THE BULLETIN OF SYMBOLIC LOGIC
Volume 13, Number 1, March 2007

2006 ANNUAL MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

Université du Québec à Montréal
Montréal, Québec, Canada
May 17–21, 2006

The Annual Meeting of the Association was held on the campus of the Université du Québec à Montréal, May 17–21, 2006. The program included the Seventeenth Annual Gödel Lecture, one tutorial, and thirteen one-hour invited talks, three of which were part of the Symposium to commemorate the centennial of the birth of Kurt Gödel. In addition to the Gödel Centennial Symposium, the program also featured four special sessions, and seventeen contributed talks. The members of the Program Committee were Solomon Feferman, Salma Kuhlmann, Donald Martin, Prakash Panangaden, Stephen Simpson, and Matthew Valeriote (Chair). The members of the Local Organizing Committee were Luc Bélair, François Lepage, Mathieu Marion (Chair), and Prakash Panangaden. There were 138 registered participants at the meeting, including 70 graduate students. Generous financial support was provided by the Faculté des Sciences and the Faculté des Sciences Humaines of the Université du Québec à Montréal, as well as the Groupe de recherche interuniversitaire de Montréal en histoire et en philosophie de la logique et des mathématiques. The ASL hosted a conference reception.

The Seventeenth Annual Gödel Lecture, *The two layers of logic*, was delivered by Per Martin-Löf (Stockholm University). The tutorial, consisting of 3 one-hour lectures and titled *Semantics of quantum computation*, was presented by Peter Selinger (Dalhousie University). The following hour-long addresses were part of the Gödel Centennial Symposium:

Jeremy Avigad (Carnegie Mellon University), *Gödel and the metamathematical tradition*.
Sy Friedman (University of Vienna), *Generalizations of Gödel's universe of constructible sets*.
Akihiro Kanamori (Boston University), *How Gödel's work transformed set theory*.

In addition, Steve Awodey (Carnegie Mellon University), John Burgess (Princeton University), and William Tait (University of Chicago) gave presentations during a special session on Gödel's Philosophy of Mathematics. The members of the Gödel Centennial Symposium subcommittee were Solomon Feferman, Warren Goldfarb, Charles Parsons (Chair), Dana Scott, and Stephen Simpson.

The other hour-long invited addresses at the meeting were:

Elisabeth Bouscaren (Université Paris-Sud), *Finitely axiomatizable theories, old and new questions*.
Philip Kremer (University of Toronto), *Dynamic topological logic*.
Claude Laflamme (University of Calgary), *Partitions of countable homogeneous structures*.
Antonio Montalbán (University of Chicago), *Equimorphism types of linear orderings*.
Justin Moore (Boise State University), *A solution to the L space problem*.
Helmut Schwichtenberg (Universität München), *Proofs with feasible computational content*.

1079-8986/07/1301-0008/$3.60

Philip Scott (University of Ottawa), *Some recent directions in categorical logic*.

Stewart Shapiro (Ohio State University), *We hold these truths to be self-evident. But what do we mean by that? Frege, Zermelo, and rationalism*.

Patrick Speissegger (McMaster University), *An ordered structure of rank 2 related to Dulac's problem*.

Bas Spitters (University of Nijmegen), *Observational integration theory*.

There were four special sessions (with the organizers given in parentheses): Categorical Logic and Quantum Computation (Michael Makkai, Prakash Panangaden, and Robert Seely), Effective Aspects of Measure Theory and Analysis (Jeremy Avigad and Reed Solomon), Model Theory (David Marker and Charles Steinhorn), and Set Theory (Juris Steprans and Stevo Todorcevic). These included a total of 36 speakers, each of whom gave a 25-minute talk.

Abstracts of the invited talks and the contributed talks (given in person or by title) by members of the Association for Symbolic Logic follow.

For the Program Committee
MATTHEW VALERIOTE

Abstract of the 17th Annual Gödel Lecture

► PER MARTIN-LÖF, *The two layers of logic*.
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Logic has two layers, one epistemological and the other ontological. Assertion (judgement) and inference (demonstration, justification) are the basic epistemological notions of logic. They are conceptually prior to the ontological notions of logic, like proposition, truth of a proposition and the relation of consequence (possibly logical) between propositions. A purely ontological logic, such as was attempted by Wittgenstein in the Tractatus, is impossible, because the basic epistemological notions are presupposed in the semantical explanations of the ontological notions. The logics of Aristotle and Bolzano have the great merit, compared with modern logic, of including both the ontological and the epistemological layer, but the order of conceptual priority between the two layers is contrary to the order in which the semantical explanations have to be given. Their logics were grandiose, but unsuccessful, attempts at subordinating epistemology to ontology.

Abstracts of the invited tutorial on Quantum Information Theory

► PETER SELINGER, *Semantics of quantum computation*.
Department of Mathematics and Statistics, Dalhousie University, Chase Building, Halifax, Nova Scotia, B3H 3J5, Canada.
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These lectures will be an introduction to quantum computing from a logical and symbolic point of view. In the first lecture, I will introduce the basic rules of quantum computing, and I will discuss the design of a simple typed quantum programming language. This language is compositional and satisfies equational laws, and is thus amenable to being studied by structural methods. I will also outline a denotational semantics for this language based on certain operations on positive complex matrices, known as superoperators.

In the second lecture, I will explain the categorical structures behind the "equational laws" mentioned above, and I will present a useful graphical language for reasoning about them. This will lead us to study the class of "strongly compact closed categories" recently introduced by Abramsky and Coecke, which is an axiomatic presentation of the category of Hilbert spaces.

In the third lecture, I will address the issue of higher-order quantum computation, and I will introduce a typed lambda calculus for quantum computation that was developed in joint work with Valiron. I will also discuss some connections between higher-order quantum computation and linear logic.

Abstracts of invited plenary talks

► ELISABETH BOUSCAREN, *Finitely axiomatizable theories, old and new questions*.
Laboratoire de Mathématiques, CNRS UMR 8628, Université Paris-Sud, Bat 425, 91405 Orsay Cedex, France.
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Questions about finite axiomatizability are nearly as old as model theory itself. Nevertheless the question of the non finite axiomatizability of totally categorical theories was at the origin of the development of geometric stability theory in the early 80's. Since that time many new results about finite axiomatizability have been proved and many new questions asked. We will give a short survey of the subject, focusing on the relationship with "geometric model theory".

► PHILIP KREMER, *Dynamic Topological Logic*.
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Dynamic Topological Logic provides a context for studying the confluence of the topological semantics for S4, based on topological spaces rather than Kripke frames; topological dynamics; and temporal logic. In the topological semantics for S4, $\Box$ is interpreted as topological interior: thus S4 can be understood as the logic of topological spaces. Topological dynamics studies the asymptotic properties of continuous maps on topological spaces. Thus, we define a *dynamic topological system* to be a topological space X together with a continuous function f that can be thought of in temporal terms, moving the points of the topological space from one moment to the next. *Dynamic topological logics* are the logics of dynamic topological systems, defined for a trimodal language with an S4 topological modality, $\Box$ (interior), and two temporal modalities, $\bigcirc$ (next) and $*$ (henceforth). In this talk, I will lay out some basic definitions and issues, state some basic theorems, and indicate some results and avenues for research.

► CLAUDE LAFLAMME, *Partitions of countable homogeneous structures*.
Department of Mathematics and Statistics, University of Calgary, 2500 University Drive N.W., Calgary, Alberta, T2N 1N4, Canada.
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We will provide an overview of the partition problem for countable universal binary homogeneous structures, with the Rado graph as a special case. Extensions to homogeneous metric spaces will be discussed.

► ANTONIO MONTALBÁN, *Equimorphism types of linear orderings*.
Department of Mathematics, University of Chicago, 5734 S. University Ave., Chicago, IL 60637, USA.
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We analyze the structure of equimorphism types of linear orderings ordered by embeddability. (Two linear orderings are equimorphic if they can be embedded in each other.)

Our analysis is mainly from the viewpoints of Computable Mathematics and Reverse Mathematics. But we also obtain results, as the definition of equimorphism invariants for linear orderings, which provide a better understanding of the shape of this structure in general.

Here are our main results: Spector proved in 1955 that every hyperarithmetic ordinal is isomorphic to a computable one. We extend his result and prove that every hyperarithmetic linear ordering is equimorphic to a computable one. From the viewpoint of Reverse Mathematics, we look at the strength of Fraïssé's conjecture.

From our results, we deduce that Fraïssé's conjecture is sufficient and necessary to develop a reasonable theory of equimorphism types of linear orderings.

► JUSTIN MOORE, *A solution to the L space problem*.
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Two natural and well studied topological properties are the hereditary separability (HS) and the hereditary Lindelof property (HL) of a given regular topological space. As both of these properties seem very closely related in practice, it is natural to ask about their relationship to one another. In 1968 Hajnal and Juhasz asked whether each implies the other. This pair of questions became known as the S and L space problems.

Early on it was observed that it is consistent with ZFC that these properties have no formal relationship. Assuming the Continuum Hypothesis, for instance, there are a variety of constructions of spaces which are HS but not HL and vice versa. Then, in the early 1980s, Todorcevic demonstrated that some assumption is necessary: assuming the Proper Forcing Axiom, HS implies HL. This solved the so-called S space problem. The dual question—the L space problem—remained open.

Recently I demonstrated—in ZFC—that HL does not, in general, imply HS in the class of regular spaces. I will give an exposition and combinatorial analysis of both the S and L space problems and a broad-strokes outline of what is involved in the construction of an L space in ZFC.

► HELMUT SCHWICHTENBERG, *Proofs with feasible computational content*.
Mathematisches Institut der Universität München, Theresienstr. 39, D-80333 München, Germany.
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We consider logical propositions concerning data structures. If such a proposition involves (constructive) existential quantifiers in strictly positive positions, then—according to Brouwer, Heyting and Kolmogorov—it can be seen as a computational problem. A (constructive) proof of the proposition then provides a solution to this problem, and one can machine extract (via a realizability interpretation) this solution in the form of a lambda calculus term, which can be seen as a functional program. We concentrate on the question of how to control at the proof level the complexity of the extracted programs.

► PHILIP SCOTT, *Some recent directions in categorical logic*.
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Category theory, and in particular techniques from categorical proof theory, have made major progress since the basic book by J. Lambek and myself some 20 years ago. On the one hand, we have witnessed extraordinary uses of categorical logic in Theoretical Computer Science (especially in the Foundations of Programming Languages), in linguistics, and even in certain parts of quantum computation and physics.

On the other hand, in these past 20 years, whole new areas of logic, with deep insights into the meaning of proofs, have developed and inspired major foundational as well as categorical interest. For example, linear logic and proof nets, introduced by Girard in 1986, gave rise to new ways of thinking about computation and the geometry of proofs, and the mathematical modelling of the dynamics of cut-elimination. Other results have moved the Curry-Howard isomorphism to higher categorical levels, towards a general model theory of proofs, with applications in both pure mathematics as well as in the above-mentioned areas.

We shall survey some of these recent developments.

► STEWART SHAPIRO, *We hold these truths to be self-evident. But what do we mean by that? Frege, Zermelo, and rationalism*.
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At the beginning of ***Die Grundlagen der Arithmetik*** (§2) [1884], Frege observes that "it is in the nature of mathematics to prefer proof, where proof is possible". This, of course, is true, but thinkers differ on why it is that mathematicians prefer proof. And what of propositions for which no proof is possible? What of axioms? This talk explores various notions of self-evidence, and the role they play in various foundational systems.

► PATRICK SPEISSEGGER, *An ordered structure of rank 2 related to Dulac's Problem*.
Department of Mathematics and Statistics, McMaster University, 1280 Main Street West, Hamilton, Ontario, L8S 4K1, Canada.
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For a vector field F on the real plane we construct, under certain assumptions on F, an ordered model-theoretic structure associated to the flow of F. We do this in such a way that the set of all limit cycles of F is represented by a definable set. This allows us to restate Dulac's Problem for F—that is, the question whether F has finitely many limit cycles—in model-theoretic terms involving the recently developed notion of thorn-rank.

This is joint work with Alf Dolich.

► BAS SPITTERS, *Observational integration theory*.
Foundations of Mathematics and Computer Science, Department of Computer Science, University of Nijmegen, P.O. Box 9010, NL-6500 GL Nijmegen, The Netherlands.
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In this talk I will present a constructive theory of measure and integration based on ideas from 'pointless topology' (locales, complete Heyting algebras). We connect this to 'pointless probability' as developed by von Neumann and Carathéodory. Von Neumann used this theory as a basis for quantum theory. Both of these pointless theories start from a basic collection of observable objects, the notion of a point in a set as then seen as a derived notion. This may be seen as a move from model theory to a direct description in terms of proof theory. The theory of integration we present is logically very simple, we do not need classical logic or the axiom of countable choice. However, when adding countable choice we can derive as a corollary generalizations of results from constructive or computable analysis, like the spectral theorem, but with a simpler and more transparent proof. In my opinion this provides a nice illustration how ideas from logic (proof theory) can be used to obtain mathematical results. Finally, I will show (by constructing a sheaf model) that in this context the reals can not be proved to be uncountable.

Abstracts of invited plenary talks in the Gödel Centennial Symposium

► JEREMY AVIGAD, *Gödel and the metamathematical tradition*.
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Early in the twentieth century, David Hilbert developed the metamathematical stance as a means of addressing foundational and philosophical issues in mathematics. It has been Kurt Gödel, however, who has had the most profound impact on our metamathematical understanding since then. I will discuss some of Gödel's early results in this light, and locate them in their historical context. I will also assess the influence these results have had on our contemporary foundational and philosophical attitudes.

► SY FRIEDMAN, *Generalisations of Gödel's universe of constructible sets*.
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Gödel's universe L of constructible sets has a beautiful and well-understood internal structure, providing us with an appealing interpretation of ZFC. The theory ZFC $+ V = L$, obtained by adding to ZFC the assertion that every set is constructible, has a corresponding appeal and is in many ways a powerful theory.

However many interesting statements in set theory imply the consistency of ZFC, a statement which is not provable in ZFC $+ V = L$. As large cardinal axioms provide an excellent tool for measuring the consistency strength of set-theoretic statements, they therefore play an important role in modern set theory, as Gödel had proposed.

Two important questions arise.

Are there universes like Gödel's L which satisfy large cardinal axioms? Much progress has been made on this question through elaborations of Gödel's inner model (constructibility) technique and of Cohen's outer model (forcing) technique.

Do large cardinal axioms arise solely out of the need to classify consistency strengths, or do they follow from more basic logical principles? We approach this question using the "inner model hypothesis", a principle which asserts that the universe is complete for demonstrating consistency via inner models.

► AKIHIRO KANAMORI, *How Gödel's work transformed set theory*.
Department of Mathematics, Boston University, 111 Cummington Street, Boston, Massachusetts 02215, USA.
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We describe how Gödel's work on the constructible universe was part of a steady intellectual development, and how this work transformed set theory and launched it as a distinctive field of mathematics, one with structured subject matter in the cumulative hierarchy and with specific methods of proof incorporating first-order logic.

Abstracts of invited talks in the Special Session on Gödel's Philosophy of Mathematics

► STEVE AWODEY, *Gödel and Carnap*.
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Recent historical research has revealed significant interaction in the late 1920s and early 1930s between the young Gödel and some other members of the Vienna Circle, particularly Rudolf Carnap. Three instances are noteworthy for the development of modern logic: Carnap's early studies in axiomatics and Gödel's dissertation on the completeness of first-order logic; Carnap's ill-fated "Gabelbarkeitssatz" and Gödel's first incompleteness theorem; and Carnap's formulation, with Gödel's aid, of a general notion of logical validity in The Logical Syntax of Language. Not only did their impact abruptly alter the course of Carnap's program, but it also transmitted to Gödel the logical momentum that Carnap had acquired from Frege.

► JOHN BURGESS, *On the outside looking in*.
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I will examine the role of characterizations of a viewpoint not graspable by those who hold that viewpoint in analyses of the Lucas-Penrose fallacy and in connection with applications of conservativeness metatheorems in proof theory.

► WILLIAM TAIT, *Hilbert's finitism, Gödel's finitary number theory, and Gödel's conception of Hilbert's finitism*.
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I want to disassociate Gödel's conception of finitary mathematics from his conception of Hilbert's finitism and, secondly, I want to at least question whether his understanding of Hilbert is correct. The latter question concerns the meaning of the term "intuition". Hilbert certainly stated that his finitism is based upon intuition; but it is possible that Gödel understands this in a way that Hilbert did not intend.

Abstracts of invited talks in the Special Session on Categorical Logic/Quantum Computation

► RICHARD BLUTE, *Deep inference and probabilistic coherence spaces*.
Department of Mathematics and Statistics, University of Ottawa, 585, King Edward Avenue, Ottawa, Ontario, K1N 6N5, Canada.
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We propose a definition of categorical model of the deep inference system BV. BV was designed as a logical framework for the consideration of Retoré's extension of multiplicative linear logic to include a self-dual noncommutative connective.

Our definition is based on Cockett and Seely's notion of a linear functor. A BV-category is a $*$-autonomous category with an additional tensor product, which when viewed as a bivariant functor, is linear with a degeneracy condition. We show that this simple definition implies the isomorphisms required of the theory.

We show that coherence spaces, with Retoré's noncommutative tensor, form a model. We then consider Girard's category of probabilistic coherence spaces, and show that it contains an additional monoidal structure making it a BV-category.

We discuss possible applications of such categories to the discrete quantum causal dynamics of Blute-Ivanov-Panangaden. The latter is ongoing work with Guglielmi, Panangaden, and Strassburger.

This is joint work with Prakash Panangaden and Sergey Slavnov.

► ROBIN COCKETT, *Programming with classical quantum datatypes*.
Department of Computer Science, University of Calgary, 2500 University Drive NW, Calgary, Alberta, T2N 1N4, Canada.
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The aim of this talk is to describe a method of implementing quantum programming languages based on a "quantum stack". In particular, the manner in which calls to (recursive) functions are managed will be described as will the details of how the stack architecture supports the implementation of datatypes (with classical control) such as list and trees.

A quantum programming language, which compiles down to quantum stack operations, shall be described (and hopefully demonstrated!). The language is based on Peter Selinger's Quantum programming language. However, this language differs somewhat in that it emphasizes the underlying "linear" semantics of the computations and the role of coproducts and datatypes. This makes it more functional in style (although, of course, it lacks higher order types).

The implementation actually entails a particular semantics and, time permitting, there are some remarks I would like to make on this.

This is joint work with Brett Giles.

► BOB COECKE, *In the beginning God created tensor*.
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In this talk we show that the part of quantum mechanics relevant to quantum informatics can be fully expressed in terms of tensor structure alone, and, without loss of expressiveness, admits a very high level of abstraction in terms of a particular kind of symmetric monoidal categories. In contrast with the usual Birkhoff-von Neumann lattice-theoretic 'non-logics' this categorical axiomatization comes with some kind of 'hyper-deductive logic', namely a degenerate version of Lambek-Barr-Seely categorical semantics for multiplicative linear logic. Relying on the work of Kelly and Laplaza and Joyal and Street it also follows that such categories admit a purely graphical calculus which turns out to be a quite substantial two-dimensional extension of Dirac's bra-ket calculus.

A first step towards this is the axiomatization of the tensor product in terms of dagger-compact categories, due to Abramsky (Oxford) and myself, which provides the theory with the notions of scalar, (self-)adjointness, compound system, Bell-states and map-state duality, unitarity, inner-product, trace, positivity, projector, a Born-rule to calculate probabilities, an abstract counterpart to complex phase, and we are in fact able impose the absence of (the redundant) global phases—the presence of which was the main reason for von Neumann to shift to a lattice-theoretic setting.

Selinger's CPM-construction extends this list with the important notions of mixed states, mixed operations and Jamiolkowski map-state duality.

More surprisingly, also quantum measurements do not need explicit sums and can be defined as a 'self-adjoint' Eilenberg-Moore coalgebra with respect to some particular kind of internal commutative comonoid, a result due to Pavlovic (Kestrel Institute) and myself. In conceptual terms, this definition of measurement exactly captures von Neumann's projection postulate together with the distinct abilities to copy and delete in the quantum world as compared to the classical world (a feature which is key to the whole quantum informatic endeavor). A compelling application of this purely multiplicative notion of quantum measurement, due to Paquette (U. of Montreal) and myself, is a simple graphical proof of Naimark's theorem, from which the definition of generalized measurement arises.

► NICOLA GAMBINO, *On relating type-theoretic and category-theoretic interpretations of constructive set theory*.
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Constructive Zermelo-Frankel set theory (CZF) admits both type-theoretic and category-theoretic models. Type-theoretic models were originally introduced by Aczel by defining an interpretation of CZF in Martin-Löf's type theory. Category-theoretic models have been defined by Moerdijk and Palmgren using the techniques of Algebraic Set Theory.

The aim of the talk is to relate these different models of CZF. This will be done by exploiting a generalisation of the type-theoretic interpretation of constructive set theories, recently introduced by Aczel and the author.

► VICTOR HARNIK AND MICHAEL MAKKAI, *A monadic adjunction between ω-categories and computads*.
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An *ω-category* has objects, called *cells*, and composition operations in each dimension $n < \omega$, where each $(n+1)$-cell has a domain and a codomain that are n-cells. The ω-categories form a category ωCat, whose arrows are called *ω-functors*. For a precise definition, see, e.g., [2].

A *computad* is an ω-category freely generated by a sequence of *indeterminates*, or *indets* (one set of indets in each dimension). A *computad map* is an ω-functor that preserves indets. The computads with their maps, form a category *Comp*, a non-full subcategory of ωCat (cf. [2]).

A cell of a computad is, actually, a *term* that indicates how it was constructed from indets. A computad can be seen as a language for denoting cells in ω-categories. Given an ω-category $\mathbf{X}$, one can build a computad $\mathcal{N}\mathbf{X}$, the *nerve* of $\mathbf{X}$, which has, for each cell of $\mathbf{X}$, an indet of $\mathcal{N}\mathbf{X}$ denoting it. $\mathcal{N}\colon \omega Cat \to Comp$ is a functor that has the inclusion functor $\mathcal{I}\colon Comp \to \omega Cat$ as a left adjoint. The adjoint pair of functors $(\mathcal{N}, \mathcal{I})$ shares an important property with familiar adjoint pairs from algebra.

THEOREM. *The nerve functor $\mathcal{N}$ is monadic.*

For a definition of monadic (or tripleable) functor, see [1].

[1] MICHAEL BARR and CHARLES WELLS, *Toposes, triples and theories*, ***Grundlehren der Mathematischen Wissenschaften***, vol. 278, Springer-Verlag, New York, 1985.

[2] MICHAEL MAKKAI, *The word problem for computads*, URL: `http://www.math.mcgill.ca/makkai`.

► DUSKO PAVLOVIC, *Bra-ket categories: An elementary language for quantum states*.
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Extending the recent proposals by Abramsky, Coecke and Selinger, we define a simple language of abstract density operators, independent of linear algebra. This provides an elementary interface for some otherwise cumbersome computations, and opens an alley towards nonstandard models of quantum theories.

This is joint work with Bob Coecke.

► PETER SELINGER, *Idempotents in dagger categories*.
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Dagger compact closed categories were recently introduced by Abramsky and Coecke (under the name "strongly compact closed categories") as an abstract presentation of the category of Hilbert spaces and linear maps. I subsequently showed that dagger compact closed categories can also describe "mixed" quantum computation, where the morphisms are completely positive maps. I introduced the CPM construction to pass from the "pure" to the "mixed" world. One slight nuisance with this construction is the following: the category CPM(C) usually does not have biproducts (even if C did), and so one must add them freely to obtain a category CPM + (C) of quantum and classical types. In this talk, I will show that CPM + (C) can be equivalently constructed from CPM(C) by splitting self-adjoint idempotents. I will discuss general properties of idempotents in dagger categories, and comment on the significance of the view that a classical type is given by a self-adjoint idempotent on a quantum type.

► SERGEY SLAVNOV, *"Physical" interpretation of linear logic: semiclassical vs. quantum*.
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It is believed by many researchers that linear logic is somehow related to quantum mechanics. However, various "quantum-mechanical" categories, such as the category of Hilbert spaces, do not seem good for modelling LL, since the two distinct multiplicative conectives both have to be interpreted as tensor product.

We consider the symplectic "category" introduced by A. Weinstein for purposes of geometric quantization and related problems. It has classical phase spaces as objects and asymptotic quantum states as morphisms. We note that in this setting a non-degenerate $*$-autonomous structure arises quite naturally, and this gives us a complete model of MLL. Then we discuss an interpretation of the exponential fragment.

Abstracts of invited talks in the Special Session on Effective Aspects of Measure Theory and Analysis

► GEORGE BARMPALIAS, *Random reals and Lipschitz continuity*.
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I will discuss the notion of Lipschitz continuous operators, an effective version of it, and the reducibility and degree structure that it induces. Also, I will present a number of results in the global and local structure of these degrees and establish a connection with the so-called sw reducibility and random reals.

This is joint work with Andy Lewis.

► MARK BRAVERMAN, *Efficient computation of sets in R^n*.
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We present a natural computability notion for subsets of R^n. Intuitively, a set is computable if one can produce "pictures" of it with arbitrarily high precision. This notion is closely related to the computability of real functions in the approximability sense. We present some of those connections.

We apply the notions to present a series of results about the computability and complexity of a family of sets that appear in complex dynamics: the Mandelbrot and the Julia sets.

► FERNANDO FERREIRA AND PAULO OLIVA, *Effective* WKL *conservation in feasible analysis*.
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The first author has shown (model theoretically) in [3] that adding a suitable version of weak König lemma to the basic theory for feasible analysis BTFA does not increase its proof theoretic strength. In particular, it follows that any Π^0_2-theorem of $\mathsf{BTFA} + \mathsf{WKL}$ can already be proven in Cook and Urquhart [2] feasible system PV^ω. Avigad and Feferman have asked in § 7 of [1] for a functional interpretation proof of Ferreira's result. This challenge has been met in [5], where the authors prove the following result.

THEOREM 1. [5] *Let* E-CPV^ω *denote an extensional version of the classical theory* CPV^ω, *and* Σ^0_1-UB *denote the uniform boundedness principle*

$$\forall k^0 \forall f \leq_1 hk\, \exists e^1 A_0(f, h, k, e) \to \exists g^{0\to 1} \forall k^0 \forall f \leq_1 hk\, \exists e \leq_1 gk A_0(f, h, k, e)$$

for any parameter $h^{0\to 1}$ *and* 0*-bounded formula* A_0. *If*

$$\text{E-CPV}^\omega + \text{bAC}^{0,1}_0 + \text{AC}^{0,\text{b}0}_{\hat{\Sigma}^b_1} + \Sigma^0_1\text{-UB} \vdash \forall x^0 \exists y^0 A_{\text{qf}}(x, y),$$

where A_{qf} *is a quantifier-free formula and* $\text{bAC}^{0,1}_0$, $\text{AC}^{0,\text{b}0}_{\hat{\Sigma}^b_1}$ *are choice principles defined in* [5], *then* $\mathsf{PV}^\omega \vdash A_{\text{qf}}(x, tx)$ *for a closed term* $t^{0\to 0}$ *of* CPV^ω.

It is shown in [5] that WKL follows directly from Σ^0_1-UB. The proof of Theorem 1 relies on a combination of two applications of functional interpretation. Initially, the bounded functional interpretation (developed in [4]) is applied in order to eliminate the extra principles. One can then apply Cook and Urquhart Dialectica interpretation of IPV^ω in order to extract the witnessing term t and a verification proof in PV^ω.

[1] J. AVIGAD and S. FEFERMAN, *Gödel's functional ("Dialectica") interpretation*, ***Handbook of proof theory*** (S. R. Buss, editor), Studies in Logic and the Foundations of Mathematics, vol. 137, North Holland, Amsterdam, 1998, pp. 337–405.

[2] S. COOK and A. URQUHART, *Functional interpretations of feasibly constructive arithmetic*, ***Annals of Pure and Applied Logic***, vol. 63 (1993), pp. 103–200.

[3] F. FERREIRA, *A feasible theory for analysis*, ***The Journal of Symbolic Logic***, vol. 59 (1994), no. 3, pp. 1001–1011.

[4] F. FERREIRA and P. OLIVA, *Bounded functional interpretation*, ***Annals of Pure and Applied Logic***, vol. 135 (2005), pp. 73–112.

[5] ———, *Bounded functional interpretation in feasible analysis*, submitted for publication, 2005.

► JEFFRY L. HIRST AND CARL MUMMERT, *Uniformity and set existence proofs (preliminary report)*.
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We are interested in sentences in the language of second-order arithmetic of the form $(*)$: $\forall X\, \exists Y \Theta(X, Y)$, where Θ is arithmetical. Each such sentence Φ has a canonical uniformization Φ^U of the form $\forall \langle X_n \rangle\, \exists \langle Y_n \rangle\, \forall n \Theta(X_n, Y_n)$. Many examples are known of natural sentences Φ of the form $(*)$ such that Φ is provable in RCA_0 but Φ^U is not provable in RCA_0. For example, we may let Φ be the sentence which says that every real number encoded by a quickly converging Cauchy sequence of rationals has a decimal expansion. Nonprovability of Φ^U seems to be strongly associated with the use of the law of the excluded middle in the proof of Φ. In particular, we are often able to obtain weak counterexamples in the sense of Brouwer from Φ^U when Φ^U is not provable in RCA_0.

This talk will describe joint work of Hirst and Mummert that seeks to relate the provability of Φ^U to the provability of Φ without the law of the excluded middle.

► JOHN M. HITCHCOCK, *Resource-bounded dimension, computational learning, and hard problems*.
Department of Computer Science, 4084 Engineering Hall, University of Wyoming, Laramie, WY 82071-3315, USA.
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Resource-bounded dimension was recently introduced by Lutz as an effective version of Hausdorff dimension. We establish a relationship between resource-bounded dimension and Littlestone's online mistake-bound model of learning. This connection is applied to obtain new lower bounds for the density of problems that are hard for complexity classes.

► JOSEPH S. MILLER, *Oscillation in initial segment complexity*.
Department of Mathematics, 196 Auditorium Road, University of Connecticut, Unit 3009, Storrs, CT 06269-3009, USA.
E-mail: joseph.miller@math.uconn.edu.

The prefix-free Kolmogorov complexity of the initial segments of a random real are bounded below by n and above by $n + K(n)$ (constants suppressed). Between those bounds, the behavior of initial segment complexity is hard to describe. We partially solve this problem by giving analytic conditions on the upward oscillations of Martin-Löf random reals and the downward oscillations of almost all reals.

► KAZUYUKI TANAKA, *Non-standard methods and reverse mathematics*.
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In this talk, I will survey some recent works of our group (Yamazaki, Sakamoto, Yokoyama and myself) on non-standard analysis accommodated to weak subsystems of second-order arithmetic. Our proof technique is essentially based on a theorem due to myself [1] that every countable non-standard model of WKL_0 has a proper initial part isomorphic to itself.

[1] K. Tanaka, *The self-embedding theorem of* WKL_0 *and a non-standard method*, ***Annals of Pure and Applied Logic***, vol. 84, 1997, pp. 41–49.

► YIMU YIN, *Model-theoretic quantifier elimination tests*.
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Quantifier elimination is a powerful tool in fields like real algebraic geometry and p-adic analysis. The study of quantifier elimination using model-theoretic methods was pioneered by Abraham Robinson. Subsequently many powerful model-theoretic quantifier elimination tests were proposed. Although in most cases nonconstructive, these tests often provide clues as to how to find an explicit algorithm for eliminating quantifiers in a theory. In this talk we investigate the logical relations among these tests. In particular we prove that, for countable theories, the stronger of two tests proposed by Shoenfield is equivalent to a test proposed by van den Dries.

► JEFFERY ZUCKER, *A network model of analog computation over metric algebras*.
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We define a general concept of a network of analog modules connected by channels, processing data from a metric space A, and operating with respect to a global continuous clock T. The inputs and outputs of the network are continuous streams $u \colon T \to A$, and the input-output behaviour of the network with system parameters from A is modelled by a function

$$F \colon C[T, A]^p \times A^r \to C[T, A]^q \quad (p, q > 0,\ r \geq 0)$$

where $C[T, A]$ is the set of all continuous streams of A-data equipped with the compact-open topology. We give an equational specification of the network, and a semantics which involves solving a fixed point equation over $C[T, A]$ using a contraction principle, based on the fact that $C[T, A]$ can be approximated by metric spaces. We analyse a case study involving a mechanical system. Finally, we introduce a concrete computation theory over $C[T, A]$ and show that if the modules are concretely computable then so is the network function F.

This is joint work with John Tucker (Swansea).

Abstracts of invited talks in the Special Session on Model Theory

► LUC BÉLAIR, *Approximation in difference and differential discrete valuation rings of characteristic p*.
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We proved before [*Equations aux différences dans les vecteurs de Witt*, ***Comptes Rendus de l'Académie des Sciences de Paris, Séries I*** vol. 340 (2005), pp. 99–102] an approximation property for solutions to difference equations in discrete valuation rings of characteristic zero satisfying an appropriate Hensel's lemma, analog to a theorem of Greenberg [*Rational points in henselian discrete valuation rings*, ***Publications Mathématiques de l'Institut des Hautes Études Scientifiques***, vol. 31 (1966), pp. 59–64], and from which we deduced a Nullstellensatz for Frobenius difference equations in Witt vectors. Much the same holds in the differential case, i.e., with a derivation instead of a fixed automorphism. We will discuss the situation in characteristic $p > 0$.

► OZLEM BEYARSLAN, *Pseudofinite fields and tournaments*.
Department of Mathematics, (M/C 249), University of Illinois at Chicago, 851 S. Morgan St., Chicgao, IL 60607, USA.
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A tournament on a set X is an irreflexive binary relation $R \subset X \times X$ such that for every $x \neq y$ in X exactly one of $R(x, y)$ and $R(y, x)$ holds. A pseudofinite field F interprets a tournament by the formula $\exists z\colon (x - y) = z^2$. The automorphism group of any field interpreting a 0-definable tournament can not have any involutions.

To generalize this observation, we will examine the effects of interpreting such structures on the automorphism groups of certain pseudofinite fields.

This is work in progress with Ehud Hrushovski.

► CLIFTON EALY, *Rosiness and the Mann property*.
Department of Mathematics, University of Illinois, 1409 W. Green St., Urbana, IL 61801, USA.
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We examine thorn independence in a variety of structures, focusing on the field of real numbers expanded with a dense multiplicative subgroup, G, which satisfies the Mann property and whose pth powers are of finite index in G. We show such structures are super-rosy and eliminate imaginaries up to codes for small sets.

This is joint work with Ayhan Gunaydin, and Alex Berenstein.

► RICHARD KRAMER, *A pointless definition of parallel*.
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In this talk we consider geometries in which the universe consists of s-flats in the n-dimensional real Euclidean space, and where the only primitive notions are geometric (i.e., preserved under similarity transformations) binary relations between s-flats. We show that in almost all cases, the notion of two s-flats being parallel is first order definable (without parameters). The exceptions, which are all associated with geometric equivalence relations other than parallel, are enumerated. (If $n > 2$, there are 4 exceptions, the logical binary notions, namely the empty, universal, identity and diversity relations, from which parallel cannot be defined. If also $2s = n$, there are 4 additional exceptions, associated with the geometric equivalence relation of being either parallel or totally perpendicular.) It should be noted that the first nontrivial case of this theorem occurs when $s = 1$ and $n = 3$, that is, in geometries of lines in the real 3-dimensional Euclidean space. Even in the case of $s = 1$ and $n = 3$, the question of whether parallel is definable from an arbirtary (non-logical) ternary geometric relation is open.

► KRZYSZTOF KRUPINSKI, *On small profinite structures and their generalizations*.
Department of Mathematics, University of Illinois, 1409 W. Green St., Urbana, IL 61801, USA and Mathematical Institute, University of Wroclaw, Pl. Grunwaldzki 2/4, Wroclaw, Poland.
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Profinite structure is a pair $(X, Aut^*(X))$ where X is a profinite space and $Aut^*(X)$ is a closed subgroup of the group of all homeomorphisms of X respecting the inverse system defining X. We say that a profinite structure $(X, Aut^*(X))$ is small if on each product X^n, $n \in \omega$, there are only countably many orbits under the action of $Aut^*(X)$. Newelski developed the model theory of small profinite structures in analogy with stability theory, but using purely topological notions (without formulas or types).

I shall define main notions considered in profinite structures (e.g., a topological notion of forking independence). I shall give some examples and formulate main conjectures and theorems describing the structure of small profinite groups regarded as profinite structures. Finally I shall say about my recent ideas concerning generalizations of small profinite structures to the case of profinite structures which are not necessarily small and more generally, to some objects which are even not profinite.

► ALICE MEDVEDEVA, *Fine structure in ACFA*.
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ACFA is the model-companion of the theory of fields with a distinguished automorphism. Chatzidakis and Hrushovski showed in their seminal paper that models of ACFA are built up from minimal sets, which come in three kinds reminiscent of Zilber's trichotomy: fieldlike, grouplike, and trivial.

Simplest minimal sets can be encoded by algebraic correspondences on curves. Among these, any grouplike minimal set is non-orthogonal to a minimal set encoded by a group correspondence on a dimension-1 algebraic group. Dimension-1 algebraic groups are elliptic curves, the multiplicative group, or the additive group. Non-orthogonality is witnessed by interalgebraicity encoded by a commutative diagram of curves.

In this talk, we present the following results concerning minimal sets encoded by rational functions (which are the simplest kind of correspondences). On a family of minimal sets, Zilber's trichotomy is definable in parameters. Non-orthogonality among grouplike minimal sets coming from elliptic curves is not definable. Non-orthogonality among grouplike minimal sets coming from a function on the multiplicative group is definable. These results are obtained by looking at the corresponding commutative diagram of tangent spaces. Then certain maps turn out to be quotients by finite groups of (algebraic group) automorphisms.

Omitting definable classes of minimal sets closed under non-orthogonality would produce other model-complete theories of difference fields. If non-orthogonality to grouplike minimal sets coming from the multiplicative group is also definable for correspondences, then we can get new model-complete theories of difference fields.

► ALF ONSHUUS, *Th-forking and equivalence relations*.
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We will prove that a theory does not have infinite (meaning arbitrarily large) chains of refining definable equivalence relations if and only if it is rosy, providing a characterization of rosy theories in terms of definable equivalence relations.

The talk was presented by Clifton Ealy.

► REHANA PATEL, *A family of universal countable H-free graphs*.
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Given a fixed graph H, a graph G is said to be H-free if H does not embed into G as a subgraph, induced or otherwise. In [1], Cherlin, Shelah and Shi give a model-theoretic condition that guarantees the existence of a universal countable H-free graph, for certain finite, connected H; in particular, for $H = K_n \dot{+} K_3$, $n \geq 3$. This talk will describe this infinite family of universal graphs, and discuss some of their model-theoretic properties.

[1] G. Cherlin, S. Shelah, and N. Shi, *Universal graphs with forbidden subgraphs and algebraic closure*, ***Advances in Applied Mathematics***, vol. 22 (1999), pp. 454–491.

► BENJAMIN ROSSMAN, *Properties preserved under homomorphism*.
MIT Computer Science and Artificial Intelligence Laboratory, The Stata Center, Building 32, 32 Vassar Street, Cambridge, MA 02139, USA.
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I will present a new compactness-free proof of the classical Homomorphism Preservation Theorem (a first-order formula is preserved under homomorphisms iff it is equivalent to an existential-positive formula) and discuss a version of this theorem for finite structures.

Abstracts of invited talks in the Special Session on Set Theory

► LILJANA BABINKOSTOVA, *$\mathsf{S}_c(\mathcal{A}, \mathcal{B})$: games, powers and groups*.
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We give a brief status report on the selection principle $\mathsf{S}_c(\mathcal{A}, \mathcal{B})$, defined by: For each sequence $(O_n : n < \infty)$ of elements of $\mathcal{A}$ there is a sequence $(T_n : n < \infty)$ such that: (i) each T_n is a pairwise disjoint collection of sets, (ii) each T_n refines O_n, and (iii) $\bigcup_{n<\infty} T_n \in \mathcal{B}$.

In the special case when $\mathcal{A} = \mathcal{B} =$ the collection of open covers of a topological space, $\mathsf{S}_c(\mathcal{A}, \mathcal{B})$ is: (a) related to covering dimension, (b) a selective version of Bing's 1950 property of screenability, (c) equivalent to the 1978 Addis-Gresham property C, and (d) a topological version of the stronger 1973 Haver property.

We report results on:

- Rohm's problem for products and examples of R. Pol;
- The corresponding game and characterizing countable dimensionality;
- This selection principle in topological groups.

► MAXIM R. BURKE, *Smooth order-isomorphisms of sets of cardinality $\aleph_1$*.
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When A and B are countable dense subsets of $\mathbb{R}$, it is a well-known result of Cantor that A and B are order-isomorphic. A theorem of K. F. Barth and W. J. Schneider states that the order-isomorphism can be taken to be very smooth, in fact the restriction to $\mathbb{R}$ of an entire function. We discuss the possibility of proving such a result for sets of cardinality $\aleph_1$. The main positive results involve the construction of models of ZFC using Shelah's oracle-cc forcing technique, incorporating generalizations due to L. Hoischen of Carleman's Theorem from approximation theory.

► JAMES HIRSCHORN, *Quasi order augmentation schemes*.
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For a quasi order $(Q, \leq)$, an augmentation scheme is a parameterization of the relation $\leq$ by subsets of Q so that $\leq_Q$ is $\leq$, and $A \subseteq B \subseteq Q$ implies $\leq_A$ is an augmentation of $\leq_B$. A subset $A \subseteq Q$ is called weakly cofinal in the augmentation scheme if A is cofinal in Q under the quasi ordering $\leq_A$. We discuss the preservation of weak cofinalities in definable augmentation schemes under the (iterated) forcing operation.

► BART KASTERMANS, (*Maximal*) *cofinitary groups*.
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We will define these groups, and show some interesting results and questions on them.

► BERNHARD KOENIG, *Forcing indestructibility of set-theoretic axioms*.
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Three types of forcing indestructibility are considered: indestructibility by forcings that are (1) strategically closed under ω_1-sequences, (2) closed under ω_1-sequences, and (3) closed under directed systems of size ω_1.

Our most quotable result among others is that MA (proper, not adding reals), PFA, and MM all have different indestructibility properties on this scale. Similar results are possible for axioms true in the Levy-Collapse of a large cardinal.

We feel that these indestructibility results are interesting in their own right, but they also give a number of applications concerning approachability sequences, Kurepa-trees and model-theoretic transfer properties.

► JORDI LOPEZ-ABAD, *Pre-compact families of finite sets and partial unconditionality*.
Equipe de Logique Mathématique, UFR de Mathématiques (case 7012), Université Denis-Diderot Paris 7, 2 place Jussieu, 75251 Paris Cedex 05, France.
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We present a somewhat general framework for studying weakly null sequences in Banach spaces using Ramsey theory of families of finite subsets of $\mathbb{N}$. We use this to prove, among other things, variants of near and convex unconditionality in subsequences of weakly-null sequences in Banach spaces.

This is a joint work with S. Todorcevic

► VOJKAN VUKSANOVIC, *Dual Ramsey theory*.
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We give an overview of what has been done so far in the so-called dual Ramsey theory.

► STUART ZOBLE, *Fragments of the Universal Baire Property for* Π^1_2 *sets below* $0^\#$.
Department of Mathematics, University of Toronto, Toronto, Ontario, M5S 2E4, Canada.
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We present some equiconsistency results (joint with H. Woodin) involving fragments of the Universal Baire property which Π^1_2 sets can have in forcing extensions of L, and discuss related issues.

Abstracts of contributed talks

► STEVE AWODEY, *Kripke semantics for the lambda-calculus*.
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The (typed) lambda-calculus can be interpreted in the style of Kripke semantics, as sets and functions indexed over a partially-ordered set. Past attempts have used a non-standard interpretation of the function types in order to ensure completeness of the calculus of beta-eta conversion. We show that this is not necessary, and that the equational theory of beta-eta equivalence is deductively complete with respect to a very simple semantics with product and function types interpreted in a standard and natural way. The proof uses a new result on the embedding of a cartesian closed category into a presheaf topos.

► STEVE AWODEY AND KOHEI KISHIDA, *Topological semantics for first-order modal logic*.
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As McKinsey and Tarski showed, the Stone representation theorem for Boolean algebras extends to algebras with operators, providing topological semantics for (classical) propositional modal logic. The "necessity" operator is interpreted as taking the open interior of an arbitrary subset of a topological space. This talk will show how to generalize this interpretation from propositional to first-order logic by replacing the space with the topos of all sheaves on it. We give a complete axiomatization of such topological models.

► SERGIO FRATARCANGELI, *The Pfaffian closure of an o-minimal structure whose underlying universe is not the real line*.
Department of Mathematics and Statistics, McMaster University, Hamilton, Ontario L8S 4K1, Canada.
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We present a notion of Pfaffian closure for o-minimal expansions of real closed fields. Under some conditions, this structure is also o-minimal. As a consequence, we obtain a strong uniformity result on the number of connected components of sets defined with Rolle leaves.

► YVON GAUTHIER, *The notion of "outer" consistency from Hilbert to Gödel*.
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In a 1972 note (see [2]), Gödel draws attention to a remark he had made in 1966 (see [3]) on "outer" or ω-consistency saying that "perhaps it has not received sufficient notice." Gödel's insistence on outer consistency refers to Hilbert's own characterization of a formal system as *"äusseres Handeln"* or external treatment of internal or contentual inference (*"inhaltliches Schliessen"*) (see [4]). The very idea of outer consistency means that internal consistency of various systems of arithmetic cannot be proved within the concrete finite resources afforded by the external treatment of a formal system. Consistency or ω-consistency must be assumed from without in order to justify the transfinite axioms, as Gödel says. That does not mean however that Hilbert's consistency programme is doomed to failure, only that it must be pursued by more internal—or more abstract—means. Gödel's *Dialectica* interpretation is an attempt in that direction. I shall explore another direction, internal consistency of arithmetic with infinite descent (see [1]), but the conclusion to be drawn in that context is the persistence of the constructivist motives in Gödel's work and the continuity of Hilbert's finitistic foundations which can be traced back to Kronecker.

[1] Y. GAUTHIER, ***Internal logic. Foundations of mathematics from Kronecker to Hilbert***, chap. 4, Synthese Library, Kluwer, Dordrecht/Boston/London, 2002.

[2] K. GÖDEL, ***Collected Works***, (Solomon Feferman, editor), Oxford University Press, New York/Oxford, 1990, vol. II, p. 305.

[3] ———, *On formally undecidable propositions*, ***From Frege to Gödel: a source book in mathematical logic, 1879–1931***, (van Heijenoort, editor), Harvard University Press, Cambridge, Massachusetts, 1967, pp. 616–617.

[4] D. HILBERT, *Über das Unendliche*, ***Mathematische Annalen***, vol. 95, pp. 161–190.

► DANKO ILIĆ, *Zermelo's well-ordering theorem in type theory*.
DCS Master Program, Chalmers University of Technology, Rännvägen 6B, 412 96 Göteborg, Sweden.
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Taking a 'set' to be a type together with an equivalence relation and adding an extensional choice axiom to the logical framework (Martin-Löf type theory restricted to Π, Σ, N_0, N_1 and N_2) it is shown that any 'set' can be well-ordered. Zermelo's first proof from 1904 is followed, with a simplification to avoid comparability of well-orderings. A formal proof is conducted in the system agdaLight.

[1] JESPER CARLSTRÖM, *EM + Ext- + ACint* is equivalent to *ACext*, ***Mathematical Logic Quarterly***, vol. 50 (2004), no. 3, pp. 236–240.

[2] ERNST ZERMELO, *Beweis, daß jede Menge wohlgeordnet werden kann*, ***Mathematische Annalen***, vol. 59 (1904), pp. 514–516.

► STEFFEN LEMPP, *Some results on the reverse mathematics of rings and vector spaces*.
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We present some results on the reverse mathematics of rings and vector spaces (joint with Downey and Mileti for rings, and with Downey, Hirschfeldt, Kach, Mileti and Montalbán for vector spaces):

THEOREM 1. *The following are equivalent over* RCA_0: (i) WKL_0 (ii) *Every commutative ring which is not a field has a proper ideal.* (iii) *Every vector space of dimension* > 1 *has a proper subspace.*

THEOREM 2. *The following are equivalent over* RCA_0: (i) ACA_0 (ii) *Every commutative ring which is not a field has a finitely generated proper ideal.* (iii) *Every vector space of dimension* > 1 *has a proper subspace of finite dimension*.

We will also discuss some related results from computable algebra.

► ROBERT S. LUBARSKY AND MICHAEL RATHJEN, *Cauchy and Dedekind reals in constructive set theories*.
Department of Mathematical Sciences, Florida Atlantic University, 777 Glades Rd., Boca Raton, FL 33431 USA.
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With powerful enough methods, such as classical logic or countable choice, it is easy to see that the Cauchy reals (equivalence classes of Cauchy sequences) and Dedekind reals (Dedekind cuts) are equivalent. Without such methods, though, this is not the case, as shown for instance by a topological or sheaf model in which there is a Dedekind real with no corresponding Cauchy sequence [1, 2, 3]. In this talk, an essentially similar Kripke model is given, which is then varied to produce a model in which the Dedekind reals do not even form a set (although the Cauchy reals still do). This model still satisfies most of the standard axioms for intuitionistic set theory (see e.g., [4]), only the strongest Power Set-like principle holding is Exponentiation.

[1] P. ACZEL, *The type theoretic interpretation of constructive set theory*, ***Logic Colloquium '77***, (A. MacIntyre, L. Pacholski, and J. Paris, editors), North-Holland, Amsterdam, 1978, pp. 55–66.

[2] M. P. FOURMAN, *Sheaf models for set theory*, ***Journal of Pure and Applied Algebra***, vol. 19 (1980), pp. 91–101.

[3] M. P. FOURMAN AND J. M. E. HYLAND, *Sheaf models for analysis*, ***Applications of sheaves***, (M. P. Fourman, C. J. Mulvey, and D. S. Scott, editors), Lecture Notes in Mathematics, vol. 753, Springer-Verlag, New York, 1979, p. 280–301.

[4] R. J. GRAYSON, *Heyting-valued models for intuitionistic set theory*, (M. P. Fourman, C. J. Mulvey, and D. S. Scott, editors), ***Applications of sheaves***, Lecture Notes in Mathematics, vol. 753, Springer-Verlag, New York, 1979, p. 402–414.

► RUSSELL MILLER, *Infinite-time computable model theory*.
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We apply the notion of an infinite-time Turing machine to study infinite-time computability questions for structures whose domains are subsets of Cantor space 2^{ω}. After a brief introduction to the notion of infinite-time computation, we discuss results for these structures, emphasizing results which are not true for finite-time computation. For instance, in the infinite-time context, the computable Completeness Theorem turns out to be independent of ZFC, as does the computable presentability of computable quotients of certain computable models. We also consider arithmetic on the real line $\mathbb{R}$, showing that there are infinite-time computable presentations not only of the entire real line (as an ordered field) but also of the ordered subfield containing all infinite-time computable real points.

This work is joint with Joel Hamkins, Dan Seabold, and Steve Warner.

[1] J. D. HAMKINS and A. LEWIS, *Infinite time Turing machines*, ***The Journal of Symbolic Logic***, vol. 65 (2000), no. 2, pp. 567–604.

► HUGH MILLINGTON, *Informal consistency of set theory*.
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The set-theoretical foundation of twentieth-century mathematics has been eminently successful. This very success has tended to obscure the fundamental difficulty of axiomatic set theory, namely, that of finding a family of individuals for which one can demonstrate that it behaves in the manner postulated by an axiomatic theory of sets. In this regard, the remarks of S. C. Kleene are still relevant (***Introduction to Metamathematics***, S. C. KLEENE, van Nostrand, 1952, p. 41): "After axiomatization, there must still be some level at which we have truth and falsity. If the axiomatics is informal, the axioms must be true. If the axiomatics is formal, at least we must believe that the theorems do follow from the axioms; and *also there must be some relationship between these results and some actuality outside the axiomatic theory* (emphasis ours), if the mathematician's activity is not to reduce to nonsense." In this paper we provide such an actuality. Specifically, we exhibit a family of individuals (defined by a predicate in a formal language) for which the elementary axioms of set theory without choice are formally derivable. The existence of such a family is relevant to ontological issues posed by axiomatic set theory.

The formal language is a "lambda calculus" extension of a three-sorted, classical, logical calculus, and uses two membership relations. The axioms are non-existential, that is, they make no assumptions as to the existence of individuals with special set-theoretic properties. Nonetheless, we exhibit two predicates, one being a "member" of the other, both of which are closed under analogues of the basic operations of set theory. It follows that the larger predicate behaves like the universe of a set theory with infinite sets. Curry's paradox and Russell's paradox are circumvented by the substitution rules. Variants of the latter are resolved by the classification axioms.

► RUSSELL O'CONNOR, *Verifying the incompleteness theorems in the Coq proof assistant*.
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A software proof assistant is a program that can verify mathematical deductions. Proof assistants typically provide users with a logical foundation, and from this, a user defines mathematical structures and proves theorems about them. The software manages the construction of the proof and verifies that each deduction is sound.

Between 2002 and 2003, I developed a proof of the Gödel-Rosser incompleteness theorem in the Coq proof assistant [1]. In this talk, I will describe the approach I took to complete this proof and discuss the difficulties I had. The development defines a Hilbert-style deduction system for first order logic and proves various logical properties of such a system. It defines the notion of a primitive recursive function, and proves that all such functions are representable. It proves that several functions are primitive recursive; in particular, proving substitution is primitive recursive was quite difficult.

This development improves upon the previous verification of the incompleteness theorem by showing the essential incompleteness of Peano arithmetic, including infinite extensions. Furthermore, the entire proof is carried out inside a constructive logic. This demonstrates that off-the-shelf software is now suitable for verifying difficult theorems in logic.

Finally, this development can serve as a stepping stone to the second incompleteness theorem. I will discuss what further work needs to be done to reach a new milestone in mathematical reasoning done proof assistants by verify the second incompleteness theorem.

[1] R. O'Connor, *Essential incompleteness of arithmetic verified by Coq*, ***Proceedings of the 18th International Conference on Theorem Proving in Higher Order Logics, 2005***, Lecture Notes in Computer Science, vol. 3603, August 2005, pp. 245–260.

► MEHRNOOSH SADRZADEH, *A two-sorted residuated algebra to reason about information flow*.
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I present a two-sorted residuated algebra with operators and its applications to reasoning about information update in multi-agent systems. One of the sorts represents modal and factual propositions in a sup-lattice with a family of endomorphisms one for each agent $(M, \{f_A^M\}_{A\in\mathcal{A}})$ and the other sort stands for actions in a quantale with a family of endomorphisms $(Q, \{f_A^Q\}_{A\in\mathcal{A}})$. The propositional lattice M is the right module of the quantale of actions Q, the actions of Q on M stands for *epistemic update*. This is a binary operation $-.-: M \times Q \to M$ that preserves arbitrary joins of both lattices and thus has two residuals. The one that arises with regard to the joins on the module $-.q \dashv [q]-$ is a dynamic modality, used to reason about dynamics of systems. The endomorphisms on the module are sup-maps, so they also have residuals that are epistemic modalities $f_A^M(-) \dashv \Box_A^M -$ and are used to reason about information state of agents. Using both dynamic and epistemic modalities, we can reason about information flow between agents. The endomorphisms on the quantale encode information of agents about actions, these are sup-maps and thus also have epistemic modalities as residuals $f_A^Q(-) \dashv \Box_A^Q -$, this time on actions. We use these to encode epistemic structure of secret and private actions and their effects that cause agents to obtain wrong information. This structure helps us encode and reason about dynamic epistemic scenarios in e-commerce, AI, and security in a semi-automatic algebraic way. We also have a sequent calculus, sound and complete with regard to algebra, which reflects this two-sortedness and has novel proof-theoretic features about occurance of context sharing and splitting in the same rule.

Traditionally, relational models such as Kripke structures are used for reasoning about these scenarios. Our algebraic axiomatics subsumes the relational model via a two level lifting of the accessibility relations on the set of states $R_A \subseteq S \times S$: first to set functions $\zeta_A : S \to \mathcal{P}(S)$ and then to sup-maps $f_A : \mathcal{P}(S) \to \mathcal{P}(S)$. A one level lifting to functions on set, would yield a power set coalgebraic view that gives rise to a modular coalgebraic logic, this constitutes the subject of my postdoc research.

This contribution is based on my Ph.D. thesis [4], previous work with B. Coecke and A. Baltag [1, 2], and my current research with Corina Cirstea [3].

[1] A. Baltag, B. Coecke, and M. Sadrzadeh, *Epistemic actions as resources*, ***Journal of Logic and Computation***, submitted, `http://www.ecs.soton.ac.uk/~ms6/JLC.pdf`.

[2] ———, *Algebra and sequent calculus for epistemic action*, ***Electrical Notes in Computer Science***, vol. 126 (2005), pp. 27–52.

[3] C. Cirstea, *Compositional approach to defining logics for coalgebras*, ***Theoretical Computer Science***, vol. 327 (2004), pp. 45–69.

[4] M. Sadrzadeh, *Actions and resources in epistemic logic*, Ph.D. Thesis, submitted to University of Quebec at Montreal, December 2005, `www.ecs.soton.ac.uk/~ms6/all.pdf`.

► KENTARO SATO, *Topos theory for modern set theory*.
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Although modern set theory, especially forcing, is one of the origins of topos theory, nowadays these two fields of logic, namely set theory and topos theory, have been developed in completely opposite directions: most set theoretical discussions emphasize combinatorics, to which topos theorists prefer more conceptual approaches.

One typical example is the treatment of forcing. While set theorists usually consider it as constructing 2-valued models $V[G]$ from a model V, topos theorists consider it as constructing the sheaf-topos $V^{\mathbb{P}} \cong \mathbf{Sh}_V(\mathbb{P}, \neg\neg)$; they do not consider the quotients $V[G] \cong V^{\mathbb{P}}/G$ or the stalks, because $(\mathbb{P}, \neg\neg)$ has no points in V, and because there is no topos theoretically nice morphism $V \to V[G]$ or $V[G] \to V$. (We cannot define the skyscraper $V[G] \to V$, since the generic filter G does not exist in V.)

It is true that, if we consider forcing as only a tool for consistency results, dealing with $V[G]$ and with $V^{\mathbb{P}}$ are equivalent. However, in modern set theory, the analogy is quite important between forcing and forcing axioms, where forcing axioms assert that particular kinds of preorders have "weakened generic filters" in V, and topos theoretic treatments conceal the analogy.

We propose a new notion, stalker, for points on the site-level, to express $V[G]$ as a stalk even if the point G exists outside of V; in this approach the analogy mentioned above remains clear. And, according to the analogy, we have a nice application of topos theory to set theory: a characterization of forcing axioms by some kinds of geometric "generic-"absoluteness, which can be seen as an extension of Bagaria's result [1].

[1] J. Bagaria, *A characterization of Martin's axiom in terms of absoluteness*, ***The Journal of Symbolic Logic***, vol. 62 (1997), pp. 366–372.

► JACK D. STECHER, *Heyting's conditions and social choice theory*.
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It has been known since the time of Condorcet that individuals with complete, transitive preferences need not have any method of forming a "social" preference with reasonable properties. This talk gives conditions under which one can aggregate preferences into a *virtual ordering*, i.e., into a partial order that respect's Heyting's conditions. Moreover, the virtual ordering has a representation as an interval on the real line. In the special case where complete, transitive social preferences exist and have a utility representation, the social preference intervals described here are isomorphic to the utility function. Thus while social welfare functions may not generally exist, it is possible to construct *approximate* social welfare, i.e., social welfare neighborhoods.

The author has greatly benefited from the remarks of Harrie de Swart, Bertil Tungodden, and Per Manne. All errors are my own.

► R. GREGORY TAYLOR, *An ordinal-theoretic account of definite relations*.
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Zermelo's late theory of systems of infinitely long propositions is extended to a theory of definite level-1 relations, wherein strongly inaccessible ordinals continue to play a central role. It is then possible to characterize the class of all sets definable relative to an arbitrary collection of Boolean tensors.

► ROGER VILLEMAIRE, SYLVAIN HALLÉ, OMAR CHERKAOUI, AND RUDY DECA, *A first-order logic with a forest of variables*.
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A logic on hierarchical structures was introduced in [1] and used to define and verify properties of communication network equipments. We show how this logic can be described as a first-order logic for which the variables form a forest (set of trees). This allows generalizing Skolem functions and Herbrand universes to this setting and provides a method to help generate network equipment configurations.

[1] Roger Villemaire, Sylvain Hallé, and Omar Cherkaoui, *Configuration logic: A multi-site modal logic*, ***Time***, IEEE Computer Society, 2005, pp. 131–137.

► MICHAEL A. WARREN, *Coalgebras in a category of classes*.
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It is a familiar result that if $\mathcal{E}$ is an elementary topos and G is a cartesian comonad on $\mathcal{E}$, then the category $\mathcal{E}_G$ of coalgebras for G is also an elementary topos. In [3] this result is extended to the setting of "algebraic set theory" by showing that if $\mathcal{C}$ is one of several kinds of categories of classes and G is a cartesian comonad which preserves small maps, then $\mathcal{C}_G$ is also a category of classes of the same kind as $\mathcal{C}$. One immediate corollary is that the set theories **BIST**, **BCST** and **CST** studied in [1] and [2] possess the disjunction and existence properties.

[1] Steve Awodey, Carsten Butz, Alex Simpson, and Thomas Streicher, *Relating set theories, toposes and categories of classes*, in preparation, 2006, A preliminary version from June 2003 is available on the AST website (www.phil.cmu.edu/projects/ast/).

[2] STEVE AWODEY and MICHAEL A. WARREN, *Predicative algebraic set theory*, ***Theory and Applications of Categories***, vol. 15 (2005), pp. 1–39.

[3] MICHAEL A. WARREN, *Coalgebras in a category of classes*, in preparation, 2006. A draft copy may be obtained from the AST website (`www.phil.cmu.edu/projects/ast/`).

► DAN E. WILLARD, *A counter-intuitive incompleteness property of the axiom system $I\Sigma_0$*.
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Consider a version of $I\Sigma_0$, called Ax-1, that corresponds to the union of the axiom system Q with Equation (1)'s induction scheme for any Δ_0 formula $\phi(x, y)$. Our article [2] showed that Gödel's Second Incompleteness Theorem is valid for Ax-1 when either Herbrand deduction or semantic tableaux corresponds to the deductive rule of inference. We recently received some inquires about whether [2]'s methodologies will generalize for an equivalent definition of $I\Sigma_0$, called Ax-2, which replaces (1)'s induction scheme with Equation (2)?

$$\forall x \, \{\{\phi(x,0) \wedge \forall y[\phi(x,y) \Longrightarrow \phi(x,y+1)]\} \Longrightarrow \forall y \, \phi(x,y)\}, \tag{1}$$

$$\forall x \, \forall z \, \{\{\phi(x,0) \wedge \forall y \leq z[\phi(x,y) \Longrightarrow \phi(x,y+1)]\} \Longrightarrow \forall y \leq z \, \phi(x,y)\}. \tag{2}$$

The simple answer is our prior formalism can be adapted also to Ax-2, with some additional work. However, this is not the issue we shall focus upon in this abstract. Rather it is that a third axiomization of $I\Sigma_0$ has been discovered, called Ax-3, that partially evades the Herbrandized version of the Second Incompleteness Theorem.

Ax-3 has been designed by us to prove the identical set of theorems as Ax-1 and Ax-2 (although not with a comparable Herbrand proof length efficiency). Consequently, Ax-3 will possesses the anomalous property that there exists an axiom system α that recognizes its own Herbrand consistency, and which is a **strict superset** of Ax-3's axioms. (The techniques from [1, 3] are used to construct this system α.) Our main theorem will state that a **strict superset** α of Ax-3's axioms can recognize its own Herbrand consistency, *but an analogous system* α likewise super-setting Ax-1 and Ax-2 is provably impossible.

The intuitive reason for these counter-intuitive opposing effects is that none of Ax-1, 2 or 3 can prove "Ax-3 $\cong$ Ax-1" or "Ax-3 $\cong$ Ax-2", although these three systems do actually prove the same theorems as each other.

[1] D. WILLARD, *Self-verifying systems, the incompleteness theorem and the tangibility reflection principle*, ***The Journal of Symbolic Logic***, vol. 66 (2001), no. 2, pp. 536–596.

[2] ———, *How to extend the semantic tableaux and cut-Free versions of the second incompleteness theorem almost to Robinson's Arithmetic Q*, ***The Journal of Symbolic Logic***, vol. 67 (2002), no. 1, pp. 465–496.

[3] ———, *An exploration of the partial respects in which an axiom system recognizing solely addition as a total function can verify its own consistency*, ***The Journal of Symbolic Logic***, vol. 70 (2005), no. 4, pp. 1171–1209.

Abstracts of papers presented by title

► ANDREW ARANA, *Descartes and the cogency of Euclidean practice*.
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In his Géométrie (1637), Descartes distinguished some curves as being acceptable for use in geometric constructions, and others as not. His geometrically acceptable curves correspond to those that we nowadays call algebraic curves. I present a new justification of Descartes' delineation of geometry, as follows. The Euclidean proof procedure uses particular geometric diagrams in establishing general results. For this to be possible, the particular diagrams must be taken to have no hidden details, lest inferences from particular diagrams to general theorems be jeopardized. Call a curve ICM (for "indefinite change under magnification") if, roughly, no matter what degree of magnification is applied to the curve, there are further details of the curve (such as points of inflection, or intersections with other curves) that could be revealed by further magnification. Then we can restate the above: the possibility of cogent Euclidean proof depends on its not considering any ICM curves. Hence, Euclidean practice contains a 'transcendental' presupposition that there are no ICM curves. But given the diversity of curves that a geometer might consider, why is this presupposition plausible? I show how Descartes' delineation of geometrically acceptable curves may be read as a defense of the Euclidean presupposition, by showing that ICM curves that would threaten this presupposition cannot arise in Cartesian geometry. To do this, I first provide and defend a rational reconstruction of Descartes' geometric curves in a contemporary setting. For a curve to be geometric, I argue that it is a necessary condition that these curves be definable in the field of real numbers. Since the setting is now definability in an o-minimal structure, results from contemporary model theory are now available. In particular, I use corollaries of the Cell Decomposition Theorem to argue that ICM curves (of two different kinds, in particular) do not arise in Descartes' geometry. In this way Descartes' delineation of geometric curves defends the Euclidean presupposition, and hence the cogency of Euclidean practice.

► ADIB BEN JEBARA, *About teleportation of elementary particles in the future*.
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The teleportation, I mean, is by others experiments than nowadays.

About what is assumed as a particular case of the axiom of choice, see web page of Andreas Blass: http://www.math.lsa.umich.edu/~ablass/dpcc.pdf CC(only 2 through m) is the countable axiom of choice for a family of sets of n elements, $n = 2$ through m.

I am conjecturing that a particle is teleported when it leaves the space $U(m)$, where CC(only 2 through m) is true, for the space $U(m+1)$, where CC(only 2 through $m+1$) is true, and then from $U(m+1)$ to $U(m+2)$ and so on until $U(m+p)$. The number of possible locations at a given time goes from m to $m+p$. The particle will be able to leap to more locations.

The particle moves until it has m possible locations for a given time. In $U(m)$, $U_1 \times U_2 \times U_3 \times \cdots$ is a set of possible paths and, the number of elements of U_i (set of urelements) being m, the Cartesian product is not the void set. The particle can move. But, if the particle moves until it has $m+1$ possible location for a given time, the number of elements of U_i being $m+1$, the set of possible paths is the void set and the particle cannot move in $U(m)$, but can in $U(m+1)$.

Mr. Andreas Blass pointed out that, in some models, the Cartesian product of some sets of urelements can very well be nonempty, even if other sets of the same cardinalities have empty Cartesian products.

To reply a question of Mr. Blass, going from $U(m)$ to $U(m+1)$ happens in the same physical space which goes from state 1 to state 2 (until $p-1$). It is an approximation to consider that several models of set theory fit within the physical space.

► JOHN CORCORAN,*Existential import*.
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This paper discusses treatments of "existential import" in the literature of standard first-order logic with identity. According to one of several usages reviewed, a universal sentence has *existential import* if it implies "the corresponding existential". Confusions arise from several sources including ambiguity of the expression 'the corresponding existential'. Examples take numbers as the universe of discourse. $\mathrm{F}(x)$, $\mathrm{S}(x)$ and $\mathrm{P}(x)$ have only x free. If 'some number is F', $\exists x\mathrm{F}(x)$, is taken as "the corresponding existential" for the universal sentence 'every number is F', $\forall x\mathrm{F}(x)$, then every universal sentence has existential import. Whatever $\mathrm{F}(x)$ is, $\forall x\mathrm{F}(x)$ implies $\exists x\mathrm{F}(x)$. For example, where $\mathrm{F}(x)$ is a conditional, taking 'some number is P if it is S', $\exists x(\mathrm{S}(x) \rightarrow \mathrm{P}(x))$, as "the corresponding existential" for 'every number is P if it is S', $\forall x(\mathrm{S}(x) \rightarrow \mathrm{P}(x))$, the latter has existential import. However, the literature does not adequately treat the more widely discussed cases in which 'some number that is S is P', $\exists x(\mathrm{S}(x)\ \&\ \mathrm{P}(x))$, is "the corresponding existential" for 'every number that is S is P', $\forall x(\mathrm{S}(x) \rightarrow \mathrm{P}(x))$. 'Some number that precedes itself is even', $\exists x(x < x\ \&\ \mathrm{B}x)$, is not implied by 'Every number that precedes itself is even', $\forall x(x < x \rightarrow \mathrm{B}x)$. However, many authors overlook the fact that the existential 'Some number that is zero is even', $\exists x(x = 0\ \&\ \mathrm{B}x)$, is implied by the universal 'every number that is zero is even', $\forall x(x = 0 \rightarrow \mathrm{B}x)$, which thus has existential import. The following easy metatheorem clarifies the issue: $\forall x(\mathrm{S}(x) \rightarrow \mathrm{P}(x))$ implies $\exists x(\mathrm{S}(x)\ \&\ \mathrm{P}(x))$ if and only if $\exists x\mathrm{S}(x)$ is tautological (logically true).

► MATTHEW JACKSON, *A framework for building and extending measure theory using the machinery of sheaf logic*.

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The key components of Measure Theory can all be found in the topos of sheaves over a sigma-algebra. In this talk, these objects will be constructed using the logic of the topos. Corresponding objects will then be investigated in an arbitrary localic topos, leading to the notion of a measure on a locale. Finally a statement of the Radon-Nikodym Theorem will be presented. This statement of the Radon Nikodym Theorem will give sufficient conditions for differentiation of measures on a locale.

► ZUNWEI ZHOU, *Second level recursion*.

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Recursion theory is a branch of mathematical logic. Recursion is extensively used in computer science. Prolog employs recursive technique. A Prolog program is as follows:

(1) parent (John, Sam).
(2) parent (Sam, Bob).
(3) ancestor (x, y): – parent (x, y).
(4) ancestor (x, z): – parent (x, y), ancestor (y, z).
(5) ?_ancestor (John, Bob).

In formula (4), the head and the body share the same predicate ancestor, this is recursion. Formula (3) is the outlet of the recursion.

Prolog can be regarded as based on the first level calculus of mutually-inversistic logic. Based on the second level calculus of mutually-inversistic logic, second level single quasi-Prolog can be constructed which employs second level recursion. A second level single quasi-Prolog program is as follows:

(6) rat (x): – int (x).
(7) real (x): – rat (x).
(8) {Q!P}: = {Q: –P}.
(9) {R!P}: – {R: –Q}, {Q!P}.
(10) ?_{ real (x) ! int (x)}.

In the program, ! represents existential connective, := represents set-theoretic or logical implicational connective. Formula (9) means that if some P are Q and all Q are R then some P are R. In formula (9), the head and the body share the same connective !, this is second level recursion. Formula (8) means that if all P are Q then some P are Q, it is the outlet of second level recursion.