbf{DTL_1}$, where "henceforth" may not appear in the scope of a topological operator. Here we show that satisfiability of any formula of our language in an Aleksandrov space implies satisfiability in $\mathbb{R}^2$ by constructing continuous, open maps from the plane into arbitrary Aleksandrov spaces which give us a type of bisimulation. We conclude that satisfiability in **S4C**, $\mathbf{DTL_1}$ and any other fragment that is complete for Aleksandrov spaces is equivalent to satisfiability in $\mathbb{R}^2$.

[1] P. Kremer and G. Mints, *Dynamic topological logic*, ***The Bulletin of Symbolic Logic***, vol. 3, pp. 371–372.

[2] S. Slavnov, *On completeness of dynamic topological logic*, ***Moscow Mathematical Journal***, vol. 5 (2005), no. 2.

► CYRUS F. NOURANI, *Frege, Hilbert, Kant, and computation: a preface to discerning epistemic illusion*.
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The project is towards a new illusion logic since Morph Gentzen (Nourani 1997) proofs are closer to human experience. In the papers diagrams for cognitive modeling and scientific techniques are applied towards a discovery and consciousness science. With Kant's Transcendental Idealism is the possibility of synthesizing a priori knowledge to possible descriptions. Experience is easily explainable, since each object must necessarily conform to the conditions under which they can become objects for us. Creative pure mathematics as a sythetical cognition a priori, is only possible by referring to no other object than those of the senses. Morph Gentzen (Nourani 1997) comes close to human experience in attaining proofs. At the base of its empirical intuition lies a pure intuition which is a priori. Frege's basic logical ideas and Hilbert's program separate carrying out pure mathematics from the physical cognition perceptions of what is carried out as an end. Frege's "concept and object" and on "sense and meaning," is where carrying out logic for objects named by a language had started being distinguished from the object sense perception. Hilbert's program, aside from its being left to reconcile with transcendental idealism on concepts, were to arithmatize the entire mathematics. We are at the language, model, arithemtatization trichotomy. The objects are described with languages as Frege intended, modeled by structures, which can be examined by Kant's transcendental Idealism, and their computability or reducibility areas Hilbert arithmetized. Hence there is a systematic basis to carryout concept-object descriptions for machine discovery. There are specific theorems proved on illusion logic with Morph Gentzen.

► GONG QIRONG, *Lin's Entailment Logic symbol system can logically represent all knowledge*.
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So far, Lin's Entailment Logic is the only logic that has explicitly described the sufficient condition relations as they actually are. The Lin-entailmental relation is in fact the sufficient condition relation described explicitly. It is not a variation of any implication relation, it is quite different from material implication, strict implication, correlation implication and entailment implication. As a matter of fact, it forms the theoretical core of a reasoning pattern by which people can acquire new information with the help of ordinary logical thinking without falling into circular argument.

Lin's Entailment Logic does not employ quantifiers. It is free from any trouble that is unavoidable in formal systems which involve quantifiers, what is more, it turns out to be even closer to the reality of ordinary logical thinking than if quantifiers are introduced into the system.

Model words such as $\Box$, $\Diamond$, of model logic is a special pure truth value. According to Lin's Entailment Logic, "if **A**, then **B**" is synonymous with "**A** necessarily **B**", "**A** Lin-entails **B**" is exactly "**A** necessarily **B**". Their expression forms are $\mathbf{A} \rightharpoonup \mathbf{B}$. The necessity relation is a binary non-pure truth values connection relation. It has the first independence, and sometimes the second independence. According to this, we can define "**A** probably $\mathbf{B}(\mathbf{A}!\mathbf{B})$", "**A** occasionally $\mathbf{B}(\mathbf{A} \bigcirc \mathbf{B})$", "**A** wind-horse-cow **B**" (i.e., **A** totally occasionally **B**, symbolized as $\mathbf{A} \,\textsc{f}\, \mathbf{B}$) as follows: $\mathbf{A}!\mathbf{B} = \mathrm{df}\neg(\mathbf{A} \rightharpoonup \neg\mathbf{B})$; $\mathbf{A} \bigcirc \mathbf{B} = \mathrm{df}\neg(\mathbf{A} \rightharpoonup \mathbf{B}) \wedge \neg(\mathbf{A} \rightharpoonup \neg\mathbf{B})$; $\mathbf{A} \,\textsc{f}\, \mathbf{B} = \mathrm{df}\, \mathbf{A} \bigcirc \mathbf{B} \wedge \neg\mathbf{A} \bigcirc \mathbf{B}$. !, $\bigcirc$ and $\textsc{f}$ are also binary non-pure truth value connection relations. Description of the connection relation is in accordance with the reality of ordinary logical thinking of human beings.

Lin's Entailment Logic established the prepositional calculus system ***Cm***, the notional calculus system ***Cn*** and the notional calculus system with equivalence terms ***Cnd***. Lin's Entailment Logic got rid of all paradox formula, material implication paradoxes and paradoxes such as $\mathbf{A} \wedge \neg\mathbf{A} \rightarrow \mathbf{B}$ that model system can't avoid; the problem has been solved in a definite and reasonable way.

In a word, compared with logic of all shades, Lin's Entailment Logic has stronger, richer and more accurate expressing ability. So Lin's Entailment Logic symbol system can logically represent all knowledge.

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By introducing a dressed epsilon symbol $\hat{I}+$, an alternative definition of , multisubset is put forward. means that x belongs to y at least n times.