THE BULLETIN OF SYMBOLIC LOGIC
Volume 13, Number 1, March 2007

IN MEMORIAM: SAUNDERS MAC LANE 1909–2005

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One of the most influential mathematicians of the 20th Century, Saunders Mac Lane died on 14 April 2005. Mac Lane began his long and distinguished career as one of America's few early logicians, before expanding his research into algebra and topology, and, together with collaborator Samuel Eilenberg, inventing category theory, a contribution to the foundations of mathematics of deep and lasting importance.

The son of a congregational minister, Mac Lane had an almost evangelical zeal for all aspects of mathematics, studying it as a devoted disciple, traveling and teaching it like a missionary, delivering fiery, sermon-like lectures, and tending the mathematical flock like a pastor. After degrees from Yale (BS, 1930), Chicago (MS, 1931), and Göttingen (PhD, 1934), and stints at Yale, Harvard (twice), and Cornell, he finally settled at the University of Chicago, where he remained for over 50 years. He succeeded Marshall Stone as chair of Mathematics at Chicago in the 1950s, when that department was perhaps the world's best. He was at times president of the American Mathematical Society and the Mathematical Association of America, vice president of the National Academy of Sciences and the American Philosophical Society, and he received the National Medal of Science. He published over 100 research papers and 6 books, and supervised some 50 dissertations.

As a student at David Hilbert's Göttingen in the early 1930s, Mac Lane picked up the new *moderne Algebra* that was developing there in the lectures of Emmy Noether. His first textbook ***A survey of modern algebra*** (1941, coauthored with Garrett Birkhoff) brought the abstract style home and became a classic. The invention of category theory shortly thereafter was driven by his research in algebraic topology. Under the influence of Noether, Birkhoff, and Stone, he and Eilenberg essentially invented the new field of "homological algebra", which intended to describe geometric objects and relations in terms of associated algebraic ones with which one could calculate—a task at which Mac Lane was particularly skilled. This description required the development of an entirely new language and set of tools, i.e., category theory, the basics of which were first published in 1945.

1079-8986/07/1301-0007/$1.50

By definition, a *category* consists of "objects" $A, B, C, \dots$ and "arrows" $f: A \to B, g: B \to C, \dots$ with each arrow associated formally as indicated to a pair of objects as its "domain" and "codomain", and with each object A having an "identity" arrow $1_A: A \to A$, and each composable pair of arrows having a formal "composite" $g \circ f: A \to C$. These operations of composition and identity arrows are postulated to satisfy the evident associativity and unit laws, making the entire collection of objects and arrows something akin to an abstract group. A simple example of a category has all finite sets as the objects and all functions between them as the arrows, with the usual identity function and composition of functions as the operations. The definition is general enough, however, to capture all manner of different structures and compare them (via "functors," which are mappings between categories) as regards those constructions and properties which can be specified in this "impoverished" language of objects and arrows—and thus only in terms of mappings between objects with the same structure, i.e., in the same category. Here we recognize the legacy of Felix Klein's *Erlanger Programm* (via Mac Lane's training at Göttingen), according to which a species of mathematical structure is determined by preservation under a preselected system of mappings.

In category theory, a diagram of the form,

$$\begin{tikzcd} A \arrow[r, "f"] \arrow[d, "a"'] & B \arrow[d, "b"] \\ C \arrow[r, "g"'] & D \end{tikzcd}$$

indicates that there are arrows a, b, f, g among the objects A, B, C, D as stated, and such a diagram is said to "commute" if the two possible composite arrows are equal, $g \circ a = b \circ f$. Using configurations of such commutative diagrams to represent interrelated equations facilitates much more elaborate equational reasoning than would be possible without this device. The real power of these methods is hardly evident from the definitions, however, and the early reaction to category theory was rather mixed; there were complaints about the reformulation of known results in a jazzy new language. The practical success of this language eventually dispelled any skepticism, however, and today wide swaths of mathematics cannot even be formulated without it. Indeed, its use has revolutionized the foundations and practice of modern algebra and related fields like algebraic topology, geometry, and number theory.

Let us now turn specifically to Mac Lane's relation to logic, as is likely to be of chief interest to the readers of this BULLETIN. Mac Lane's doctoral thesis in logic was written under Paul Bernays and Hermann Weyl (the latter taking over after the former was dismissed as a Jew in Hitler's Germany).

When Bernays rejected his first proposals, Mac Lane wrote home of his frustration with the "utter lack of philosophic grasp of the local professors toward my thesis," saying that he was "prepared to transfer to the University of Vienna, where there are many who think as I do"—presumably a reference to the logicians Rudolf Carnap and Kurt Gödel. Bernays was eventually persuaded after a meeting in which Mac Lane "told him of the philosophical aim" of the thesis.[1] The only evidence of this "aim" in the dissertation, entitled ***Abbreviated proofs in logical calculus***, is in the concluding section, where he briefly discusses the notion of a "guiding idea," which is something like an overarching conception that is both precise and general, and which, he says, should serve as the basis for the determination of the particulars, not only of proofs, but of "all mathematical processes", such as theories and operations. This conception, he says, "suggests a vast and important field of study for mathematical logic: the study of the structure of the elements of mathematics and the determination of this structure through guiding ideas." Moreover, such an investigation, he says:

> should strengthen the connection between mathematical Logic and Mathematics; for a successful consideration of the structure of Mathematics must obviously begin with Mathematics itself. From this point of view, mathematical Logic should not become a separate and highly specialized field in itself. (Mac Lane 1934, p. 61)

At his first posts at Yale and Harvard, while still looking for a permanent position, Mac Lane found that Göttingen-style algebra was in much greater demand than Göttingen-style logic, and he was quick to oblige. He nonetheless maintained an active interest in logic: he was friendly with Church and his students Kleene and Rosser at Princeton; was one of the founding members of the Association for Symbolic Logic; and he wrote numerous reviews of important recent works in logic.

In one such review, of Carnap's ***Logical syntax of language***, he observed that there was a crucial flaw in the all-important definition of logical validity. Carnap had attempted to define the "logical symbols" as the largest collection of symbols such that every sentence constructed only from them is logically determinate, and Mac Lane observed that there need be no *unique* such maximal set. The situation is, of course, similar to one arising often in algebra, as he surely recognized. Mac Lane concluded:

> Such technical points might raise doubts as to the philosophical thesis Carnap wishes to establish here: that in any language whatsoever one can find a uniquely defined "logical" part of the language, and that "logic" and "science" can be clearly distinguished. (Mac Lane 1938, p. 174; see also pp. 173–175).

[1]The foregoing three quotations are from Mac Lane's autobiography.

This point would later be the nub of an influential critique of Carnap by Mac Lane's Harvard colleague and friend W. v. O. Quine, leading to much logical research and philosophical debate.

At around the time the "Stone Age" in Chicago was coming to an end, a group of graduate students there began looking for someone to teach them logic. As Anil Nerode recalls:

> In about 1952 there were several students in mathematics at Chicago interested in logic: Anil Nerode, William Howard, Ray Smullyan, and Stanley Tenenbaum. They persuaded Saunders to sponsor an informal logic seminar, which he attended and in which he delivered polished lectures on the latest papers. The first two, and then [Michael] Morley, took their PhD's with him, respectively in recursion theory, proof theory, and model theory. He was a very broad mathematician. Ray Smullyan went to Princeton to get a degree with Church, Stanley Tenenbaum never got a Ph.D., but contributed to independence proofs in set theory. So this was a very successful seminar. Robert Solovay did a topology thesis with Saunders, but jumped into independence proofs in set theory as soon as Cohen gave his Princeton lecture.

Mac Lane kept a close (if sometimes critical) eye on developments in logic throughout his career: he served on the Council of the Association for Symbolic Logic from 1944 to 1948; as editor of the Carus Monographs Series he encouraged his friend Steve Kleene to write what eventually became the milestone ***Introduction to metamathematics***; and, of course, he took a special interest in the foundational aspects of the theory of categories, devising solutions to the foundational problems it raised. As a member of the National Academy of Sciences, he nominated Gödel for the National Medal of Science, and even accepted it for him from President Ford when Gödel was too ill to accept it in person.

A new chapter in Mac Lane's involvement with logic was opened in the early 1960's by F. W. Lawvere's pioneering work on applying category theory to logic, thereby relating it to other fields like algebraic geometry. Mac Lane took a keen interest in this work and became a great supporter of it. When topos theory arrived on the scene a few years later through the work of A. Grothendieck, Lawvere and M. Tierney, Mac Lane was back into logic with both feet: conducting research; organizing seminars; writing expositions; and lecturing on the subject around the world. He continued (into his 80s!) to pursue the subject with burning interest, promoting the work of categorically-minded logicians and toposophers like Dana Scott and Peter Johnstone, supervising research (including my own), and eventually writing a book on toposes (with I. Moerdijk), which has become the standard textbook in the field.

In sum, Mac Lane not only started out in logic and maintained a life-long interest in it, he also returned to it later to make contributions using the tools he had forged in the meantime. Even his perhaps greatest work, the development of category theory at the height of his powers, might properly be regarded as a contribution to logic—and one of a deeper and more lasting kind than would have been, say, some further theorems in proof theory. As a conceptual tool of the first magnitude, category theory has proven its value in articulating the structure of arguments, operations, and constructions throughout Mathematics. Indeed, this sort of conceptual organization matches Mac Lane's own declared ideal of the proper role of logic, not as a "separate and highly specialized field in itself," but rather as providing the "guiding ideas" for the purpose of studying "the structure of the elements of Mathematics" in order to "strengthen the connection between mathematical Logic and Mathematics".

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