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2005–06 WINTER MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

The Hilton New York Hotel
New York, NY
December 27–29, 2005

A Winter Meeting of the Association of Symbolic Logic was held December 27–29, 2005 in New York at the Hilton New York Hotel, in conjunction with the annual meeting of the Eastern Division of the American Philosophical Association. The Program Committee consisted of Melvin Fitting, Haim Gaifman (Chair), and Alasdair Urquhart. The ASL hosted a reception on the evening of December 28th.

The program included six invited one-hour talks, divided into two sessions:

Recent Developments in Computer Science and Set Theory and their Philosophical Implications
Sergei Artemov (CUNY Graduate Center), *Computer-aided proofs and their significance*.
Peter Koellner (Harvard University), *Foundational aspects of modern set theory*.
Michael Rabin (Harvard University), *Randomization and non-transferable proofs*.

Foundational Issues in Modern History of Mathematics and Logic
Jeremy Avigad (Carnegie Mellon University), *Methodology and metaphysics in the development of Dedekind's theory of ideals*.
Wilfried Sieg (Carnegie Mellon University), *Hilbert's finitism and reductive structuralism*.
William Tait (University of Chicago), *The concept of intuition in Hilbert's program and its later extensions*.

The program also included twenty-four contributed papers presented by logicians from the US and abroad.

Abstracts of the invited talks and contributed talks given (in person or by title) by members of the Association for Symbolic Logic follow.

For the Program Committee
HAIM GAIFMAN

Abstracts of invited talks in the session on Foundational Issues in Modern History of Mathematics and Logic

► JEREMY AVIGAD, *Methodology and metaphysics in the development of Dedekind's theory of ideals*.
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The philosophy of mathematics has long been concerned with identifying the appropriate grounds for mathematical knowledge, and clarifying the standards by which proofs are judged to be correct. But in mathematical practice one finds a much broader range of value judgments expressed; questions can be *natural*, definitions can be *fruitful*, and an appropriate conceptual framework can make it possible to *understand* complex phenomena. The history of mathematics can help us come to terms with judgments like these, even if the ultimate goal is a philosophical account that avoids historical terms: characterizing the practice before and after a "conceptual advance" can provide insight as to the types of resources that are mathematically important, and the goals they achieve.

The development of Dedekind's theory of ideals provides a good case in point. Between 1871 and 1894, Dedekind presented three distinct approaches to proving unique factorization, each intended to offer specific methodological improvements over its predecessors. In this talk, I will characterize the changes, and the goals that were addressed.

► WILFRIED SIEG, *Hilbert's finitism and reductive structuralism*.
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It is a curious fact of intellectual history that the *Grundlagenstreit* of the 1920s still colors our perspectives on the foundations of mathematics and beyond. Brouwer remarked in his *Intuitionistic Reflections on Formalism* from 1927, "The disagreement over which is correct, the formalistic way of founding mathematics anew or the intuitionistic way of reconstructing it, will vanish, and the choice between the two activities be reduced to a matter of taste, as soon as the following insights ... are generally accepted." These insights have been accepted as far as they were still defended by Brouwer in 1953.

I sketch first the evolution to Hilbert's finitist consistency program and point to the deep connections to Dedekind's logicist views. Second, I describe the problematic of articulating the epistemologically distinctive aspects of finitist mathematics. An analysis of the informal ideas underlying this approach motivates, however, the formulation of *reductive structuralism*. The goal in this third part is to give an integrating perspective for philosophical work and to formulate focused problems for mathematical investigation.

[1] L. E. J. Brouwer, *Intuitionistische Betrachtungen über den Formalismus* translated in ***From Frege to Gödel*** (van Heijenoort, editor), Cambridge 1967, pp. 490–492.

[2] W. Sieg, *Beyond Hilbert's Reach?*, ***Reading Natural Philosophy***, (Malament, editor), Chicago, 2002, pp. 363–405.

Abstracts of invited talks in the session on Recent Developments in Computer Science and Set Theory and their Philosophical Implications

► SERGEI ARTEMOV, *Computer-aided proofs and their significance*.
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Computer-assisted proofs are playing an increasingly visible role in mathematics. Their scope ranges from routine uses of computational algebra packages to dramatic new discoveries which make newspaper headlines, such as the four-color problem, Keppler's conjecture, Robbins' conjecture, etc. Some of these proofs rely heavily upon modules that compute numerical values or decision procedures which do not produce formally certifiable proof codes.

Proof assistants have also reached the stage where they can be used for formalization and certification of real mathematical theorems. Theorems which have been formally verified in this way include the Jordan curve theorem, the prime number theorem, the first Gödel incompleteness theorem, the aforementioned four-color problem, and other famous mathematical results.

These developments have become possible due to the gradual improvements made by computer scientists in the fields of automated deduction and verification. There are a variety of systems now available ranging from model checkers and fully automated provers for propositional logic to sophisticated proof assistants for higher-order theories; the latter provide user-friendly and theoretically well-founded comprehensive tools for creating, handling, and using formal proofs. Some of these techniques have been used successfully in industry for hardware and software verification. In the formal study of programming languages the state of the art is almost at the point where mechanized metatheoretical tools can be used routinely, and an electronic appendix with machine-checked proofs accompanying papers is fast becoming the norm there.

In this talk we will discuss some methodological and philosophical questions concerning these developments. To what extent can one trust computer-generated proofs? Are such proofs more trustworthy than conventional hand-checked proofs? Can computer proofs be "elegant," or provide insights about the structure of mathematical objects? Do incompleteness phenomena extend to the foundations of verification? For that matter, can a verifier be verified? What kinds of reflection mechanisms are needed to justify the use of computational and decision procedures in formal proofs? What else should be done to make proof assistance more attractive to users?

► PETER KOELLNER, *Foundational aspects of modern set theory*.
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The proliferation of independence results in set theory have led some to despair of the hope of finding axioms that settle the classical questions undecided by the standard axioms of ZFC. However, a cluster of results show that there is a good deal of structure and unity beyond ZFC. In particular, there is a precise sense in which every question of complexity strictly below that of the Continuum Hypothesis is settled by large cardinal axioms. Furthermore, a strong argument can be made that the Continuum Hypothesis admits of an answer. Many issues concerning new axioms revolve around the status of an outstanding open problem in modern set theory: the Ω conjecture. We will underscore its foundational significance and examine ways in which the subject might unfold given its positive or negative resolution. We will close by examining recent results of Hugh Woodin and a scenario for axioms extending ZFC that are as complete for the universe of sets as the axioms of PA are for the natural numbers.

Abstracts of contributed talks

► HORACIO ARLÓ-COSTA AND ROHIT PARIKH, *Some non-normal logics of knowledge without logical omniscience*.
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We study a weakening [4] of a notion of knowledge initially proposed by R. Nozick [3]: agent I knows that A if and only if A is true, I believes A and if A were true, I would believe A. Van Benthem has recently remarked that this account of knowledge constitutes '[. . .] an open challenge to logicians. It is easy to see that its adoption will block laws such as Distribution or Introspection. But are there any valid inference patterns left? Given some plausible background logic of belief and counterfactuals, what is the complete set of validities of Nozick's K? [5].' We answer this question parametrically by considering various such plausible background logics.

Our richest account of conditionals defines them in the context of a branching time structure (CTL). The resulting logic of knowledge is a non-normal modal logic, which we model by appealing to neighborhood semantics [1]. The knowledge operator is not closed under entailment and therefore there is no known technique to determine a normal logic that can simulate it [2]. Nevertheless other important axioms, like Adjunction, are obeyed.

[1] H. ARLÓ-COSTA AND E. PACUIT, *First Order Classical Modal Logic: Applications in logics of knowledge and probability*, ***TARK'05***, (R. van der Meyden, editor), NUS, Singapore, pp. 262–79, 2005.

[2] M. KRACHT AND F. WOLTER, *Normal Monomodal Logics can simulate all others*, ***The Journal of Symbolic Logic***, vol. 64 (1999), no. 1, pp. 99–138.

[3] R. NOZICK, ***Philosophical explanations***, Oxford University Press, 1981.

[4] R. PARIKH, *Logical omniscience and common knowledge: WHAT do we know and what do WE know?*, ***TARK'05***, (R. van der Meyden, editor), NUS, Singapore, pp. 62–78, 2005.

[5] J. VAN BENTHEM, *Epistemic logic and epistemology: the state of their affaires*, ***Philosophical Studies***, forthcoming.

► ARNON AVRON, *A purely syntactic framework for formalizing set theories*.
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We present a new framework for formalizations of axiomatic set theories of different strength. It allows the use of abstract terms, and like the inconsistent "ideal calculus" for set theory is based on just two set-theoretical principles: extensionality and comprehension (to which one may add the axiom of choice and/or the axiom of foundations). Comprehension is formulated as: $x \in \{x \mid \varphi\} \leftrightarrow \varphi$, where $\{x \mid \varphi\}$ is a valid term of the theory (the formula φ itself may contain other valid abstract terms). In order for $\{x \mid \varphi\}$ to be a valid term, φ should be *safe* with respect to $\{x\}$, where safety is a relation between formulas and finite sets of variables. The various safety relations we employ are all purely syntactically defined (using some induction on the logical structure of formulas). Hence our framework allows for a static check of validity of terms.

The core of each of the systems we consider is the safety relation it employs. Our basic system is RST (for "Rudimentary Set Theory"). It is based on the safety relation which implicitly underlies commercial query languages for relational database systems (like SQL). We show that the corresponding comprehension principle is equivalent to closure under Jensen's rudimentary functions. We then show that over RST, each of the other comprehension axioms of ZF (separation, replacement, powerset, infinity) is equivalent to some syntactic safety condition.

► WALTER CARNIELLI, *Deconstructing logics by means of possible-translations semantics*.
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Possible-translations semantics were proposed in [2] for the purpose of assigning semantic interpretations to non-classical logics. The intuition behind possible-translations semantics is to assign an interpretation for a given logic by means of translating its formulas into formulas of other well-known (possibly simpler) logics and to use semantics of the later to explain the former. More on possible-translations semantics, including pertinent references, can be found at [3].

This talk addresses the question of factoring a logic into families of less complex components, which we call *splitting*, versus the bottom-up compositional procedure, which we call *splicing*. The method of fibring, introduced by D. Gabbay in [5] is a prototypical splicing technique, combining logics so as to produce new systems. Though, arguably, there is no intrinsic distinction between splitting and splicing, the fact is that fibring always produces a new logic, and does not allow to start from a desired logic and analyze it, while possible-translations semantics work on both directions.

We show that the nondeterministic semantics of [1] and the plain fibring and direct union of matrices of [4] can be seen as particular instances of possible-translations semantics. This makes the possible-translations semantics a general tool for composing and decomposing logics. Examples of deconstructing logic systems in this way are sketched. This is a joint work with Marcelo E. Coniglio.

[1] A. Avron and I. Lev, *Non-deterministic multiple-valued structures*, ***Journal of Logic and Computation***, vol. 15 (2005), no. 3, pp. 241–261.

[2] W. Carnielli, *Many-valued logics and plausible reasoning*, ***Proceedings of the XX International Congress on Many-Valued Logics, University of Charlotte, USA***, pp. 328–335, IEEE Computer Society, 1990.

[3] W. A. Carnielli, M. E. Coniglio, and J. Marcos, *Logics of Formal Inconsistency*, ***Handbook of Philosophical Logic***, vol. 14, (D. Gabbay and F. Guenthner, editors), Kluwer Academic Publishers, 2005, In print. vol. 14, Preliminary version available at *CLE e-Prints*, vol. 5 (2005), no. 1. `http://www.cle.unicamp.br/e-prints/articles.html`.

[4] M. E. Coniglio and V. L. Fernández, *Plain fibring and direct union of logics with matrix semantics*, ***Proceedings of the 2nd Indian International Conference on Artificial Intelligence (IICAI 2005), Pune, India***, (B. Prasad, editor), IICAI, 2005.

[5] D. Gabbay, *Fibred semantics and the weaving of logics: Part 1*, ***The Journal of Symbolic Logic***, vol. 61 (1996), no. 4, pp. 1057–1120.

► MARCELO E. CONIGLIO, *Meta-translations and fibring*.
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Usually, a translation $f : \mathcal{L}_1 \to \mathcal{L}_2$ from a logic $\mathcal{L}_1$ into a logic $\mathcal{L}_2$ is conceived as a mapping $f : For_1 \to For_2$ between the respective algebra of formulas preserving the consequence relation, that is, if $\Gamma \vdash_{\mathcal{L}_1} \varphi$ then $f(\Gamma) \vdash_{\mathcal{L}_2} f(\varphi)$. This approach has negative consequences for the logics obtained by categorial fibring (in the sense of [2]): the fibring of fragments of a given logic cannot recover in general the original logic, because the loss of some meta-properties of the given logics by means of the usual notion of translation.

A solution to this failure, as we pointed out in [1], is to consider a stronger notion of translation between logics (that we call *meta-translations*) such that, if $\mathcal{L}_1$ satisfies a meta-property of the form

$$\texttt{If } \Gamma_1 \vdash_{\mathcal{L}_1} \varphi_1 \texttt{ and} \ldots\ \Gamma_n \vdash_{\mathcal{L}_1} \varphi_n \texttt{ then } \Gamma \vdash_{\mathcal{L}_1} \varphi$$

then $\mathcal{L}_2$ must satisfy the translated meta-property:

$$\texttt{If } f(\Gamma_1) \vdash_{\mathcal{L}_2} f(\varphi_1) \texttt{ and} \ldots \texttt{ and } f(\Gamma_n) \vdash_{\mathcal{L}_2} f(\varphi_n) \texttt{ then } f(\Gamma) \vdash_{\mathcal{L}_2} f(\varphi)$$

In this talk we discuss the categories **Mcon** and **Seq** of multiple-conclusion consequence relations and sequent calculi, respectively, introduced in [1]. In these categories, which encompass a wide class of logics (including linear logic), the morphisms are meta-translations. Constrained and unconstrained fibring are defined in these categories, and it is shown that a given logic can be recovered by the fibring of its fragments in some cases in which the usual fibring fails. A general semantics is proposed, and a theorem of preservation of completeness by fibring is obtained.

[1] M. E. Coniglio, *The meta-fibring environment: Preservation of meta-properties by fibring*, CLE e-Prints vol. 5 (2005), no. 4, `http://www.cle.unicamp.br/e-prints/vol_5,n_4,2005.htm`.

[2] A. Sernadas, C. Sernadas, and C. Caleiro, *Fibring of logics as a categorical construction*, ***Journal of Logic and Computation***, vol. 9 (1999), no. 2, pp. 149–179.

► WALTER DEAN, *Are procedures logical objects*?

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Yiannis Moschovakis and Yuri Gurevich have recently put forth theories according to which individual mathematical procedures of the sort customarily referred to as *algorithms* in computer science (and as canonically exemplified by Euclid's algorithm or Mergesort) may be regarded as individual mathematical objects. These programmes promise to be a substantial boon for theoretical computer science as they offer a uniform means by which particular algorithms can be identified with particular combinatory objects. These theories thus allow a purely mathematical interpretation to be assigned to statements such as "Mergesort has optimal running time complexity among all comparison sorting algorithms" which might otherwise be interpreted as expressing quantification across a non-mathematical domain of procedures. In this paper, I will argue that both theories reflect a fundamental misunderstanding about the ontological status of procedures. In particular, I will argue that if algorithms are to be regarded as mathematical objects, the appropriate means of doing so is not via a straightforward reduction of procedures to mathematical objects of the sort exemplified by the canonical identification of natural numbers with finite ordinals. Instead, I will argue that the most promising way to proceed is to attempt to view procedures as *logical objects* in sense originally proposed by Frege and recently rehabilitated by the so-called neo-Fregean school in the philosophy of mathematics—i.e., as the items in the range of an appropriate abstraction operation $\mathcal{F}$ corresponding to the equivalences classes of a grounding relations $\sim$ defined over domain of objects D. I will argue that in this context an appropriate choice for D is given by a generalized definition of *machine model* and that $\sim$ ought to be taken to be a relation resembling that of *bisimulation* familiar from modal logic and process algebra. I will finally argue that even in this setting there remain substantial concerns that appropriate choices for $\sim$ may give rise to identity conditions for procedures with deviate from standard intuitions about when two algorithms are the same and when they are different.

► VICTORIA GITMAN, *Can the proper forcing axiom provide a partial solution to the Scott set problem*?
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The problem of the relationship between the notion of a standard system of a model of Peano Arithmetic and that of a Scott set has thwarted logicians for the past forty years. It is easy to show that every standard system is a Scott set. In 1962 Dana Scott, the discoverer of Scott sets, proved that the notions are equivalent for countable models of PA. They also turn out to be equilvalent for models of size ω_1. After that nothing is known and no progress has been made towards a solution. Once we get beyond models of size ω_1, the problem assumes a set theoretic character and it is only natural, judging from the developments in set theory in the past half century, to assume that the solution might be independent of ZFC. The problem can then be approached using techniques of forcing and/or strengthening of ZFC by large cardinal notions. I will talk about my work on how using PFA, the proper forcing axiom, and hence both forcing and strengthening of ZFC might lead to getting at least a partial solution to the Scott set problem. This work was motivated by recent work of Fredrik Engstrom in the same direction.

► JOEL DAVID HAMKINS, *The modal logic of forcing*.
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The subject of set theory has to a great extent become the study of the models of set theory, the possible mathematical worlds, in a sense, that we might inhabit. The method of forcing has been instrumental in revealing the enormous diversity of such models; we can now often use forcing to construct models with highly specific features. Because a model of set theory has some access to the objects and truths of a forcing extension, there is a natural affinity with modal logic and Kripke models. Specifically, consider the models of set theory as possible worlds and define that a set theoretic assertion is possible if it holds in some forcing extension and necessary if it holds in all forcing extensions. With this modal interpretation, what are the valid principles of forcing? We aim to do for forceability what Solovay did for provability. In joint work, Benedikt Löwe and I have established that the provably valid principles of forcing are exactly those in the modal theory S4.2.

► THOMAS JOHNSTONE, *Indestructibility of strongly unfoldable cardinals*.
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Large Cardinals may get destroyed or stay preserved after forcing with a particular poset. It has been of major interest in modern set theory to investigate which cardinals can be made indestructible by various classes of forcing. For instance, such theorems often provide a way of obtaining new independence results. While lots of research has dealt with cardinals stronger than measurable cardinals, in this talk I will address the issue for strongly unfoldable cardinals. These are weaker than measurable cardinals and closely related to weakly compact cardinals, with some of the flavor of strong cardinals. Given a strongly unfoldable cardinal, I will provide a forcing extension in which the cardinal is indestructible.

► SREĆKO KOVAČ, *Impossibilities, identities, and belief*.
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Some main problems of first-order logic of belief include modelling classically impossible propositions as possible beliefs, and also the preservation of the identity of objects through distinct beliefs. First-order non-standard semantics is proposed where classically inconsistent *de re* beliefs are allowed, the identity of objects of belief being preserved, and where intensional objects of belief are generated through the presentation of objects by individual constants. In a model, each world w has an associated set of worlds $q(w)$ by means of which the satisfaction of an atomic formula is defined. Members of $q(w)$ agree on properties of objects if the objects are taken in the same mode of presentation. The proposed semantics is compared with some other first-order modal semantics (e.g., Aloni, Fitting, Gerbrandy, etc.). An appropriate tableau system for the logic of belief is also proposed.

[1] Maria Aloni, *Individual concepts in modal predicate logic*, ***Journal of Philosophical Logic***, vol. 34 (2005), pp. 1–64.

[2] , Melvin Fitting, *First-order intensional logic*, ***Annals of Pure and Applied Logic***, vol. 127 (2004), pp. 171–193,

[3] Jelle Gerbrandy, *Identity in epistemic semantics*, ***Logic, Language and Computation***, (P. Blackburn and J. Seligman, editors), vol. 3, CSLI, Stanford, CA, pp. 1–36, 2000.

► HIDENORI KUROKAWA, *Cut-elimination for a hypersequent calculus of intuitionistic logic with classical atoms*.
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In this presentation, we introduce a hypersequent calculus for intuitionistic logic with classical atoms, i.e., intuitionistic logic augmented with a special class of propositional variables for which we postulate the decidability property. This system combines classical logical reasoning with constructive and computationally oriented intuitionistic logic in one system. Our main result is the cut-elimination theorem with the subformula property for this system. We show this by a semantic method, namely via proving the completeness theorem of the hypersequent calculus without the cut rule. The cut-elimination theorem gives a semantic completeness of the system, finite model property, decidability, and some form of the disjunction property.

► ROMAN KUZNETS, *On self-referentiality in modal logic*.
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One of the applications of modal logic is provability logic with $\Box F$ read as 'formula F is provable (in a theory $\mathcal{T}$).' Another application is epistemology logic where $\Box F$ is understood as 'formula F is known (by an agent a).' Recently developed logic LP (see [1]) provides an explicit version for both applications. Using $t{:}F$ to mean 'F is provable and t represents its proof' in provability case or 'F is known and t justifies this knowledge' in epistemology case allows for a finer grained discussion. It was shown ([1]) that LP is an explicit counterpart of modal logic S4, which can be viewed both as an informal provability logic and a logic of knowledge.

In principle, one can imagine a proof or a justification that refers to itself. For example, it would make sense to reserve one uniform justification for all circular reasoning instances; such a justification, in particular, would apply to every circular reasoning about this very justification. The language of modal logic is too poor to capture this notion of self-referentiality. Its explicit counterpart, on the contrary, can distinguish between self-referential and non-self-referential justifications.

We tried to answer the following question: Is self-referentiality crucial for the provability/knowledge encompassed by modal logic S4? The following theorem affirms that it is.

THEOREM 1. *Some theorems of* S4 *require self-referential proofs to be given an explicit realization in* LP, *e.g.*, $\neg\Box\neg(S \to \Box S)$.

[1] SERGEI N. ARTEMOV, *Explicit provability and constructive semantics*, ***The Bulletin of Symbolic Logic***, vol. 7 (2001), no. 1, pp. 1–36.

► BILLY JOE LUCAS, *The logic of categorical moral reasoning*.
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Let a moral (legal) code be a set of sentences (at least one deontic). One then studies rationality conditions on moral codes, as in [2]. K_0 imposes three such conditions: $O\alpha, O\beta \vdash O(\alpha\cdot\beta)$, $O(\alpha\cdot\beta) \vdash O\alpha$, $\exists\alpha\colon \vdash O\alpha$. Not following Kant, call a moral code basic categorical (BCM in [3]) if in addition to K_O it satisfies: $O\alpha \vdash OP\alpha$ and $OP\alpha \vdash P\alpha$ (call α in '$OP\alpha$' a categorical right, for α a 'right' in '$\sim O\sim\alpha$'). Now consider theories of categorical moral reasoning that extend BCM via three conditions: categorical completeness ($OO\alpha \vdash O\alpha$), categorical soundness ($O\alpha \vdash OO\alpha$), categorical non-contingency (no (deontic) moral claim is morally contingent, for α morally contingent iff $\sim O\alpha \cdot \sim O\sim\alpha$). *Fact*: No extension of K_O proves non-contingency and completeness yet allows the existence of rights which are not categorical rights. BCM + $PO\alpha \vdash OO\alpha$ is NCM (see [4]), the minimal extension of BCM (even of K_O) that proves categorical non-contingency; NCM is also categorically sound. BCM + both soundness and completeness is KD4! in [1] and S&CCM in [2]. Thus, for categorical moral reasoning that distinguishes rights from categorical rights, NCM and S&CCM are the mutually exclusive maximal extensions of BCM by any of these three conditions.

[1] BRIAN F. CHELLAS, ***Modal logic***, Cambridge University Press, Cambridge, 1980.
[2] BILLY JOE LUCAS, ***The logic of moral reasoning***, Jones and Bartlett Publishers, Boston, forthcoming.

[3] ———, *The logic of categorical rights*, ***The Bulletin of Symbolic Logic***, vol. 6 (2000), p. 385.
[4] ———, *Systems in which MLA implies LLA and deontic logics with no morally contingent obligations*, ***The Journal of Symbolic Logic***, vol. 56 (1991), pp. 366–367.

► JOÃO MARCOS, *Generalizing truth-functionality*.
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A common standard for the interpretation of classical propositional logic is set by the functionally complete collection of 2-valued truth-tables. The structure of the free algebra of formulas is faithfully mirrored in the semantics: In each admissible model, each sentential letter freely ranges over the set of truth-values, and to each n-ary logical constant there corresponds a convenient n-ary operator over those same truth-values. The whole approach is easily generalizable so as to define the class of *truth-functional logics*, i.e., *many-valued* logics whose operators can be characterized by *truth-tables* over some convenient set of truth-values.

A further interesting generalization of the above idea is produced by the so-called *non-deterministic truth-tabular semantics* (*N-truth-tables*), where each n-ary operator is allowed to choose in between a number of possible outputs for each given n-tuple of inputs. The set of admissible models may thus be enlarged, allowing for the natural adequate interpretation of more generous classes of non-classical logics. Yet another generalization of the same idea is given by the so-called *possible-translations semantics with many-valued ingredients* (Many-valued PTS), where each model of a logic $\mathcal{L}$ is given by an admissible translation of $\mathcal{L}$ into an appropriate many-valued logic $\mathcal{L}_k$ coupled with a standard many-valued valuation from $\mathcal{L}_k$.

In the finite-valued case, logics with truth-tabular or N-truth-tabular semantics share interesting meta-properties such as *compactness* and *decidability*. Finite-valued PTS are also guaranteed to share those properties as soon as all the corresponding admissible translations are recursively defined. However, several logics known to be uncharacterizable by finite-valued truth-tables can be adequately characterized by finite-valued N-truth-tables, and several logics that have no finite-valued N-truth-tabular characterization can be characterized by finite-valued PTS.

The present contribution will examine and illustrate the multiple relations between the three above mentioned alternative styles of semantics.

► JOHN MUMMA, *A formal analysis of Euclid's diagrammatic constructions*.
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Euclid's diagrams are not typically thought of as constituting the formal syntax of his theory. They are instead understood as providing, or at least suggesting, an interpretation for his terms. In this talk, I offer an alternative account, where the diagrams are understood as part of the formal deduction. Specifically, I present a formal proof system termed Eu which possesses a discrete, syntactic type for diagrams as well as sentences, and formulates rules of inference in terms of both. It is relatively straightforward to define the syntax of Euclid's diagrams. The central difficulty is fitting the diagrams into a system whose proofs have the generality of Euclid's propositions. The rules must restrict diagrammatic inferences to those which hold not just of the particular diagram on display in a proof but of all diagrams resulting from the diagram's construction. I explain how the rules of Eu do this. A definite picture of how geometric construction works in the Elements thus results. I close by pointing out how this picture serves to make some of Kant's observations on the structure of Euclid's demonstrations formally explicit.

► G. MINTS, *Extracting algorithms from non-effective cut elimination proofs*.
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Non-effective cut elimination proof uses Koenig's lemma to obtain a non-closed branch of a proof-search tree T_A (without cut) for a formula A, if A is not cut-free provable. A partial model (semi-valuation) corresponding to this branch and verifying $\neg A$ is extended to a total model for $\neg A$ using arithmetical comprehension. This contradicts soundness, if A is derivable with cut. The same argument works for Herbrand Theorem. We discuss algorithms of obtaining cut-free proofs corresponding to this schema and quite different from complete search through all possible proofs.

A primitive recursive bound for the height of T_A is obtained depending on a given proof of A with cut.

The simplest case is the cut on an atomic formula C in a Tait-style calculus: A, C; $A, \neg C \vdash A$. The cut-free proof of A, C provides an h such that all non-closed branches of T_A of the length $\geq h$ contain $\neg C$. Similar bound h' exists for C. Hence every branch of T_A of the length $\geq \max(h, h')$ contains $C, \neg C$, hence is an axiom.

► CATARINA DUTILH NOVAES, *Logic, symbols and icons*.
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What is *symbolic* logic? The very use of the adjective implies that there are other kinds of logic. This (purported) distinction is largely based on the languages used: artificial notations vs. 'ordinary' language. But, is there a principled, clear-cut division between symbolic and non-symbolic logic? Obviously, it all depends on one's definition of symbol.

I propose to examine the consequences of Peirce's concept of symbol to the status of symbolic logic. According to Peirce [1], signs are of three kinds: symbols, icons and indexes. Symbols are signs established by pure convention, by the acceptance of rules governing the relation between signs and their objects; icons are signs whose relations with the corresponding objects are founded on similarity—icons depict their objects. Now, following this trichotomy, artificially created symbols *as well as* ordinary words both belong to the same category, i.e., symbols, and thus systems of logic expressed with 'plain' words are just as symbolic.

What then (if anything) gives a special status to what we usually refer to as 'symbolic logic'? I argue that what is special about the symbols we use in logic is that they are actually less symbolic and more iconic than ordinary words. But the signs used in so-called 'symbolic' logic are iconic not in the same way that a painting depicts a person. More than individual objects, logical systems of signs depict **structures**—the mutual configuration of objects—by means of the concatenation of signs (their syntax). Thus, I conclude that the grounds for the expressivity of so-called 'symbolic' logical languages is, ironically, their iconic nature.

[1] C. S. PEIRCE, *Logic as semiotic: the theory of signs*, ***Philosophical Writings of Peirce***, Dover Publications, New York, 1955, pp. 98–119.

► KENNETH A. PRESTING, *Embedding a cylindrical set algebra into a Boolean algebra*.
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Cylindrical algebras in several forms were introduced by Henkin, Monk, and Tarski as algebraic analogues of first-order languages. A CSA is a concrete structure obtained from the extensions of first-order relations, using Boolean operations along with two new operations, cylindrification and substitution, and distinguished 'diagonal' elements which are identity relations. Cylindrification is the algebraic analogue of existential quantification, as substitution is a change of variables.

We show that cylindrical operations can be reduced to Boolean operations. By slicing the graph of an n-ary relation along lines defined by all combinations of $n - 1$ domain elements, we obtain a matrix with n columns and (in general) infinitely many rows. Components of this matrix are subsets of the domain, and the row indices are $(n-1)$-place vectors of domain elements. The slicing construction maps every relation to a unique matrix representation, and an inverse mapping takes equivalence classes of matrices to well-defined relations. Component-wise operations on the matrices are Boolean operations in the Cartesian product of the originating first-order model.

Diagonal elements are matrix images of identity assertions. Cylindrification is the Boolean union of multiple matrix images, each image being the result of restricting the target variable to a single instance (an algebraic analogue of substitutional quantification). Substitution of variables is a Boolean combination of diagonalization with cylindrification (interestingly, also an elementary matrix operation—swapping column vectors).

The mapping defined by the slicing construction is a unique homomorphism, and the Boolean operations on the product algebra determine the cylindrical structure of the CSA. Applications to probability theory are discussed.

► JONAS REITZ, *The Ground Axiom*.
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The technique of forcing was invented by Paul Cohen in 1962 in order to prove the independence of the Axiom of Choice and the Continuum Hypothesis. In the decades since, set theorists have demonstrated the flexibility of the technique by using it to produce an enormous variety of models of set theory. The models thus obtained, though diverse, do have a common feature: they are all forcing extensions. This suggests a natural question—under what circumstances is the universe *not* a forcing extension? A new axiom is proposed, the Ground Axiom (GA), asserting that the universe is not a forcing extension of any inner model. While this axiom is second order in nature, I will show that it is expressible as a first order statement. That GA is consistent follows from observing that the axiom holds in L, Gödel's constructible universe. I will discuss a flexible method for constructing models of GA, and show that any model of ZFC has an extension which satisfies GA.

► BRYAN RENNE, *Games, strategies, and explicit knowledge*.
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Fitting has described LP, Artemov's Logic of Proofs, as a logic of explicit knowledge. To say that knowledge is explicit means that something is known for a specific reason, as opposed to the Hintikka notion of knowledge, which is best described as knowability. LP introduces symbols for reasons along with natural operations to combine and manipulate them. What results is a joint calculus of reasons and formulas, achieved by admitting new formulas of the form $t{:}\varphi$, which may be understood as "φ for reason t."

An evaluation game is a zero-sum game between two players with perfect information. Given a model for a logical theory, the game is played on the parse tree of a formula, but the rules of play are rigged so that the Verifier player has a winning strategy exactly when the formula holds in the model. Interpreting a specific winning strategy for Verifier as a reason why Verifier can always win an evaluation game on an LP formula in some LP model, we study the connection between reasons in logics of explicit knowledge and winning strategies in evaluation games.

► SAMER SALAME, ERIC PACUIT, AND ROHIT PARIKH, *Some results on* Adjusted Winner.
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We study the *Adjusted Winner* procedure of Brams and Taylor for dividing goods fairly between two individuals, and prove several results. In particular we show rigorously that as the differences between the two individuals become more acute they both benefit. We study some rather odd knowledge-theoretic properties of strategizing. We introduce a geometric approach which allows us to give alternate proofs of some of the Brams-Taylor results and which gives some hope for understanding the many-agent case also. We also point out that while honesty may not always be the best policy, it *is* as Parikh and Pacuit [3] point out in the context of voting, the only *safe* one. Finally, we also show that provided that the assignments of valuation points are allowed to be real numbers, the final result is a continuous function of the valuations given by the two agents.

[1] STEVEN BRAMS AND ALAN TAYLOR, *Fair Division*, Cambridge University Press, 1996.

[2] ———, *The win-win solution: Guaranteeing fair shares to everybody*, W. W. Norton & Company, New York, 1999.

[3] ROHIT PARIKH AND ERIC PACUIT, *Safe votes, sincere votes, and strategizing*, presented at the ***Workshop on Uncertainty in Economics***, Singapore, 2005.

► LUIS A. URTUBEY, *Coping with context effects on knowledge in a multi-agent setting*.
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I present an analysis of the notion of "contextual knowledge" in the framework of epistemic logic. Borrowing from a paper by Lomuscio and Sergot, I apply their semantical deontic model to get a formalization of contextual reasoning. To cope with the effects of context on knowledge, I introduce into this formal framework an operator representing the knowledge that an agent would have if it reasons under the assumption that a group of agents share a context C.

Instead of reifying context integrating them in a formal system, as previous works on context logic have done, I follow Keith Devlin's approach, where contexts point to real life contexts. Moreover, in defining contexts, there have been two deferent approaches. Either the context is viewed as the set of assumptions that the agent supposes itself to share with other agents or it is held to be the set of assumptions that agents actually do share. This ambivalence reflects both, a normative framework that underlies context and a sort of uncertainty over the kinds of norms involved. These norms could be fairly internal, or they could be somewhat less under the agent cognitive control.

This normative flavour of contexts also paves the way for applying deontic-interpreted systems to take account of context effects on reasoning about knowledge. In particular, the deontic distinction between allowed and disallowed labelling states, could be used to model a distinction that renders possible to pick out the normal states from the abnormal ones, which do not fit into the assumed context.

[1] K. DEVLIN, *A framework for modelling evidence-based, context-influenced reasoning*, ***CONTEXT03***.

[2] A. LOMUSCIO AND M. SERGOT, *Deontic interpreted systems*, ***Studia Logica***, vol. 75 (2003).

► IVAN WELTY, *Truth or logical consequence: Frege and Tarski on indirect proof*.
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Frege insists that the counterfactual assumption at the start of an indirect proof (reductio ad absurdum) must appear as the antecedent of a material conditional. This looks bizarre

to anyone schooled on Tarski's conception of logical consequence, according to which the counterfactual assumption is a premise in a formal (uninterpreted) argument. I show that Frege's account of indirect proof amounts to a restatement of the Tarskian account from a higher—viz. second-order—point of view. I then show how this ascent is motivated by Frege's conception of logic, as a language for making explicit certain relations among true-or-false commitments.

► YIMU YIN, *Quantifier elimination for the reals with a predicate for the powers of two*.
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In [1], van den Dries showed that the theory of the reals with a predicate for the powers of two admits quantifier elimination in an expanded language, and is hence decidable. He gave a model-theoretic argument, which provides no apparent bounds on the complexity of a decision procedure. We provide a syntactic argument that yields a procedure that is primitive recursive, although not elementary. In particular, we show that it is possible to eliminate a single block of existential quantifiers in time $2^0_{O(n)}$, where n is the length of the input formula and 2^x_k denotes k-fold iterated exponentiation. We also discuss some decidability results on recurrent relations to which our decision procedure can be readily applied.

[1] LOU VAN DEN DRIES, *The field of reals with a predicate for the powers of two*, ***Manuscripta Mathematica***, vol. 54 (1985), pp. 187–195.

Abstracts presented by title

► ADIB BEN JEBARA, *The actual infinite and a cell escaping death*.
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I am using a particular case of the axiom of choice, the countable axiom of choice for a family of sets of n elements, n 2 through m, notation : CC(only 2 through m). see web page of Andreas Blass http://www.math.lsa.umich.edu/ ablass/dpcc.pdf.

A cell goes through living states when alive. We apply the set theory ZFU with urelements and take a particular U to be a set of living states, which are urelements. We consider $U(m)$ by considering CC(only 2 through m) in U.

A cell lives until it has $m+1$ possible living states at a given time. The set of possible living states is (Cardinal product) $U_1 \times U_2 \times U_3 \times \cdots$ and is the void set in $U(m)$ as the number of elements is $m + 1$ (no possible sequence of living states).

So, the cell should die, but, if it goes to $U(m + 1)$, it will survive. When CC(only 2 through m) holds, there is an hyper-state 1 of the living states and when CC(only 2 through $m + 1$) holds, there is an hyper-state 2 of the living states. An hyper-state is something in common to living states. Hyper-state 1 and hyper-state 2 are known from the existence or not of possible sequences of living states.

Mr. Andreas Blass pointed out about another text that, in some models, the Cartesian product of some sets of urelements can very well be nonempty, even if other sets of the same cardinalities have empty Cartesian products.

Then, the same thing will happen from $U(m+1)$ to $U(m+2)$, and so on until $m+p$. So, we will end up with the cell more able to survive as it goes from hyper-state 1 to hyper-state $(p-1)$.

I assume that living states are in infinite number as I think that there is a link between life and the actual infinite.

► JOHN CORCORAN, *Revisiting Boole's principle of wholistic reference*.
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In its strongest unqualified form, the principle of wholistic reference is the proposition that each and every proposition, regardless how limited the referents of its non-logical or content terms, refers to the whole of its universe of discourse. According to this principle every proposition of number theory, even an equational proposition such as "$5 + 7 = 12$", refers not only to the individual numbers that it happens to mention but to the whole universe of numbers. The relation verb 'refers' is being used in its broad sense (loosely "is about") and not as a synonym for 'names' in the sense of "is a name of". George Boole (1815–1864) introduced this principle into modern logic. Even though he changed *from* a monistic fixed-universe framework in his 1840s writings *to* a pluralistic multiple-universe framework in 1854, he never wavered in his frank avowal of the principle of wholistic reference. Indeed, he took it as an essential accompaniment to his theory of concept formation and proposition formation. For Boole, the essential first step in the process of conceiving of a proposition preliminary to making a judgement of its truth or falsity—or even using it in a deduction, however hypothetically—was to conceive of the universe of discourse. See ***Boole 1854***, xxi, 27, 42, 43. Similar views, perhaps not similarly motivated, are found in later logicians, including Gottlob Frege (1848–1925). Some recent formulations of standard one-sorted first-order logic seem to be in accord with a form of it, if they do not actually imply the principle itself. This paper surveys the current and classic literature of logic for discussions of related views, thus supplementing my article *Principle of Wholistic Reference*, ***Manuscrito***, vol. 27 (2004), pp. 155–166 and my Introduction to George Boole's 1854 ***Laws of Thought***, reprinted Buffalo: Prometheus Books, 2003.

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In [1] and [2] Per Martin-Löf has introduced his Type Theory (hereafter MLTT). I have built an algorithm, which automates inference in MLTT. Any proposition can be written as a MLTT type, namely, the type of objects that are proofs or solutions of the proposition in question. My system with dialog applied to a type constructs an object of that type, which can be used as an algorithm that solves the proposition. Based on the algorithm a computer program can be written. To summarize, my system with dialog can be used as a skeleton of a software that automatically synthesizes computer programs.

[1] Per Martin-Löf, *Constructive mathematics and computer programming*, ***Sixth International Congress for Logic, Methodology, and Philosophy of Science***, North–Holland, Amsterdam, 1982, pp. 153–175.

[2] ———, *An Intuitionistic Theory of Types: Predicative Part*, ***Logic Colloquium '73***, (H. E. Rose and J. C. Shepherdson, editors), North–Holland, Amsterdam, 1973, pp. 73–118.

► XUNWEI ZHOU, *Adequate set of logical operators in mutually-inversistic logic*.
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In mutually-inversistic logic, there are 12 logical operators, of which four are compounders: $\neg$, $\wedge$, $\vee$, $\oplus$, eight are connectives: $/\wedge^{-1}$ (mutually inverse conjunction), $\vee/^{-1}$ (mutually inverse disjunction), $\leq^{-1}$ (mutually inverse implication), $=^{-1}$ (mutually inverse equivalence), $<^{-1}$ (mutually inverse proper implication), $\vee^{-1}$(mutually inverse inclusive disjunction), $\oplus^{-1}$ (mutually inverse exclusive disjunction), $\times^{-1}$ (mutually inverse intercross). The adequate set of logical operators in mutually-inversistic logic is $\{\neg, \wedge, \leq^{-1}\}$. $P \vee Q$ can be represented by $\neg\{\neg P \wedge \neg Q\}$, $P \oplus Q$ can be represented by $\neg\{\neg\{P \wedge \neg Q\} \wedge \neg\{\neg P \wedge Q\}\}$, $P =^{-1} Q$ can be represented by $\{P \leq^{-1} Q\} \wedge \{Q \leq^{-1} P\}$, $P <^{-1} Q$ can be represented by $\{P \leq^{-1} Q\} \wedge \neg\{Q \leq^{-1} P\}$, $P/\wedge^{-1} Q$ can be represented by $\neg\{P \leq^{-1} \neg Q\}$, $P \times^{-1} Q$ can be represented by $\neg\{P \leq^{-1} \neg Q\} \wedge \neg\{P \leq^{-1} Q\} \wedge \neg\{\neg P \leq^{-1} \neg Q\} \wedge \neg\{\neg P \leq^{-1} Q\}$, $P \vee /^{-1} Q$ can be represented by $\neg P \leq^{-1} Q$, $P \vee^{-1} Q$ can be represented by $\{\neg P \leq^{-1} Q\} \wedge \neg\{Q \leq^{-1} \neg P\}$, $P \oplus^{-1} Q$ can be represented by $\{\neg P \leq^{-1} Q\} \wedge \{Q \leq^{-1} \neg P\}$.