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REVIEWS

The Association for Symbolic Logic publishes analytical reviews of selected books and articles in the field of symbolic logic. The reviews were published in ***The Journal of Symbolic Logic*** from the founding of the JOURNAL in 1936 until the end of 1999. The Association moved the reviews to this BULLETIN, beginning in 2000.

In a review, a reference "JSL XLIII 148," for example, refers either to the publication reviewed on page 148 of volume 43 of the JOURNAL, or to the review itself (which contains full bibliographical information for the reviewed publication). Analogously, a reference "BSL VII 376" refers to the review beginning on page 376 in volume 7 of this BULLETIN, or to the publication there reviewed. "JSL LV 347" refers to one of the reviews or one of the publications reviewed or listed on page 347 of volume 55 of the JOURNAL, with reliance on the context to show which one is meant. The reference "JSL LIII 318(3)" is to the third item on page 318 of volume 53 of the JOURNAL, that is, to van Heijenoort's *Frege and vagueness*, and "JSL LX 684(8)" refers to the eighth item on page 684 of volume 60 of the JOURNAL, that is, to Tarski's *Truth and proof*.

References such as 49***5*** or 280***1*** are to entries so numbered in *A bibliography of symbolic logic* (the JOURNAL, vol. 1, pp. 121–218).

LARRY WOS and GAIL W. PIEPER. ***A fascinating country in the world of computing—your guide to automated reasoning.*** World Scientific, Singapore, New Jersey, London, Hong Kong, 1999, 608 pp.

Larry Wos invented the term Automated Reasoning (henceforth, just AR) for what used to be called "(Mechanical) Theorem Proving." The present volume is a survey of where the area stood just before the turn of the millennium; clearly Wos was its principal author, with valuable assistance from Pieper.

"Tarski used to torment his graduate students with that problem!" So Urquhart reflected (in conversation) about the Robbins problem, disposed of electronically by methods akin to those investigated in this book. "It is the sort of problem," he continued, "for which one *wants* a computer—lots of messy cases, with irksome details to keep track of."

Be that as it may, the success of William McCune (one of the heroes of the book) and his Argonne (and other) colleagues in knocking off Robbins is *not* to be sneezed at. It was, at the time, worth a write-up in the NY Times. Like the vanquishing of former World Chess Champion Kasparov at the hands (or rather the circuits) of the CMU–IBM program Deep Blue, Robbins was one more triumph for the emerging powers of Computerdom against the limited rationality of humankind.

Or was it? In the book in focus, Wos and his co-author Pieper go to considerable lengths to insist that AR programs are *assistants* to human beings, doing mathematics and other respectable human stuff. The program on which the book concentrates is McCune's OTTER,

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the penultimate in a series produced by the (sadly, now closed) Automated Reasoning project at the Argonne National Laboratory in Chicago. We note in passing that McCune's improved AR program Prover9 is available from its author (with copious documentation) at his new website, `mccune@cs.unm.edu`.

The Wos–Pieper book consists of a foreword (by the authors' old Argonne colleague Ross Overbeek), a preface, 12 chapters and an appendix. It is written in the usual chatty, brash and entertaining style of Larry Wos, the modern day Homer of Automated Reasoning. The book is accompanied by a CD, which contains a copy of OTTER and much supplementary material. (In fact there are *three* copies of the program, for Unix machines, PC's and Mac's respectively; there are also input files that both serve as samples and also save readers the laborious task of entering code by hand.)

The underlying metaphor for the book is that of a travel guide—a sort of Lonely Planet survey of the AR nation. Like any such guide, the book is intended for readers of various levels of familiarity with the terrain and interest in exploring it. So one is offered the choice of various routes through Logic, Circuit Design, Mathematics and associated beauty spots, with the aid of Argonne paradigms for AR. There are *no* prerequisites!

In the (skippable) introductory Logic chapter 2, the usual special pleading occurs for the truth-functional analysis of the connectives. Most distressing in the case of *if–then* and its (purported) synonym *implies*. The clause language is introduced with simple examples. By chapter 3 the principal characteristics of the Argonne approach have been discussed. These include an introduction to unification and resolution, with equality handled via demodulation and paramodulation. Here and later Wos is big on *strategies*, many of which he was a key figure in developing.

There is no doubt that *some* strategy is necessary for *any* AR program to achieve its goal. (Otherwise, as that other Argonne pioneer J. Alan Robinson once put it, one could simply load up axioms for number theory and sit back until Fermat's Last Theorem came trundling out. The reader will recall that Andrew Wiles did *not* do it that way! There is not world enough and time, since the combinatorics become too intimidating.) But it is an open question *how much* human intervention is in order to guide a search for proofs. Eyeballing even a fairly simple set of clauses—say with only 50 or 100 symbols—who among us would be so bold as to suggest what to do *next*?

Still, Wos's strategies have often *delivered*. An early one discussed again here was set-of-support. This may be thought of as a timely injection of *relevance*, into an otherwise classical context. Certain clauses are chosen as "having support". It is then laid down that a formal conclusion "has support" if one of its premisses does. It is further laid down that the only conclusions permitted are those that have support.

What, readers may wonder, is the *goal* of an OTTER program? Most often it is to derive a contradiction, after assuming the *negation* of the desired theorem. Such "proofs by contradiction" are a well-known classical technique. They are described in traditional AR parlance as "deriving the empty clause." And the authors do show that their underlying first order logic with equality is (classically) refutation complete, on an appropriate choice of strategies.

But this is *not* the only possible outcome of a proof search. More disturbing and all too frequent are the proof searches which do not terminate. The *undecidability* of classical first order logic assures that no general guarantees rule these out. On the other hand we may make our search for contradictions explicit by putting clauses C on the "passive list". With good management (or simply good luck) we may arrive anyway at success.

OTTER (and its successor Prover9) are written in the highly portable C programming language. This means that (unlike previous Argonne programs) you can have your own copy, joining the AR community in pursuit of your own goals. What might those goals be? The authors make numerous suggestions. You might, for example, be searching for more efficient formulations of a (classical or non-classical) logical system. Or you might be looking for fixed-point combinators (in the sense of Haskell Curry and Raymond Smullyan). You could be doing group theory (or Moufang loop theory). In all of these areas (and many more), the successes of OTTER and its cousins are dwelt upon.

What, however, is an appropriate *philosophy* for the tools explored here? The reviewer is much impressed by that espoused by the authors. They do *not* consider Automated Reasoning AR to be a branch of Artificial Intelligence AI. They see it as a task of AI to "emulate person-oriented reasoning" (p. 441). There is *no reason*, they claim, to see this as among the aims of AR, whose goal is after all to draw logical conclusions as flawlessly and as efficiently as possible.

Put otherwise—it is as silly to take *human* reasoning as our model of good logical reasoning as it is to suppose that the wings of an airplane should flap like a *bird's*. Our interest in airplanes is to use them to get from place to place. Just so, our interest in AR logics is to use them, as *assistants*, to get from premisses to conclusions. As *consumers* of these logics, the details of what is merely *implementation* need not interest us.

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JOHN L. BELL. ***The continuous and the infinitesimal in mathematics and philosophy.*** Polimetrica, International Scientific Publisher, Monza-Milano, 2005, 349 pp.

According to the author, "[t]his book has a double purpose. First, to trace the historical development of the concepts of the continuous and the infinitesimal; and second, to describe the ways in which these two concepts are treated in contemporary mathematics" (2005, p. 11). Although the book falls short of realizing either of these lofty goals, it does provide synopses of some of the important episodes in the historical development of the concepts of the continuous and the infinitesimal as well as overviews of some of the important ways in which these two concepts are treated in contemporary mathematics. In so doing, the book weaves together selected fruit from a large body of research and expository work carried out by a wide array of authors during the twentieth century, the selected fruit (together with the presentation thereof) often being shaped and colored by the research interests and philosophical predilections of the book's distinguished author.

The historico-philosophical part of the book consists of five chapters: Chapter 1 covers the continuous and the discrete in Ancient Greece, the Orient and the European Middle Ages; Chapter 2 treats the founding of the infinitesimal calculus in the 16th and 17th centuries; and Chapter 3 deals with the philosophies of the continuum and/or the infinitesimal of a host of 18th- and early 19th-century mathematicians and philosophers from Euler to Hegel. Chapter 4 begins with Bolzano's and Cauchy's clarifications of the concepts of continuous function and limit, moves on to Riemann's work in differential geometry, and then to the arithmetization of analysis and accompanying banishment of infinitesimals therefrom via the work of Cantor, Dedekind and Weierstrass. Bertrand Russell's rejection of infinitesimals is also discussed as is the rejection of infinitesimals in analysis by E. W. Hobson, the latter being based more on matters of convenience than on a claim of inconsistency, as was Russell's following in the footsteps of Cantor. Finally, Chapter 5 foreshadows the non-classical theories of continua that are discussed in second part of the book with sketches of the views of two groups of philosophers and mathematicians who did not fully embrace the Cantor-Dedekind philosophy of the continuum—du Bois-Reymond, Veronese, and Peirce, who pursued infinitesimalist conceptions, and Brentano, Poincaré, Brouwer and Weyl, who had phenomenological and/or constructivist predilections.

Given the breadth of material Bell attempts to cover in the five historical chapters, the discussions are inevitably sometimes rushed and incomplete. Still, given Bell's frequent use of citations and quotations from original sources, readers seeking more on topics that are covered will often learn where to look, though the references to the important secondary literature are sometimes dated and incomplete. Moreover, Bell's choices of which topics to omit, which to include, and of those included which to give more attention than others are sometimes puzzling, given his stated goals. For example, whereas Hegel's obscure remarks on the continuum and the calculus are given eight pages of discussion—three more than accorded Newton's calculus together with his philosophy thereof—the method of exhaustion and its underlying Eudoxian theory of proportions are treated in less than one page. Similarly, Bell devotes merely two sentences to the discovery of incommensurable magnitudes (p. 21), and never mentions that this and related discoveries, coupled with ancient conception of number as a multitude of units, convinced the ancients that it was impossible to bridge the gap between the discrete domain of number and the continuous domain of geometry. Having not mentioned this critical component of the standard ancient philosophy of the continuum, Bell further fails to mention the revolutionary rethinking of the number concept and its relation to the geometrical continuum that began to emerge when mathematicians such as Simon Stevin (1548–1620) argued that not only is 1 also a number but there is a complete correspondence between (positive) number and continuous magnitude, as well as a parallelism between certain geometrical constructions and the now familiar arithmetic operations on numbers. Thus, while there is much that is useful in Bell's sweeping historical account, these and other historical shortcomings would make this author hesitant to recommend it as a comprehensive overview of the development of the concepts of the continuous and the infinitesimal.

The mathematical part of the book is likewise divided into five chapters, the first two of which provide preliminaries for the subsequent three, the vast bulk being for the chapter on *Smooth Infinitesimal Analysis/Synthetic Differential Geometry*—SIA/SDG. Chapter 6 covers continuity in topological spaces and the concept of a smooth manifold, and Chapter 7 provides introductions to categories, functors and pointless topology as well as overviews of Grothendieck topologies, sheaves, and elementary topoi. Though brief, these chapters cover a good deal of material in a clear and revealing fashion. Chapter 8 covers *non-standard analysis*—NSA; Chapter 9 treats the continuum from Bishop's constructivist and Brouwer's intuitionist standpoints, and offers an account of Vesley's intuitionistic theory of infinitesimals, and Chapter 10 covers SIA/SDG and provides a brief comparison of SIA and NSA.

Here Bell's apology early on that "it is inevitable that a number of topics have not received the attention they deserve" (p. 11) is an understatement. Bell's five-page discussion of NSA (including non-standard topology) is perfunctory and appears to play no other role than serve as a basis for the aforementioned comparison with SIA. Moreover, while descriptions of the roles played by infinitesimals in NSA and SIA are undoubtedly indispensable to an understanding of some of the important ways infinitesimals are treated in contemporary mathematics, such accounts neither provide a substitute for, nor obviate the need of, descriptions of the in-depth investigations of the various types of systems of finite, infinite and infinitesimal elements undertaken by contemporary model theorists, order-algebrists and geometers alike, investigations that are neither referred to nor referenced by Bell. Nor is there mention of the distinctive infinitesimal conceptions that arise in Alain Connes's *noncommutative geometry* and in *p-adic analysis*. J. H. Conway's theory of *surreal numbers* is referred to, but only in a footnote (p. 209). Still, the treatment of SIA is informative and provides, for the most part, a revealing account of its nilpotent-infinitesimalist theory of continua and category-theoretic, intuitionistic-logical framework.

During his discussion of SIA, Bell complements C. S. Peirce for his "awareness, even before Brouwer, of the fact that a faithful account of the truly continuous would involve abandoning the unrestricted applicability of the law of excluded middle ... " (p. 295; also see pp. 184, 208, and 327). One wonders if this "fact" explains the short shrift given NSA and the absence of any reference to Paolo Giordano's important work in which he develops an alternative category-theoretic, nilpotent-infinitesimalist approach to analysis and differential geometry that is compatible with classical logic (*Nilpotent infinitesimals and synthetic differential geometry in classical logic* in ***Reuniting the antipodes—constructive and nonstandard views of the continuum***, Kluwer Acad. Publ., 2001, pp. 75–92; *Infinitesimal differential geometry*, ***Acta Mathematica Universitatis Comenianae, New Series***, vol. 73 (2004), no. 2, pp. 235–278). Also absent are references to the vast literature on geometries over rings containing nilpotent infinitesimals that has its roots in the classical work of J. Hjelmslev (1898; 1929; 1949), C. Segre (1911) and W. Klingenberg (1954; 1955) (see *Geometry over Rings* by F. D. Veldkamp in ***Handbook of Incidence Geometry***, edited by F. Buekenhout, Elsevier, 1995 for references) and to the work of F. Bachmann that unifies a remarkable array of geometries including geometries over division rings that do not employ infinitesimals, non-Archimedean geometries over division rings that employ invertible infinitesimals and geometries over rings that employ nilpotent infinitesimals (***Ebene Spiegelungsgeometrie: eine Vorlesung über Hjelmslev-Gruppen***, B. I. Wissenschaftsverlag, 1989). It is perhaps worth noting that these geometrical works, which are grounded in classical logic, along with the work of Giordano indicate how misleading Bell's claim is that non-zero nilpotent infinitesimals are "possible" in SIA, unlike in NSA, because whereas "[t]he logic of SIA is intuitionistic ... the logic of NSA is classical" (p. 308).

Much as Bell fails to discuss many of the important ways that infinitesimals are employed in contemporary mathematics, there is a host of alternative contemporary mathematical theories of continua Bell likewise ignores. Included among these are the predicative theory of Feferman, the Russian constructivist theory of Markov, and the fuzzy theory of Zadeh, to name only a few.

As some of our earlier remarks suggest, the references to these shortcomings are not intended to imply that the book is without merit. Indeed, there is much that is interesting and stimulating in the book. Still, at least to this reviewer, the book appears to misrepresent what it is. It is not that the book does not accomplish the ambitious historical and expository goals it sets for itself—that is understandable—but it all too often reads like a celebration of SIA. In any case, when viewed from this perspective, it is a welcome addition to the literature that can be profitably mined for an array of interesting ideas including the most comprehensive treatment to date of John Bell's own interesting and provocative thinking on the continuous and the infinitesimal in mathematics and philosophy.

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PETER G. HINMAN. ***Fundamentals of mathematical logic.*** A K Peters, Wellesley, Massachusetts, 2005, xvi + 878 pp.

This book contains a few semesters worth of mathematical logic at the graduate level, from introductory material on propositional and first-order logic (assuming no prior training in logic), up through Gödel's incompleteness theorems and topics in set theory, model theory, and recursion theory. Most sections include several exercises. It can be used quite well as a textbook in an introductory graduate logic course, depending on the student population. Students who are casually interested in taking just one course in graduate logic or students who would be intimidated by the level of abstraction may be better served by another textbook. However, students more inclined toward the subject and students who will appreciate a more thorough treatment should be pleased with Hinman's text.

Chapter 1, on propositional logic, includes a section on intuitive notions of decidability and effective enumerability, a proof of the compactness theorem, and four extra proofs of the compactness theorem, including proofs using ultrafilters and Boolean algebras. (The reader or teacher will probably skip three or four of the extra proofs the first time through, and come back to them as desired.) The reason for studying all these topics in a chapter on propositional logic is stated in the preface: "One of the central expositional tenets of the book is the introduction of difficult ideas in the simplest possible context." Thus, for example, compactness is studied in the context of propositional logic as a preview of the first-order version, and more examples of this tenet put into practice will be mentioned throughout this review. It is not the most efficient way to present the material, but the pedagogical idea is reasonable and I have found it to be effective.

Hinman makes a careful notational distinction between formal languages and the metalanguage. For example, $\dot{+}$ is one of the symbols in the first-order language of arithmetic, whereas the dotless $+$ is a symbol in the metalanguage of "mathematical English" which we use for addition. Hence, a sentence in the formal language of arithmetic might look like "$\forall x\, \forall y\, [x \dot{+} \dot{\mathsf{S}}\, y \doteq \dot{\mathsf{S}}(x \dot{+} y)]$." This sort of notation is not unusual, but the consistency with which it is used in this book is striking. The result is quite a lot of dots (especially in the chapter on set theory, but perhaps none in the chapter on recursion theory). Much of the time the reader tunes them out. On occasion, especially for the student encountering new material, it is quite helpful to have such precise notation.

The consistent use of dots is just one example of a thorough, detail-oriented approach which distinguishes the textbook. Technical issues such as the difference between formal languages and the metalanguage mentioned above are laid out in the open, and are difficult to forget about. I think this is extremely useful for properly learning the material; however, there is danger of students feeling bogged down or losing sight of the main ideas (even though main ideas tend to be explicitly identified). Thus, the approach to technical details is pedagogically friendly, while at the same time demanding some sophistication on the part of the student.

First-order logic is presented in Chapter 2 with a model theoretic emphasis. Before the compactness and completeness theorems are stated and proved (in Chapter 3), Chapter 2 already has, for example, the countable downward Löwenheim–Skolem theorem, nonstandard models of arithmetic, and a proof by way of quantifier elimination that the theory of the natural numbers with $<$ and $+$ (but not multiplication) is decidable. (Quantifier elimination is first introduced in the simpler context of the language of equality with no non-logical symbols.) The study of formal proofs is de-emphasized; logical axioms and rules of inference are not introduced until after the proof of the compactness theorem in Chapter 3.

Chapters 4 and 5 are mostly about Gödel's incompleteness theorems and notions of recursive and recursively enumerable sets. The first section of Chapter 4 is a nice, long overview, stating versions of both incompleteness theorems in terms of the intuitive notion of effectiveness introduced in Chapter 1. Two claims about effective sets are made (the "Definability Hypothesis" and the "Representability Hypothesis"), and the incompleteness theorems are proved assuming these claims. A proper treatment begins in the next section, with a formal definition of recursive function, and so on. The presentation is relatively thorough, containing elements which might be skipped on first reading (or a set of classroom lectures), but are nice to have as supplements. For example, there is a nearly seven page outline of what goes into the proof of the Hilbert–Bernays derivability conditions. When I first encountered this material as a student, I was a bit disappointed with the typical handwaving justification of the derivability conditions; I would have been quite satisfied with this book's treatment.

Chapter 6 is on set theory. There is no attempt to treat set theory as "normal math" which happens to be about sets; rather, it is a study of a particular first-order theory, ZF. It requires a solid understanding of the material in Chapters 2 and 3; given such understanding, the presentation should be reasonably clear, even though the chapter is relatively dense. After suitable development, complete proofs of the basic results about L and forcing are presented. The chapter is long, also including sections introducing large cardinals (up to measurable) and determinacy.

Chapter 7 on model theory includes Morley's theorem and Lindström's first and second theorems. Chapter 8, on recursion theory, begins with many-one reducibility and introduces priority arguments in that context, and then moves to the topic of Turing reducibility. The chapter ends with the jump interpolation theorem. I suspect that model theorists and recursion theorists will find the book insufficient as a sole text for courses on their respective subjects, and it is probably not the author's intention for it to serve as such. Each chapter would be a useful introduction to the material, supplementing or supplemented by other sources, and would be especially useful in a program that uses this same text for multiple courses.

Teachers of graduate logic courses and students studying independently should consider using Hinman's textbook, especially where it is expected that students will continue for more than one semester. At the least, there should be a copy in every library where graduate logic is taught. Even when not deemed the best choice as a primary text, it would make a good secondary source for a thorough treatment of introductory material or the various other topics.

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Richard Tieszen. ***Phenomenology, logic, and the philosophy of mathematics.*** Cambridge University Press, Cambridge, 2005, x + 357 pp.

Through his creation of phenomenology, Edmund Husserl (1859–1938) was one of the most original and influential thinkers of the 20th century. Not only did he play a pivotal role in the establishment of modern continental philosophy, but he also anticipated key issues in contemporary philosophy of mind and in cognitive science—fields where his insights into the nature of consciousness keep resurfacing with added significance. However, what is overlooked by most commentators, is that Husserl had initially been trained in the natural sciences and in mathematics, at a time when those subjects underwent decisive transformations, and that his philosophy developed out of a concern for the philosophical ramifications of those transformations.

The book under review consists of a collection of essays, written over the last 15 years, in which the author argues for the enduring importance of Husserlian ideas by setting them in relation to current debates in the foundation of mathematics. For the most part, the arguments focus on the nature of objectivity in mathematics, and on philosophical questions concerning such notions as evidence, knowledge, truth, being and reality. But the main conclusions are also put to work by the author in shedding light on related issues such as the mind-machine problem, in reflecting on the scope of logic and in critisizing a purely naturalistic conception of science. By and large the book is written from an anti-reductive, realistic point of view, and apparently one of its goals is to use phenomenological analyses to support, or at least to help make sense of, Gödel's ideas about mathematical intuition and the objective existence of mathematical objects. Moreover, the author contrasts Gödel's philosophical standpoint with that of other influential figures such as Quine. The final part of the book contains enlightening comparisons of the philosophy of logic and mathematics found in Husserl with the traditional schools in the foundation of mathematics. In view of the scope and the depth of the issues discussed it is impossible here to review individually all the essays meriting serious consideration. I therefore confine myself to outlining some of the main ideas in Husserl which form the backbone of Tieszen's arguments and to making a few critical comments.

The hallmark of a phenomenological approach to the philosophy of mathematics is the shift of focus away from the purported objects to their *Gegebenheitsweise*, that is the manner in which they present themselves in our mathematical experiences. Phenomenologists, however, are not concerned with the empirical detail of those experiences, but rather with the ideal possibility of specific kinds of mental acts, and it is in terms of the latter that they aim to clarify the meaning of existence claims in mathematics. One of the basic dichotomies exploited by Husserl in his early writings is that between *significative* and *intuitive* intentions. The former are linked to the peculiar aspect of consciousness that we can be directed to objects in their 'absence'. For example, I can identify a physical object by phrases like 'the house at the end of the street' without actually having it in front of me. Or one may name a complex number by means of an algebraic expression such as $-e^{i\pi}$ without realizing that one is in fact referring to an integer. In an intuition, by contrast, the intended object is present *in propria persona*. In the case of a physical object, this amounts to sense perception. But what could this possiby mean for mathematical objects and other ideal entities?

Husserl's answer is articulated in terms of the Kantian notion of *synthesis* that lies at the heart of any conscious experience. Already in the perception of a material thing diverse contents of consciousness need to be co-instantiated *as* properties or aspects of an identical something. Nevertheless this synthesis is *passive* in the sense that, from the outset, one is directed to a fully-fledged object: there are no sides or perspectives floating around which are subsequently united by consciousness in order to infer that we are dealing with an object. Thus in a perception the whole is prior to the parts. Ideal objects, on the other hand, can only be experienced by means of a higher-order synthesis that involves more basic acts having their own objects. For example, a set of things is brought to awareness in a two-tiered synthesis whose final step consists in the *active* re-grasping of a preceding act of collection by which individual objects have been put aside (cf. Cantor's 'definition' of *set* as a Many comprehended as a One).

The axioms of set theory, to be sure, are not concerned with this or that particular set, but rather with the *essence* of set. In Husserlian parlance an essence is an entity that is common to several objects, as for example the triangularity of a triangle or the pitch of a tone. Our awareness of essences rests on the active synthesis that Husserl calls *free variation*. His own examples of this procedure are mainly concerned with the inexact (or 'morphological') essences of physical objects and of the essential structures of consciousness which form the subject matter of phenomenology. He did, however, regard mathematics itself as the most highly developed science of essence. Free variation starts with an arbitrary example of a class of objects which is subsequently modified in imaginative consciousness up to a point where further variation becomes imcompatible with membership in the class one set out to delimit. (The shift to imaginative consciousness is necessary for producing counterfactual examples.) According to Husserl, we intuit the essence once we grasp a 'rule' allowing us in principle to go on indefinitely producing additional variants—even if we cannot do so in actuality. It is in virtue of this 'rule' that the variants are recognized as examples of the same type and not merely as random individuals. Tieszen illustrates this in connection with developments in modern pure geometry, namely the idea originating with F. Klein of finding invariants under groups of transformations.

The foregoing also yields a characterization of mathematical intuition in phenomenological terms. An intuition is the *fulfillment* of a significative intention: the significative intention names an object in its absence by specifying a synthesis the performance of which will result in the experience of the object as itself-given; fulfillment consists in the performance of that synthesis. (Thus, in particular, whether or not a mental act is intuitive can only be decided by consulting the corresponding significative intention.)

For his account of how we come to know mathematical objects the author essentially adopts this model. Following Husserl, he maintains that *knowledge* always involves a situation where the things judged are present 'in person', and consequently, their features and relationships will be *evident*. The things in question are invariants or identies against a background of variation, e.g., the concept of a group as the invariant in diverse manifestations of symmetry, the continuum against its different axiomatizations in algebra, analysis, topology and set theory, or the linearity of the hierarchy of strong axioms of infinity against the ostensible disparity of their motivations.

> We *know* about these invariants in the way indicated earlier, that is, through sequences of acts in which we have evidence for their existence, even if we know about them only partially or incompletely. Otherwise, they are merely intended, in the sense that an intention directed toward an object exists even if we do not exprerience the object itself, and these intentions may have some stability in our experience and admit of further clarification (p. 64).

The author observes that this account of mathematical knowledge does not presuppose that we are causally related to mathematical objects. And neither does it attempt to reduce mathematical intuition to patterns of neuronal activity, or otherwise.

These phenomenological descriptions, though evidently incomplete, seem to me to have the plainest introspective warrant. And the author is entirely correct in emphasizing that, unlike the more esoteric viewpoints in the foundation of mathematics (according to which we are systematically misled by mathematical language), Husserl aims to do justice to mathematics as it is practised.

On a critical note, I would like to mention briefly two points. First I had difficulties grasping the author's views on the nature of fulfillment. In his discussion of the cognitive aspect of proofs, he argues that the distinction between the absence and the presence of mathematical entities to consciousness is appropriately modeled by the constructivist dogma that we must exhibit the objects of our existence claims (p. 279). But elsewhere (p. 62) he suggests that one can account for non-constructive parts of mathematics on the basis of Husserl's views about meaning clarification, idealization, reflection, abstraction and formalization. Presumably, the reader would like to see an implementation of this suggestion by way of an actual case study shedding light on the viability of impredicative set theory or, for the more ambitious, on the status of new *axioms* in connection with classically undecidable problems.

Another issue where the author could have been more explicit concerns the peculiar species of realism that Husserl adopted after his transcendental turn and that also attracted the attention of Gödel in his later years. As the author remarks (e.g., in his excellent account of the philosophical background of H. Weyl's constructivism), one finds in Husserl an intriguing interplay of both platonist elements, namely that reality is experienced as independent of us, and idealist elements, that we structure or *constitute* reality. But in contrast with 'metaphysical idealism', for Husserl constitution is not about generating things but meanings, and it is only by way of a methodological separation of the meanings from the objects of experience (the *transcendental reduction*) that he arrives at a clarification of the meaning of—as opposed to a verification of—our claims concerning knowledge, truth, being and reality.

The point of this analysis is that even though all meanings constitute themselves 'within' consciousness, this subjectivity is not merely subjective but presents objectivity to itself. Phenomenology as a systematic study of the subjective perspective of the mathematician thus provides an alternative to the established viewpoints in the foundations of mathematics as well as to philosophically sterile versions of contemporary naturalism. In my opinion the primary importance of Tieszen's essays lies in the fact that they offer a wealth of insight into a philosophical approach to mathematics that is yet to receive the attention and recognition it deserves.

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The architecture of modern mathematics: Essays in history and philosophy, edited by José Ferreirós and Jeremy J. Gray, Oxford University Press, Oxford 2006, xii + 442 pp.

In this volume, Ferreirós and Gray have brought together a selection of 14 new essays (only one of them published before) that share a commitment to integrating the philosophical reflection of mathematics with the close historical study of the development of mathematical practices and ideas. In the resulting volume, philosophy and history do not play exactly symmetrical roles: All contributions (with the possible exception of the introduction) present themselves as *historical essays* that highlight some philosophical aspects rather than the other way round. The papers are concerned with mathematical developments between 1800 and 1970, and the majority of them focus on the contributions of one or two great mathematicians within a specific area of mathematics.

The book opens with a long and spirited introduction by the editors, in which they adopt a critical stance towards contemporary mainstream philosophy of mathematics. They see it dominated by attempts to apply "systematic philosophy" to mathematics. The resulting paradigm is criticized for various shortcomings: It tends to ignore the conceptual and methodological *dynamics* of mathematics; it over-emphasizes one element of mathematical practice (viz., proving theorems within the framework of a given axiomatic system) at the expense of other, philosophically no less interesting ones (like conjecturing, explaining, or visualizing); it is obsessed with its pet theories, logic and set theory, and therefore often fails to acknowledge the richness of mathematical concepts and methods; it has suffered from "excessive attention to all-inclusive claims" (p. 18). Such deficiencies, the editors suggest, can be overcome by means of a closer integration of the history and philosophy of mathematics, geared to an ideal of "philosophical reflection" rather than "systematic philosophy". To be sure, similar criticisms have been raised before (for example in Kitcher and Aspray's introduction to their *Minnesota Studies* volume of 1988), and the editors explicitly acknowledge this. Yet one can hardly contest their diagnosis that, by and large, the received picture of the philosophy of mathematics has not changed much during the last 50 years (p. 4 f.). This is despite the many valuable investigations along the lines suggested by Ferreirós and Gray that have of course already been undertaken (among others, by many of the contributors to their volume). Given this, the editors' initiative to present a good selection of recent samples of this work is all the more welcome.

Many of the contributions focus on the philosophical ideas developed by great mathematical practitioners in the context of their work. For example, Jamie Tappenden alerts his readers to the fact that during the time of Frege's mathematical upbringing, which occurred in a context where analysis was widely perceived as a core discipline of mathematics, there was no consensus about what exactly analysis was. In particular, a Weierstrassian tradition with an emphasis on rigor and avoidance of appeal to geometric intuition clashed with a Riemannian one that permitted the application of geometric tools to analysis and pursued conceptual clarity and simplicity as a primary goal. Frege's siding with Riemann's understanding of complex analysis is then shown to be continuous with some of his logico-philosophical insights—e.g., that a function is not an expression and that definitions should be representation-independent.

Riemann himself and his philosophical ideas are subjected to a close investigation in another contribution. José Ferreirós argues that the key to Riemann's philosophy of mathematics is his involvement with physics, which for a while was so serious that he regarded his quest for a unified theory of physical forces as his main occupation. According to Ferreirós, this led Riemann to view mathematics as a part of the natural sciences, which implied the abandonment of its a priori status.

Michael Beaney's paper invites reflections on the historicity of philosophical ideas in mathematics itself, as he reviews the role of historical references in Frege's work and how they relate to his pursuit of the elucidation ("*Erläuterung*") of expressions. The use of historical reference-points like Kant is shown often to be an important part of Fregean elucidation, which is itself an indispensable task as not every expression can be defined in terms of more basic ones.

The volume also contains papers on more unusual and idiosyncratic philosophical ideas of mathematicians. For example, Erhard Scholz portrays Hermann Weyl as an advocate of a "symbolic realism", according to which mathematics acquires meaning from contexts of scientific, technical and social practices, while questions concerning its fundamental ontology should be regarded in an agnostic way. Moritz Epple juxtaposes Felix Hausdorff with his literary alter ego Paul Mongré, under which name he published several pieces of fiction and philosophical reflections in a broadly Nietzschean vein. Epple tries to unearth the common epistemological concerns of the mathematician and the literary writer. Alongside such investigations of individual mathematicians and their philosophical agenda, the volume also presents some interesting and more encompassing efforts to correlate and compare the philosophical reactions of several different mathematicians to certain mathematical and scientific developments. An example is Jeremy Gray's closing essay on the profound changes that transformed mathematics after 1880. He concentrates on geometry and identifies a Leibnizian and a Kantian strategy of coping with the deep philosophical questions about mathematical truth raised by, among other things, the advent of non-Euclidean geometry. How can such historical studies of mathematicians' philosophical ideas (there are several more among the papers within this volume) support the philosophical reflection on mathematics demanded by the editors? I think they can play very helpful roles in this respect, especially if they are concerned with influential, established, or at least coherently defensible philosophical positions. They can serve to demonstrate the contingency of the ideas that dominate the philosophical terrain, in the sense that they show how positions have emerged from changes and tensions within mathematics at certain points in time (as in the case of Frege). They can even highlight alternative paths (as, arguably, in the case of Riemann).

However, if the proposed convergence of history and philosophy of mathematics is to show its whole potential, it must go beyond a mere history *of* philosophy of mathematics. It must also prove its potential to use the history of mathematics in order to lead to convincing, contemporary philosophical insights. The actual mathematical practices of the historical protagonists often offer more important material than their own philosophical reflections. As an illustration of this, Jeremy Avigad has contributed a paper on Dedekind's theory of ideals that exemplifies the great promise of an integrated history *and* philosophy of mathematics. Among the more fascinating aspects of mathematical practice that has received too little attention from traditional philosophy is its considerable freedom: Not only are there myriads of new concepts and structures that could in principle be introduced and investigated, but each of them can also be defined and approached in many very different ways. A case in point is the theory of ideals and its different early treatments, most importantly by Kronecker and Dedekind. Avigad traces the development of Dedekind's several versions of the theory in order to identify the mathematical values that informed his methodological choices. In unexpressed analogy to the so-called "epistemic values" that are familiar from the philosophy of the natural sciences, Avigad gives a precise description of principles of generality, uniformity, and familiarity as they appear in Dedekind's work. The insights enabled by this investigation promise to give rise to many interesting and potentially fruitful historical and philosophical questions—e.g., how did certain mathematical values become established in the mathematical community rather than others, how do they relate to the values of other sciences and even to other ("extra-scientific") values?

Most of the many interesting discussions in this volume offer *impulses* for philosophical reflection rather than their own philosophical conclusions. An exception is the introduction, where Ferreirós and Gray put forward a tentative "hypothetical conception of mathematical practice", which is designed to take the historical richness of the subject into account. It is meant to reflect an insight that most mathematical propositions cannot be taken to be “true in the world we live in”, but must be seen as parts of systems based on hypotheses. How exactly this is to be spelled out—and, in particular, how it relates to views that have been discussed at length under the name “deductivism” or “if-thenism”—is not clear. Prima facie, the hard problem for a hypothetical conception is to explain how a whole mathematical theory “is erected on” its hypothetical basis (to use the authors’ expression), without loading all the explosive philosophical freight onto an uncritical use of the notion of logical consequence (as deductivism has been criticized of doing). Clearly, systematic philosophizing about mathematics will have to be part and parcel of an integrated enterprise of the history and philosophy of mathematics. Ferreirós and Gray’s volume shows that the real convergence has yet to occur, but it also offers enough samples of interesting and insightful work to make the endeavor look promising.

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HARRY J. GENSLER. ***Historical dictionary of logic.*** Historical Dictionaries of Religions, Philosophies, and Movements, vol. 65. The Scarecrow Press, Lanham (Maryland), Toronto, and Oxford, 2006, xliv + 307 pp.

The book under review is a sort of an encyclopedia of logic. The author declares in the Preface that he understands logic as a discipline analyzing arguments and helping us to see which ones are valid. The aim of the book is to introduce the central concepts of logic in a series of brief, nontechnical articles. The book emphasizes deductive logic but there are also entries on areas like inductive logic, fallacies, definitions as well as key concepts from epistemology, mathematics and set theory.

Logic is nowadays developed by philosophers, mathematicians and computer scientists. In the book under review the philosophical branch of logic is emphasized. Therefore the areas in which philosophers are interested and which they teach are first of all taken into account. The mathematical and computer-science items as more technical and more specialized have not been treated so extensively.

The main part of the book is a dictionary section consisting of 352 entries. They are arranged alphabetically and are devoted to fundamental concepts and problems of logic. The most significant theories and issues arising out of Western philosophy and other traditions, the more common fallacies and argument forms are presented and explained. There are entries on historical periods and figures, including ancient logic, medieval logic, Buddhist logic, Aristotle, Ockham, Boole, Frege, Russell, Gödel, Peano, Quine, etc. One finds there information on propositional logic, modal logic, deontic logic, temporal logic, set theory, many-valued logics, mereology, paraconsistent logic. There are also entries concerning applications of logic to various fields such as philosophy, mathematics or computer science. The presupposed character of the book excluded very technical expositions but the author succeeded to explain technical problems—as, for example, Gödel’s incompleteness theorems—in an adequate way not using complicated technical apparatus. The entries vary in length from a sentence or two to several pages.

One should stress that the book under review provides the potential reader not only with knowledge about well established canon of logical notions and concepts but it indicates and explains also many modern, still being developed and studied theories and achievements in logic, in particular those connected with computer science or fuzzy logic as well as with cognitive science.

The front of the book has three important parts: (1) a short notation section, (2) a chronology starting from the ancient Greece and going to 2001, (3) an introduction where an overall view of logic is given (it should provide a broader context for the dictionary entries).

At the end of the book an extensive bibliography (50 pages!) is included. Its scope is broad: it includes items on topics like the history of logic, its various branches, its controversies and its relationships to other areas. First of all items that are readable and understandable for beginners have been chosen. Nonetheless one finds there also some highly technical but classical items. Except for a couple of important works that have not yet been translated only works available in English were included. The material was organized in seven sections: general material, history of logic, textbooks, beyond text (logic videos, software, interactive websites), interdisciplinary, individual figures and topics, others. Being aware that such an extensive bibliography may be not fully useful for many beginners, the author suggests also a list of eight fundamental works for them.

The book was published in a very nice and careful way. In particular all Polish names are printed in a right way using all diacritical signs—what happens not so often. The rich variety of parts of logic covered in the book makes it useful both for specialists (especially logicians, not only philosophers but also mathematicians and computer scientists) looking for information in areas outside their subspecialties as well as for a non-professional reader, in particular students and generally laypersons.

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S. AWODEY. ***Category theory.*** Oxford Logic Guides, vol. 49. Oxford Sciences Publications, Oxford, 2006, xi + 256 pp.

Since its creation in 1945, category theory has spread into many areas of Mathematics and Logic and the interested reader can find a number of textbooks offering its presentation. For those who are well prepared, Mac Lane's ***Categories for the working mathematician*** (Springer–Verlag 1998) is still the best choice but S. Awodey intends to serve readers just beginning the serious study of mathematics. He succeeded and his book can be very well used to teach Category Theory to undergraduate and graduate students of Computer Science, Mathematics and Logic (or even Cognitive Science, Linguistic and Philosophy). Simplifying a little bit, one can say that it is more elementary than M. Barr and C. Wells, ***Category Theory for computing science*** (Prentice Hall International 1990) and more advanced than F. W. Lawvere and S. Schanuel, ***Conceptual mathematics: a first introduction to categories*** (Cambridge University Press 1997).

The book presents a standard core consisting of categories, functors, natural transformations, limits and adjoints and, as optional topics, it offers cartesian closed categories, presheaves and monads. Some readers will miss toposes and, especially, monoidal categories. The author recommends to read these in Mac Lane's monograph after mastering his book (one can use Barr and Wells, as well). The book does not assume a reader to know concepts like groups, vector spaces, modules or topological spaces which are traditionally used to give the first examples of categories. Instead, the author introduces posets and monoids and treats them as categories. Everyone does it because, formally, categories are a mixture of posets and monoids but the author's intention is to take this fact as a guiding principle. Unfortunately, he misses some opportunities provided by this approach. For readers coming from Logic, it might be interesting to explain the passage from posets to categories as the passage from provability to proofs. Although the category associated to a deductive system is presented in example 9. in 1.4, the author does not use it later. For instance, he could explain that products in this category are conjunctions. It is somewhat hidden in the section devoted to the λ-calculus but an explicit presentation would be helpful. Similarly, one could serve readers coming from Computer Science by relating computability to computations. Again, it is only mentioned in example 10. Concerning the first examples of categories, I would relate the category of integer matrices from example 4. to the usual matrices of real numbers and, later, to linear maps of finitely dimensional vector spaces. In the same example, the author's introduces concrete categories. But his later remark 1.7 is confusing because there exist categories which are not isomorphic to concrete ones. The first example, due to P. Freyd, is the homotopy category of topological spaces. And, even in 7.2, the author avoids the definition of a concrete category as a category equipped with a faithful functor to sets.

In a connection with the poset and monoid strategy, one can regret that monoidal categories are excluded. I would replace groups in categories in 4.1 by monoids and relate strict monoidal categories to monoids in the category of categories. Monoidal categories lead to Linear Logics and to concurrency in Computer science. One could even introduce quantales and mention the Lambek calculus in Linguistics. Also, one could explain that monads are monoids in a suitable monoidal category. In the section devoted to monads, I miss more examples of algebras for a monad. For instance, the example 10.5 (repeated in 10.6) does not mention that the algebras for the powerset monad are complete join semilattices. Finally, even in a very rough history of category theory, one should not forget D. M. Kan who discovered adjoint functors in 1958 and who has been using categories with great success in algebraic topology.

But each author has to make his choice and the book under review is certainly a very needed and useful addition to the existing literature which will be welcome by all teaching a course of category theory.

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Nonstandard methods and applications in mathematics, edited by Nigel J. Cutland, Mauro Di Nasso and David A. Ross, Lecture Notes in Logic, vol. 25. Association for Symbolic Logic, A.K. Peters, Ltd. Wellesley, Massachusetts, 2006. x + 248 pp.—therein:

VIERI BENCI, MARCO FORTI, and MAURO DI NASSO. *The eightfold path to nonstandard analysis*. Pp. 3–44.

SERGIO FAJARDO and H. JEROME KEISLER. *Neoclosed forcing*. Pp. 45–79.

KAREL HRBACEK. *Nonstandard objects in set theory*. Pp. 80–120.

PETER A. LOEB. *The microscopic behavior of measurable functions*. Pp. 123–126.

DAVID A. ROSS. *Nonstandard measure constructions—solutions and problems*. Pp. 127–146.

RENLING JIN. *Inverse problem for upper asymptotic density II*. Pp. 147–173.

ANGUS MACINTYRE. *Nonstandard analysis and cohomology*. Pp. 174–191.

NIGEL J. CUTLAND. *Loeb space methods for stochastic Navier–Stokes equations*. Pp. 195–223.

MANFRED P. H. WOLFF. *Discrete approximation of compact operators and approximation of their spectra*. Pp. 224–231.

RICHARD O'DONOVAN and JOHN KIMBER. *Nonstandard analysis at pre-university level: Naive magnitude analysis*. Pp. 235–248.

This volume is a collection of peer-reviewed papers solicited from some of the participants in the conference on Nonstandard Methods and Applications in Mathematics held in Pisa, Italy, June 12–16 2002. The editors state their aim as "providing something more timely than a textbook, but less ephemeral than a conventional proceedings."

The first three papers are on foundational topics. The article by Benci, Forti and di Nasso opens with a brief introduction to nonstandard techniques, including basic principles such as the $*$ map, transfer and saturation as well as the ultrapower construction. The remainder of the article is concerned with eight approaches to nonstandard analysis (NSA), the first being the widely used superstructure framework of Robinson and Zakon (JSL XL 509(15), XLVII 204(8)) while the remaining seven are approaches due to one or more of the authors of the paper. These vary in flavour from topological to algebraic.

A common pattern in applications of NSA is to construct a standard object by pushing down a hyperfinite approximation of some kind. In his paper *From discrete to continuous time*, ***Annals of Pure and Applied Logic***, vol. 52 (1991), pp. 99–141, Keisler showed that such results could be proved in a uniform manner in the context of stochastic processes, using a notion of forcing. The paper by Fajardo and Keisler generalizes the earlier work in the light of their subsequent work on neometric spaces. A language of neoclosed formulas is introduced; the authors' main result, the Neoclosed Forcing Theorem, reduces the proof of certain existence claims to showing that certain formulas are forced. Applications are described in game theory (existence of Nash equilibria) and stochastic integral equations.

The superstructure approach is widely used, and adequate for many applications of NSA. However, it amounts to doing mathematics in a cumulative type structure of order ω. This has led several authors to attempt the construction of extensions of standard set theory that accommodates both standard and nonstandard objects. The paper by Hrbacek surveys these constructions, including **HST**, or "Hrbacek Set Theory," based on his own work of 1978; this theory is the main topic of the recent monograph ***Nonstandard analysis, axiomatically*** (Springer 2004) by Kanovei and Reeken. Hrbacek also discusses Edward Nelson's Internal Set Theory JSL XLVIII 1203, the favourite tool of the Alsatian school of nonstandard analysts, as well as lesser known systems such as those of Andreev-Gordon and David Ballard.

The next four papers are devoted to pure mathematics. Peter Loeb's short note shows that for an appropriate measure space of finite measure, the nonstandard extension of a measurable function is essentially constant on the modified monad of each point in a standard set of full measure (the modification removes from each monad a thin set of points).

Early work on measure theory in the context of NSA faced the following problem. If $(X, \mathcal{B})$ is a standard measurable space, and $({}^*X, {}^*\mathcal{B})$ its nonstandard extension, then the finitely-additive measure defined on the original space by taking the standard part of an internal, *finitely-additive measure is not necessarily σ-additive. This difficulty was overcome in the well known construction of Loeb (JSL L 243) on which most of the subsequent applications of nonstandard measure theory have been based. However, Loeb's construction does not cover all measures to which one might wish to apply nonstandard methods. Ross's article returns to some of the pre-Loeb-space techniques, refines and extends them, and applies them in giving new proofs of both standard and nonstandard results.

The classical problems in additive number theory are direct problems, concerned with determining the properties of the sets A, $A+A$, $A+A+A$ etc., starting from a set of integers A. Inverse problems are concerned with the opposite implication, deducing properties of A from a sumset $A+A+\cdots+A$. Renling Jin's paper is concerned with the particular case of deducing properties of A from those of $A+A$. The results are all of the form: if $A+A$ is sparse, then A must have some structure. Nonstandard analysis enters here as a simplifying tool, replacing asymptotic arguments by *finite arguments in a nonstandard world.

Angus Macintyre's article is concerned with the interaction between model theory and cohomology theory in the context of algebraic geometry. One of his main themes is the question of the appropriate framework for nonstandard cohomology theories. Macintyre argues persuasively that a category-theoretic approach is preferable to the more traditional set-theoretic approach, and discusses category-theoretic versions of the ultraproduct construction.

Two articles follow on applied mathematics. Cutland's contribution is a survey of the use of Loeb space methods in the study of stochastic Navier–Stokes equations. The Navier–Stokes equations, a pair of partial differential equations describing the flow of an incompressible fluid in a region of space, have a long history, and recently have been in the news, since the Clay Mathematics Institute offers a \$1m prize for an answer to certain problems concerning the existence and uniqueness of solutions to the equations. NSA is useful in the study of these equations for two reasons, first, the use of hyperfinite-dimensional equations to simplify the classical infinite-dimensional Hilbert space framework, second, the possibility of adapting Keisler's nonstandard approach to stochastic differential equations.

Manfred Wolff uses nonstandard methods to give a necessary and sufficient conditions for an operator A to be compact in terms of the sequence $\{A_n\}$ of operators approximating A discretely.

The volume ends with a very interesting article on the use of NSA in teaching calculus on the pre-university level. Richard O'Donovan and John Kimber teach at the Collège André-Chavanne in Geneva (in Switzerland, the Collèges are four-year institutions intermediate between middle school and university, roughly corresponding to the upper grades in North American high schools). They have developed a calculus course using basic nonstandard analysis, apparently with success. Their article outlines their syllabus, together with some of the exercises that they give their students.

In the reviewer's opinion, the editors have succeeded admirably in assembling a useful and readable collection of articles both surveying the field of NSA, and giving some pointers to current work.

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