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2006 SUMMER MEETING
OF THE ASSOCIATION FOR SYMBOLIC LOGIC
LOGIC COLLOQUIUM '06

Nijmegen, The Netherlands
July 27–August 2, 2006

Logic Colloquium '06, the 2006 European Summer Meeting of the Association for Symbolic Logic, was hosted by the Radboud University Nijmegen from 27 July to 2 August 2006. Lectures were held in the University Auditorium (Aula) on the University Campus.

The sponsors of the meeting include: the Association for Symbolic Logic, the Netherlands Organisation for Scientific Research (NWO), the Royal Dutch Academy of Science (KNAW), the E. W. Beth Foundation, and the Radboud University Nijmegen. We thank them for their support.

The success of the meeting was due largely to the hard work of the Organizing Committee under the leadership of its Chair, Herman Geuvers, the other members being N. Messink, B. Spitters, and F. Wiedijk from the Foundations group of the Institute for Computing and Information Science of the Radboud University Nijmegen. The Program Committee consisted of B. Cooper (Leeds), S. Friedman (Vienna), H. Geuvers (Nijmegen), D. Macpherson (Leeds), G. Mints (Stanford), A. Pillay (Urbana-Champaign), M. Rathjen (Columbus Ohio), R. Schindler (Münster), H. Schwichtenberg (Munich, Chair), R. Shore (Cornell), W. Sieg (Carnegie Mellon), G. Sundholm (Leiden), and J. Väänänen (Helsinki).

The main conference topics were: *Computability Theory, Computer Science Logic, Model Theory, Proof Theory and Type Theory, and Set Theory*. The program included three short tutorial courses, fourteen invited plenary lectures, and thirty invited lectures in five special sessions corresponding to the five main topics. There were 94 contributed talks and 185 participants from 29 different countries. Forty-three student or postdoctoral participants were assisted by ASL grants, and a further 7 participants received grants from other sources.

The following short tutorial courses were delivered:

Rodney Downey (Wellington), *Algorithmic randomness and computability*.
Ieke Moerdijk (Utrecht), *Introduction to algebraic set theory*.
Boban Velickovic (Paris), *Forcing axioms*.

The following invited plenary lectures were presented:

Samson Abramsky (Oxford), *Categorical quantum logic*.
Marat Arslanov (Kazan), *Problems of definability and structural differences among elementary theories of the Ershov difference hierarchy*.
Harvey Friedman (Ohio), *Search for consequences*.
Martin Goldstern (Vienna), *Applications of mathematical logic in algebra: the lattice of clones*.
Ehud Hrushovski (Jerusalem), *Model theory of valued fields*.
Jochen Koenigsmann (Freiburg), *Axiomatizing fields via Galois Theory*.
Andy Lewis (Leeds), *On constructing strong minimal covers*.

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Antonio Montalban (Chicago), *Embeddability and decidability in the Turing Degrees*.
Erik Palmgren (Uppsala), *Intuitionism, Bishop constructivism and point-free thinking*.
Wolfram Pohlers (Münster), *Iterations of ordinal operators*.
Ernest Schimmerling (Pittsburgh), *Coherent sequences and threads*.
John Steel (Berkeley), *Some correctness results for extender models*.
William Tait (Chicago), *Cut-elimination for impredicative systems of Second-Order Number Theory*.
Frank Wagner (Lyon), *Amalgamation, fusion and collapse: Exotic fields and vector spaces*.

The proceedings of Logic Colloquium '06, edited by H. Geuvers (Chair), B. Cooper, A. Pillay, and J. Väänänen, will contain many of the invited lectures and will be published in the ASL series *Lecture Notes in Logic*.

More information about the meeting can be found at the conference webpage `http://www.cs.ru.nl/lc2006/`.

Abstracts of invited and contributed talks given in person or by title by members of the Association follow.

For the Program Committee
HELMUT SCHWICHTENBERG

Abstracts of invited tutorials

► RODNEY DOWNEY, *Algorithmic randomness and computability*.
School of Mathematics, Statistics and Computer Science, Victoria University, PO Box 600, Wellington, New Zealand.
E-mail: `rod.downey@mcs.vuw.ac.nz`.

In these lectures, I will examine questions such as: what does it mean for a real to be random? what does it mean for one real to be more random than another? Does a real being random mean that it has high or low information content?

There has been amazing progress in this area in recent years through that work of many authors. I plan to cover the basic ideas behind the theory of algorithmic randomness (randomness means (i) not having rare properties (ii) having initial segments that are hard to compress and (iii) random means hard to predict, for example). I will then look at recent work clarifying the relationship between randomness and computational power, as well as the classification of reals by initial segment complexity. Finally I plan to look at the deep work on triviality and lowness with its important implications in classical computability.

Background material and details can be found in the references below.

[1] R. DOWNEY, *Some computability-theoretical aspects of reals and randomness*, ***The Notre Dame Lectures*** (P. Cholak, editor), Lecture Notes in Logic, vol. 18, pp. 97–148, Association for Symbolic Logic, 2005.

[2] ———, *Five lectures on algorithmic randomness*, ***Proceedings of Computational Prospects of Infinity***, World Scientific, to appear.

[3] R. DOWNEY and D. HIRSCHFELDT, ***Algorithmic randomness and complexity***, Springer-Verlag Monographs in Computer Science, to appear.

[4] R. DOWNEY, D. HIRSCHFELDT, A. NIES, and S. TERWIJN, *Calibrating randomness*, ***The Bulletin of Symbolic Logic***, to appear.

[5] A. NIES, *Computability and randomness*, to appear.

► BOBAN VELICKOVIC, *Forcing axioms*.
Équipe de Logique Mathématique, UFR de Mathématiques (case 7012), Université Paris 7, Denis-Diderot 2 place Jussieu, 75251 Paris Cedex 05, France.
E-mail: `boban@logique.jussieu.fr`.

Forcing axioms are natural combinatorial statements which decide many of the questions undecided by the usual axioms ZFC of set theory. The study of these axioms was initiated in the late 1960s by Martin and Solovay who introduced Martin's Axiom, followed by the formulation of the Proper Forcing Axiom by Baumgartener and Shelah in the early 1980s and Martin's Maximum by Foreman, Magidor and Shelah in the mid-1980s. In the mid 1990s Woodin's work on Pmax extensions established deep connections between forcing axioms and the theory of large cardinals and determinacy. Nevertheless, some of the key problems remained open. In 2003 Moore formulated the Mapping Reflection Principle (MRP) which seems to be the missing ingredient needed in order to resolve many of the remaining open problems in the subject and a number of important developments in the subject followed.

In this series of lectures we first survey the theory of forcing axioms and then present some recent results: Moore's work on MRP, my work with A. Caicedo on definable well-orderings of the reals, Viale's result that the Proper Forcing Axiom implies the Singular Cardinal Hypothesis, etc.

Abstracts of invited plenary talks

► SAMSON ABRAMSKY, *Categorical quantum logic*.
Oxford University Computing Lab., Wolfson Building, Parks Road, Oxford OX1 3QD, UK.
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We describe a novel approach to axiomatizing Quantum Mechanics in the setting of strongly compact closed categories. This axiomatization captures all the features which are significant for Quantum Information and Computation, and provides a basis for effective reasoning about quantum informatic processes. It can be presented in terms of a diagrammatic calculus which can be seen as a proof system for a logic, leading to a new perspective on what the right logical formulation for Quantum Mechanics should be.

► MARAT ARSLANOV, *Problems of definability and structural differences among elementary theories of the Ershov difference hierarchy*.
Kazan State University, 420008, Kazan, Russia.
E-mail: `Marat.Arslanov@ksu.ru`.

In my talk I will give a survey on the current state of the research on differences among elementary theories of the Ershov difference hierarchy, including a recent negative solution of well-known Downey's Conjecture on elementary equivalency of n- and m-c.e. degree structures for $n, m > 1$ and $n \neq m$.

Problems on definability of the relation "m-computably enumerable" in n-c.e. degree structures $(n > m)$, and on m-c.e. degrees as Σ_1-substructures of n-c.e. degree structures will be also considered, including recent solutions of a number of open questions in this area. New open questions will be raised, and trends of the research development in this area will be discussed.

► HARVEY FRIEDMAN, *Search for consequences*.
Ohio State University, Room 754, Mathematics Building, 231 West 18th Avenue, Columbus, OH 43210, USA.
E-mail: `friedman@math.ohio-state.edu`.

We will discuss some new examples of necessary uses of set theory with large cardinals that fit squarely into finite graph theory. We will also discuss prospects for the future.

► MARTIN GOLDSTERN, *Applications of mathematical logic in algebra: the lattice of clones*.
DMG/Algebra, TU Wien, Wiedner Haupstrasse 8-10/104, A-1040 Wien, Austria.
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A clone on a set X is a family of finitary operations on X which contains all projections and is closed under substitution (composition), such as the family of term functions on any universal algebra. The set of all clones on a fixed set X forms a complete lattice, whose structure is rather complicated already for finite sets X.

I will discuss some old (1960s), intermediate and recent (2006) results about the structure (and more often: non-structure) of this lattice on infinite sets, as well as open questions. It turns out that methods and results from set theory (forcing, descriptive set theory, infinite combinatorics) are essential for this analysis.

► EHUD HRUSHOVSKI, *Model theory of valued fields*.
Institute of Mathematics, Hebrew University (Giv'at Ram), Jerusalem 91904, Israel.
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I will survey some of the developments in the subject, with emphasis on connections to abstract model theory and to geometry.

► JOCHEN KOENIGSMANN, *Axiomatizing fields via Galois theory*.
Institut für mathematische Logik und Grundlagen der Mathematik, Universität Freiburg, Eckerstrasse 1, 79104 Freiburg, Germany.
E-mail: Jochen.Koenigsmann@unibas.ch.

Grothendieck's program of "anabelian geometry" is designed to recover a field K from its absolute Galois group $G_K := Gal(K^s/K)$. This has been achieved, e.g., for the class of finitely generated fields, but it doesn't work in general. However, replacing G_K by $G_{K(x)}$, the absolute Galois group of the rational function field $K(x)$ over K, does the trick: almost all perfect fields K are determined up to isomorphism by $G_{K(x)}$.

The model theoretic version of this theorem proves that K and $G_{K(x)}$ are biinterpretable where we consider the field K in the usual first order language for rings, and where $G_{K(x)}$ is understood as a structure in the language for profinite groups developed by Cherlin, van den Dries, Macintyre and Chatzidakis. We will also discuss the impact on decidability questions for fields like $\mathbf{C}(t)$ or $\mathbb{F}_p((t))$.

► ANDREW LEWIS, *On constructing strong minimal covers*.
Dipartimento di Scienze Matematiche ed Informatiche Roberto Magari, University of Siena, Pian dei Mantellini 44, 53100 Siena, Italy.
E-mail: andy@aemlewis.co.uk.

After giving an introduction to the splitting tree technique of minimal degree construction we shall discuss the generality of this approach and applications to the construction of strong minimal covers.

► ANTONIO MONTALBÁN, *Embeddability and decidability in the Turing Degrees*.
Department of Mathematics, University of Chicago, 5734 S. University Avenue, Chacago, IL 60637, USA.
E-mail: antonio@math.uchicago.edu.

One of the main goals of computability theory is to understand the shape of the structure of the Turing degrees. Two common ways of studying this is by looking at structures that can be embedded into the Turing Degrees and by looking a the decidability of the theory of this structure varying the language or taking substructures. We describe the author's results in this area.

► ERIK PALMGREN, *Intuitionism, Bishop constructivism and point-free thinking*.
Department of Mathematics, Uppsala University, PO Box 480, S-571 06 Uppsala, Sweden.
E-mail: palmgren@math.uu.se.

A consequence of the Heine-Borel theorem is that a point-wise continuous function on the closed unit interval is uniformly continuous. Brouwer incorporated this theorem into intuitionism by developing new mathematical axioms. Topology could then be developed using essentially the classical point-based theory. These axioms, incompatible with the interpretation of logic in terms of computable functions, were given up in the later development of constructive mathematics by the followers of Errett Bishop. Instead various techniques to provide necessary information about (local) uniformity were developed.

A parallel development was that within locale theory, where a point is a derived concept of the topology, while the notion of cover is basic. This made it possible to naturally extend the notion of being locally uniformly continuous to general maps between topological spaces. We show how the theory of Bishop and the point-free theory can be reconciled. The foundational problems of point-free topology are interesting from a predicativistic point of view, and have been addressed by many researchers since the mid 1980s. We discuss some recent developments in this area. We also discuss examples of point-free methods in measure theory.

► WOLFRAM POHLERS, *Iterations of ordinal operators*.
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E-mail: pohlers@math.uni-muenster.de.

An ordinal operator is a function mapping sets of ordinals to sets of ordinals. Iterations of ordinal operators play an important role in the ordinal analysis of impredicative theories. We define the iterations of an ordinal operator and give an example how iterated ordinal operators are used in ordinal analysis. Finally we discuss their importance ot the design of ordinal notation systems.

► ERNEST SCHIMMERLING, *Coherent sequences and threads*.
Department of Mathematical Sciences, Carnegie Mellon University, 6113 Wean Hall, Pittsburgh, PA 15213, USA.
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I will discuss results that connect forcing axioms such as PFA, core models and combinatorial principles such as Jensen's "square".

► JOHN STEEL, *Some correctness results for extender models*.
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Every real in an iterable extender model M is ordinal definable, by the basic canonicity results for such models. However, it is not always the case that M is sufficiently correct to satisfy the statement "every real is ordinal definable". We shall discuss this situation, and show that if M is sufficiently small, in that its extender sequence does not have extenders overlapping Woodin cardinals, then M satisfies the statement "there is a real from which every real is ordinal definable". The key to this result is an analysis of the extent to which such an M knows how to iterate itself. That analysis also has applications in the infinitary combinatorics of extender models.

Part of this work is joint with Ralf Schindler.

► WILLIAM TAIT, *Cut-elimination for impredicative systems of second-order number theory*.
5522 S. Everett Avenue, Chicago, IL 60637, USA.
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The main content of the talk will be a quite simple proof of Takeuti's result, the eliminability of cuts in Π^1_1 Analysis with the Omega-Rule. The proof uses essentially only the well-known proof theory of predicative systems. Although we will obtain ordinal bounds on the length of cut-free proofs, the computations are simple and there will be no discussion of ordinal notations. I will discuss the possibility of extending the method of proof to stronger systems of analysis.

► FRANK WAGNER, *Amalgamation, fusion and collaps: Exotic fields and vector spaces*.
Institut Camille Jordan, Université Claude Bernard (Lyon-1), 21 av. Claude Bernard, 69622 Villeurbanne Cedex, France.
E-mail: `wagner@math.univ-lyon1.fr`.

Modifying an old method of Fraisse for the construction of countable universal homogeneous structures, Ehud Hrushovski has developed a powerful tool to build exotic structures and geometries out of finitely generated substructures. His methods have recently been extended to constructions over a vector space over a finite field, and even over the rationals. This has in particular yielded a field of finite Morley rank of positive characteristic with a distinguished proper infinite additive subgroup, and a field of finite Morley rank of characteristic zero with a distinguished proper infinite multiplicative subgroup. The latter construction uses non-trivial results from algebraic geometry.

I shall survey the method, and sketch the proofs.

Abstracts of invited talks in the Special Session on Computability Theory

► NOAM GREENBERG, *Definability in the lower regions of the c.e. degrees*.
School of Mathematics, Statistics and Computer Science, Cotton Building, Victoria University, Kelburn Campus, Wellington, New Zealand.
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The nether regions of the c.e. degrees have yielded a variety of interesting classes (e.g., the array computable, almost deep, and K-trivial degrees). Using Ershov's difference hierarchy, we obtain a transfinite hierarchy of classes in the low_2 degrees. Two levels of this hierarchy are now known to be naturally definable, by appealing to embeddings of non-distributive lattices in initial segments. An application to higher computability theory will also be given.

► PETER HERTLING, *Is the Mandelbrot set computable*?
Institut für Theoretische Informatik und Mathematik, Fakultät für Informatik, Universität der Bundeswehr München, 85577 Neubiberg, Germany.
E-mail: `peter.hertling@unibw.de`.

First, we discuss the question in the title shortly in the framework of the computation model by Blum, Shub and Smale. Then we turn to the Turing machine model and the computability notions studied in computable analysis. We show that the exterior of the Mandelbrot set, the boundary of the Mandelbrot set, and the hyperbolic components satisfy certain natural computability conditions. We conclude that the two-sided distance function of the Mandelbrot set is computable if the famous hyperbolicity conjecture is true. We also formulate the question whether the distance function of the Mandelbrot set is computable in terms of the escape time.

► BAKHADYR KHOUSSAINOV, FRANK STEPHAN, AND YUE YANG, *Computable categoricity and Ershov hierarchy*.
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Let A be a structure. A computable presentation of A is a structure A' isomorphic to A such that the domain of A' is the set of natural numbers and the relations of A' are computable (= recursive). It is understood that all structures considered in the talk have computable presentations.

The main topic of this talk is the following question: given two computable presentations A' and A'' of A what is the complexity of the easiest isomorphism between these two presentations?

A structure is called Δ-0-2-categorical iff there is always an isomorphism between any two computable presentations which is limit-recursive, that is, computable relative to the halting problem as an oracle. A structure is called computably categorical iff there is always a computable, that is, recursive, isomorphism between any two computable presentations. In the present talk, an Ershov-type hierarchy is investigated between these two levels of categoricity.

Call the structure A Fn-categorical iff there is for any two computable presentations A' and A'' of A an isomorphism f from A' to A'' such that f is the limit of a uniformly recursive approximation a with $f(x) = a(x, t)$ for almost all t and there are at most n numbers t with either $t = 0$ or $a(x, t)$ being different from $a(x, t - 1)$. Call A Gn-categorical iff there is for any two computable presentations A' and A'' an isomorphism f from A' to A'' such that its graph $\{(x, y) \colon f(x) = y\}$ is an n-r.e. set in the Ershov-hierarchy. That is, there is a uniform approximation $a(x, y, t)$ to the graph such that for all pairs (x, y), $a(x, y, 0) = 0$ and all but at most n positive natural numbers t satisfy $a(x, y, t) = a(x, y, t - 1)$. These notions can easily be generalized to recursive ordinals n.

One has the trivial implication that every computable categorical structure is Fn-categorical, every Fn-categorical structure is Gn-categorical and every Gn-categorical structure is Δ-0-2-categorical. The following results have been obtained.

THEOREM 1. *The hierarchy is proper, that is, for any recursive ordinal n there is a Fn-categorical structure which is not Gm-categorical for any* $m < n$.

THEOREM 2. *Let A be the disjoint union of finite graphs with edge-relation* R. (a) *If there is a constant bound on the size of all involved finite graphs then* (A, R) *is G2-categorical.* (b) *The disjoint union of infinitely many singletons and infinitely many pairs of nodes with one connection is G2-categorical but not Fn-categorical for any n.* (c) *The disjoint union of all finite graphs in which every graph occurs infinitely often is not Gn-categorical for any recursive ordinal n.* (d) *Every disjoint union of finite graphs is* Δ*-0-2-categorical.*

THEOREM 3. *If a Boolean algebra is Gn-categorical for a number n then it is already computable categorical. The same is true for linear orders.*

► BJØRN KJOS-HANSSEN, *Complex oscillations and the law of the iterated logarithm*.
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Algorithmic randomness is most often studied in the setting of the Cantor space. However, it has also been considered for the space of continuous functions on the unit interval. There, the analogue of a Martin-Löf random real is a complex oscillation.

In response to a question of Fouché, we show that the law of the iterated logarithm holds almost everywhere for each complex oscillation. The main idea of the proof is to take advantage of Carathéodory's theorem on isomorphism of measure algebras.

► JOSEPH S. MILLER, *Lowness notions, measure and domination*.
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An oracle A is *low* for a given computability theoretic class if relativizing to A does not change the class. The oracles that are low for ML-randomness are a particularly interesting example. They were introduced by Zambella in 1990 and several characterizations were given in papers of Nies, Hirschfeldt, Stephan and others. Recently, Downey, Nies, Weber and Yu proved that every oracle low for weak 2-randomness is low for ML-randomness. We see that the reverse is also true.

This work is related to two domination properties introduced by Dobrinen and Simpson: *almost everywhere domination* and its uniform variant. Following up on work of Kjos-Hanssen, we explain how lowness properties relate to measure regularity properties and thus to domination properties. We can use this relationship to prove that the domination properties of Dobrinen and Simpson are equivalent.

► JAN REIMANN, *The metamathematics of algorithmic randomness*.
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We will discuss some recent work with Theodore Slaman (Berkeley) on the logical structure of random reals. By relativizing Martin-Löf's test concept to representations of measures one can define a notion of randomness with respect to arbitrary (probability) measures on Cantor space. Instead studying the properties of random reals with respect to Lebesgue measure, we can now study a 'reverse' problem: Which properties of a real ensure randomness with respect to a reasonably well-behaved, e.g., non-atomic measure?

The investigation of this problem reveals interesting relations between the logical complexity of a real and its randomness properties. Furthermore, we will address the question whether these relations cannot be proved without transcending the theory they are formulated in (e.g., analysis).

Abstracts of invited talks in the Special Session on Computer Science Logic

► ULRICH BERGER, *Strong normalisation via domain-theoretic computability predicates*.
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It is well-known that intersection types on the one hand characterise the strongly normalisable lambda terms and on the other hand give rise to a syntactically defined domain model for the lambda calculus. Recently, Coquand and Spiwack combined and extended these facts to give criteria for strong normalisability for the lambda calculus with recursively defined constants, thus extending my previous domain-theoretic normalisation results. In the talk I will show that the normalisation proof can also be carried out in an abstract axiomatic setting where the computability predicates are indexed by the elements of a (not syntactically presented) domain.

► VENANZIO CAPRETTA AND AMY FELTY, *Higher order abstract syntax in type theory*.
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The use of higher-order abstract syntax is an important approach for the representation of binding constructs in encodings of languages and logics in a logical framework. Formal metareasoning about such object languages is a particular challenge. We present a mechanism for such reasoning, formalized in Coq, inspired by the Hybrid tool in Isabelle.

At the base level, we define a de Bruijn representation of terms with basic operations and a reasoning framework. At a higher level, we can represent languages and reason about them using higher-order syntax. We take advantage of Coq's constructive logic by formulating many definitions as Coq programs.

We illustrate the method on two examples: the untyped lambda calculus and quantified propositional logic. For each language, we can define recursion and induction principles that work directly on the higher-order syntax.

Work in progress consists in generalizing the tool to provide an implementation of higher-order universal algebra in type theory. It will allow to specify multi-sorted algebras with binding operations and automatically obtain algebras of terms over it together with higher-order recursion and induction principles. We plan to apply the method to the formalization of derivation systems with hypothetical judgements, using our tool in a two-level approach. We will also tackle the POPLmark challenge to test the practicality of our approach.

► JOHN HARRISON, *Taking theorem proving mainstream*.
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In recent years, theorem proving has made notable progress, with many impressive new applications both in formal verification and the formalization of pure mathematics. However, theorem proving is still the preserve of specialists, and has a reputation, not entirely undeserved, for being difficult and painful. We will discuss how we might bring forward the day when theorem proving becomes a mainstream activity for a much larger community of mathematicians and computer scientists.

► MARTIN HOFMANN, *A proof system for the linear time mu-calculus*.
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We present a simple sound and complete infinitary proof system for the linear time mu-calculus and then give two decision procedures for provability in the system, hence validity of formulas. One uses nondeterministic Büchi automata, the other one a generalisation of size-change termination analysis (SCT) known from functional programming.

This is joint work with Christian Dax and Martin Lange.

► BRANIMIR LAMBOV, *Exact and efficient computation over the real numbers*.
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Floating point numbers have been used for a long time as the simple solution to the problem of efficiently using real numbers in computers. Most of the time they give us meaningful results, but since they such a crude representations of the theoretical objects they are used to replace, floating point computations are prone to cause difficult to detect problems.

As computers are becoming more and more powerful, it becomes increasingly feasible to use exact real computations, where real numbers are given by objects that are a mathematically precise representation of the real numbers and the functions that operate on them do not try to approximate the results, but exactly represent the operation of interest. Implementations of the concept exist, but they are very rarely used in practice, and even specialized tools such as Mathematica or MatLab do not offer this machinery.

This presentation will demonstrate an implementation of exact real arithmetic in C++ which attempts to solve the main problems that still make exact arithmetic unsuitable for wide use: the performance of exact computations and their ease of use. To make exact computations easy to use, one has to construct libraries in such a way that the behaviour of the objects that represent real numbers differs as little as possible from the behaviour of the floating point types available in the language. On the other hand, to make them efficient, one needs to introduce some unconventional behavour in (at least parts of) the user's programs. We will describe an implementation of an approach used to combine two different approaches to exact computability in one tool, where the user code has two levels of access to real numbers each complete with its own exact representation for every function provided by the library or implemented by the user, and which has all necessary machinery to easily permit very fast exact computations in an easy to use environment.

► ANDREW PITTS, *Names, equations and freshness*.
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Although equational logic is perhaps a little tame compared with other branches of mathematical logic, it has proved very useful to several areas of computer science—especially in its "many-sorted" form. Here I focus on meta-logic and meta-programming, where sorts may represent the various syntactical data structures associated with a formal language. The syntax of such "object-level" languages, be they programming languages or formal logics, nearly always involves various kinds of name and various constructs that bind names to a particular scope. When describing the properties of such languages, we very often employ notions of "freshness" of a name with respect to a structure. Nominal Logic [***Theoretical Computer Science***, vol. 323 (2004), pp. 473–497] is a meta-logic that axiomatises the notion of freshness of names within first-oder logic. Recently Murdoch Gabbay and Aad Mathijssen have studied "nominal algebra". It is, roughly speaking, an equational part of nominal logic, in which one considers both equations and "freshnesses", conditioned by freshness assumptions. It seems to provide a natural way of expressing many meta-theoretical properties of syntax to do with meta-variables, substitution and quantification.

In this talk I will give a simplified presentation of this kind of nominal equational logic, emphasising its relationship to ordinary, many-sorted equational logic. The main result is a completeness theorem for nominal equational theories with respect to the nominal sets model (also known as the Schanuel topos). Unlike the completeness theorem for ordinary equational logic, this is a non-trivial result that sheds some light of the role of explicit versus implicit parameters in meta-mathematics.

This is joint work with Ranald Clouston.

Abstracts of invited talks in the Special Session on Model Theory

► RAF CLUCKERS, *b-minimality*.
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We present joint work with F. Loeser on a new variant on the notion of minimality, generalizing o-minimality, p-minimaliy, and other notions. It is suited for all Henselian valued fields. In fact, the only ingredients are a notion of balls and points, the possibility of having auxiliary sorts, and some axioms using these notions. There is a plentitude of examples and there are promising expansions by nontrivial entire analytic functions. There is a dimension theory and Euler characteristics of the main sort relative to the auxiliary sorts can be studied. Henselian valued fields of char 0 are b-minimal in a natural language. Henselian valued fields in characteristic p are not b-minimal in this language: some expansion is needed but yet unknown.

This is joint work with F. Loeser.

► CLIFTON EALY, *Equivalence relations and the existence of a notion of independence*.
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I will isolate several properties that one may reasonably demand of a notion of independence in models of a theory T. We show that the only obstacle to the existence of a notion satisfying these properties is the interpretability in models of T of arbitrarily long chains of equivalence relations. Alternatively, one may view this result as a characterization of non-rosiness in terms of equivalence relations.

This is joint work with Alf Onshuus.

► ASSAF HASSON, *Strongly minimal sets definable in expansions of RCF.*
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We prove the following

THEOREM. *Let $\mathcal{R}$ be an o-minimal expansion of a real closed field R. Let $f : R^2 \to R^2$ be a $\mathcal{R}$-definable function such that the reduct $\mathbb{C}_f := (R^2, +, f)$ is strongly minimal and not locally modular. There exists $A \in \mathrm{GL}_2(\mathbb{R})$ such that $\mathbb{C}_{A\circ f\circ A}$ is interdefinable with $(R(\sqrt{-1}), +, \cdot)$. In particular, up to conjugation by a real matrix, f is a constructible function.*

This is joint work with P. Kowalski.

► PIOTR KOWALSKI, *Ax Theorem in arbitrary characteristic*.
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Recall a theorem of Ax, which is a differential version of the Schanuel Conjecture.

THEOREM. *Let (K, ∂) be a differential field of characteristic 0 and $C = \partial^{-1}(0)$ its field of constants. Assume $x, y \in K^n$ satisfy*

$$\partial(x_1) = \frac{\partial(y_1)}{y_1}, \dots, \partial(x_n) = \frac{\partial(y_n)}{y_n}$$

and $\partial(x_1), \dots, \partial(x_n)$ are linearly independent over $\mathbb{Q}$. Then

$$\mathrm{trdeg}_C(x, y) \geqslant n + 1.$$

The above statement clearly does not hold when K has positive characteristic, since K is algebraic over C then. If we replace a derivation ∂ with a Hasse derivation D, then K is not algebraic over C anymore, but there are still problems caused by the lack of the exponential function.

Nevertheless, we formulate a characteristic-free version of the Ax Theorem in terms of Hasse derivations and logarithmic derivatives of commutative algebraic groups and discuss its connections with the Conjecture on the Intersection of Tori.

► SONIA L'INNOCENTE, *Model theory and Lie algebras: infinite dimensional representations over $sl_2(K)$*.
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This report aims at illustrating the main results of two joint works, the former with Mike Prest [3] and the latter with Ivo Herzog [2], both devoted to a model theoretic investigation of certain simple infinite dimensional representations over $sl_2(K)$-the Lie algebra of trace zero 2×2 matrices over an algebraically closed field K of characteristic zero—and inspired by Herzog's paper [1] showing several remarkable results about the ring of definable scalars of finite-dimensional representations of $sl_2(K)$.

In detail, we deal with

- Verma modules,
- pseudo-finite dimensional representations of $sl_2(K)$

(meaning infinite dimensional representations satisfying the same first order sentences as finite dimensional representations of $sl_2(K)$).

First we examine the ring of definable scalars of a Verma module and, among other things, we show that it is Von Neumann regular; also, we obtain some information on the Ziegler spectrum of a Verma module.

Then we consider a pseudo-finite dimensional representation V over $sl_2(K)$. It is known that its endomorphism ring is a field extension K_V of K, so we want to characterize all elements of this field K_V. Here some intriguing connections with exponentiation arise. We also discuss some open problems in this framework.

[1] I. HERZOG, *The pseudo-finite dimensional representations of $sl(2,k)$*, ***Selecta Mathematica***, vol. 7 (2001), pp. 241–290.

[2] S. L'INNOCENTE and I. HERZOG, *Pseudo-finite dimensional representation over $sl_2(K)$ and exponentiation*, in progress.

[3] S. L'INNOCENTE and M. PREST, *Rings of definable scalars of Verma modules*, submitted.

► TIM MELLOR, *An Euler characteristic for real closed valued fields*.
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We highlight some ideas in the proof that real closed valued fields admit a strong Euler characteristic. One utilizes a link between the definable sets of a real closed valued field and the semi algebraic sets of an overfield. This allows the use of theorems from o-minimality.

Abstracts of invited talks in the Special Session on Proof Theory and Type Theory

► KLAUS AEHLIG, *Non-interleaved polymorphic types and iterated inductive definitions*.
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Consider the following restriction of the polymorphically typed lambda calculus ("System F"). All quantifications are parameter free. In other words, in every universal type $\forall\alpha.\tau$, the quantified variable α is the only free variable in the scope τ of the quantification. This fragment characterises precisely the provably recursive functions of finitely iterated inductive definitions.

► ANDREY BOVYKIN, *New results on unprovability and logical strength*.
Department of Computer Science, University of Liverpool, Liverpool, L69 7ZF, UK.
St. Petersburg Department of Steklov Mathematical Institute, Fontanka 27, St. Petersburg, Russia.
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I shall present recent results on unprovability in Peano Arithmetic and on logical strength of infinitary ramseyan statements. In particular I shall discuss

- model-theoretic treatment of Infinitary Canonical Ramsey Theorem and Regressive Ramsey Theorem, using strong cuts in models of arithmetic;
- a reduction of logical strength of Ramsey Theorem for pairs to logical strength of some first-order theory (the density scheme) by a model-theoretic construction;
- an unprovable statement about the sine-function and a method of encoding Ramsey-theoretic statements in the language of dynamical systems;
- $I\Sigma_2$-unprovable statements about braids;
- exact unprovability results (threshold results).

► NICOLA GAMBINO, *Sheaf models for constructive set theories*.
Laboratoire de Combinatoire et d'Informatique Mathématique, Université du Québec à Montréal, Case Postale 8888 – Succarsale Centre-Ville, Montréal (Québec) H3C 3P8, Canada.
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Constructive Zermelo-Frankel set theory (CZF) was introduced by Peter Aczel in order to provide a set-theoretical foundation for the development of Bishop's constructive mathematics.

I will describe sheaf models for CZF. These models provide a general tool to prove relative consistency and independence results, analogous to the classical method of forcing. Furthermore, I will show how these models fit into the framework of Joyal and Moerdijk's Algebraic Set Theory.

► JOOST JOOSTEN, *Lowerbounds in proof complexity*.
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E-mail: Joost.Joosten@phil.uu.nl. *URL*: http://www.phil.uu.nl/~jjoosten/.

There exist various relations between lower bounds on lengths of proofs of a sequence of propositional tautologies and seperations of computational complexity classes. In this talk we shall see some non-trivial examples and some results on the lowerbounds involved. In particular, we shall see a relation where propsitional encodings of finite consistency statements play a crucial role.

The author was partly supported by the Netherlands Organisation for Scientific Research (NWO).

► THOMAS STUDER, *Cut-free systems for the propositional modal μ-calculus*.
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The propositional modal μ-calculus extends standard (multi-)modal propositional logic by operators for least and greatest fixed points of positive operators. We begin with setting up a very natural infinitary calculus for the propositional modal μ-calculus which we collapse to a finite cut-free system afterwards. We show soundness and completeness of both axiomatizations. Completeness is established by constructing a canonical counter-model to any non-provable formula using the method of saturated sequents. Soundness is a consequence of the small model property and well-known results about monotone operators.

This is joint work with Gerhard Jäger and Mathis Kretz.

► HENRY TOWSNER, *Ordinal analysis of recursive weak compactness*.
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We outline an approach to the ordinal analysis of systems at the strength of a recursively weakly compact ordinal (usually characterized as Kripke-Platek set theory with reflection on Π_3 formulas) using an ordinal notation system derived from the equivalent notion of 2-admissibility. The result is an ordinal notation system based on a notion of collapsing functions to ordinals and higher order infinitary rules whose premises range over all suitable transformations on derivations.

Abstracts of invited talks in the Special Session on Set Theory

► NATASHA DOBRINEN, *Co-stationarity of the ground model, internal consistency, and new sequences*.
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All but the very last portion of this talk is joint work with Sy-David Friedman. Given $V \subseteq W$ models of ZFC with the same ordinals and $\kappa < \lambda$ cardinals in W with κ regular, let $\mathcal{P}_\kappa(\lambda)$ denote the collection of subsets of λ of size less than κ in W. We say that the ground model is *co-stationary* if $\mathcal{P}_\kappa(\lambda) \setminus V$ is stationary in $\mathcal{P}_\kappa(\lambda)$. Gitik showed that if κ is a regular cardinal in W, $\lambda \geq (\kappa^+)^W$, and there is a real in $W \setminus V$, then the ground model is co-stationary in $\mathcal{P}_\kappa(\lambda)$.

We consider problems of generalizing Gitik's Theorem to forcing extensions in which no reals are added. Let $(^*)$ denote "If $\mathbb{P}$ is an $\aleph_2$-c.c. forcing which adds a new subset of $\aleph_1$ (but no new ω-sequences), then for all $\lambda > \kappa \geq \aleph_2$ with κ regular, $\mathcal{P}_\kappa(\lambda) \setminus V$ is stationary in $V^{\mathbb{P}}$". $(^*)$ is equiconsistent with a proper class of ω_1-Erdös cardinals. For regular $\kappa \geq \aleph_2$ with $\aleph_\kappa > \kappa$, the co-stationarity of the ground model in the $\mathcal{P}_{\kappa^+}(\aleph_\kappa)$ of a κ-Cohen forcing extension is equiconsistent with κ measurable cardinals.

Using large cardinal hypotheses past a superstrong, we prove that $(^*)$ holds in some inner model. From the same methods, we can obtain a model of $(^*)$ in which there are class many measurable cardinals.

A related problem is the following: Suppose $\mathbb{P}$ adds a new ν-sequence but no new subsets of ν. When does $\mathbb{P}$ make the ground model co-stationary? For $\nu \geq \omega_1$, some consistency results are provided. Some work due to the speaker on the case when $\nu = \omega$ will also be included.

► JOHN KRUEGER, *Some consistency results related to internal approachability*.
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E-mail: `jkrueger@logic.univie.ac.at`.

I will sketch a proof that under suitable forcing axioms, various weakenings of internal approachability, namely internally unbounded, internally stationary, and internally club, are not equivalent.

► PAUL B. LARSON, *Martin's Maximum and definability in $H(\aleph_2)$*.
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We will discuss a coding method invented by Woodin which has been used to solve several problems in the study of $H(\aleph_2)$, the collection of sets whose transitive closures have cardinality $\aleph_1$. In a paper appearing in 2000, we used this method to show that from a supercompact limit of supercompact cardinals one could force Martin's Maximum to hold while the $\mathbf{P}_{max}$ axiom $(^*)$ fails. Recently we modified that argument to prove a stronger fact, that Martin's Maximum is consistent with the existence of a wellordering of the reals definable in $H(\aleph_2)$ without parameters, from the same large cardinal hypothesis. In doing so we give a much simpler proof of the original result.

► JORDI LOPEZ-ABAD, *Partial unconditionality and barriers*.
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We survey a combinatorial framework for studying subsequences of a given sequence in a Banach space, with particular emphasis on weakly-null sequences. We base our presentation on the crucial notion of barrier introduced long time ago by Nash-Williams. In fact, one of the purposes of this work is to isolate the importance of studying mappings defined on barriersas a crucial step towards solving a given problem that involves sequences in Banach spaces. We focus our study on various forms of "partial unconditionality" present in arbitrary weakly-null sequences in Banach spaces. We give a general notion of partial unconditionality that covers most of the known cases such as, for example, Elton's near unconditionality, convex unconditionality, and Schreier unconditionality, but we also add some new cases.

This is joint work with S. Todorcevic.

► CHRISTIAN ROSENDAL, *Classifying Borel transformations in Kakutani style*.
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A classical problem of dynamics and ergodic theory dating back to Poincaré is to classify one-parameter flows up to some suitable notion of time change. One of the main tools in the ergodic theory part of this theory is a reduction of the problem to the notion of Kakutani equivalence of measure preserving automorphisms. In recent decades there has been an attempt to transfer the problems and techniques of ergodic theory into a purely descriptive setting and we will completely answer the analogue of Poincaré's question for Borel measurable one-parameter flows. We also present results on the much harder problem of classifying not necessarily injective Borel transformations up to a notion of Kakutani equivalence.

This is joint work with Benjamin D. Miller of UCLA.

Abstracts of contributed talks

► ANDREW ABERDEIN, *Combining Toulmin layouts*.
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Stephen Toulmin's 'layout of arguments' can represent deductive inference, but also encompasses defeasible reasoning [1]. Its simplest form is a derivation of claim C, from data D, in accordance with warrant W. The layout is usually represented graphically, but may be thought of as a triple, $\langle D, W, C\rangle$. The greater generality is made explicit by the additional elements: a further triple $\langle B, Q, R\rangle$ associated with the warrant. The warrant is justified by appeal to backing B, possible exceptions or rebuttals R are accommodated, and qualifier Q expresses the resultant strength. Hence the full layout may be understood as 'Given that D, we can Q claim that C, since W (on account of B), unless R'. We shall write this as $\langle D, W\langle B, Q, R\rangle, C\rangle$, where each component represents a set of propositions, except Q, a single term.

A significant limitation of this approach is that every argument is treated as a single step, although the fine-grained analysis of complex arguments would require the combination of multiple steps. We propose treating networks of layouts $\langle D_i, W_i\langle B_i, Q_i, R_i\rangle, C_i\rangle$ as digraphs in which D_i and C_i label vertices and W_i label edges. We show that any such network is reducible to a single layout, defined as follows:

$$\left\langle \bigcup_{\mathrm{in}(D_i)=0} D_i, \bigcup_i W_i \left\langle \bigcup_i B_i, \mathrm{lub} \bigcup_i Q_i, \bigvee_i R_i \right\rangle, \bigcup_{\mathrm{out}(C_i)=0} C_i \right\rangle$$

(Presuming that qualifiers may be partially ordered by strength, and that every pair has a least upper bound.) We also show how this account may be extended to include layouts as data (or claims) in other layouts.

[1] STEPHEN TOULMIN, ***The uses of argument***, Cambridge University Press, 1958.

► ARNON AVRON, JONATHAN BEN-NAIM, AND BEATA KONIKOWSKA, *Information sources logic*.
School of Computer Science, Tel-Aviv University, Tel-Aviv, Israel.
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We consider a framework for information collecting and processing which consists of a set of sources providing information about formulas, and a processor. For *each* formula φ (not only an atomic one, like in [3]!), a source can say that φ is true (1), or false (0), or that it knows nothing about φ. The processor collects information from the sources, and combines it using some strategy to form a new valuation v_P. This valuation uses four logical truth values, corresponding to the four possible sets of the classical values provided by the sources: $\mathbf{t} = \{1\}$ (truth), $\mathbf{f} = \{0\}$ (falsity), $\top = \{0, 1\}$ (inconsistency), $\bot = \emptyset$(uncertainty). This framework can give rise to many scenarios. We explore the most basic cases, e.g., the one in which the only assumption concerning v_P is that the processor observes the most general rules compliant with the properties of classical connectives, like assigning 0 to $\varphi \vee \psi$ iff it assigns 0 to both φ and ψ, and 1 if (but not only if) it assigns 1 to either of them. We show that in this case the resulting logic corresponds to a four-valued logical *non-deterministic* matrix [2, 1]. Other cases lead to other non-deterministic matrices. We provide complete, cut-free sequent calculi for the various logics we get, using a general method we have developed.

[1] A. Avron and B. Konikowska, *Multi-valued calculi for logics based on non-determinism*, ***Journal of the IGPL***, vol. 10 (2005), pp. 365–387.

[2] A. Avron and I. Lev, *Non-deterministic multiple-valued structures*, ***Journal of Logic and Computation***, vol. 15 (2005), pp. 241–261.

[3] N. D. Belnap, *A useful four-valued logic*, ***Modern uses of multiple-valued logic***, (G. Epstein and J. M. Dunn, editors), pp. 7–37, Reidel, 1977.

► STEVE AWODEY, *Predicative Algebraic Set Theory*.
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The basic results of Algebraic Set Theory are extended to the case of constructive set theories. Specifically, we identify a constructive set theory CST that is sound and complete with respect to a specified class of algebraic models. Moreover, we show how such models can be constructed in the category of "ideals" (special sheaves) over systems of constructive type theory.

This is joint work with Michael Warren.

► I. I. BATYRSHIN, *Quasi-reducibility and functions without fixed-points*.
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Shoenfield [1] defined the quasi-reducibility of sets: $A \leq_Q B$ if there is a computable function f such that $x \in A \Leftrightarrow W_{f(x)} \subseteq B$. Arslanov (see, for example, [2]) proved the following criterion for the T-reducibility: a c.e. set X is T-complete iff there is a Turing computable in X function f without fixed-points, i.e., $(\forall x)(W_x \neq W_{f(x)})$. We prove a similar criterion for the quasi-reducibility and use this criterion to obtain the quasi-completeness of Chaitin Ω numbers.

We say that a function f is quasi-reducible to a set A ($f \leq_Q A$), if there are computable functions a, b and g such that

$$f(x) = \begin{cases} a(x), & \text{if } W_{g(x)} \subseteq A, \\ b(x), & \text{if } W_{g(x)} \nsubseteq A. \end{cases}$$

Denote the set $B_{f,A} = \{x | W_{g(x)} \subseteq A\}$.

Theorem 1. For each set $A \subseteq \omega$ following conditions are equivalent:

(1) All c.e. sets are Q-reducible to the set A,

(2) There is a function $f \leq_Q A$ without fixed-points, i.e., $(\forall x)(W_x \neq W_{f(x)})$, and $B_{f,A}$ is a c.e. set.

Fix a universal Chaitin computer U, denote by Σ^* the set of all finite binary strings and for arbitrary $x \in \Sigma^*$ denote by $H(x)$ the program-size (Chaitin) complexity induced by Chaitin's computer U. The halting probability of U is denoted by Ω.

We say that a set C is quasi-reducible to the real Ω ($B \leq_Q \Omega$) if $C \leq_Q A$, where $A = \{q \in (0,1) \cap Q \mid q < \Omega\}$.

THEOREM 2. The set $\mathcal{H} = \{(x, n) \mid x \in \Sigma^*, n \in \omega, H(x) \leq n\}$ is Q-complete.

THEOREM 3. The set $\mathcal{H}$ is Q-reducible to Ω.

[1] J. R. SHOENFIELD, *Quasicreative sets*, ***Proceedings of the American Mathematical Society***, vol. 8 (1957), pp. 964–967.

[2] R. I. SOARE, ***Recursively enumersable sets and degrees***, Springer-Verlag, Berlin, 1987.

► LIBOR BĚHOUNEK, *An alternative justification of the axioms of fuzzy logics*.
Academy of Sciences of the Czech Republic, Institute of Computer Science, Pod Vodárenskou věží 2, 182 07 Prague 8, Czech Republic and Charles University in Prague, Faculty of Arts, Department of Logic, Celetná 20, 116 42 Prague 1, Czech Republic.
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Fuzzy logics are usually introduced by imposing some reasonable constraints on the [0, 1] truth functions of logical connectives, which lead to the notion of (left-)continuous t-norm [3]. However, the Hilbert-style calculi and algebraic semantics of the resulting logics (MTL, BL, Łukasiewicz logic, Gödel logic, etc.) make no reference to the [0, 1] interval, and their completeness w.r.t. the real-valued semantics is often hard to prove. Thus it is reasonable to search for a [0, 1]-independent justification of the axioms of these logics.

The present talk gives a justification which is based on two kinds of fuzzy consequence relations, viz. the "global" (full-truth preserving) one and the "local" (partial-truth preserving) one. The account proceeds in four steps of formalizing their pre-theoretical properties: (i) the intuitive properties of full-truth preservation are those of Tarski structural consequence relations; (ii) the properties of full partial-truth preservation lead to Cintula's class of weakly implicative fuzzy logics [2]; (iii) the properties of partially valid partial-truth preservation, internalized by implication, generate the implicational fragment of MTL (possibly fine-tuned by additional assumptions); finally (iv) by adding further propositional connectives we arrive at usual fuzzy logics.

This account clarifies the position of fuzzy logics in the logical landscape: they can be characterized as intuitionistic non-contractive prelinear logics: intuitionistic and non-contractive, since the laws of double negation and contraction are often inappropriate for fuzzy propositions; and prelinear, since the property of prelinearity corresponds to "measuring" truth along linear scales, which is the distinctive feature of fuzzy logics [1].

[1] L. BĚHOUNEK and P. CINTULA, *Fuzzy logics as the logics of chains*, ***Fuzzy Sets and Systems***, vol. 157 (2006), pp. 604–610.

[2] P. CINTULA, *Weakly implicative (fuzzy) logics*, To appear in ***Archive for Mathematical Logic***.

[3] P. HÁJEK, ***Metamathematics of fuzzy logic***, Kluwer, Dordrecht 1998.

► JOSEF BERGER, *A fan-theoretic equivalent to the antithesis of Specker's theorem*.
Department of Mathematics and Statistics, University of Canterbury, Private Bag 4800, Christchurch, New Zealand.
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We give a classification of the statement "every sequence of real numbers bounded away from each point of [0, 1] is eventually bounded away from [0, 1]" in constructive reverse mathematics.

This is a joint result with Douglas Bridges.

► MARTA BÍLKOVÁ, *Uniform interpolation in provability logics*.
Department of Logic, Faculty of Philosophy and Arts, Charles University, Celetná 20, 116 42 Prague 1, Czech Republic and Institute of Computer Science, Academy of Sciences of the Czech Republic, Pod Vodàrenskou věží 2, 182 07 Prague 8, Czech Republic.
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The uniform interpolation property is a strengthening of the Craig interpolation property. It states that for every formula A and any choice of its propositional variables there exists a post-interpolant I in these variables, such that it serves as an uniform interpolant for any formula B implied by A. Symetrically, for any B there exists a pre-interpolant. The existence of such interpolants can be seen as a simulation of certain quantifiers ranging over propositional variables.

We investigate uniform interpolation property in propositional modal logics from the proof-theoretical point of view. We prove the uniform interpolation theorem for two modal logics having arithmetical interpretation—GL and Grz. So far, only proofs based on semantics have been presented. Our proof uses terminating sequent calculi for which structural rules are admissible. It provides an explicit algorithm constructing the interpolants.

► PIOTR BORODULIN-NADZIEJA, *Measures on small Boolean algebras*.
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We investigate measures on small Boolean algebras, i.e., Boolean algebras not containing an uncountable independent sequence. We prove that most well-known subclasses of small Boolean algebras (e.g., interval, superatomic) admit only separable measures. It is not the case of retractive algebras: under certain set theoretic axioms every retractive algebra carries only separable measures but it is also consistent to assume that there is a retractive Boolean algebra without this property. We exhibit some connections to topology, e.g., the existence of tree π-bases.

► OLIVIA BREYSSE AND MICHEL DE GLAS, *On boundaries and contact*.
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Mereotopology, which results from the adjunction to mereology of topology, aims at providing a formal framework for the analysis of spatial and spatiotemporal entities that exhibit a topological structure. However, one may show that most of the various mereotopologies cannot deal conjointly with the concepts of boundary, continuity and contact. Although eminently topological in nature and intimately interrelated—the discontinuous nature of boundaries presupposes continuity and contact occurs at boundaries—these three concepts are mutually incompatible in a mereotopology. We actually prove that the so˛called boundary˛based mereotopological theories are bound to be either paradoxical or inconsistent and that this is essentially due to the modelling of some basic intuitive notions by the means of topology. We show that substituting locology for topology leads to a satisfactory solution to the above˛mentioned problems and provides the tools for an alternative to mereotopology.

► ANDREW BROOKE-TAYLOR, *Forcing while preserving 1-extendibles*.
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Given a universe containing a 1-extendible cardinal, one may force the global square principle while keeping the cardinal 1-extendible, whereas it is known that global square is incompatible with subcompact cardinals, which are only slightly stronger. The question arises as to whether one may force other combinatorial principles while preserving a 1-extendible cardinal. In certain cases, such as when forcing a morass or breaking GCH, there are subtleties that arise because the domain of the defining embedding for the 1-extendible is only an initial segment of V. We will present partial results regarding overcoming these difficulties.

► M. W. BUNDER AND W. J. M. DEKKERS, *Are there Hilbert-style pure type systems*?
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For many a natural deduction style logic there is a Hilbert-style logic that is equivalent to it in that it has the same theorems (i.e., valid judgements $\Gamma|\!\!-\!\!-P$ where $\Gamma = \emptyset$). For intuitionistic implicational logic, the axioms of the equivalent Hilbert-style logic can be propositions which are also known as the types of the combinators **I**, **K** and **S**.

Natural deduction versions of illative combinatory logics have an equivalent formulation with axioms that are actual type statements for **I**, **K** and **S**. As pure type systems (PTSs) are, in a sense, equivalent to systems of illative combinatory logic, it might be thought that Hilbert style PTSs (HPTSs) could be based in a similar way.

This paper shows that some PTSs have very trivial equivalent HPTSs, with only the axioms as theorems and that for many PTSs no equivalent HPTSs can exist. Most commonly used PTSs belong to these two classes.

For some PTSs however, including λ^* and the PTS at the basis of the proof assistant Coq, there is a nontrivial equivalent HPTS, with axioms that are type statements for **I**, **K** and **S**.

► PAOLA CATTABRIGA, *How to release Frege's system from Russell's antinomy*.
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The conditions for proper definitions in mathematics are given, in terms of the theory of definition, on the basis of the criterions of eliminability and non-creativity. As a definition, Russell's antinomy is a violation of the criterion of eliminability (Behmann, 1931; Bochvar, 1943). Following the path of the criterion of non-creativity, this paper develops a new analysis of Comprehension schema and, as a consequence, proof that Russell's antinomy argumentation, despite the words of Frege himself, does not hold in Grundgesetze der Arithmetik. According to Basic Law (III), the class of classes not belonging to themselves is a class defined by a function which can not take as argument its own course of value,

$$\top\left(\forall = \grave{\varepsilon}\left(\top\!\!\overset{\mathfrak{g}}{\smile}\!-\left(\grave{\varepsilon}(-\!\!-\mathfrak{g}(\varepsilon)) = \varepsilon \ \rightarrow \mathfrak{g}(\varepsilon)\right)\right)\right),$$

in other words, the class of classes not belonging to themselves is a class whose classes are not identical to the class itself.

► C. DIMITRACOPOULOS AND Z. SIROKOFSKICH, *On a problem of J. Paris*.
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A question asked by J. Paris (see Problem 34 in the list in [1]) is whether or not Δ_n induction implies Σ_n collection. We sketch alternative proofs of results of T. Slaman [3] and N. Thapen

[4] concerning this problem, especially for the case $n = 1$. Our proofs depend on results of C. Dimitracopoulos and J. Paris [2] concerning relationships between Σ_n collection and (versions of) Σ_n pigeonhole principle.

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[1] P. CLOTE AND J. KRAJÍČEK, *Open problems*, ***Arithmetic, proof theory, and computational complexity*** (***Prague, 1991***), Oxford Logic Guides, vol. 23, pp. 1–19, Oxford University Press, New York, 1993.

[2] C. DIMITRACOPOULOS AND J. PARIS, *The pigeonhole principle and fragments of arithmetic*, ***Zeitschrift für mathematische Logik und Grundlagen der Mathematik***, vol. 32 (1986), pp. 73–80.

[3] T. A. SLAMAN, *Σ_n-bounding and Δ_n-induction*, ***Proceedings of the American Mathematical Society***, vol. 132 (2004), pp. 2449–2456.

[4] N. THAPEN, *A note on Δ_1 induction and Σ_1 collection*, ***Fundamenta Mathematicae***, vol. 186 (2005), no. 1, pp. 79–84.

► IOANNA MATILDE DIMITRIOU AND BENEDIKT LÖWE, *Inaccessibility without choice*.

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In ZFC, the concept of a cardinal being inaccessible, i.e, a regular strong limit, is well studied and also well defined. The axiom of choice is essential when we define a strong limit cardinal in the usual way, i.e.,

$$\kappa \text{ is a strong limit} \iff \forall\lambda < \kappa(2^\lambda < \kappa)$$

where we read "$2^\lambda < \kappa$" as "some (any) ordinal in bijection with the powerset of λ is smaller than κ". In the ZFC context, this ordinal always exists, if 2^λ is not wellorderable, it may not. As it turns out, this definition is equivalent in ZFC to four other definitions (where $<$ is replaced by relations $<_\mathbf{s}$, $<_\mathbf{i}$, $<_{\bar{\mathbf{s}}}$, $<_{\bar{\mathbf{i}}}$) that are more appropriate for an investigation without the axiom of choice. These relations are defined as follows:

$X <_\mathbf{i} \alpha$ if there is some $\beta < \alpha$ and an injection from X into β,
$X <_\mathbf{s} \alpha$ if there is some $\beta < \alpha$ and a surjection from β onto X,
$X <_{\bar{\mathbf{s}}} Y$ if there is no surjection from X onto Y, and
$X <_{\bar{\mathbf{i}}} Y$ if there is no injection from Y into X.

Using these four relations, we give four definitions for inaccessibility, $\mathbf{x}$-inaccessibility for $\mathbf{x} \in \{\mathbf{i}, \mathbf{s}, \bar{\mathbf{s}}, \bar{\mathbf{i}}\}$. We show that the implications between these inaccessibilities are as follows:

$$\mathbf{i} \Leftrightarrow \mathbf{s} \Rightarrow \bar{\mathbf{s}} \Rightarrow \bar{\mathbf{i}},$$

and by using the technique of taking symmetric submodels of forcing extensions we show that in ZF the one way arrows cannot be reversed. One of the nonreversal directions was proved by Andreas Blass.

► HENRIK FORSSELL, *Ideal models of algebraic set theory*.

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We construct algebraic models of intuitionistic set theory in categories of "ideals", which are special sheaves. First, a Basic Intuitionistic Set Theory (BIST) is stated, and the categorical semantics are given. Second, a notion of an ideal over a topos is given, using which one can build a model of BIST in which a given topos occurs as the sets. Thinking of topoi as higher-order theories, this amounts to constructing, for any higher-order theory, a set theoretic universe incorporating the types of that theory as sets, in such a way that the set theory of the universe is a conservative extension of the higher-order theory we started out with. The set theory BIST is shown to be complete with respect to such universes. The approach extends the results of Awodey, Butz, Simpson, and Streicher by introducing a new and perhaps more natural notion of ideal.

► MICHELE FRIEND, *The relationship between the practice of mathematics and the philosophy of mathematics*: *or, should mathematicians listen to philosophers at all*?
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As rational agents we are sometimes forced to accept a conclusion which does not agree with us, simply because this is where the argument leads us. Much to my chagrin, I have to conclude that a practicing mathematician should not heed the advice of the philosopher too much. Of course, this is not to say that the same person cannot engage in philosophical discussion, and practice mathematics.

What leads to this conclusion? Let us begin with the simplistic construal of the relationship between mathematicians and philosophers. Either philosophers dictate to mathematicians or they listen to mathematicians. In the first the philosopher prescribes mathematical practice, in the second the philosopher describes.

These simplistic ways of construing the relationship make the relationship unstable, but each relationship is unstable for different reasons. The first is unstable because it varies with the vagaries and findings of philosophy. The second is unstable because the compass of mathematical practice varies; so, we might say that the subject of description changes. The problem is that the instability of the relationship is enough to suggest to the mathematician that he, or she, not pay attention to the philosopher.

There are more complicated relationships between practicing mathematicians and philosophers of mathematics. The more complex relationship is characterised by a mix of description and prescription. The fruitful complex relationship is piecemeal and local.

In conclusion, the best we might say is that the relationship between the mathematician and the philosopher does exist, and can locally lead to changes and modifications in both mathematics and philosophy. However, unfortunately, a wide-spread and stable relationship is not in the offing. In general, the mathematician should not pay too much attention to the philosopher.

► GUNTER FUCHS, *Degrees of rigidity for Souslin trees and changing the heights of automorphism towers*.
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Various strong notions of rigidity for Souslin trees are investigated and separated, assuming the diamond principle, into a hierarchy. Most of these rigidity properties state that a tree has a certain rigidity property in any model obtained by forcing with the tree itself.

An application to the automorphism tower problem is given, showing that, again assuming diamond, there is a group the height of the automorphism tower of which is highly malleable by forcing with certain Souslin trees. Carrying out the construction at higher cardinality levels gives the full statement on changing the heights of automorphism towers, that was realized by Hamkins and Thomas using proper class forcing, but this time in L, the minimal inner model of set theory: For any cardinal κ, and any ordinal $\alpha < \kappa$, there is a centreless group G such that its automorphism tower has height α, but for any β with $1 \leq \beta < \kappa$, there is a notion of forcing P, preserving cofinalities and cardinalities, such that in any forcing extension by P, the height of the automorphism tower of the very same group is β.

This is a joint work with Joel Hamkins.

► NIKOLAOS GALATOS, *Glivenko theorems for substructural logics*.
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Glivenko's theorem states that a formula ϕ is provable in classical propositional logic iff its double negation is provable in intuitionistic logic, in symbols $\vdash_{\mathbf{Cl}} \phi$ iff $\vdash_{\mathbf{Int}} \neg\neg\phi$. Nevertheless, $\vdash_{\mathbf{Int}}$ is not the only relation that has this property relative to $\vdash_{\mathbf{Cl}}$, nor is $\vdash_{\mathbf{Cl}}$ the only relation that admits a double negation interpretation in another logic. For example, it is shown in [1] that, for every formula ϕ, $\vdash_{\mathbf{Cl}} \phi$ iff $\vdash_{\mathbf{SBL}} \neg\neg\phi$, and that, for every formula ϕ, $\vdash_{\mathbf{Lu}} \phi$ iff $\vdash_{\mathbf{BL}} \neg\neg\phi$, where **SBL** is the extension of Hajek basic logic **BL** by the axiom $\psi \wedge \neg\psi = 0$ and **Lu** is Łukasiewicz infinite valued logic.

All the logics discussed so far are substructural logics. In this talk, we extend Glivenko's theorem and discuss pairs of substructural logics for which a double negation translation (in one of four different forms) holds. Moreover, we show that for every involutive substructural logic there exists a minimum substructural logic that contains the first via a double negation interpretation. Our presentation is algebraic (we use the fact that substructural logics over the full Lambek calculus are algebraizable) and is formulated in the context of residuated lattices.

The contents of the talk are part of joint work, [2] and [3], with H. Ono.

[1] R. Cignoli and A. Torrens, *Hájek basic fuzzy logic and Lukasievicz infinite-valued logic*, ***Archive for Mathematical Logic***, vol. 42 (2003), 361–370.

[2] N. Galatos and H. Ono, *Algebraization, parameterized local deduction theorem and interpolation for substructural logics over FL*, ***Studia Logica***, vol. 83 (2006), pp. 279–308.

[3] ———, *Glivenko theorems for substructural logics over FL*, ***The Journal of Symbolic Logic***, to appear.

► GEORGI GEORGIEV AND TINKO TINCHEV, *Monadic second order logic on equivalence relations*.
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We study the MSO logic over various special kinds of structures and find some results concerning its decidability and expressive power. For instance, we consider the structures with only unary predicates and equivalence relations in local agreement.

Let $\mathfrak{A} \leftrightharpoons (|\mathfrak{A}|, P_1, \ldots, P_k, R_1, \ldots, R_n)$ be a structure where $P_1, \ldots P_k$ are unary predicates, and $R_1, \ldots R_n$ are equivalence relations. Let $[x]_{R_i}$ denote the equivalence class of x according to the relation R_i. We say that $R_1, \ldots, R_n$ are in local agreement if for every $x \in |\mathfrak{A}|$ the set $\{[x]_{R_1}, \ldots, [x]_{R_n}\}$ is linearly ordered according to the set theoretic inclusion.

It is a well-known fact, that over the class of structures with one equivalence relation the MSO logic is decidable. We show that it is decidable over the class of all structures with unary predicates and equivalence relations in local agreement. Moreover, we show that over these structures, every MSO formula has a translation in the first order language, which has exactly the same models. The translated FO formula is very complex, compared to the original MSO formula.

► NINA GIERASIMCZUK AND JAKUB SZYMANIK, *Hintikka's Thesis revisited.*
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We discuss Hintikka's Thesis [1] that there exist natural language sentences which require non-linear quantification to express their logical form. For example:

1. Some relative of each villager and some relative of each townsman hate each other.
2. Most of circles and most of squares are connected by a line.

In order to express the meaning of these sentences Hintikka would propose following formulae:

3. $\begin{matrix}\forall x \exists y \\ \forall z \exists w\end{matrix} ((V(x) \wedge T(z)) \Rightarrow (R(x,y) \wedge R(z,w) \wedge H(y,w)))$.
4. $\begin{matrix}\mathsf{MOST}\ x\colon C(x) \\ \mathsf{MOST}\ y\colon S(y)\end{matrix}\ L(x,y)$.

We present idea to identify the logical form of sentences 1–2 with the formulae 5–6, respectively. We call such logical forms "conjunctional readings":

5. $\forall x \exists y \forall z \exists w((V(x) \wedge T(z)) \Rightarrow (R(x,y) \wedge R(z,w) \wedge H(y,w))) \wedge$
$\wedge \forall z \exists w \forall x \exists y (V(x) \wedge T(z)) \Rightarrow (R(x,y) \wedge R(z,w) \wedge H(y,w)))$.
6. $\mathsf{MOST}\ x\ (C(x), \mathsf{MOST}\ y\ (S(y), L(x,y))) \wedge$
$\wedge\ \mathsf{MOST}\ y\ (S(y), \mathsf{MOST}\ x\ (C(x), L(y,x)))$.

Moreover, our empirical research shows what follows. There is no statistically significant preference to interpret Hintikka-like sentences as having conjunctional logical forms. There are statistically significant differences between understanding Hintikka-like sentences with *most* quantifiers and these with proportional quantifiers. The first are more often understood by people as having conjunctional reading, when second are usually treated as branching sentences. This difference demands theoretical explanation.

[1] J. HINTIKKA, *Quantifiers vs. quantification theory*, ***Dialectica***, vol. 27 (1973), pp. 329–358.

► SERGEY S. GONCHAROV, *Some new spectrum of computable models*.
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We will discuss the problems about characterization of spectrums of computable models for **Uncountably Categorical theories**. The basic notions can be find in [1] and [5].

In general, existence of a computable model for an uncountably categorical theory T does not imply that all models of T have computable copies. There are a series of papers about this spectrum $SCM(T) = \{0\}$ [2], $SCM(T) = \{0, 1 \dots, n\}$ [4], $SCM(T_1) = \omega$, and $SCM(T_2) = \omega \bigcup \{\omega\} \setminus \{0\}$ [3], $Spec(T) = \{1\}$ [5].

A. Nies find some upper bound for these spectrums. He proved that for a uncountable categorical theory T $Spec(T) \in \Sigma^0_{\omega+3}$.

THEOREM 1. For any $n \in \omega$ there exists a uncountably categorical theory T_n for which $SCM(T_n) = \{n, n+1, n+2, \dots, \omega\}$.

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[1] YU. L. ERSHOV and S. S. GONCHAROV, *Constructive models*, ***Siberian school of algebra and logic***, vol. 6, Kluwer Academic/Plenum Press, Consultants bureau, New York, 2000.

[2] S. S. GONCHAROV, *Constructive models of ω_1–categorical theories*, ***Mathematical Notes***, vol. 23 (1978), pp. 486–488.

[3] B. KHOUSSAINOV, A. NIES and R. SHORE, *Computable models ot theories with few models*, ***Notre Dame Journal of Formal Logic***, vol. 38 (1997), pp. 165–178.

[4] K. ZH. KUDAIBERGENOV, *Constructivizable models of undecidable theories*, ***Sibirskiĭ Matematicheskiĭ Zhurnal***, vol. 21 (1980), no. 5, pp. 155–158.

[5] A. NIES, *A new spectrum of recursive models*, ***Notre Dame Journal of Formal Logic***, vol. 40 (1999), pp. 307–314.

► ALEX HELLSTEN, *Saturation of the weakly compact ideal*.
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The weakly compact ideal on a weakly compact cardinal is a normal ideal that extends $\mathrm{NS}_\kappa \restriction \mathrm{Reg}$, the non-stationary ideal of κ restricted to regular cardinals. In [1] results on the saturation of $\mathrm{NS}_\kappa \restriction \mathrm{Reg}$ for Mahlo cardinals κ are obtained. We use similar methods for investigating the saturation of the weakly compact ideal.

[1] THOMAS JECH and W. HUGH WOODIN, *Saturation of the closed unbounded filter on the set of regular cardinals*, ***Transactions of the American Mathematical Society***, vol. 292 (1985), no. 1, pp. 345–356.

► DAISUKE IKEGAMI, *Projective absoluteness under Sacks forcing*.
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It is known that the Sacks measurability of every Δ^1_2 set of reals, that of every Σ^1_2 set of reals, and the statement that V has more reals than $L[r]$ for any real r, are all equivalent.

It is shown that the Σ^1_3 absoluteness under Sacks forcing is also equivalent to each of them. Also, Σ^1_3 absoluteness under Sacks forcing is the weakest of all the non-trivial Σ^1_3 absoluteness. We will see these results and their generalization.

[1] TOMEK BARTOSZYŃSKI AND HAIM JUDAH, ***Set theory. On the structure of the real line***, A K Peters Ltd., Wellesley, MA, 1995.

[2] JÖRG BRENDLE, *How small can the set of generics be?*, ***Logic Colloquium 1998*** (***Prague***), Lecture Notes in Logic, vol. 13, Association for Symbolic Logic, Urbana, IL, 2000, pp. 109–126.

[3] JÖRG BRENDLE AND BENEDIKT LÖWE, *Solovay-type characterizations for forcing-algebras*, ***The Journal of Symbolic Logic***, vol. 64 (1999), no. 3, pp. 1307–1323.

[4] HAIM JUDAH AND SAHARON SHELAH, *Δ^1_2-sets of reals*, ***Annals of Pure and Applied Logic***, vol. 42 (1989), no. 3, pp. 207–223.

[5] GERALD E. SACKS, *Forcing with perfect closed sets*, ***Axiomatic Set Theory***, American Mathematical Society, Providence, R.I., 1971, pp. 331–355.

[6] ROBERT M. SOLOVAY, *On the cardinality of Σ^1_2 sets of reals*, ***Foundations of Mathematics*** (***Symposium Commermorating Kurt Gödel, Columbus, Ohio, 1966***), Springer, New York, 1969, pp. 58–73.

[7] W. HUGH. WOODIN, *On the consistency strength of projective uniformization*, ***Proceedings of the Herbrand symposium*** (***Marseilles, 1981***) (***Amsterdam***), Studies in Logic and the Foundations of Mathematics, vol. 107, 1982, pp. 365–384.

► ALEKSANDER IVANOV AND KRZYSZTOF MAJCHER, *Nice enumerations of ω-categorical groups*.
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Nice enumerations of ω-categorical structures have been introduced by G. Ahlbrandt and M. Ziegler as a tool for quasifinite axiomatizability of almost strongly minimal ω-categorical structures. E. Hrushovski has refined this notion as follows. An ordering $\prec$ of a countable structure M of order type ω is an *AZ-enumeration* if for any n and any sequence $(\bar{a}_i : i \in \omega)$ of n-tuples from M there are $i < j$ and some $\prec$-preserving elementary map $f : M \to M$ such that $f(\bar{a}_i) = \bar{a}_j$.

We find an example of an ω-categorical expansion of a vector space over $GF(2)$ without AZ-enumerations. We study ω-categorical groups having nice (and AZ-)enumerations.

► TOM JOHNSTONE, *Weakly compact cardinals made indestructible*.
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Large Cardinals may get destroyed or stay preserved after forcing with a particular poset. Determining which cardinals can be made indestructible by various classes of forcing has been of major interest in modern set theory, in part, because such theorems often provide a way of obtaining new independence results. In 1978 Richard Laver proved that given a supercompact cardinal, it can be made indestructible by a very large class of forcings. While lots of indestructibility research has dealt with cardinals stronger than measurable cardinals, in this talk I will address the issue for weakly compact cardinals and for strongly unfoldable cardinals. Both are consistent with V = L. Weakly compact cardinals resemble miniature measurable cardinals, and strongly unfoldable cardinals resemble miniature supercompact cardinals. Given a strongly unfoldable cardinal kappa, I will provide a forcing extension in which the cardinal is indestructible by all kappa-closed, kappa-proper forcing. In particular, the weak compactness of kappa is indestructible. This result sheds light on the major open question of the consistency strength of a fully indestructible weakly compact cardinal.

► PAWEŁ KAWA AND JANUSZ PAWLIKOWSKI, *Extending Baire property and Lebesgue measure by ω_1 sets*.
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We prove that if ZFC is consistent so is ZFC + "for any σ-algebra $\mathcal{A}$, where $\mathcal{A} \subseteq P(2^\omega)$ is generated by at most ω_1-sets and contains all Borel sets, the Baire property can be extended to $\mathcal{A}$". This is a category analogue of the problem of extending Lebesgue measure to any collection of ω_1 sets.

The results of this paper are a continuation of a work of Carlson and Zakrzewski. We study extensions of Baire Property and Lebesgue Measure in Cohen and Solovay models.

We also sketch how to generalize this result to other cardinals in place of ω_1.

► RUAAN KELLERMAN, *Logical theories of orthogonality structures*.
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The geometric relation of line orthogonality in Euclidean space has a rich and interesting elementary theory that captures a substantial and non-trivial fragment of Euclidean geometry. We investigate elementary theories of classes of line orthogonality structures of dimension $n \geq 2$ and establish some axiomatic and model-theoretic results for them. In particular, we analyze some completeness properties of the axiomatic systems provided, prove that the elementary theory of orthogonality structures for dimension $n \geq 3$ is not countably categorical, and investigate the expressive power of the orthogonality relation when used as a primitive for the formalization of Euclidean geometry.

► VINCENT KIEFTENBELD, *Solovay objects from Blackwell determinacy*.
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Solovay's work of the late 70's showed that the consistency strength of the axiom of real determinacy is considerably higher than that of the axiom of determinacy. We follow the lines of Solovay's analysis and present analoguous theorems for the axiom of (real) Blackwell determinacy.

This is joint work with David de Kloet (Vrije Universiteit) and Benedikt Löwe (Universiteit van Amsterdam).

► BYUNGHAN KIM, *Stable definability and generic relations*.
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Criteria for stable definability/determinability and relationship with CM-triviality are given. Those are used to verify structures equipped with generic relations (over stable structures), pseudo-finite fields, and ACFA are neither stably definable nor determinable.

This is joint work with R. Moosa.

► BYUNGHAN KIM, *On weak dividing*.
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We study the notion of weak dividing introduced by S. Shelah. In particular we prove that weak dividing is symmetric if and only if the theory is stable.

This is joint work with N. Shi.

► ALEXEI KOLESNIKOV, *Generalized types and heir bases in simple theories*.
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The study of multi-dimensional amalgamation properties in simple theories is motivated by the question of whether or not a model of a theory fails to interpret a cycle-free hypergraph. More generally, the question can be restated as whether any finite Morley sequence can be extended to an infinite one.

The notions of generalized types and generalized Morley sequences arise naturally in the study of such amalgamation properties. The notion of an "heir base" model for a finite Morley sequence is an important tool in the study of amalgamation properties.

This is joint work with A. Tsuboi and B. Kim.

► ANTON V. KULAKOV, *Some results on Medvedev degrees of separability*.
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DEFINITIONS.

1. (Medvedev) Any class $\mathcal{A} \subseteq \omega^\omega$ of total functions is called a *mass problem*.

Let $\mathcal{A}$ and $\mathcal{B}$ be mass problems. Then $\mathcal{A} \leqslant_M \mathcal{B}$ if and only if there is a computable functional Φ, such that $(\forall g \in \mathcal{B})(\exists f \in \mathcal{A})(graph(f) = \Phi^{graph(g)})$.

2. Let $S_{A,B} = \{\chi_R \mid (A \subseteq R) \quad \& \quad (B \cap R = \emptyset)\}$ be mass problem of a separation of a disjoint pair of sets (A, B). Then $(A, B) \leqslant_S (C, D) \iff S_{A,B} \leqslant_M S_{C,D}$. The class $\mathbf{a} = \{(C, D) \mid (C, D) \equiv_S (A, B)\}$ is called *the S-degree of the pair* (A, B). For S-degrees $\mathbf{a}$ and $\mathbf{b}$ define $\mathbf{a} \leqslant_S \mathbf{b}$ if and only if there are pairs $(A, B) \in \mathbf{a}$ and $(C, D) \in \mathbf{b}$, such that $(A, B) \leqslant_S (C, D)$. Let D_S denotes the structure of all S-degrees of disjoint pairs (A, B) and

let D_T denotes the structure of all Turing degrees.

3. A pair (A, B) is called a *computable enumerable* (*c.e.*) *pair* if A and B are c.e. sets. Accordingly, S-degree **a** is *c.e.*, if there is a c.e. pair $(A, B) \in \mathbf{a}$. Let D_S^0 denote the structure of all c.e. S-degrees and let D_T^0 denote the structure of all c.e. Turing degrees.

4. For any set A, the pair $(A, \overline{A})$ is called *regular*.

Denote $C_0 = \{x \mid \phi_x(x) = 0\}$, $C_1 = \{x \mid \phi_x(x) = 1\}$.

FACTS.

1. D_S and D_S^0 are upper semilattices under the relation $\leqslant_S$.

2. There is an isomorphic embedding $i\colon D_T \longrightarrow D_S$, $i(deg_T A) = deg_S(A, \overline{A})$, because $A \leqslant_T B \Longleftrightarrow (A, \overline{A}) \leqslant_S (B, \overline{B})$.

3. For any c.e. pair (A, B) we have $(A, B) \leqslant_S (C_0, C_1)$.

THEOREM 1. Any non-zero c.e. S-degree does not contain a regular pair.

THEOREM 2 (lower density). For any non-zero S-degree **b** there is an S-degree **a**, such that $\mathbf{0} <_S \mathbf{a} <_S \mathbf{b}$.

COROLLARY 2.1. There are no minimal S-degrees.

COROLLARY 2.2. D_T is not isomorphic to D_S.

THEOREM 3 (splitting). For any c.e. pair $(X, Y) \notin \mathbf{0}$ there are c.e. sets A_0 and A_1, such that $A_0 \cup A_1 = X$, $A_0 \cap A_1 = \emptyset$ and $(X, Y) \nleqslant_S (A_i, Y)$, where $i = 0, 1$.

COROLLARY 3.1. For any non-zero c.e. S-degree **b** there are non-comparable c.e. S-degrees $\mathbf{a}_0$ and $\mathbf{a}_1$, that $\mathbf{b} = \mathbf{a}_0 \cup \mathbf{a}_1$.

COROLLARY 3.2. There are no minimal c.e. S-degrees.

COROLLARY 3.3. D_S^0 is infinite.

THEOREM 4 (density). Let **a**, **b** be any non-zero c.e. S-degrees and $\mathbf{a} <_S \mathbf{b}$. Then exists c.e. S-degree **c**, such that $\mathbf{a} <_S \mathbf{c} <_S \mathbf{b}$.

► OLIVER KULLMANN, *Finding the error in propositional translations*.
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Consider checking the consistency of some specification, for example whether a specification of some component produced by some manufacturing process can actually be realised. There are interesting (and successful) applications where the increasing power of SAT solvers is utilised for this purpose, using some form of propositional translation. If now we find a "bug", that is the propositional translation is inconsistent, then we need to translate this back into the original problem domain, which is problematic, since the translation maps the high level specification into a low level propositional formula, and the inconsistency found will use bits and pieces of different aspects of the original formulation.

In my talk I want to report about the joint work [1], where we studied the algorithmic problems related to the fundamental task of computing for an inconsistent propositional formula in conjunctive normal form (a clause-set) a "good" sub-clause-set explaining the inconsistency.

[1] OLIVER KULLMANN, INÊS LYNCE, AND JOÃO MARQUES-SILVA, *Categorisation of clauses in conjunctive normal forms: Minimally unsatisfiable sub-clause-sets and the lean kernel*, Technical Report CSR 3-2006, University of Wales Swansea, Computer Science Report Series, March 2006.

► ROBERT S. LUBARSKY, *On the Cauchy completeness of the constructive Cauchy reals*.
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Classically it can be shown that, if the reals are taken to be equivalence classes of Cauchy sequences, then they are Cauchy complete (that is, every Cauchy sequence of reals has a limit). However, the same is not necessarily true in intuitionistic mathematics. In fact, the question itself splits into several variants. In this talk, some of these variants are shown to be independent of standard intuitionistic set theories.

► KERKKO LUOSTO, *Graphs and finitely many variables*.
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Consider the k-equivalence of structures, i.e., the equivalence of structures with respect to sentences (either in first order logic or infinitary finite variable logic) with at most k variables. Given a finite structure $\mathbf{A}$ which is k-equivalent to another, nonisomorphic structure, B. Poizat (1982) asked if it is true that $\mathbf{A}$ is k-equivalent also to an infinite structure. S. Thomas (1986) showed this is not necessarily the case. Call a structure $\mathbf{A}$ like this *dubious*.

We analyze the properties of dubious graphs with the aim of finding the dubious graph that is smallest in the size. We show that there is a dubious graph with 16 vertices, and all dubious graphs have at least 10 vertices.

► BARBARA MAJCHER-IWANOW, *Polish group actions and admissible sets*.
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In the book "Classification and Orbit Equivalence Relations" G. Hjorth has introduced a kind of countable invariants for analysing orbits of G-actions on Polish spaces where G is a closed subgroup of S_∞. The method generalizes the Scott analysis of models. Using these ideas we study orbit equivalence relations inside admissible sets. For example we obtain appropriate versions of some results of M. Nadel on Scott sentences and admissible sets.

Our approach naturally leads us to some refinement of the notion of the canonical partition of a Polish G-space. It can be defined for any ordinal α (we call it the α-canonical partition).

► LARISA MAKSIMOVA, *Weak interpolation in equational logic*.
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Let us fix an arbitrary signature consisting of functional symbols and constants including the constants $\bot$ and $\top$. We consider varieties of algebras such that $\mathbf{A} \models \bot = \top$ iff $\mathbf{A}$ is a one-element algebra. We call such algebras *bounded*; one can take bounded lattices, boolean algebras with operators and Heyting algebras as examples. We define a weak variant of interpolation property and find its algebraic equivalent.

For any algebra $\mathbf{A}$, a valuation in $\mathbf{A}$ is any homomorphism from the algebra of terms into $\mathbf{A}$. If $t = t'$ is an equation and v is a valuation, we write $\mathbf{A} \models (t = t')[v]$ whenever $v(t) = v(t')$. If Γ is a set of equations and γ is an equation, we define for any variety V:

$\Gamma \models_V \gamma \iff (\forall \mathbf{A} \in V)(\forall v)(\mathbf{A} \models \Gamma[v] \Rightarrow \mathbf{A} \models \gamma[v])$.

For any set $\mathbf{x}$ of variables, denote by $\Gamma(\mathbf{x})$ any set of equations whose all variables are in $\mathbf{x}$. We define the *weak interpolation property*:

WIP. If $\Gamma(\mathbf{x}, \mathbf{y}), \Delta(\mathbf{x}, \mathbf{z}) \models_V \bot = \top$, then there exists $\Gamma'(\mathbf{x})$ such that $\Gamma(\mathbf{x}, \mathbf{y}) \models_V \gamma$ for all $\gamma \in \Gamma'(\mathbf{x})$ and $\Gamma'(\mathbf{x}), \Delta(\mathbf{x}, \mathbf{z}) \models_V \bot = \top$.

We prove that WIP is equivalent to some weak variant of amalgamation. A class V has the *amalgamation property* if it satisfies the condition

AP. For any $\mathbf{B}, \mathbf{C} \in V$ with a common subalgebra $\mathbf{A}$, there exist an algebra $\mathbf{D}$ in V and monomorphisms $\delta\colon \mathbf{B} \to \mathbf{D}$ and $\varepsilon\colon \mathbf{C} \to \mathbf{D}$ such that $\delta(x) = \varepsilon(x)$ for all $x \in \mathbf{A}$.

A bounded algebra is *non-degenerate* if it contains at least two elements. We define

PRAP. For each $\mathbf{B}, \mathbf{C} \in V$ with a common subalgebra $\mathbf{A}$ there are $\mathbf{D}$ in V and homomorphisms $\delta\colon \mathbf{B} \to \mathbf{D}$, $\varepsilon\colon \mathbf{C} \to \mathbf{D}$ that coincide on $\mathbf{A}$, and $\mathbf{D}$ is non-degenerate whenever $\mathbf{A}$ is non-degenerate.

For a class K of algebras, $FG(K)$ denotes the class of finitely generated algebras of K. An algebra is said to be *simple* if it has exactly two congruences.

THEOREM 1. *Let V be a variety of bounded algebras. If V has WIP then it has PRAP. If V has CEP and $FG(V)$ has PRAP, then V has WIP.*

THEOREM 2. *Let a variety V of bounded algebras have CEP. Then the following are equivalent*:

1. *V has WIP*,
2. *the class $Sim(V)$ of simple algebras of V has AP*,
3. *the class $FG(Sim(V))$ of finitely generated and simple algebras of V has AP.*

► GIANARTURO MARCO, *The dynamics of reference: orbits and points*.
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Aim of this paper is to outline a mathematical foundation of the notion of reference, ubiquitous to various areas of mathematics, logic, philosophy of language and natural language semantics.

In particular, we introduce the notion of "orbital reference": this rises in the realm of automorphism group actions on the space of interpretations for a given countable language.

In this setting the classical notion of reference unravels its functional structure: namely, it appears as a variety of definable point-re-identification maps within a suitable space of recognition (i.e., a Polish G-space X, where X is Polish space and G is a closed group of transformations acting continuosly on X [1]. The complexity of the orbital reference for a given referring term t heavely depends on the algebraic and topological properties of the group stabilizer of the term t. One of the main consequences of our approach is that the Fregean notion of compositionality, accordingly with the referential variety, spreads out into a variety of patterns of compositionality. Such a variety of patterns rests upon the complexity spectrum of the invariant complete codes reducing the orbital spaces induced by the group actions.

Examples from Model Theory, like coordinatization for relively categorical theories over a predicate, give evidence for the independence of our notion of oprbital reference from the Russellian notion of ostensive denotation. It is just on its going beyond the denotational character that our concept of orbital reference turns out to be of crucial interest in the determinacy of semantic games for anaphoric co-reference: as pointed out by G. Sandu [2], the demonstrative, deictic notion of reference is unfit for semantic games on anaphoric co-reference whenever the anaphoric link occurs within the scope of a negation, We define a point-reidentification game that settles the problem of determinacy of coreference anaphoric link in negative contexts.

Last, but not least, we give evidence also for the fact that "rigidity" is not a property intrinsic to the pure denotational "nature" of reference: rather it belongs to the automorphism group of a given structure.

[1] H. BECKER AND H. S. KECHRIS, ***The descriptive set theory of Polish group actions***, Cambridge University Press, 1996.

[2] G. SANDU, *On the theory of anaphora: dynamic predicate logic vs. game-theoretical semantics*, ***Linguistics and Philosophy***, vol. 20, 1997.

► ALBERTO MARCONE, *Interval orders and reverse mathematics*.
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A partial order $(P, \leq_P)$ is an interval order if the elements of P can be mapped to intervals of a linear order so that $p \leq_P q$ holds iff every element of the interval associated to p precedes every element of the interval associated to q. Interval orders occur quite naturally in many different areas.

We study interval orders from the viewpoint of reverse mathematics, establishing the logical strength of the equivalences of various formalizations in second order arithmetic of the notion of interval order. We also establish the proof-theoretic strength of the basic characterization theorem for interval orders: a partial order is an interval order if and only if it does not contain a copy of the partial order $\mathbf{2} \oplus \mathbf{2}$.

Our proofs yield also results that can be viewed as examples of computable mathematics.

► ALEXANDER G. MELNIKOV, *Lempp's question for torsion free abelian groups of finite rank*.
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Steffen Lempp asked (unpublished): Does a structure with representations in all non-recursive degrees have a recursive representation? T. Slaman [4] and S. Wehner [5] independently built a structure, that has all non-recursive representations, but has no recursive one. In the other hand, R. Downey and C. Jockusch [1] showed that every low boolean algebra is isomorphic to recursive one (J. Knight and M. Stob [2] generalized this for low_4). In this context the wish to get such results for torsion free abelian groups (TFAG) is rather natural. In [3] we showed that the result of R. Downey and C. Jockusch does not hold for TFAG: there exists low non-recursive TFAG. Our present result is:

THEOREM 1. *If TFAG of finite rank has all non-recursive copies, then it must have a recursive copy.*

[1] R. DOWNEY and G. JOCKUSCH, *Every low Boolean algebra is isomorphic to a recursive one*, ***Proceedings of the American Mathematical Society***, vol. 122 (1994), pp. 871–880.

[2] J. F. KNIGHT and M. STOB, *Computable Boolean algebras*, ***The Journal of Symbolic Logic***, vol. 65 (2000), no. 4, pp. 1605–1623.

[3] A. G. MELNIKOV, *Nizkie nevychislimye abelevy gruppy bez krucheniya* (Translation: *Low non-computable torsion free abelian groups*), ***Algebra i Logika***, to appear.

[4] T. SLAMAN, *Relative to ani non-recursive set*. ***Proceedings of the American Mathematical Society***, vol. 126 (1998), pp. 2117–2122.

[5] S. WEHNER *Enumerations, countable structures and Turing degrees*, ***Proceedings of the American Mathematical Society***, vol. 126 (1998), pp. 2131–2139.

► RUSSELL MILLER, *Computable categoricity and fields*.
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We present work in progress on the question of which fields are computably categorical. A computable structure A is computably categorical if, for every computable structure B isomorphic to A, there exists a computable isomorphism from B onto A. Algebraic characterizations of this property have been found for such theories as linear orders, Boolean algebras, trees, and ordered Abelian groups, by researchers including Dzgoev, Goncharov, Lempp, McCoy, Miller, Remmel, and Solomon. Fields have proven to be more challenging in this regard, but we will offer partial results relating the computable categoricity of a field (and also its $\emptyset'$-computable catgoricity) to its normality and its transcendence degree as an extension of its prime field.

Portions of this work are joint with Hans Schoutens.

► SHEILA MILLER, *A division algorithm for the free algebra on many generators*.
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The left distributive algebras (algebras satisfying the LD-law $a(bc) = (ab)(ac)$) which occur in classical math are idempotent, hence not free. The free left distributive algebra $\mathbf{A}$ on one generator is representable as the span of an elementary embedding $j\colon V_\lambda \to V_\lambda$ (Laver) and is a subset of the braid group under a certain operation (Dehornoy). Ways in which $\mathbf{A}$—and more generally the free algebra $\mathbf{A}_\kappa$ on κ-many generators—are known or conjectured to be "well-founded" will be discussed. We will present a division algorithm theorem for the elements of $\mathbf{A}_\kappa$. The algorithm, improving one of Laver for the one generator case, takes place not in $\mathbf{A}_\kappa$ but in $\mathbf{P}_\kappa$—the result of enlarging $\mathbf{A}_\kappa$ to freely add a composition operation. It is hopped that this will be useful in solving a conjecture of J. Moody on $\mathbf{A}_\kappa$?

► RYSZARD MIREK, *Algebraization of non-classical logics*.
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According to Blok and Pigozzi [1] a deductive system S is algebraizable with equivalent semantics K, that is a quasivariety, iff for every algebra A and the Leibniz operator $\mathbf{\Omega}_A$ is an isomorphism between the lattices of *S-filters and K-congruences* of A. For instance let $\mathsf{A} = (A, \to)$ be the 4-element algebra on $A = \{\bot, t, f, \top\}$. One may show that $\mathsf{R}_\to$—implicational fragment of relevance logic R—is not algebraizable. There is a filter $F_1 = \{\top, t\}$ that is closed under *Modus Ponens* and hence is a $\mathsf{R}_\to$-filter. According to matrix for implication one obtains that $\top \to \top = \top$ and $\top \to t = \bot$, so in fact, to the congruence on A generated by $\langle \top, t\rangle$ belongs $\langle \top, \bot\rangle[\langle \top, \bot\rangle \in \Theta(\top, t)]$. What is more, $\bot \to \bot = \top$ and $f \to \bot = \bot$, so one has $\langle \top, \bot\rangle \in \Theta(\bot, f)$. The conclusion is that $\mathbf{\Omega}_A F_1 = I_A$, it means that the identity relation I_A is the only congruence compatible with F_1. Let $F_2 = \{\top, t, f\}$. F_2 is closed under *Modus Ponens* as well and hence is a $\mathsf{R}_\to$-filter. Again $\mathbf{\Omega}_A F_2 = I_A$ because $\langle \top, \bot\rangle \in \Theta(\top, t) \cap \Theta(\top, f) \cap \Theta(t, f)$. So $\mathbf{\Omega}_A$ is not injective on $\mathsf{R}_\to$-filters of A. It is of interest, that the system R of relevance logic is algebraizable, what shows that the role of conjuction is essential.

My aim is to establish the algebraizability or non-algebraizability of some non-classical logics, especially relevance logics. For instance I dwell on Perry's analytic implication or a system $\mathsf{C}_\to$.

[1] DON PIGOZZI and W. J. BLOK, ***Algebraizable logics***, American Mathematical Society, Providence, Rhode Island, 1989.

► WOJCIECH MOCZYDLOWSKI, *Normalizing Curry-Howard isomorphism for intuitionistic set theory*.
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We define a lambda calculus corresponding to proofs in a non-extensional version of intuitionistic set theory IZF. We show weak normalization of the calculus and use it to prove the disjunction, numerical existence and term existence properties of the theory.

► MARCIN MOSTOWSKI AND JAKUB SZYMANIK, *Computational complexity of some Ramsey quantifiers in finite models*.
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The problem of computational complexity of semantics for some natural language constructions—considered in [2]—motivates an interest in complexity of Ramsey quantifiers in finite models. In general a sentence with a Ramsey quantifier R of the following form $\mathsf{R}x, yH(x, y)$ is interpreted as $\exists A$ (A is big relatively to the universe $\wedge A^2 \subseteq H$). In the paper cited the problem of the complexity of the Hintikka sentence is reduced to the problem of computational complexity of the Ramsey quantifier for which the phrase "A is big relatively to the universe" is interpreted as containing at least one representative of each equivalence class, for some given equvalence relation.

In this work we consider quantifiers R_f, for which "A is big relatively to the universe" means "*card*$(A) > f(n)$, where n is the size of the universe". Following [1] we call R mighty if $\mathsf{R}x, yH(x, y)$ defines *NP*–complete class of finite models. Similarly we say that R_f is *NP*–hard if the corresponding class is *NP*–hard. We state the following:

THEOREM 1. *Let $f(n)$ be the integral part of $r \times n$, for some rational r such that* $0 < r < 1$. *Thus* R_f *is mighty.*

CONJECTURE. Let f be such that $\lim_{n\to\infty} f(n)/n = a$ exists and $0 < a < 1$. Then R_f is *NP*–hard.

THEOREM 2. *If f is such that* $\lim_{n\to\infty} f(n)/n = a$ *exists and* $0 < a < 1$ *and f is PTIME computable then* R_f *is mighty.*

[1] A. BLASS and Y. GUREVICH, *Henkin quantifiers and complete problems*, ***Annals of Pure and Applied Logic***, vol. 32 (1986), pp. 1–16.

[2] M. MOSTOWSKI and D. WOJTYNIAK, *Computational complexity of the semantics of some natural language constructions*, ***Annals of Pure and Applied Logic***, Vol. 127 (2004), no. 1–3, pp. 219–227.

► RAJA NATARAJAN, *First-order frameworks of mobility*.
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In the design space of models for concurrency, there are two paradigms that are known to possess a wide ranging expressive power, namely the "concurrent constraint paradigm" and the "pi-calculus." The expressive power of these paradigms essentially derives from their ability to model the key notion of "mobility" elegantly, within a first-order framework. These two paradigms appear to be completely divergent from each other, since they evolved from distinct sub-disciplines of computer science. The differences in the primitives of the two paradigms gives them their distinct flavour, but also makes the task of relating them a non-trivial one. We relate these two models of mobile computation, by defining a minimal calculus which contains the core features of the concurrent constraint paradigm, and by constructing a semantic-preserving embedding from it to the pi-calculus.

► CYRUS F. NOURANI, *Functorial models, generic sets, and recursion Urlements*.
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Fragment consistent models and Urlements are applied to computable KPU recursion and Peano models refering to Platek's 1966, towards computability. Admissible and descriptive sets indicate that recursive definitions lifted to other domains than integers, for example admissible sets show that there are equivalence breakdowns, e.g., there being two competing notions of r.e. and admissible sets. Author's functorial models 1996, and the Keisler fragment $\mathrm{K_A}$, we have models on admissible sets and rudimentary fragment, SLK, Leeds, 1997 and a functorial computing on generic models, ICM, Berlin 1996, ASL-LA 1998. There are L-admissible models for PA at which well-known computational complexity questions are not independent.

K_A is admissible iff it is definable with an admissible fragment $S \subseteq K_A$. The genericity concept in the Ambos-Spies et al. 1984 was on unary sets and there were no forcing properties stated nor were there forcing mentions. This author had presented forcing and applications to predictive computing at SLK-Manchester 1984, inspired he wrote Positive Forcing and Complexity there.

Based only on the arithmetic hierarchy:

LEMMA 1. P-generic sets are definable with K_A.

LEMMA 2. P-generic sets definable with θ in K_A such that θ is Σ_1 is closed.

LEMMA 3. P-generic sets are rudimentary.

THEOREM 1. P-generic sets are K_A admissible.

COROLLARY 1. P-generic sets are L-addmissible.

COROLLARY 2. P-generic sets are admissible computable.

THEOREM 2. There are models, in particular PA models obtained definable by P-generic sets, where the r.e. set independence questions can be addressed.

The author posed hierarchy questions since 1999, Hattingen-www.esf.org. The model questions as far as the P-generic sets are in part answered here. The above On Admissible Computable sets and P-genericity, written SLK, Helsinki, communicated to Heidelberg 2003 not published.

► DAVID PIERCE, *Vector-spaces over unspecified fields*.
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In the *Elements*, Euclid of Alexandria gives a geometric formulation of certain field-theoretic identities; for example, the identity

$$x^2 - y^2 = (x + y)(x - y)$$

is Euclid's Proposition II.5, but Euclid expresses in terms of the squares and rectangles bounded by certain lines. At the beginning of the *Geometry*, René Descartes shows how lines can be multiplied to produce *lines* rather than rectangles, if one line is chosen as unit. The construction justifies Descartes's algebraic treatment of geometry.

A model-theoretic version of Descartes's construction is that the theory of vector-spaces (as one-sorted structures, but over unspecified fields) can be axiomatized in the signature of abelian groups with a predicate for *parallelism* (binary linear dependence). The existentially closed models of this theory are two-dimensional spaces (over algebraically closed fields). If a predicate for n-ary linear dependence is introduced to the language, then the existentially closed models of the expanded theory are n-dimensional. In particular, a vector-space of dimension greater than n embeds in a space of dimension n so as to preserve independence in all n-element sets of vectors. This might be compared with the observation that existentially closed field-extensions have transcendence-degree one.

► S. YU. PODZOROV, *Disrtributive upper semilattices with Σ^0_3-representations*.
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Let $\mathcal{L}$ be at most countable distributive upper semilattice with the least and the greatest elements. Suppose that $\leqslant_{\mathcal{L}}$ is a Σ^0_3-preorder on $\mathbb{N}$ and $u\colon \mathbb{N}^2 \to \mathbb{N}$ is a binary computable functions. We say that the structure $\langle \mathbb{N}, \leqslant_{\mathcal{L}}, u\rangle$ is a Ω_1-*representation* of $\mathcal{L}$ if the partially ordered structure associated with $\langle \mathbb{N}, \leqslant_{\mathcal{L}}\rangle$ is isomorphic to $\mathcal{L}$ and u represents the union operation on it. We say that the structure $\langle \mathbb{N}, \leqslant_{\mathcal{L}}\rangle$ is a Ω_2-*representation* of $\mathcal{L}$ if the partially ordered structure associated with $\langle \mathbb{N}, \leqslant_{\mathcal{L}}\rangle$ is isomorphic to $\mathcal{L}$. It is obvious that any Ω_1-representable semilattice has a Ω_2-representation.

In 1972 Lachlan [1] proved that an upper semilattice is isomorphic to a principal ideal of c.e. m-degrees (without m-degrees of $\varnothing$ and $\mathbb{N}$) if and only if it is isomorphic to a direct limit of $\sqrt{\ }$ sequence of finite preordered structures with six special conditions. We call such semilattices *Lachlan's semilattices*. Lachlan's semilattices play an important role in the theory of numberings: they are exactly the segments of Rogers semilattices of Σ^0_1-computable numberings and also can be embedded as an ideals in any Rogers semilattice of Σ^0_n-computable numberings for $n \geqslant 2$ (the first is rather obvious, the second was proved in [2]).

It is easy to obtain from Lachlan's description that any Lachlan's semilattice has a Ω_1-representation. Now we proved [3] that converse is also true. Our main results concerned Lachlan's semilattices are listed below.

1. Any Lachlan's semilattices has a Ω_1-representation. Moreover, if $\langle \mathbb{N}, \leqslant_{\mathcal{L}}, u\rangle$ is a Ω_1-representation of some Lachlan's semilattice $\mathcal{L}$ than there exists a computable function f such that the mapping $n \mapsto \deg_m(W_{f(n)})$ establish the isomorphism between the quotient structure of $\langle \mathbb{N}, \leqslant_{\mathcal{L}}\rangle$ and some principal ideal of c.e. m-degrees.
2. There exists Ω_2-representable upper semilattice which is not Lachlan's one.

[1] A. H. Lachlan, *Recursively enumerable many-one degrees*, ***Algebra and Logic***, vol. 11 (1972), no. 3, pp. 326–358.

[2] S. Yu. Podzorov, *On a local structure of Rogers semilattices of Σ^0_n-computable numberings*, ***Algebra and Logic***, vol. 44 (2005), no. 2, pp. 148–172.

[3] ———, *Numbered distributive semilattices*, ***Siberian Advances in Mathematics***, to appear.

► TOMASZ POŁACIK, *Partially-elementary extension Kripke models and subtheories of Heyting Arithmetic*.
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A Kripke model for a first order language can be viewed as a family of classical structures for that language, called the worlds of the model, partially ordered by the (weak) submodel relation. However, in case of certain intuitionistic theories the relationship between the worlds of their models can be stronger. We study partially-elementary extension Kripke models that is the models whose usual ordering relation is replaced by a stronger relation of being an elementary submodel with respect to some class of formulae. We offer characterizations of partially-elementary extension Kripke models both in terms of forcing certain intuitionistically meaningfull schemata and the relationship of forcing and classical satisfiability at the worlds of a model. We also present some applications of partially-elementary Kripke models in case of subtheories of Heyting Arithmetic and show that the properties of their models differ much from the properties of models of the full theory.

► GIUSEPPE PRIMIERO, *A constructive type-theoretical interpretation of the philosophical methods of abstraction and instantiation*.
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The methods of abstraction and instantiation are a classical example of philosophical procedures finding an interesting interpretation in typed systems. In such systems there are different ways of proceding in abstraction, e.g., polytypic abstraction and data types abstraction; the procedure of instantiation is instead represented in terms of the introduction of constructors, representation of primitive data-types for computation or as the instance of a program within object-oriented programming. Nevertheless, basic differences can be drawn with the philosophical methods considered above. Constructive Type Theory introduces a rather precise formalization of the mentioned philosophical procedures. Starting by the switch from polymorphic to monomorphic versions of the theory, the methods of abstraction and instantiation are analysed by considering the extensional and the intensional levels of the theory: type as a meaning-object is a rather precise representation for the product of abstraction; the objectual predication is on the intensional level the corresponding form of individual instantiation. The epistemic clarification provided is based on the distinction between knowledge and information.

[1] L. Cardelli and P. Wegner, *On understanding types, data abstraction and polymorphism*, ***Computing Surveys***, vol. 17 (1985), no. 4, pp. 471–522.

[2] M. E. Maietti and G. Sambin, *Toward a minimalist foundation for constructive mathematics*, ***From sets and types to topology and analysis: practicable foundations for constructive mathematics***, (L. Crosilla and P. Schuster, editors), Oxford University Press, 2005.

[3] P. Martin-Löf, *An intuitionistic theory of types*, ***25 years of constructive type theory***, (Sambin and Smith, editors), Clarendon Press, Oxford, 1998, pp. 127–172.

[4] G. Primiero, *Information and knowledge – A constructive approach to the philosophy of information and a solution to the problem of analyticity*, PhD Thesis, Department FIERI, University of Palermo, 2006.

► V. G. PUZARENKO, *One reducibility on admissibles*.
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In [1] Yu. Ershov introduced a notion of Σ-definability for admissible sets as some generalization of constructivizability. He described all the structures which are Σ-definable in $\mathbb{HF}_{\mathrm{L}}$ where L is dense linear ordering. Recall that a hereditarily finite set $\mathbb{HF}_{\mathcal{M}}$ over structure $\mathcal{M}$ is the least by inclusion admissible set containing $\mathrm{dom}(\mathcal{M})$ as a subset. However, Σ-definability does not preserve computable invariants. Here we introduce some reducibility which preserves such computable invariants as effective subsets of natural numbers and computable collections of such subsets. We say that an admissible set $\mathbb{A}$ is Σ-*reducible* to an admissible set $\mathbb{B}$ if there a surjection map $\nu\colon \mathrm{dom}(\mathbb{B}) \to \mathrm{dom}(\mathbb{A})$ such that $\nu^{-1}(C)$ is $\mathbb{B}$-c.e., for any $\mathbb{A}$-c.e. predicate C. As usual, C is called $\mathbb{A}$-c.e. if it is definable by some Σ formula in $\mathbb{A}$.

We build a transformation which corresponds any admissible set to a hereditarily finite set, Σ-equivalent to one. Moreover, this transformation preserves a certain list of descriptive set theoretical properties. As corollary, the structure $\mathcal{D}$ of all Σ-equivalent classes of admissible sets is an upper semilattice. It is shown that the semilattice $\mathcal{L}_e$ of enumeration degrees is embedded into $\mathcal{D}$. This embedding is natural namely it corresponds any enumeration degree to some admissible set having this e¸degree. Furthermore, Σ-reducibility is determined by using Σ-definability. In [2, 3, 4, 5] we study computability properties of the least elements of some natural segments of $\mathcal{D}$.

We also introduced the jump operation $\mathcal{J}$ on Admissible Sets. It is proved that any enumeration degree $\mathbf{a}$, $\overline{\varnothing'} \leqslant_e \mathbf{a}$, is in the image of $\mathcal{J}$. Moreover, a locally constructible structure can be chosen as some pre-image of $\mathbf{a}$.

This work is partially supported by INTAS grant YSF 05-109-4919 and grant of RFBR 06-01-00358.

[1] Yu. L. Ershov, Σ-*definability in Admissible Sets*, ***Doklady RAN***, vol. 285 (1985), no. 4, pp. 792–795 (in Russian).

[2] A. S. Morozov and V. G. Puzarenko, Σ-*subsets of natural numbers*, ***Algebra and Logic***, vol. 43 (2004), no. 3, pp. 162–178.

[3] V. G. Puzarenko, *Computability in special models*, ***Siberian Mathematical Journal***, vol. 46 (2005), no. 1, pp. 148–165.

[4] I. S. KALIMULLIN and V. G. PUZARENKO, *Computability principles on admissible sets*, ***Siberian Advances in Mathematics***, vol. 15 (2005), no. 4, pp. 1–33.
[5] ———, *Computability on collections*, to appear.

► GEMMA ROBLES, JOSÉ M. MÉNDEZ AND FRANCISCO SALTO, *Relevance logics, paradoxes of consistency and the K rule*.
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As it is known, implicative paradoxes are divided into two classes: paradoxes of relevance and paradoxes of consistency. A characteristic exemplar of the former is the K axiom

$$\vdash A \to (B \to A)$$

or the K rule

$$\vdash A \Rightarrow \vdash B \to A$$

A typical member of the latter is the ECQ axiom

$$\vdash (A \wedge \neg A) \to B$$

We shall study the effect of the K rule on the basic positive logic B_+ and the relevance logics including it when constructive negations of the sort defined in [1] and [2] are added. We shall employ the ternary relational semantics.

[1] G. ROBLES, J. M. MÉNDEZ, AND F. SALTO, *Minimal negation in the ternary relational semantics*, ***Reports on Mathematical Logic***, vol. 39 (2005), pp. 47–65.
[2] G. ROBLES, F. SALTO AND J. M. MÉNDEZ, *A constructive negation defined with a negation connective for logics including* BP+, ***Bulletin of the Section of Logic***, vol. 34 (2005), no. 3, pp. 177–190.

► WIM RUITENBURG, *Ways to build intuitionistic theories that admit quantifier elimination*.
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We present methods by which to construct new intuitionistic theories that admit quantifier elimination, from known ones. These methods involve the construction of special Kripke models for these new theories, from Kripke models of known ones. The intuitionistic theories can be made so that they are in some sense very intuitionistic.

[1] W. RUITENBURG, *Very intuitionistic theories and quantifier elimination*, ***The Review of Modern Logic***, vol. 10, no. 1–2.

► SAEED SALEHI, *Modal logic of cut-free provability in weak arithmetics*.
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Cut-free provability is not equivalent to the usual (Hilbert-style) provability in weak arithmetics, for the fact that the cost of cut-elimination is (super–)exponential. Though, Hilbert–Bernays–Löb provability conditions have been proved to hold for Hilbert provability in theories as weak as $I\Delta_0 + \Omega_1$, little is known about cut-free provability in weak arithmetics (proper sub-theories of $I\Delta_0 + \exp$). Nevertheless, unprovability of cut-free consistency in those arithmetics have been proved by several authors, most notably by Z. Adamowicz (and P. Zbierski) and D. Willard.

In [1] the author proved several theorems for Herbrand Consistency of a few weak theories. In particular, it was shown that $I\Delta_0 \nvdash HCon(\overline{I\Delta_0})$, where $HCon$ is the predicate of Herbrand Consistency, and $\overline{I\Delta_0}$ is a variant of $I\Delta_0$, axiomatized by the conventional axioms of $I\Delta_0$ augmented with two $I\Delta_0$-provable sentences. The proof was based on a weak Σ_1-completeness for $I\Delta_0$, a special result of which is (for $T = \overline{I\Delta_0}$)

$$T \vdash HCon(T) \to \Big(HPr^*_T(\ulcorner\varphi\urcorner) \to HCon^*_T(\ulcorner HPr^*_T(\ulcorner\varphi\urcorner)\urcorner)\Big),$$

where HPr^* and $HCon^*$ are, respectively, the restriction of HPr and $HCon$ to a definable initial segment. This can be translated to a modal language:

$$(\mathtt{S})\ T \vdash \Diamond^0 T \to (\Box\varphi \to \Diamond\Box\varphi),$$

where $\Box^0$ and $\Diamond^0$ stand for HPr_T and $HCon_T$, and $\Box$ and $\Diamond$ for HPr^*_T and $HCon^*_T$. By diagonalization we have $(\mathtt{F})$ $T \vdash \tau \leftrightarrow \neg\Box\tau$ for some formula τ, and the following rules

$$(\mathtt{RM})\ \frac{T \vdash \varphi \to \psi}{T \vdash \Box\varphi \to \Box\psi}, \quad (\mathtt{RN})\ \frac{T \vdash \varphi}{T \vdash \Box\varphi},$$

can be proved to be valid in T.

Now, the proof of $T \nvdash \Diamond^0 T$ is as follows. If on the contrary $T \vdash \Diamond^0 T$, then $T \vdash \Box\tau \to \Diamond\Box\tau$ by $(\mathtt{S})$, so $T \vdash \neg\tau \to \Diamond\Box\tau$ by $(\mathtt{F})$. On the other hand, $T \vdash \Box\tau \to \Box\neg\Box\tau$ by $(\mathtt{RM})$ and $(\mathtt{F})$, and so $T \vdash \neg\tau \to \neg\Diamond\Box\tau$. Thus, we have $T \vdash \tau$, which implies $T \vdash \Box\tau$ by $(\mathtt{RN})$ and $T \vdash \neg\Box\tau$ by $(\mathtt{F})$. Hence, it follows that T is inconsistent; contradiction.

Axiomatizing the Herbrand Provability of $I\Delta_0$ (in $I\Delta_0$) as a modal logic is an open problem; all we know so far is that e.g., the axiom $(\mathtt{S})$ and the rules $(\mathtt{RM})$ and $(\mathtt{RN})$ are valid in it.

[1] SAEED SALEHI, ***Herbrand consistency in arithmetics with bounded induction***, Ph.D. Thesis, Institute of Mathematics of the Polish Academy of Sciences (2002). URL: `http://staff.cs.utu.fi/staff/saeed/thesis.pdf`

► PAVEL SCHREINER, *Recognition of the Craig Interpolation Property and Projective Beth Property in the locally tabular positive calculus.*
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L. Maksimova in [1] found all positive logics with Craig Interpolation Property (CIP). In [2] she described all positive logics with Projective Beth Property (PBP).

Denote by $L + A$ the extension of logic L by an extra axiom scheme A.

Let us denote for $n \leq 1$

$\mathbf{Z_n} = \{1, \dots n\}$ with natural ordering relation;

$\mathbf{V_n} = \{0, 1, \dots n\}$ where $0Rx$ for all x and $\neg xRy$ for $1 \leq x, y \leq n$, $x \not\leq y$;

$\mathbf{W_{I\times n}} = \bigcup_{k=1}^{I} Z_{\min(I,n)} \cup \{-1\}$ where $-1Rx$ for all x.

$\mathbf{Y_n} = V_n \cup \{-1\}$ where $-1Rx$ for all x.

PROPOSITION 1. *Let $L = Int^+ + \{A_1, \dots, A_k\}$, A is not deducible in Int^+, $A = A_1 \wedge \dots \wedge A_k$, A contain n variables, r be a total number of instances of '$\to$' in A and $I = max(1, r)$. Then*

1. *L has CIP if and only if one of the following conditions holds*:
   (a) *Formula A is refutable on the Z_1.*
   (b) *Formula A is valid on the Z_1 and it is refutable on the Z_2.*
   (c) *Formula A is valid on the Z_{n+1} and it is refutable on the V_2.*
2. *L has PBP if and only if it satisfies one of the above mentioned conditions* (a)–(c) *or one of the following conditions holds*:
   (d) *Formula A is valid on the Z_2 and it is refutable on the both Z_3 and V_2.*
   (e) *Formula A is valid on the $V_{min(2^n,I)}$ and it is refutable on the Z_3.*
   (f) *Formula A is valid on the $W_{I\times n}$ and it is refutable on the Y_2.*

The author creates the program realizing above described algorithm verification of the CIP and PBP in the local tabular positive calculus.

This work has been supported by INTAS grant 04-77-7080 and grant of Russian Foundation for Basic Research 06-01-00358.

[1] L. Maksimova, *Craig's interpolation theorem and amalgamable varieties*, ***Doklady AN SSSR***, vol. 237 (1977), pp. 1281–1284.

[2] ———, *Implicit definability and positive logic*, ***Algebra and Logic***, vol. 42 (2003), pp. 65–93.

► PETER SCHUSTER, *The shrinking principle and the axiom of choice*.

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The axiom of choice is equivalent to the principle that every indexed cover of a set has a refinement with the same index set which is a partition.

This is joint work with Bernhard Banaschewski.

► MARINA SEMENOVA, *Embedding lattices into subsemigroup lattices*.

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For an algebraic structure A, let $\operatorname{Sub} A$ denote the substructure lattice of A. The problem of characterizing of the class $\mathbf{S}(\operatorname{Sub}\mathcal{K})$ of lattices embeddable into substructure lattices of structures belonging to some class $\mathcal{K}$; then $\mathcal{K}$ is *lattice-universal*, if $\mathbf{S}(\operatorname{Sub}\mathcal{K})$ coincides with the class of all lattices.

We consider this problem restricted on different classes of semigroups. V. B. Repnitskiĭ proved in [1] that several classes of semigroups are lattice-universal. Problem 28.14.2 in [3] asks about a description of the class $\mathbf{S}(\operatorname{Sub}\mathcal{N}_n)$, $n < \omega$, where $\mathcal{N}_n$ denotes the class of n-nilpotent semigroups. We provide a syntactic descriprion of this class.

Theorem 1. *For any $n < \omega$, the class $\mathbf{S}(\operatorname{Sub}\mathcal{N}_{n+1})$ is a finitely based variety. The class of finite members of $\mathbf{S}(\operatorname{Sub}\mathcal{N}_{n+1})$ is defined by a finite set of identities within the class of finite lattices, whence it is a pseudovariety.*

Problem 28.14.2 in [3] asks about a description of the class $\mathbf{S}(\operatorname{Sub}\mathcal{K})$, where $\mathcal{K}$ is the class of free (commutative) (2-nil)semigroups or the class of free semilattices. V. B. Repnitskiĭ provided in [2] a description of finite lattices belonging to the above classes. We present the following

Theorem 2. *For any infinite cardinal κ, the follwing statements are equivalent*:

(1) *$L \in \mathbf{S}(\operatorname{Sub} F(\kappa))$, where $F(\kappa)$ is a free semigroup of rank κ;*
(2) *$L \in \mathbf{S}(\operatorname{Sub} FC(\kappa))$, where $FC(\kappa)$ is a free commutative semigroup of rank κ;*
(3) *$L \in \mathbf{S}(\operatorname{Sub} FN(\kappa))$, where $FN(\kappa)$ is a free 2-nilsemigroup of rank κ;*
(4) *$L \in \mathbf{S}(\operatorname{Sub} FCN(\kappa))$, where $FCN(\kappa)$ is a free commutative 2-nilsemigroup of rank κ;*
(5) *$L \in \mathbf{S}(\operatorname{Sub} FSL(\kappa))$, where $FSL(\kappa)$ is a free semilattice of rank κ;*
(6) *L embeds into $\prod_{\xi<\kappa} L_\xi$, where L_ξ is a finite lower bounded lattice for any $\xi < \kappa$.*

In particular, $\mathbf{S}(\operatorname{Sub}\mathcal{F}) = \mathbf{S}(\operatorname{Sub}\mathcal{FC}) = \mathbf{S}(\operatorname{Sub}\mathcal{FN}) = \mathbf{S}(\operatorname{Sub}\mathcal{FCN}) = \mathbf{S}(\operatorname{Sub}\mathcal{FSL}) = \mathbf{SP}(LB_{\text{fin}})$.

[1] V. B. Repnitskiĭ, *On the representation of lattices by lattices of subsemigroups*, ***Russian Mathematics*** (***Izvestiya VUZ. Matematika.***), vol. 40 (1996), pp. 55–64.

[2] ———, *On finite lattices which are embeddable in subsemigroup lattices*, ***Semigroup Forum***, vol. 46 (1993), pp. 388–397.

[3] L. N. Shevrin and A. J. Ovsyannikov, ***Semigroups and their subsemigroup lattices***, Kluwer Academic publishers, Dordrecht, 1996.

► ALEXANDRA A. SOSKOVA, *Properties of relative spectra*.
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The Degree spectrum $\mathrm{DS}(\mathfrak{A})$ of a countable partial structure $\mathfrak{A}$ is the set of all enumeration degrees generated by all enumerations of $\mathfrak{A}$. The notion is introduced by Richter and studied by *Knight, Ash, Jockush, Downey and Soskov*. The Co-spectrum $\mathrm{CS}(\mathfrak{A})$ of the structure $\mathfrak{A}$ is the set of all enumeration degrees which are lower bounds of $\mathrm{DS}(\mathfrak{A})$. In [4] Soskov shows that the Degree spectra behave with respect to their Co-spectra very much like the cones of enumeration degrees. Each Degree spectrum is closed upwards. The elements of an upwards closed set of enumeration degrees $\mathcal{A}$ with arbitrary high jumps determine completely the co-set of $\mathcal{A}$. The elements of the degree spectrum $\mathrm{DS}(\mathfrak{A})$ with low jumps also determine it's co-set $\mathrm{CS}(\mathfrak{A})$. Other typical specific properties of the Degree spectra and their Co-spectra are the Minimal Pair type theorem and the existence of Quasi-Minimal degree.

We will present a generalized notion of Degree spectrum of the structure $\mathfrak{A}$, relatively given structures $\mathfrak{A}_1, \dots, \mathfrak{A}_n$, inspired by the notion of relatively intrinsic on $\mathfrak{A}$ sets. An internal characterization of the relatively intrinsic on $\mathfrak{A}$ sets is presented in [1], [2] and in [3] with respect to the infinite sequence of sets.

The Relative spectrum $\mathrm{RS}(\mathfrak{A}, \mathfrak{A}_1, \dots, \mathfrak{A}_n)$ of $\mathfrak{A}$ with respect to the structures $\mathfrak{A}_1, \dots, \mathfrak{A}_n$ is the set of all enumeration degrees generated by all enumerations of $\mathfrak{A}$, such that the structure $\mathfrak{A}_k$ is relatively k-intrinsic on $\mathfrak{A}$, i.e., $\mathfrak{A}_k$ is admissible in the kth jump of $\mathfrak{A}$. We will show that this generalized notion of Degree spectra posses all general and specific properties of the Degree spectra of a structure. And we will compare this notion with the notion of Joint Spectrum of $\mathfrak{A}$ with respect to the structures $\mathfrak{A}_1, \dots, \mathfrak{A}_n$, considered in [5], [6].

[1] C. J. Ash, J. F. Knight, M. Manasse, and T. Slaman, *Generic copies of countable structures*, ***Annals of Pure and Applied Logic***, vol. 42 (1989), pp. 195–205.

[2] J. Chisholm, *Effective model theory vs. recursive model theory*, ***The Journal of Symbolic Logic***, vol. 55 (1990), pp. 1168–1191.

[3] I. N. Soskov and V. Baleva, *Ash's theorem for abstract structures*, ***Proceedings of Logic Colloquium '2002*** (to appear).

[4] I. N. Soskov, *Degree spectra and co-spectra of structures*, ***Annuaire de l'Université de Sofia***, vol. 96 (2004), pp. 45–68.

[5] A. A. Soskova and I. N. Soskov, *Co-spectra of joint spectra of structures*, ***Annuaire de l'Université de Sofia***, vol. 96 (2004), pp. 35–44.

[6] A. A. Soskova, *Minimal pairs and quasi_minimal degrees for the joint spectra of structures*, ***New Computational Pradigms***, (S. B. Cooper and B. Löwe, editors), Lecture Notes in Computer Science, vol. 3526, pp. 451–460, Springer-Verlag, 2005.

► JACK STECHER, *Using Brouwer's continuity principle to pick stocks*.
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The purpose of this talk is to show that intuitionistic reasoning in general, and Brouwer's continuity principle in particular, can provide insights into the behavioral sciences that are classically unattainable. Our setting is an economic application.

In financial economics, many formulae have been proposed for estimating the intrinsic worth of a share of stock. These formulae all involve the present value an infinte series, such as expected future dividends (favored by researchers in finance or economics) or expected future earnings (favored by those in accounting). It has been argued since at least the 1930s that these formulae are logically equivalent, and therefore that researchers should get the same estimated value for every company at every date, irrespective of which approach they choose. In practice, empirical researchers invariably report different results, with the earnings-based approach generally outperforming the dividends-based approach (measured by closeness to observed market prices).

One explanation for this empirical finding is that the logical equivalence depends critically on reasoning about infinite sequences: for arbitrarily large finite sequences, the dividends-based valuation need not be even remotely close to the earnings-based valuation. Thus the equivalence argument depends on a stronger than intuitionistic logic.

We show that these two approaches to valuation are intuitionistically quite different, and that only the earnings approach satisfies Brouwer's continuity principle. Thus the better observed performance of the earnings-based approach reflects the fact that estimated values are insensitive to assumptions about an infinitely remote future.

Lastly, we show how the continuity principle can address additional puzzles concerning dividends. We view the omission of a promised dividend as the lifting of a provisional restriction; this enables us to explain the market's reaction to dividend omissions, which has previously been seen as anomalous.

This is joint work with Mark van Atten.

► ALEXEY STUKACHEV, *Presentations of structures in admissible sets and on natural numbers*.
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Following [1], for a countable structure $\mathfrak{M}$ of computable signature we consider the set of all possible presentations of $\mathfrak{M}$ on natural numbers. The set of characteristic functions of the atomic diagrams of such presentations forms the mass problem $\underline{\mathfrak{M}}$—the problem of presentability of $\mathfrak{M}$. For arbitrary countable structures $\mathfrak{M}$ and $\mathfrak{N}$, we consider the following properties:

(1) $\mathfrak{M}$ is Σ-definable in $\mathbb{HF}(\mathfrak{N})$;
(2) $\underline{\mathfrak{M}} \leqslant \underline{(\mathfrak{N}, \bar{n})}$ for some $\bar{n} \in N^{<\omega}$ ($\leqslant$ is the Medvedev reducibility);
(3) $\underline{\mathfrak{M}} \leqslant_w \underline{\mathfrak{N}}$ ($\leqslant_w$ is the Muchnik reducibility).

It is easy to see that, for any $\mathfrak{M}$ and $\mathfrak{N}$, 1 implies 2 and 2 implies 3. Yet in general these conditions are not equivalent. We introduce a series of examples showing that. On the other hand, we introduce some sufficient conditions under which 1, 2 and 3 are equivalent, and study the situation for some natural classes of structures.

We also study degree specrta of countable structures in admissible sets of the form $\mathbb{HF}(\mathfrak{N})$, where $\mathfrak{N}$ is countable. A characterization of structures having a degree is found, some examples are presented.

[1] A. I. STUKACHEV, *On mass problems of presentability*, ***TAMC20006: Theory and Applications of Models of Computation***, (J.-Y. Cai, S. B. Cooper, and A. Li Editors), Lecture Notes in Computer Science, vol. 3959 (2006), pp. 774–784.

► CARLO TOFFALORI, *Decidability of the theory of modules over commutative valuation domains*.
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We consider modules over an "effectively given" commutative valuation domain V. It is well known that, if the value group of V is isomorphic to the ordered group of integers, then V-modules have a decidable theory. We deal with an opposite case, when the value group is dense and archimedean. We show that the theory of V-modules is decidable even under this assumption. This is a joint work with Gena Puninski and Vera Puninskaya.

► GEERT VERNAEVE AND ALBERT HOOGEWIJS, *A tale of two substitutions*.
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We have been extending a two-valued first order sequent calculus for predicate logic with identity [1] to handle partial functions, essentially by generalizing Hilbert and Bernays's approach [2] to partially defined iota terms of the form $\iota x_\psi(\varphi)$ which represent the unique x satisfying φ whenever the condition ψ is fulfilled.

Substitution in calculi handling undefinedness can sometimes be subtle. We obtain a sound and complete calculus using a syntactically simple definition of substitution which however is semantically rather 'crude'. We define a second kind of substitution which is syntactically more complicated but behaves better semantically.

In our talk, we will illustrate the interplay between syntax, semantics and deduction rules of both kinds of substitution.

(Supported by Wuytack Fonds UGent)

[1] H. Hermes, ***Introduction to mathematical logic***, Universitext, Springer-Verlag, 1973.
[2] D. Hilbert and P. Bernays, ***Grundlagen der Mathematik I***, 2nd edition, Springer, 1968.

► ANDREAS WEIERMANN, *Renormalization and universality for Gödel incompleteness*.
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The talk is about phase transition phenomena for first order Peano Arithmetic. The idea is to study true assertions which are provable for small values of the order parameter and unprovable for large values. In this situation there will be a natural phase transition threshold which locates the jump from provability to unprovability. Inspired from ideas stemming from theoretical physics we investigate renormalization and universality phenomena in this context. We present a selection of corresponding results which have been obtained so far. It will turn out, for example, that the phase transition thresholds in various examples have quite often the same form.

► ROMAN WENCEL, *On the property of strong cell decomposition for weakly o-minimal structures*.
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A totally ordered structure $\mathbf{M} = (M, \leq, \dots)$ is said to be weakly o-minimal iff all subsets of M definable in $\mathbf{M}$ are finite unions of convex sets. As weak o-minimlity is not preserved under elementary equivalence, we cannot hope for a reasonable cell decomposition theorem in the weakly o-minimal setting.

Strong cell decomposition is a property of weakly o-minimal structures generalizing the cell decomposition for o-minimal structures (i.e., by definition every o-minimal structure has the strong cell decomposition). If $\mathbf{M} = (M, \leq, +, \dots)$ is a weakly o-minimal expansion of an ordered group such that the only subgroups of $(M, +)$ definable in $\mathbf{M}$ are $\{0\}$ and M, then $\mathbf{M}$ has the strong cell decomposition property. During my talk I am going to discuss some consequences of the strong cell decomposition property related to the structure of definable sets and definable groups.

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A set $A \subseteq \omega$ is called 2-c.e. if there are computable enumerable (c.e.) sets A_1 and A_2 such that $A = A_1 - A_2$. A set D is n-c.e. if $D = D_1 - D_2$ for a c.e. set D_1 and a $(n-1)$-c.e. set D_2. A Turing degree is called n-c.e. *degree* if it contains a n-c.e. set; for $n \geq 2$ it is called *properly* n-c.e. if it is n-c.e. but not $(n-1)$-c.e. degree.

A set A is relatively c.e. in a set B if we can computable enumerate A using B as an oracle, in this case we write $A \in \Sigma_1^B$. In my talk I will consider sets and degrees which are relatively c.e. in c.e. sets and degrees.

It is well-known that any 2-c.e. set is relatively c.e. in a c.e. degree below it. Also, obviously, any set is relatively c.e. in any degree above it. Therefore, it is natural to ask the following question: is there for any 2-c.e. set A a c.e. set B such that A is relatively c.e. in B, and A is Turing incomparable with B? It is easy to see that for 2-c.e. sets the answer is negative: for any c.e. set B just take $\omega - B$. The most interesting case is that when the given set A has a properly 2-c.e. degree. The following theorem gives an answer.

Theorem 1. *For any 2-c.e. set A from a properly 2-c.e. degree there exists a c.e. set B such that $A \in \Sigma_1^B$ and $A|_T B$.*

Also, there are two well-known theorems.

Theorem (Arslanov, LaForte, Slaman). *Let A and C be sets such that C is c.e., A is c.e. in C, and A is ω-c.e. Then deg(A) is 2-c.e.*

Theorem (Downey, LaForte, Lempp). *There exists a 3-c.e. set A such that for all c.e. sets W, A has Σ_1^W degree if and only if $W \equiv_T 0'$.*

The first theorem says that we can't consider for properly n-c.e. $(n \geq 3)$ sets relative c.e. in c.e. sets below it. The second theorem which is proved by a $0'''$-priority argument establishes a stronger result on sets which are relatively c.e. in c.e. sets.

Using $0'$-priority argument we proved that

Theorem 2. *There exists a 3-c.e. set A such that for all c.e. sets W, $A \in \Sigma_1^W$ if and only if $W \equiv_T 0'$.*

Since the proof of this theorem uses only $0'$-priority argument it can be combined to obtain 3-c.e. sets with this and some additional properties. For instance, it can be made of a low degree, it can be found above any noncomplete c.e. degree, it can be found in any jump c.e. class and so on. Also in this theorem we can require that the degree of A is properly 3-c.e.

► ANNA ZALEWSKA, *Formalized system of algorithmic logic and procedures*.
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A certain formalized system of algorithmic logic (AL) with procedures is presented. In the system algorithmic properties of programs with procedures can be proved.

The system is an extension of a certain Gentzen-like system for AL with program variables. It uses the notion of a tree of sequents as a basis tool. Algorithmic language is extended by new constructs: block and procedures. A procedure text is treated as a text constant. The declaration of procedure is treated as an assignment of the text constant to the name of the procedure. New Gentzen-like inference rules for the language are proposed.

Soundness and completeness of the system (extended in this way) are discussed from the point of view of procedure complexity.

► MAXIM ZUBKOV, *Strongly η-representable sets*.
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I will talk about a classification of strongly η-representable Δ_3 degrees. In particular, I'll construct a Δ_3 set A such that any noncomputable set $B \leq_T A$ is not strongly η-representable. In another way, I'll show that a set-theoretical difference of two Σ_2 sets is a strongly η-representable.

A infinite set $A = \{a_0 < a_1 < \cdots\}$ is called a strongly η-representable, if there is a computable linear ordering of order type $\eta + a_0 + \eta + a_1 + \cdots$. A degree $\mathbf{a}$ is called a strongly η-representable, if $\mathbf{a}$ contains a strongly η-representable set.

Abstracts of Contributed papers on the Panel on Frege's Begriffsschrift

► SANFORD SHIEH, *Quantification and the nature of Frege's logic*.
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In Frege's Logic, Danielle Macbeth argues against the widespread assumption that Frege is one of the discoverers of modern polyadic quantificational logic, and in favor of taking Frege's logical language to subserve an expressivist and inferentialist project similar in spirit to that of Robert Brandom.

In this paper I examine some of the evidence in Frege's writings for Macbeth's interpretation, focusing on her claims about the non-quantificational status of Frege's "Begriffsschrift" symbols, and on her account of the subject matter of logic for Frege. I raise some difficulties for Macbeth's interpretive arguments based on Frege's explanations of "Begriffsschrift" and his mathematical practice in Grundgesetze der Arithmetik. Specifically, I argue that attempts to interpret Frege's explanations of his symbols as non-quantificational are implausible, and that some of Frege's derivations are difficult to account for non-quantificationally. I conclude that Frege's writings fail to support Macbeth's interpretation unequivocally.

However, I also argue that there is an aspect of Frege's thought about logic that might provide motivation for Macbeth's attempt to account for his logic in non-quantificational terms. This is Frege's repeated characterization of the laws of logic as the most general standards of correctness for judgment. There are difficulties for explaining such a characterization quantificationally, which supports the attempt to find a non-quantificational explanation. But on my reading these difficulties may simply indicate tensions in Frege's commitments on the nature of logic.

Abstracts presented by title

► ADIB BEN JEBARA, *About the attributes of Good and the attributes of a soul*.
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I am using a particular case of the axiom of choice, CC(only 2 through m), the countable axiom of choice for a family of sets of n elements, $n = 2$ through m. See web page of Andreas Blass:

http://www.math.lsa.umich.edu/~ablass/dpcc.pdf

In "All things are numbers" of Logic Colloquium 2001, I wrote that the numbers of attributes of Good are Dedekind Cardinals.

Mr. Andreas Blass thinks that, in the model of the above web page, the set of Dedekind cardinals is of cardinality the cardinality of the continuum.

Evil has as numbers of attributes only aleph zero and the cardinality of the continuum, see "About the strength of Evil" in ASL Winter Meeting 2004–2005. Good has more numbers of attributes than Evil. That could be one of the strengths of Good, as the numbers of attributes are related to strength, and a superiority to Evil.

One way to think of the attributes of Good is to think of the patterns of good behavior associated to most historical characters. Already so, the numbers of attributes of Good are very numerous.

In "An axiom to settle the continuum hypothesis?" in Logic Colloquium 2004, it was stated that the cardinality of the continuum could be the number of attributes of God related to the immortality of souls. It could be also the number of attributes related to immortality for the soul itself. Attributes of the soul other than immortality, like resisting Evil, make the number of attributes of the soul greater or equal than the cardinality of the continuum.

It is therefore either that that number of attributes is the cardinality of the continuum or between the cardinality of the continuum and 2 power that cardinality (negation of the generalized continuum hypothesis).

► JOHN CORCORAN, *Different counterexample sets for logically equivalent propositions*.
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This lecture sharpens discussions in this BULLETIN vol. 11 (2005), p. 460 and pp. 554–5. Two propositions are [*logically*] *equivalent* if each is a consequence of the other. A suitable first-order language is interpreted in the set N of [natural] numbers as universe of discourse. A number n is a *counterexample* for a universal proposition $\forall x\, \mathrm{P}(x)$ iff n satisfies $\sim\mathrm{P}(x)$. Let n^* be a numeral for n. Given any false $\forall x\, \mathrm{P}(x)$ and any n, the universal proposition $\forall y\ (y = n^* \rightarrow \forall x\, \mathrm{P}(x))$ is logically equivalent to $\forall x\, \mathrm{P}(x)$ and has n as sole counterexample—regardless of which counterexamples $\forall x\, \mathrm{P}(x)$ has. Using disjunctive antecedents as in $\forall y\ ((y = n^* \vee y = m^*) \rightarrow \forall x\, \mathrm{P}(x))$, the result generalizes to arbitrary non-empty finite sets of numbers. More generally, we have the First Counterexample Theorem: given any false proposition F and any non-empty finite set S of numbers, there is a universal proposition logically equivalent to F and having S as counterexample set. A similar result holds for N: since every proposition P is equivalent to $\forall x\ (x = x \rightarrow \mathrm{P})$, every falsehood is logically equivalent to a false universal proposition having the entire set N as counterexample set. Corollary: given any false proposition F and any two finite sets A and B, there are two universal propositions both logically equivalent to the given falsehood, one having A as its counterexample set and the other B. Analogous results evidently apply with other languages interpreted in other universes. Other results will be presented if time permits. Special thanks to Mark Brown (Syracuse), Elizabeth Compton (Buffalo), and Frango Nabrasa (Tampa).

► FRANQUI SOLIS CARDENAS POLOCHE, *Simplified gap 2 morasses and some applications*.
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I am going to give a short account about my PhD thesis where it was proved the consistency of the existence of simplified gap 2 morasses using a little simpler definition of simplified gap 2 morasses due to R. Jensen, and prove some old combinatorial results which were proved with the classical morasses.

► ALEXANDER DENISOV, *The constructive homogeneous extensions on the plane*.
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Let $\mathcal{M} = \langle M, \sigma\rangle$ be an arbitrary countable model of an effective signature σ, $H(\mathcal{M})$ be the set of all numerations of $\mathcal{M}$. Consider $\mathcal{M}$ together with some $\nu \in H(\mathcal{M})$, write $(\mathcal{M}, \nu)$. If we add new constant symbols c_n in σ for all elements νn of M we obtain the complete theory $Th(\mathcal{M}, \nu)$ and $S(Th(\mathcal{M}, \nu))$—the family of all complete m-types of that theory, $m \in N$. An arbitrary set $S \subseteq S(Th(\mathcal{M}, \nu))$ can satisfy to the possible properties $1, 2, 3^e, 4, 5$ from [3] and 5^M from [1].

THEOREM. *Let $\mathcal{M}$ be arbitrary **O**-element of the signature σ. Then the diagram obtained from the diagram in the formulation of theorem* [2] *by replacing of the thick **O**$'$ through a thick **O** can be complemented in $\mathcal{M}_{Th\mathcal{M}}^{(d_2,d_3)}$ by the indicated way iff there exists a computable $S \subseteq S(Th(\mathcal{M}, v))$ satisfying to the conditions* $1, 2, 3^e, 4, 5^M$.

PROOF. "If". The constants c_n realizing variables of m-types of S can be represented by a circles or strings of the cartesian plane. The list of the conditions on S allows extend a types up to necessary limits.

The structural diagram for that process consists of two parts. The first one contains horizontal string continued by the point with correlated symbols L_m and x under them and slightly inclinated string continued by the point with corelated symbols M_m and d_m over them. The string M_m and the point d_m lie also over the string L_m and the point x. The second part consistes of the little ellipse with correlated symbol $\Delta_{g(m)}$ and the string with correlated symbol Ξ to right of it, all inside of the big punctuated ellipse with correlated symbol $\Delta_{h(m)}$.

"Only if". Obviously.

[1] A. S. DENISOV, *The constructive homogeneous extensions*, ***Siberian Mathematical Journal***, vol. 25 (1984), no. 6, pp. 60–69. [Russian]

[2] ———, *The most left branch in computable tree* (abstract), ***The Bulletin of Symbolic Logic***, vol. 6 (2000), no. 1, pp. 111–112.

[3] YU. L. ERSHOV, ***The problems of decidability and constructive models***, Nauka, Moscow, 1980. [Russian]

► ANDREY FROLOV, *Incomputable linear orderings*.
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In my talk I will show that there is a low η-like linear ordering with no a computable copy. A linear ordering L is called a η-like, if order type of L has the form $\{f(x) \mid x \in \mathbf{Q}\}$, where $f : \mathbf{Q} \to \mathbf{N} - \{0\}$. Also I'll talk about Downey question: is there a linear ordering whose spectrum is exactly all nonzero Turing degrees.

► I. SH. KALIMULLIN, *Uniform reducibilities of the families of c.e. sets. and degree spectra of the structures*.
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Let $\mathcal{A} = \langle \omega, \Sigma_{\mathcal{A}} \rangle$ be an algebraic structure under a relational signature Σ. We say that a structure $\mathcal{B} = \langle \omega, \Sigma^*_{\mathcal{B}} \rangle$ under the signature $\Sigma^* = \Sigma \cup \{\equiv, \not\equiv\}$ is a *presentation* of $\mathcal{B}$, if $\equiv_{\mathcal{B}}$ is a congruence relation on $\mathcal{B}' = \langle \omega, \Sigma_{\mathcal{B}} \rangle$ and $\mathcal{A} \cong \mathcal{B}' / \equiv_{\mathcal{B}}$.

Following Soskov we define the *degree spectrum* $DS(\mathcal{A})$ of a structure $\mathcal{A} = \langle \omega, \Sigma_{\mathcal{A}} \rangle$ as the class of enumeration degrees of $D^+(\mathcal{B})$ for all presentations of $\mathcal{A}$, where $D^+(\mathcal{B})$ consists of all atomic sentences which holds in $\mathcal{B}$.

Answering the question posed by Lempp, Slaman and Wehner constructed examples of algebraic structures with the degree spectrum, which consists of all nonzero total enumeration degrees.

In my talk these examples will be improved by constructing an algebraic structure whose degree spectrum consists of all nonzero enumeration degrees. Also some connected problems on uniform reducibilies on the structures and on the families of subsets of ω will be considered.

[1] I. N. SOSKOV, *Degree spectra and co-spectra of structures*, ***Annuaire de l'Université de Sofia***, vol. 96 (2003), pp. 45–68.

► GRIGORI MINTS, *A simplified cut elimination for ID1ϵ.*
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A cut elimination proof sketched here allows to simplify the definition of the ϵ-substitution process for ID1 compared to [1] along the lines of [4]. We assume one Π^0_1 inductive definition $\forall x\,[x \in I^{<\Omega} \leftrightarrow \forall y\, \beta(I^{<\Omega}, x, y)]$. ϵ-substitutions with components corresponding to

$$\epsilon x F, \epsilon\xi F, (\epsilon\xi < \alpha)F, n \in I^{\alpha}, n \in I^{<\alpha}, n \in I^{<\Omega}$$

ranks, computations, application of the H-rule are defined as in [1] except the definition of an H-ordinal which is changed drastically. Derivable objects (*sequents*) are finite ϵ-substitutions. Axioms are AxF (computationally inconsistent sequents), AxS (solutions), AxH (sequents to which H-rule is applicable).

Rules of inference (Cut rules). Let $e(\gamma, n) := \epsilon y \neg\beta(I^{<\gamma}, n, y)$.

From $(e, v), \Theta$ for all $v \in V(e)$ infer Θ, where e is an ϵ-term.

From $(n \in I^{<s}, \top), \Theta$ and $(n \in I^{\alpha}, \top), \Theta$ for all $\alpha < s$ infer Θ, where $s \leq \Omega$.

From $(n \in I^{\alpha}, \top), \Theta$ and $(e(n, \alpha), k), \Theta$ for all k infer Θ.

From $(e(n, \Omega), ?)$, $(n \in I^{<\Omega}, ?), \Theta$ and $(e(n, \Omega), k), \Theta$ for all k and $(n \in I^{\alpha}, \top), \Theta$ for all $\alpha < \Omega$ infer Θ.

Empty sequent $\emptyset$ expresses existence of a solution for a given system Cr of critical formulas. *Original derivation* of $\emptyset$ of ordinal height $\Omega \cdot 2 + \omega$ and cut rank $< \Omega + \omega$ is constructed in a standard way.

THEOREM. Cut elimination holds for derivations of $\emptyset$. The proof uses standard technique of locally predicativity [5]. Introducing H-rules into the cut elimination process and using Buchholz notations [2] along the lines of [3] to compute H-ordinals, we get a termination proof for an ϵ-substitution process.

[1] TOSHIYASU ARAI, *Epsilon substitution method for* $\mathrm{ID}_1(\Pi^0_1 \vee \Sigma^0_1)$, ***Annals of Pure and Applied Logic***, vol. 121 (2003), no. 2–3, pp. 163–208.

[2] W. BUCHHOLZ, *Notation systems for infinitary derivations*, ***Archive for Mathematical Logic***, vol. 30 (1991), no. 5–6, pp. 277–296.

[3] G. MINTS, *A termination proof for epsilon substitutions using partial derivations*, ***Theoretical Computer Science***, vol. 303 (2003), pp. 167–214.

[4] ———, *Cut elimination for a simple formulation of* PAϵ, submitted.

[5] W. POHLERS, ***Proof Theory***, Springer Lecture Notes in Mathematics, vol. 1407, 1989.

► MICHAL PELIŠ, *Inferences in logics of questions*.
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Logic of questions is the study of interrelationships between interrogative, imperative and declarative sentences. (See [2] and [3].) Special parts of this field are focused on the analysis of logical relations of questions (inferences, evocation of questions from declaratives etc.).

Two systems seem to be very promising for a further study in the logical "course": the theory of inferences proposed by Wiśniewski [4] and Groenendijk-Stokhof's intensional approach [1]. Both of them deal with similar notions (e.g., declarative—interrogative and interrogative—interrogative entailments, answerhood, reduction of questions etc.).

First, some basic definitions will be introduced for a comparison of the systems. Secondly, possibilities of further developments will be mentioned.

[1] J. GROENENDIJK and M. STOKHOF, *Questions*, ***Handbook of Logic and Language*** (J. van Benthem and A. ter Meulen, editors), Elsevier/MIT, 1997, pp. 1055–1124.

[2] D. HARRAH, *The logic of questions*, ***Handbook of Philosophical Logic*** (D. Gabbay and F. Guenthner, editors), Kluwer, vol. 8, 2002, pp. 1–60.

[3] A. WIŚNIEWSKI, ***The posing of questions: Logical foundations of erotetic inferences***, Kluwer, 1995.
[4] ———, *Questions and inferences*, ***Logique and Analyse***, vol. 173–174–175 (2001), pp. 5–43.

► V. V. RYBAKOV, *Decidability variants of linear temporal logic LTL w.r.t. logical consecutions*.
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We study linear temporal logic LTL in standard language with operations **U** (Untill) and **N** (Next) and its extension by operations **S** (Since) and $\mathbf{N^{-1}}$ (Previous). $LTL^{\mathbf{U}}_{\mathbf{N}}(\mathcal{N})$ is the logic with **U** and **N** generated by the frame $\mathcal{N}$ of all natural numbers as Kripke/Hinttikka structure, $LTL^{\mathbf{U}}_{\mathbf{N}}(\mathcal{Z})$ is the linear logic based on the frame $\mathcal{Z}$ of all integer numbers, and $LTL^{\mathbf{U,S}}_{\mathbf{N,N^{-1}}}(\mathcal{Z})$ is the similar logic which uses **S** and $\mathbf{N^{-1}}$. Logical consecution is a structural inference rule $\mathbf{A_1}, \dots, \mathbf{A_n}/\mathbf{B}$ (where $\mathbf{A_i}$ and **B** are some formulas in the language of given logic).

THEOREM 1. *Linear temporal logic* $LTL^{\mathbf{U}}_{\mathbf{N}}(\mathcal{N})$ *is decidable w.r.t. admissible logical consecutions.*

THEOREM 2. *Linear temporal logics* $LTL^{\mathbf{U,S}}_{\mathbf{N,N^{-1}}}(\mathcal{N})$ *and* $LTL^{\mathbf{U,S}}_{\mathbf{N,N^{-1}}}(\mathcal{Z})$ *do not have the finite model property.*

THEOREM 3. *There is an algorithm recognizing inference rules admissible in* $LTL^{\mathbf{U,S}}_{\mathbf{N,N^{-1}}}(\mathcal{Z})$. *So,* $LTL^{\mathbf{U,S}}_{\mathbf{N,N^{-1}}}(\mathcal{Z})$ *is decidable w.r.t. admissible inference rules, as a consequence,* $LTL^{\mathbf{U,S}}_{\mathbf{N,N^{-1}}}(\mathcal{Z})$ *itself is decidable.*

These theorems extends results from [1] and [2] to LTL with standard operations **U** (Untill) and **N** (Next) and its extension by operations **S** (Since) and $\mathbf{N^{-1}}$ (Previous).

[1] V. V. RYBAKOV, *Logical consecutions in discrete linear temporal logic*, ***The Journal of Symbolic Logic***, vol. 70 (2005), no. 4, pp. 1137–1149.
[2] ———, *Logical consecutions in intransitive temporal linear logic of finite intervals*, ***Journal of Logic Computation***, vol. 15 (2005), no. 5, pp. 633–657.

► XUNWEI ZHOU, *Second level hypothetical inference is better than deduction theorem*.
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In classical logic, we cannot prove a theorem directly. Deduction theorem transforms theorem proving into fact proving. For example, if we we want to prove $\mathrm{int}(x) \to \mathrm{real}(x)$ from $\mathrm{int}(x) \to \mathrm{rat}(x)$ and $\mathrm{rat}(x) \to \mathrm{real}(x)$, then we regard $\mathrm{int}(x)$ as additional premise and prove $\mathrm{real}(x)$. This is done as follows (to simplify the inference, we do not consider quantifiers):

(1)	$\mathrm{int}(x)$	additional premise
(2)	$\mathrm{int}(x) \to \mathrm{rat}(x)$	*P*
(3)	$\mathrm{rat}(x)$	*T*, (1), (2), *MP*
(4)	$\mathrm{rat}(x) \to \mathrm{real}(x)$	*P*
(5)	$\mathrm{real}(x)$	*T*, (3), (4), *MP*
(6)	$\mathrm{int}(x) \to \mathrm{real}(x)$	*CP* rule

In mutually-inversistic logic, however, we have as inference rule the second level hypotheitcal inference (SLMP), which takes logical theorem, say, $\{P \leq^{-1} Q\} \wedge \{Q \leq^{-1} R\} \leq^{-1} \{P \leq^{-1} R\}$ ($\leq^{-1}$ means sufficient condition) as the major premise, takes mathematical theorems, say, $\{\mathrm{int}(x) \leq^{-1} \mathrm{rat}(x)\} \wedge \{\mathrm{rat}(x) \leq^{-1} \mathrm{real}(x)\}$ as the minor premise, infers mathematical theorem, say, $\mathrm{int}(x) \leq^{-1} \mathrm{real}(x)$. The inference mentioned above is done as follows:

(7)	$\mathrm{int}(x) \leq^{-1} \mathrm{rat}(x)$	P
(8)	$\mathrm{rat}(x) \leq^{-1} \mathrm{real}(x)$	P
(9)	$\{\mathrm{int}(x) \leq^{-1} \mathrm{rat}(x)\} \wedge \{\mathrm{rat}(x) \leq^{-1} \mathrm{real}(x)\}$	T, (7), (8), Conjunction rule
(10)	$\{P \leq^{-1} Q\} \wedge \{Q \leq^{-1} R\} \leq^{-1} \{P \leq^{-1} R\}$	P
(11)	$\mathrm{int}(x) \leq^{-1} \mathrm{real}(x)$	T, (9), (10), SLMP

From this example we observe that in mutually-inversistic logic, a theorem can be proved directly. So, mutually-inversistic logic is better than classical logic in this respect.