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2005–06 WINTER MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

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January 14–15, 2006

A Winter Meeting of the Association for Symbolic Logic was held on January 14–15, 2006 at the San Antonio Henry B. Gonzalez Convention Center, in conjunction with the annual Joint Mathematics Meetings. The members of the Program Committee were Su Gao, Valentina Harizanov (Chair), and Christopher Miller. The program included seven invited 50-minute talks and seventeen contributed talks. The ASL hosted a welcoming reception on January 14th in the Alamo B Room of the Marriot Riverwalk Hotel. In addition, on January 12–13, prior to the ASL meeting, there was a joint AMS-ASL Special Session, **Interdisciplinary Research Involving Analysis and Logic**, organized by Itay Ben-Yaacov, Su Gao, and Jose Iovino.

The 50-minute delivered invited addresses were:

Douglas Cenzer (University of Florida), *Effectively closed sets*.

C. Ward Henson (University of Illinois at Urbana-Champaign), *Model theory for metric structures*.

Michael C. Laskowski (University of Maryland), *Strictly stable theories and descriptive set theory*.

Alain Louveau (CNRS and University of Paris 6), *Countable quotients of combinatorial structures*.

Russell Miller (Queens College, CUNY), *Spectra of Turing Degrees*.

Itay Neeman (University of California at Los Angeles), *Finite state automata and monadic definability of ordinals*.

Sergei Starchenko (University of Notre Dame), *Complex analytic geometry over o-minimal structures*.

Abstracts of the invited talks and contributed talks given (in person or by title) by members of the Association for Symbolic Logic follow.

For the Program Committee
VALENTINA HARIZANOV

Abstracts of invited talks

► DOUGLAS CENZER, *Effectively closed sets*.
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This will be a survey of recent progress on the structure and applications of effectively closed sets (Π^0_1 classes). Π^0_1 classes are important in the study of computable functions

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and computably enumerable (c.e.) sets. In particular, there are many interesting results which compare and contrast the inclusion lattice of Π^0_1 classes with the inclusion lattice of c.e. sets. Fundamental questions here concern automorphisms, invariance and definability. The (Medvedev) degrees of difficulty of Π^0_1 classes form a lattice and can be related to the difficulty of proving certain theorems in subsystems of second order arithmetic. Π^0_1 classes have wide-ranging applications in computable analysis, algebra and other areas of computable mathematics. The notion of a random real based on Kolmogorov complexity is defined in terms of Π^0_1 classes. We propose the notion of a random closed set and give a few basic results.

► C. WARD HENSON, *Model theory for metric structures*.
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A metric structure is a many-sorted structure with each sort a metric space, which for convenience is assumed to have finite diameter. Additionally, there are functions (of several variables) between sorts, assumed to be uniformly continuous. Examples arise in all areas of mathematics; they include metric spaces themselves, measure algebras (with the metric given by measure of symmetric difference), and structures based on Banach spaces (where one interprets the sorts as balls), including Banach lattices, algebras of operators, etc. Every ordinary structure from first order model theory is canonically a metric structure, taking every distance between distinct elements to be 1.

The usual first-order logic does not work very well for non-discrete metric structures, and several good alternatives have been developed. One is the logic of positive bounded formulas with an approximate semantics (see [5]). Another is the setting of compact abstract theories (cats) (see [1]). A recent development is the convergence of these two points of view and the realization that they are also connected to the [0, 1]-valued continuous logic that was studied extensively in the 1960s and then dropped (see [4]). This leads to a new formalism for the logic of metric structures in which the operations of sup and inf play a central role.

The analogy between this logic for metric space structures and the usual first order logic for ordinary structures is far reaching. Continuous logic satisfies the compactness theorem, Löwenheim-Skolem theorems, diagram arguments, existence of saturated and homogeneous models, characterizations of quantifier elimination and model completeness, Beth's definability theorem, Craig's interpolation theorem, the omitting types theorem, fundamental results of stability theory, and appropriate analogues of essentially all results in basic model theory of first order logic. There is a rich theory of definability for predicates and operations.

The development of this continuous logic for metric structures is an ongoing project being carried out as a collaboration among Itay Ben-Yaacov, Alex Berenstein, Ward Henson, and Alex Usvyatsov. See [2, 3] for expositions of some of this work.

In this talk the speaker will present this logic for metric structures and indicate a few of its application areas, with emphasis on probability spaces and certain linear spaces from functional analysis.

[1] I. Ben-Yaacov, *Positive model theory and compact abstract theories*, ***Journal of Mathematical Logic***, vol. 3 (2003), pp. 85–118.

[2] I. Ben-Yaacov and A. Usvyatsov, *Continuous first order logic and local stability*, preprint. Available at http://www.math.wisc.edu/~pezz/.

[3] A. Berenstein and C. W. Henson, *Model theory for metric structures*, lecture notes for a workshop given in March/April 2005 at the Newton Institute, University of Cambridge, England. Available at http://www.math.uiuc.edu/~henson/.

[4] C. C. Chang and H. J. Keisler, ***Continuous model theory***, Princeton University Press, 1966.

[5] C. W. Henson and J. Iovino, *Ultraproducts in analysis*, ***Analysis and Logic***, London Mathematical Society Lecture Notes 262, Cambridge University Press, 2002, pp. 1–113.

► MICHAEL C. LASKOWSKI, *Strictly stable theories and descriptive set theory*.
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In joint work with Saharon Shelah we investigate strictly stable (i.e., stable, unsuperstable) theories in a countable language. If we throw away the 'hopeless ones' (those having DOP or those which are deep) then, by using results from classical descriptive set theory, we prove a decomposition theorem for the class of (fully) saturated models of the theory. Curiously, using unsuperstability as a **positive** quality, we show that any model of such a theory interprets an abelian group, together with an infinite, descending sequence of subgroups, each of infinite index in its predecessor.

These theorems allow us to prove 'nonstructure' results for all strictly stable theories by considering each of the three cases separately. As an example we prove that the Karp complexity of the class of models of any strictly stable theory is uncontrolled.

Finally, we discuss what more would be needed to prove the fabled 'Main gap for $\aleph_1$-saturated models' which remains open.

► ALAIN LOUVEAU, *Countable quotients of combinatorial structures*.
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In this talk, I will address the following questions: What are the conditions for a model of a theory T (in some finite relational language) to admit a finite, resp. a countable quotient which is also a model of T?

In the particular case of a theory given by a finite list of forbidden configurations, I will present an answer in terms of generalized chromatic numbers for associated hypergraphs.

I will also present some applications, in finite combinatorics, in countable combinatorics to the existence of universal countable structures, and in descriptive set theory to dichotomies of type "countable vs perfect" for general analytic structures.

► RUSSELL MILLER, *Spectra of Turing degrees*.
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We present a survey of results and open questions on various different notions of a *spectrum* which have been investigated by computable model theorists. The two best-known notions are the spectrum of a structure and the spectrum of a relation on a computable structure. In both cases, the idea is to use Turing degrees to measure the complexity intrinsic to the structure or the relation. We also introduce a third notion, the *automorphism spectrum*, which is the set of all Turing degrees of nontrivial automorphisms of a fixed computable structure.

For researchers in computability, the most common question about spectra has been to ask which sets of Turing degrees can be realized as the spectrum of a structure, or as the spectrum of a relation. We will review a number of specific results along these lines, mostly without proof. Often the question is restricted to models of a particular theory, or to relations on such models. For relations, one can also ask how the definability of the relation is connected to its spectrum; the Ash-Nerode Theorem is the most famous result in this area.

For researchers from other fields, the interest in spectra is more in results that distinguish one theory from another, particularly theories which appear widely in mainstream mathematics. As an example, we will consider the theorem of Hirschfeldt, Khoussainov, Shore, and Slinko, stating that the theory of graphs is universal for spectra, in the sense that any spectrum of a structure (or of a relation on a structure) can also be realized as the spectrum of a (relation on a) graph. They prove the same result for a number of other theories—groups, rings, partial orders, etc.—but on the other hand, the theories of linear orders, Boolean algebras, and trees are known not to have this universality, essentially because they do not admit the sort of coding used in the theorem.

Finally, the notion of an automorphism spectrum applies the same approach to the set of symmetries of a computable structure. This more recent concept is the subject of current work by Harizanov, Morozov, and the speaker, and we present certain early results. An extra dimension to this investigation lies in the possibility of composing two automorphisms to get another; for structures and relations there is no analogous operation. In some cases results on automorphism spectra follow from or give rise to results on other spectra. As an example, we will show how our study of single-degree automorphism spectra led to the first known example of a structure whose spectrum contains exactly the high$_n$ Turing degrees (those **d** whose n-th jump computes $\emptyset^{(n+1)}$).

► ITAY NEEMAN, *Finite state automata and monadic definability of ordinals*.
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A formula φ is **monadic second order** (**monadic** for short) if each of its variables is assigned a type, either the type "first order" or the type "second order". In defining the truth value of a formula in a structure $\langle A; \dots \rangle$ we take the first order variables to range over elements of A, and take the second order variables to range over subsets of A. Note that monadic formulae do not allow, at least not directly, talking about sets of *pairs* of elements of A. In particular they need not introduce Gödel sentences, and they need not allow defining cardinality and cofinality.

Let ON be the class of all ordinals. It is not hard to see that ω_n is definable over $\langle \mathrm{ON}; < \rangle$ by a monadic formula, for each $n < \omega$. Assuming large cardinals it is consistent that $\omega_{\omega+1}$ is definable (Magidor). I will show that ω_ω is not definable, and in fact no singular cardinal is definable. The proof uses finite state automata, acting on infinite strings, to convert monadic formulae over the ordinals to formulae of a specific kind that allows talking about clubs, stationary sets, and reflection, but does not allow quantifying over individual ordinals. The connection between these finite state automata and monadic formulae leads to a precise analysis of monadic definability of singular ordinals, assuming knowledge of the definable regular cardinals.

► SERGEI STARCHENKO, *Complex analytic geometry over o-minimal structures*.
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Every algebraically closed field of characteristic zero K can be viewed as $R(i)$, where R is a maximal real closed subfield of K and $i = \sqrt{-1}$. K is then identified with R^2 and subsets of K^n are viewed as subsets of R^{2n} (just like the relationship between complex and reals numbers). In this talk we will discuss as some basic principles of holomorphic functions in one and several variables can be developed for definable functions on K^n, in o-minimal expansions of R. Because every germ of a complex analytic function is definable in the o-minimal structure $\mathbb{R}_{an}$, much of the local theory of complex analytic functions can be treated in this o-minimal framework. It follows that compact analytic manifolds, with their analytic subsets, are definable in $\mathbb{R}_{an}$. Nonstandard models give rise to new examples of K-holomorphic functions and manifolds. As it turns out, o-minimality yields strong finiteness results and new theorems on the removal of singularities of analytic sets. We will also discuss some of these as time permits.

This is a joint work with Y. Peterzil.

Abstracts of contributed talks

► JASON AUBREY, *Pseudo-dominating families of functions*.
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Let $\omega \nearrow \omega$ denote the collection of nondecreasing functions from ω to ω. A family of functions $\mathcal{X} \subseteq {}^{\omega}\omega$ is pseudo-dominating if for every $f \in {}^{\omega}\omega$ there is a partition Π of ω into finite intervals so that for every union Q of intervals from Π there is a function $g \in \mathcal{X}$ so that $g \geq^* f$ on Q or on $\omega \setminus Q$. $\mathcal{X}$ is finitely pseudo-dominating if the set $\{\max_{f \in F} f : F \in [\mathcal{X}]^{<\omega}\}$ is pseudo-dominating. In this talk I will prove that (1) Filter Dichotomy is equivalent to the statement that every unbounded subset of $\omega \nearrow \omega$ is finitely pseudo-dominating and (2) $\mathfrak{u} < \mathfrak{g}$ is equivalent to the statement that every unbounded subset of $\omega \nearrow \omega$ either contains a superperfect set or is simultaneously pseudo-dominating and not 3-dominating. I will also discuss some consequences of these results.

► STEFAN BOLD AND BENEDIKT LÖWE, *Simple measure analyses for more than two measures*.
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We work throughout in the theory $\mathsf{ZF} + \mathsf{AD} + \mathsf{DC}$. In the mid 1980's Steve Jackson solved the fifth Victoria Delfino problem by computing the values of the projective ordinals $\boldsymbol{\delta}^1_n$. A key part of this analysis was the concept of descriptions, finitary objects that "described" how to build ordinals less than a projective ordinal. In the mid 1990's Steve Jackson and Farid T. Khafizov used this theory to provide a full measure analysis for cardinals below $\boldsymbol{\delta}^1_5$. So far a full measure analysis for cardinals below higher projective ordinals is still desirable.

In 2004/5 we developed a simple inductive argument avoiding Jackson's description theory for a measure analysis with just two measures that reaches the first ω^2 cardinals after a projective ordinal. In this talk we will present these results and their generalization to more than two measures, as well as some applications of this measure analysis.

Both authors were funded by a DFG-NWO Bilateral Cooperation Grant (DFG KO 1353/3-1; NWO DN 61-532).

► ROSS BRYANT, *Ehrenfeucht-Fraïssé games and generic coding*.
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The theory of generic codes, under AD, provides an alternative to forcing arguments to prove statements about $L(\mathbb{R})$. A generic coding function gives codes for countable ordinals which can then be used to act like Cohen forcing. Using Ehrenfeucht-Fraïssé games on certain ordinal structures, we analyze the strength of a condition in Cohen forcing. The ultimate goal of this investigation is to strengthen and possibly extend theorems proven with generic coding arguments to higher projective ordinals.

► WESLEY CALVERT, VALENTINA HARIZANOV, AND ALEXANDRA SHLAPENTOKH, *Turing degrees of isomorphism types of algebraic objects*.
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The Turing degree spectrum of a countable structure $\mathcal{A}$ is the set of all Turing degrees of isomorphic copies of $\mathcal{A}$. The Turing degree of the isomorphism type of $\mathcal{A}$, is the least Turing degree in its degree spectrum, if such a minimum exists. Richter was the first to explore these notions, in her Ph.D. thesis supervised by Carl Jockusch. She showed, for instance, that there are Abelian torsion groups whose isomorphism types have arbitrary degree, and also some whose isomorphism types have no degree. She proved this and other results by systems of embedding finite structures.

By extending her methods to use embedding systems of infinite structures, we prove analogous results to additional mathematically interesting structures. We show that there are countable fields, rings of algebraic numbers and functions, and torsion-free Abelian groups of arbitrary rank, whose isomorphism types have arbitrary Turing degrees. We also show that there are structures in each of these classes whose isomorphism types do not have Turing degrees.

► NATHAN C. CARTER, *Reflexive intermediate propositional logics*.
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Which intermediate propositional logics can prove their own completeness? I call a logic reflexive if a second-order metatheory of arithmetic created from the logic is sufficient to prove the completeness of the original logic. It is well known that classical logic is reflexive in this sense, and that intuitionistic logic is not. Where is the boundary?

Given the collection of intermediate propositional logics, I prove that the reflexive logics are exactly those that are at least as strong as testability logic, i.e., intuitionistic logic plus the scheme $\neg\varphi \vee \neg\neg\varphi$. I show that this result holds regardless of whether Tarskian or Kripke semantics is used in the definition of completeness.

► JOHN CHISHOLM, *A non-computable perfect tree T containing no non-computable branches that are wtt-reducible to T*.
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THEOREM 1. *There exists a noncomputable perfect tree $\mathcal{T} \subset 2^{<\omega}$ that contains no noncomputable branch* $\mathbf{b}$ *that is* wtt-*reducible to* $\mathcal{T}$.

It is an open question whether Theorem 1 is true when “wtt-reducible” is replaced by “Turing-reducible”.

► RUMEN D. DIMITROV, *Automorphisms of certain filters of $\mathcal{L}^*(V_\infty)$*.
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The vector space V_∞ is an $\aleph_0$-dimensional space over a computable field F. The lattice $\mathcal{L}(V_\infty)$ of c.e. subspaces of V_∞ is an interesting modular counterpart of the distributive lattice $\mathcal{E}$. Guichard [3] proved that the automorphisms of $\mathcal{L}(V_\infty)$ are countably many because they are induced by computable semilinear transformations. The question about the number of automorphisms of the lattice $\mathcal{L}^*(V_\infty)$ ($\mathcal{L}(V_\infty)$ modulo finite dimensional spaces) has remained open for more than twenty years. We will present structural results about some specific filters in the lattice $\mathcal{L}^*(V_\infty)$ and the automorphisms of these filters.

[1] R. D. DIMITROV, *Quasimaximality and principal filters isomorphism between $\mathcal{E}^*$ and $\mathcal{L}^*(V_\infty)$*, ***Archive for Mathematical Logic***, vol. 43 (2004), pp. 415–424.

[2] ———, *A class of Σ^0_3 modular lattices embeddable as principal filters in $\mathcal{L}^*(V_\infty)$*, submitted to ***Archive for Mathematical Logic***.

[3] D. R. GUICHARD, *Automorphisms of substructure lattices in recursive algebra*, ***Annals of Pure and Applied Logic***, vol. 25 (1983), no. 1, pp. 47–58.

► NATASHA DOBRINEN AND SY FRIEDMAN, *Co-stationarity of the ground model*.
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Given $V \subseteq W$ models of ZFC with the same ordinals and κ a regular cardinal in W, let $\mathcal{P}_\kappa\lambda$ denote the collection of subsets of λ of size less than κ in W. We say that *the ground model is co-stationary* if $\mathcal{P}_\kappa\lambda \setminus V$ is stationary in $\mathcal{P}_\kappa\lambda$. Gitik showed the following: Let $V \subseteq W$ be two models of ZFC with the same ordinals, κ a regular cardinal in W, and $\lambda \geq (\kappa^+)^W$. If there is a real in $W \setminus V$, then $\mathcal{P}_\kappa\lambda \setminus V$ is stationary in $\mathcal{P}_\kappa\lambda$ [1].

We consider problems of generalizing Gitik's Theorem to forcing extensions in which no reals are added. Assuming the existence of an ω_1-Erdős cardinal, we construct a model W of ZFC such that for each partial ordering $\mathbb{P} \in W$ which adds a new subset of ω_1 and is $(\omega_3, \omega_3, < \omega_2)$-distributive, $\mathcal{P}_{\omega_2}(\lambda) \setminus V$ is stationary in $\mathcal{P}_{\omega_2}(\lambda)$ in the forcing extension $W^{\mathbb{P}}$ for all $\lambda > \omega_2$. By a covering theorem of Magidor [2], this cannot be the case over a model V whose core model K has no ω_1-Erdős cardinal, for any $\mathbb{P} \in V$ which adds no new sequences of length ω. If there is a class of ω_1-Erdős cardinals, then we can construct a model of ZFC in which a generalization of Gitik's Theorem holds for partial orderings which add a new subset of ω_1. We discuss these and other related results.

[1] Moti Gitik, *Nonsplitting subsets of $\mathcal{P}_\kappa(\kappa^+)$*, ***The Journal of Symbolic Logic***, vol. 50 (1985), no. 4, pp. 881–894.

[2] Menachem Magidor, *Representing sets of ordinals as countable unions of sets in the core model*, ***Transactions of the American Mathematical Society***, vol. 317 (1990), no. 1, pp. 91–126.

► DAVID FERNÁNDEZ DUQUE, *A polynomial translation of S4 into intuitionistic logic*.
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The translation we present is based on Kripke semantics and simulates model checking in $S4$ within the Intuitionistic Propositional Calculus.

We show that every non-valid formula φ containing exactly N box symbols can be disproved in a model $M = \langle W, R, V, \mathbf{0}\rangle$ where each world $x \in W$ has been assigned a natural number $L(x) < N$ indicating distance from the root world $\mathbf{0}$ and $i(x) < N$ indexing it within a cluster.

We then assign to variables the corresponding indices. A propositional variable p is replaced by a set of variables p^n_m, while a boxformula $\Box\alpha$ is replaced by a set of variables b^n_α. A formula ψ^n_m denotes a copy of ψ where all variables and $\Box$-formulas have been substituted accordingly.

Define

$$\begin{aligned}
Lev(\varphi) &= l^0 \wedge \neg l^N \textstyle\bigwedge_{k=0}^{n-1} \left(l^{k+1} \to l^k\right);\\
Mid^n(\varphi) &= l^n \to \left(\textstyle\bigwedge_{\psi \in sub_\Box(\varphi)} \left(b^n_\psi \vee \neg b^n_\psi\right)\right) \wedge \left(\textstyle\bigwedge_p \bigwedge_{0 \leq m \leq N-1} \left(p^n_m \vee \neg p^n_m\right)\right);\\
A^n_\alpha(\varphi) &= \textstyle\bigwedge_{k \geq n} \left(l^k \to \left(l^{k+1} \vee \bigwedge_{m=0}^{N-1} \alpha^{(k,m)}\right)\right);\\
Box^n_\alpha(\varphi) &= l^n \to \left(b^n_\alpha \to l^{n+1} \vee A^n_\alpha(\varphi)\right) \wedge \left(A^n_\alpha(\varphi) \to l^{n+1} \vee b^n_\alpha\right);\\
\\
P(\varphi) &= Lev(\varphi) \textstyle\bigwedge_{0 \leq n \leq N-1} Mid^n(\varphi) \bigwedge_{0 \leq n \leq N-1} \bigwedge_{\alpha \in sub_\Box \varphi} Box^n_\alpha(\varphi).
\end{aligned}$$

Then set $\varphi^{Int} = P(\varphi) \to \varphi^0_0$.

Our main result is the following

THEOREM.

$$S4 \vdash \varphi \Leftrightarrow Int \vdash \varphi^{Int}.$$

This is proven by translating a model M disproving φ into a model M^{Int} disproving φ^{Int}, along with an inverse translation.

► JOEL DAVID HAMKINS AND BENEDIKT LÖWE, *The modal logic of forcing*.
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Consider the ordinary modal propositional language $\mathcal{L}_\Box$ with operators $\Box$ and $\Diamond$. Given a function S that assigns to each propositional variable a sentence of the language of set theory, we can recursively translate a modal formula φ into a sentence of the language of set theory φ^S by replacing each $\mathtt{p}$ by $S(\mathtt{p})$ and interpreting $\Diamond\psi$ as "there is a forcing extension such that ψ^S". It is easy to see that $\mathbf{S4}^S$ is true for all assignments S.

Hamkins (JSL 2003) has started the investigation of these translations, in particular, he constructed a model of set theory in which $\mathbf{S5}^S$ is true for all assignments S.

Against this background, it is now very natural to ask: What is the class Force := $\{\varphi; \mathsf{ZFC} \vdash \varphi^S$ for all assignments $S\}$? We answer this question and compute that Force = **S4.2**.

► JEFFRY L. HIRST, *Representations of reals in reverse mathematics*.
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In reverse mathematics and computable analysis, real numbers are usually represented by rapidly converging Cauchy sequences. In RCA_0, it is possible to convert any single real in this format to any of a number of other representations including decimal expansions and Dedekind cuts. This conversion is not uniform; the situation for converting sequences of reals is more problematic. For example, the assertion that every sequence of rapidly converging Cauchy sequences can be converted to a corresponding sequence of decimal expansions is equivalent to WKL_0. This work extends results of Mostowski [1] in computable analysis.

[1] A. MOSTOWSKI, *On computable sequences*, ***Fundamenta Mathematicae***, vol. 44 (1957), pp. 37–51.

► TETSUYA ISHIU, *A fine structure construction of a perfectly normal non-realcompact space*.
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We shall present a new construction of a perfectly normal non-realcompact space of size $\aleph_1$ in L. The constructed space is very different from the previously known examples of such spaces in L. Our construction shares the same idea with the forcing construction by Hernández-Hernández and the speaker, but uses the fine structure theory developed by Jensen instead of forcing. Some similar constructions will also be discussed.

► GOHAR MARIKYAN, *Consistency of Martin-Löf's intuitionistic Type Theory*.
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In [1] and [2] Per Martin-Löf has introduced his Type Theory (hereafter MLTT). I prove that the set of all terms of MLTT is a continuous lattice. From which follows that lattices of Scott are model of MLTT. Therefore, Martin-Löf's type theory is consistent.

[1] PER MARTIN-LÖF, *Constructive mathematics and computer programming*, ***Sixth International Congress for Logic, Methodology, and Philosophy of Science***, North-Holland, Amsterdam, 1982, pp. 153–175.

[2] ———, *An intuitionistic theory of types: Predicative part*, ***Logic Colloquium '73***, (H. E. Rose and J. C. Shepherdson, editors) North-Holland, Amsterdam, 1973, pp. 73–118.

► CARL MUMMERT, *Reverse mathematics and Π^1_2 comprehension*.
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Let $\mathrm{MF}(P)$ denote the set of maximal filters on a poset P, and let $N_p = \{F \in \mathrm{MF}(P) \mid p \in F\}$ for each $p \in P$. An MF space is a space of the form $\mathrm{MF}(P)$, with the topology generated by $\{N_p \mid p \in P\}$. If P is countable then $\mathrm{MF}(P)$ is called countably based. The class of countably based MF spaces may be defined in the subsystem ACA_0 of second order arithmetic. ACA_0 proves that every complete separable metric space is homeomorphic to a countably based regular MF space. The converse, "Every countably based regular MF space is homeomorphic to a complete separable metric space," is equivalent to Π^1_2-CA_0 over Π^1_1-CA_0. This is the first example of a theorem of core mathematics which is provable in second order arithmetic and implies Π^1_2-CA_0.

This is joint work with Stephen G. Simpson.

► ERMEK S. NURKHAIDAROV, *Interstitial and pseudo gaps in models of PA*.
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[1] defines an *intersticial gap*. [2] gives the definition of a *pseudogap*. We show that if M is a recursively saturated model of PA and Ω is an interstice with 3-indiscernible elements, then for Ω these definitions coincide.

Using this result we prove that if M is an arithmetically saturated model of Peano Arithmetic such that $(\omega, \mathrm{Rep}(\mathrm{Th}(M))) \models \mathrm{RT}^3_2$, then for every $a \in \Omega_\omega$: $\mathrm{igap}(a) = \mathrm{pseudogap}(a)$.

We also show that if M is a recursively saturated model of PA, Ω_ω is the smallest interstice, and for every $a \in \Omega_\omega$: $\mathrm{igap}(a) = \mathrm{pseudogap}(a)$, then if $b \in \Omega_\omega$ realizes a selective type then G_b is a maximal subgroup of $\mathrm{Aut}(M)$.

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[2] T. BIGORAJSKA, H. KOTLARSKI, AND J. H. SCHMERL, *On regular interstices in countable arithmetically saturated models of Peano Arithmetic*, ***Fundamenta Mathematicae***, vol. 158 (1998), pp. 125–146.

► ERIC UFFERMAN, JENNIFER CHUBB, VALENTINA HARIZANOV, ANDREI MOROZOV, AND SARAH PINGREY, *Semigroups of partial automorphisms of countable structures*.
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We consider only countable structures for finite languages. Such a structure is computable if its domain is computable and its relations and operations are computable. For a structure, the set of all finite partial automorphisms forms a semigroup under function composition.

We show that for certain classes of important structures, the elementary equivalence of these semigroups implies the elementary equivalence of associated structures, and, in some cases, even the isomorphism of corresponding structures.

For a computable structure, we also consider the semigroup of all partial automorphisms which are partial computable. We establish that for certain computable structures, the isomorphism of these semigroups implies that the corresponding structures are also isomorphic, even by a computable isomorphism.

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► BRIAN WYNNE, *The model theory of some non-projectable l-groups*.
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In [1] Tarski gives numerical invariants that classify Boolean algebras with respect to elementary equivalence. In [2] Weispfenning classifies all divisible projectable Abelian l-groups with weak unit by reducing the elementary-equivalence problem for such groups to that of Boolean algebras. We discuss an analogous classification of a special collection of non-projectable l-groups.

A point in a completely regular space is called a P-point if every continuous, real-valued function on the space is constant on a neighborhood of the point. A P-space is a completely regular space all of whose points are P-points and an essential P-space is a completely regular space with at most one non-P-point. The set of all continuous, real-valued functions on a P-space, when viewed as an l-group, is projectable: For every continuous function $f \geq 0$ the interval $[0, f]$ is pseudocomplemented. The set of all continuous, real-valued functions on an essential P-space, however, need not be projectable.

We isolate a special class of essential P-spaces and describe how the elementary-equivalence problem for l-groups of continuous functions on such spaces may be reduced to that of an associated class of Boolean algebras with distinguished ideal. Employing a result of Touraille from [3], we obtain numerical invariants that classify these Boolean algebras with distinguished ideal up to elementary equivalence.

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[2] A. Touraille, *Théories d'algèbres de Boole munies d'idéaux distingués. I. Théories élémentaires*, ***The Journal of Symbolic Logic***, vol. 52 (1987), no. 4, pp. 1027–1043.

[3] V. Weispfenning, *Model Theory of Abelian l-groups*, ***Lattice-ordered groups***, (A. M. W. Glass and W. C. Holland, editors), Kluwer Academic Publishers Group, Dordrecht, 1989, pp. 41–79.

Abstracts presented by title

► ADIB BEN JEBARA, *About immortality of some souls*.
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Let CC(only 2 trough m) be the countable axiom of choice for a family of sets of n elements, n being 2 through m. About what is assumed as a particular case of the axiom of choice, see web page of Andreas Blass : http://www.math.lsa.umich.edu/~ablass/dpcc.pdf. Let ZFU be a set theory with urelements. Let us apply it to U being the set of unethical acts by a person.

A soul has a set of possible sequences of unethical acts by a person, Which is a cardinal product XU_i, U_i a subset of U, which is either 1. the void set (for a number of elements of U_i greater than m) or 2. an infinite set.

For 1 the soul will have a tendency to disappear at death. Not for 2. So, all souls are not immortal.

Immortal souls could be joining a community of souls, at death. These are the approximately honest souls. Honesty is participating to the Good of Plato. The dishonest will get the hell of nothingness for the eternity.

To answer two questions of Mr. Andeas Blass:

– An existing soul has to keep having possible unethical acts by the person. It is possible unethical acts which have to be there and not actual ones.

– If the numbers of elements of the U_i are not the same, there will be also a void set.

For people half honest, the disappearing of the soul depends on the Cartesian product being the void set or not. Note that unethical acts by a person are not sets but elements of origin, urelements. I assume that the unethical acts are in infinite number as I assumed infinite numbers of attributes of Evil in "About the strength of Evil" in ASL Winter Meeting 2004–2005.

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The concept of schema (plural: schemata) has played important roles in logic since Aristotle: the syllogistic figures and moods can be taken to be argument schemata. Likewise, the axioms and rules of the Stoic propositional logic can be taken to be sentence schemata and argument schemata, respectively. Modern philosophers know the role of schemata in explications of the semantic conception of truth through Tarski's Convention T. Mathematical logicians recognize schemata in first-order number theory: lack of finite-axiomatizability is finessed by the Induction Schema. In first-order set theory the inconsistency associated with the second-order comprehension or abstraction principle is finessed by the Separation Schema. In some of several closely related senses a schema is a complex system having several components one of which is a template-text or scheme-template, a syntactic string composed of one or more "blanks" and usually also significant words and/or symbols. In accordance with a side-condition specifying how the blanks are to be filled, the template-text of a schema is used as a "template" to specify a multitude of linguistic expressions such as phrases, sentences, or argument-texts, called instances of the schema. The side-condition is a second component. The collection of specified instances of the schema may be regarded as another of its components. The instances are considered to be in a previously identified language (whether formal or natural), which is often considered to be another component. This article reviews the often-conflicting uses of the expression 'schema' and its synonyms in the literature of logic. It discusses the different definitions presupposed by those uses. And it examines the ontological and epistemic presuppositions circumvented or mooted by the use of schemata, as well as the ontological and epistemic presuppositions engendered by their use. In short, this paper is an introduction to the history and philosophy of schemata.

► XUNWEI ZHOU, *Theorem proving: from formalization to automation*.
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In a proving system based on classical logic, there are many inference rules. It is hard to choose among the numerous rules when making inference. Therefore, theorem proving can only be done formally, not automatically. In mutually-inversistic logic, all of the inference

rules are transformed into logical theorems with the second level hypothetical inference being the sole inference rule. Every time the system wants to make inference, it resorts to the only rule. For the numerous theorems, the system matches them top-down, from left to right, like the way Prolog does. So, theorem proving can be done automatically.

To give you a concrete idea of how mutually-inversistic logic and classical logic differ in proving theorems, I'll show you the example of "all integers are rationals, all rationals are real numbers, therefore all integers are real numbers".

In classical logic, it goes as follows:

(1)	$\forall x(\mathrm{int}(x) \to \mathrm{rat}(x))$	P
(2)	$\mathrm{int}(y) \to \mathrm{rat}(y)$	T, (1), US
(3)	$\forall x(\mathrm{rat}(x) \to \mathrm{real}(x))$	P
(4)	$\mathrm{rat}(y) \to \mathrm{real}(y)$	T, (3), US
(5)	$\mathrm{int}(y) \to \mathrm{real}(y)$	T, (2), (4), hypothetical syllogism
(6)	$\forall x(\mathrm{int}(x) \to \mathrm{real}(x))$	T, (5), UG

In mutually-inversistic logic, it goes as follows:

(11)	$\mathrm{int}(x) \leq^{-1} \mathrm{rat}(x)$	P
(12)	$\mathrm{rat}(x) \leq^{-1} \mathrm{real}(x)$	P
(13)	$\{\mathrm{int}(x) \leq^{-1} \mathrm{rat}(x)\} \wedge \{\mathrm{rat}(x) \leq^{-1} \mathrm{real}(x)\}$	T, (11), (12), Conjunction rule
(14)	$\{P \leq^{-1} Q\} \wedge \{Q \leq^{-1} R\} \leq^{-1} \{P \leq^{-1} R\}$	P
(15)	$\mathrm{int}(x) \leq^{-1} \mathrm{real}(x)$	T, (13), (14), Second level hypothetical inference

The difference is that in classical logic, hypothetical syllogism is an inference rule, while in mutually-inversistic logic it is a logical theorem numbered (14).

In program verification, western computer science has formal program verification, mutually-inversistic computer science has automated program verification. In software development, western computer science has formal software development, mutually-inversistic computer science has semi-automated software development. In derivation of function dependencies of relational database, western computer science has Armstrong axiomatic system, mutually-inversistic computer science has automated derivation system. In program semantics, western computer science has formal semantics, mutually-inversistic computer science will have automated semantics.