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2005 SUMMER MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC LOGIC COLLOQUIUM '05

Athens, Greece
July 28–August 3, 2005

Logic Colloquium '05, the 2005 European Summer Meeting of the Association for Symbolic Logic, was hosted by the University of Athens from July 28 to August 3, 2005. The opening session was held in the impressive Great Hall of the University's Main Building and was followed by a Welcome Reception in the adjoining Courtyard. Thereafter, lectures were held in the Mathematics Building on the University Campus.

The sponsors of the meeting include: the Association for Symbolic Logic, Athens Chamber of Commerce and Industry, the Bank of Greece, Graduate Program in Logic and Algorithms, IVI Loutraki Water, Hellenic Parliament, Katoptro Publications, Kleos S.A., the Ministry of National Education and Religious Affairs, Mythos Beer, National and Kapodistrian University of Athens, National Bank of Greece, and the Sigalas Wine Co. Ltd. We thank them for their support.

The success of the meeting was due largely to the hard work of the Organizing Committee under the leadership of its Chair, Costas Dimitracopoulos, the other members being D. Anapolitanos, L. Kirousis, G. Koletsos, M. Mytilinaios, S. Papastavridis, Th. Pheidas, P. Rondogiannis, G. Stavrinos, A. Synachopoulos, A. Tzouvaras, and S. Zachos. The Program Committee consisted of C. T. Chong, C. Dimitracopoulos, H. Field, G. Jaeger, G. Metakides, L. Newelski, D. Normann, R. Parikh, J. Steel, S. Todorcevic, J. Tucker, F. Wagner, and S. Wainer (Chair).

The main conference topics were: *Computability Theory, Computer Science Logic, Model Theory, Philosophy of Logic, Proof Theory and Constructivism, and Set Theory*. The program included four short tutorial courses, twelve invited plenary lectures, and eight invited lectures in four special sessions corresponding to the first four main topics. There were 100 contributed papers and just over 200 participants from 32 different countries. Forty-one student or postdoctoral participants were assisted by ASL grants, and a further 39 participants received grants from other sources.

The following short tutorial courses were delivered:

P. Aczel, *Constructive set theory*.
I. Ben Yaacov, *Model theory in positive and continuous logics*.
P. Kolaitis, *Constraint satisfaction, complexity and logic*.
G. Restall, *The shape of proofs: inference and meaning in classical and non-classical logics*.

The following invited plenary lectures were presented:

J. Bergstra, *Decision problems for pushdown threads*.
S. Goncharov, *Isomorphisms and definable relations on computable models*.
D. Haskell, *Independence for types in algebraically closed valued fields*.
E. Jaligot, *Simple groups of finite Morley rank*.

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J. Moore, *Uncountable linear orders*.
A. Nies, *Algebras with finite descriptions*.
C. Parsons, *Paul Bernays' later philosophy of mathematics*.
H. Schwichtenberg, *Minimal logic for computable functionals*.
M. Sheard, *A transactional approach to the logic of truth*.
S. Tupailo, *Monotone inductive definitions and consistency of New Foundations*.
K. Weihrauch, *Computable analysis*.
J. Zapletal, *Forcing idealized*.

The proceedings of Logic Colloquium '05, edited by C. Dimitracopoulos, L. Newelski, D. Normann, and J. Steel, will contain many of the invited lectures and will be published in the ASL series *Lecture Notes in Logic*.

More information about the meeting can be found at the conference webpage `http://www.math.uoa.gr/lc2005/`.

Abstracts of invited and contributed talks given in person or by title by members of the Association follow.

For the Program Committee
STAN S. WAINER

Abstracts of invited plenary talks

► JAN A. BERGSTRA, INGE BETHKE, AND ALBAN PONSE, *Decision problems for pushdown threads*.
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A challenging question in language theory is to decide whether the languages accepted by two different machines in some given class are the same. This question is called the *equivalence problem*. Another important question, known as the *inclusion problem*, is that of determining whether one language is a subset of another. The connection between these two problems is that the latter implies the former, that is, that any algorithm that decides inclusion for some family of languages can also be used to decide equivalence. The question then arises whether the converse also holds, i.e., whether every language family with a decidable equivalence problem does also have a decidable inclusion problem.

In the early 70's the languages accepted by two-tape Rabin and Scott machines, and deterministic finite-turn and one-counter pushdown automata were proved to possess a decidable equivalence problem and an undecidable inclusion problem. More recently, e.g., erasing and nonerasing pattern languages and deterministic context-free languages have been added to this growing list.

In this talk we shall investigate yet another class: *pushdown threads*, i.e., regular threads that may use an internal stack—a form of processes describing sequential program behaviour and using the services offered by a single stack. we shall show that equivalence between pushdown threads is decidable by reducing the equivalence problem for deterministic pushdown automata as treated by Sénizergues to our equivalence problem. Moreover, we shall prove that inclusion is undecidable for pushdown threads. Here we shall make an appeal to the halting problem for Minsky machines.

► SERGEY S. GONCHAROV, *Isomorphisms and definable relations on computable models*.
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We will discuss the problems about characterization of theoretical-model complexity of computable models up to isomorphism and definable relations on computable models. It is connected with problems of algorithmic dimensions and complexity index sets for different classes of models. We will consider the connection of complexity definable relations of computable models with computable numberings in general sense relative to classification of sets.

Partially supported by grant RFBR–05-01–00819.

[1] C. J. ASH AND J. F. KNIGHT, ***Computable structures and the hyperarithmetical hierarchy***, Elsevier, Amsterdam, 2000.

[2] YU. L. ERSHOV AND S. S. GONCHAROV, ***Constructive models***, Siberian School of Algebra and Logic, Kluwer Academic/Plenum Publishers, New York, 2000.

[3] YU. L. ERSHOV, ***Theory of numberings***, Nauka, Moscow, 1977 (Russian).

[4] S. S. GONCHAROV, ***Countable Boolean algebras and decidability***, Plenum, Consultants Bureau, New York, 1997.

► DEIRDRE HASKELL, *Independence for types in algebraically closed valued fields*.
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The property of non-forking yields a notion of independence for types in a stable theory. In a particular stable theory, for example, the theory of algebraically closed fields, there may be several ways to define an independence notion, all of which are equivalent to non-forking. In an unstable theory, one expects there to be competing notions of independence, each of which captures different properties of non-forking. In this talk, I will describe ways in which different notions of independence in an algebraically closed field can be modified to make them appropriate for a language with a valuation. I will give examples illustrating how the notions differ in an algebraically closed valued field, and a general theorem characterising when they coincide.

► ERIC JALIGOT, *Simple groups of finite Morley rank*.
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The Morley rank appeared in model theory when M. Morley proved is famous theorem on the categoricity in any uncountable cardinal of first-order theories categorical in one uncountable cardinal. This rank can be seen as an abstract version of the Zariski dimension in algebraic geometry over an algebraically closed field. One of the most striking problem arising is the classification of simple groups of finite Morley rank, motivated since the seventies by the Algebricity Conjecture which postulates that they are algebraic. There is now a huge amount of ideas coming from the classification of finite simple groups which have been transfered in this context. I will try to give a panoramic view on what is known and what is unknown.

► JUSTIN MOORE, *Uncountable linear orders*.
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When studying the class of linear orderings, it is natural to consider the quasi-order of embeddability. Since the countable linear orderings are well understood—they can all be embedded into the rationals—one is led to ask about the uncountable linear orders and their relationship to each other in this quasi-order. In particular: Do the uncountable linear orderings have a finite basis? If so, then to what extent are the elements canonical? Since any non-minimal element of a finite basis is redundant, a finite basis can be assumed to contain only minimal elements.

By an old result of Sierpinski, the Continuum Hypothesis implies that (1) any basis for the uncountable separable orders—the real types—must have cardinality $2^{2^{\aleph_0}}$ and (2) there are no minimal real types. On the other hand, building on work of Abraham, Baumgartner, Shelah, and Todorcevic, I recently proved the following result, which confirms a conjecture of Shelah.

THEOREM 1. *The Proper Forcing Axiom implies that the uncountable linear orders have a five element basis consisting of X, ω_1, ω_1^*, C, and C^*, where X is any set of reals of cardinality $\aleph_1$ and C is any Countryman line.*

While ω_1 and ω_1^* are minimal assuming only ZFC, existence of minimal real types requires some additional set theoretic assumption. One is led to speculate about the minimality of the other members of the basis in ZFC. Recently I proved that they also require set theoretic assumptions for their minimality.

THEOREM 2. *It is relatively consistent with ZFC that the only minimal uncountable order-types are ω_1 and ω_1^*.*

One can consider the latter result as indicating that the X, C, and C^* are somewhat less canonical than ω_1 and ω_1^* in that they may consistently not be minimal (hence redundant in a basis). I will discuss both of these results and give them some context.

► ANDRE NIES, *Algebras with finite descriptions*.
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How can one determine a countably infinite algebra in a finite signature by a finite amount of information? I discuss two ways to do so that have received considerable attention recently. The first approach is via finite automata, and is mostly of interest for infinitely generated algebras. The second is restricted to finitely generated algebras, where the description is a single first order sentence. For both approaches, I discuss interesting examples of groups which can be described that way.

1. To be finite automaton (FA)-presentable seems to be a rather strong restriction: the domain can be represented by a regular set, in a way that finite automata can also verify that the atomic relations hold (where the strings representing elements are written one below the other, using symbols in a power alphabet). This notion goes back to Khoussainov and Nerode (1995), extending work of Hodgson, 1982. Examples include the integers with addition, and the Boolean algebra B of finite or co-finite subsets of N. The isomorphism problem for FA-presentable associative semigroups is Sigma-1-1 complete, so that in this case the class is quite rich after all [1]. However, for several classes of interest, the FA-presentable structures are quite limited: for instance, the only FA-presentable infinite BA's are finite powers of B [1]. Nies and Thomas have shown that the f.g. subgroups of each FAP group have an abelian subgroup of finite index, and that each FAP ring with 1 is locally finite [5]. On the positive side, Nies and Semukhin have found FA-presentable torsion free abelian groups of arbitrary finite rank which are not a proper direct product. It is still unknown if the group of rationals with addition is FA-presentable.

2. We say that a f.g. algebra A is quasi-finitely axiomatizable (q.f.a.) if there is a first order sentence such that A is, up to isomorphism, the only f.g. algebra (in the same signature)

satisfying the sentence [3]. There are no obvious examples of q.f.a. groups; for instance the group of integers with addition is not q.f.a. The simplest known example is the Baumslag-Solitaer group $\langle a, d \mid d^{-1}ad = a^2 \rangle$ [4]. Morozov and Nies have shown that a q.f.a. algebra has hyperarithmetic word problem, and that there are q.f.a. groups arbitrarily high in the hyperarithmetic hierarchy [2]. A f.g. algebra is a prime model of its theory iff there is a definable orbit of generators. This condition looks very similar to being q.f.a. In fact Oger and Sabbagh have shown that being prime and being q.f.a. are equivalent for nilpotent groups. However, I have proved that there are continuum many f.g. prime groups, so not all are q.f.a. [4]. Recently researchers have started to look at q.f.a. commutative rings with 1. The conjecture is that each f.g. (and hence finitely presented) ring is q.f.a.

[1] B. KHOUSSAINOV, A. NIES, S. RUBIN, AND F. STEPHAN, *Automatic structures: richness and limitations*, ***Proceedings of LICS 04***, Helsinki, 2004.

[2] A. MOROZOV, AND A. NIES, *Finitely generated groups and first-order logic*, ***Journal of London Mathematical Society***, vol. 71 (2005), no. 3, pp. 545–562.

[3] A. NIES, *Separating classes of groups by first order sentences*, ***International Journal of Algebra and Computation***, vol. 13 (2003), no. 3, pp. 287–302.

[4] ———, *More on q.f.a. and prime groups*, to appear.

[5] A. NIES AND R. THOMAS, *FA-presentable groups and rings*, to appear.

All papers are available under `cs.auckland.ac.nz/~nies`.

► CHARLES PARSONS, *Paul Bernays' later philosophy of mathematics*.
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Paul Bernays (1888–1977) is well known for his contributions to mathematical logic and for his collaboration in David Hilbert's proof-theoretic program culminating in the monumental [5].

His philosophical writings can be divided into several periods: (1) essays written as a student under the tutelage of Leonard Nelson, a champion of the Kantian philosophy of J. F. Fries (1773–1843), (2) items written 1922–34 during his active collaboration with Hilbert, (3) transitional writings (1934–45), and (4) post-war writings reflecting the influence of and collaboration in organizational matters with his Zürich colleague Ferdinand Gonseth (1890–1975).

By his later philosophy I mean chiefly (4), although two essays from (3), his well known [2] and his almost unknown [3], are significant. I will argue that Bernays by the late 1930s had concluded that the traditional notion of the a priori is not helpful in understanding mathematical knowledge. Gonseth probably helped him to articulate an alternative. Characteristic of Bernays' later position are reserve toward the a priori but giving a place to "intellectual experience" (*geistige Erfahrung*) that would be incompatible with empiricism, although he allows experience a role in the formation of conceptual schemes. Bernays also had his own form of structuralism, differing from the views that have gone by that name more recently. The view of mathematical objects expressed in [4] also has points of contact with Carnap's.

[1] PAUL BERNAYS, ***Abhandlungen zur Philosophie der Mathematik***, Wissenschaftliche Buchgesellschaft, Darmstadt, 1976.

[2] ———, *Sur le platonisme dans les mathématiques*, ***L'enseignement mathématique***, vol. 34 (1935), pp. 52–69. Translation in ***Philosophy of Mathematics***: *Selected readings*, (Paul Benacerraf and Hilary Putnam, editors), 2d edition, Cambridge University Press, 1983.

[3] ———, *Grundsätzliche Betrachtungen zur Erkenntnistheorie*, ***Abhandlungen der Fries'schen Schule N. F.***, vol. 6 (1937), pp. 279–290.

[4] ———, *Mathematische Existenz und Widerspruchsfreiheit*, Reprinted in [1], pp. 92–106. First published 1950.

[5] David Hilbert and Paul Bernays, ***Grundlagen der Mathematik***, 2 vols. Springer, Berlin, 1934, 1939. 2d ed., 1968, 1970.

► HELMUT SCHWICHTENBERG, *Minimal logic for computable functionals*.
Mathematisches Institut der Universitaet Muenchen, Theresienstr. 39, D-80333 Muenchen, Germany.
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We present a logical system which has the Scott-Ershov partial continuous functionals as its intended model. It is based on minimal (not classical or intuitionistic) logic, and is well suited for program extraction (via modified realizability). Moreover, it is easy to identify a fragment which ensures polynomial time complexitiy of the extracted programs.

More precisely, constants denote computable functionals, and quantifiers range over the partial continuous functionals, with free algebras as base types. However, the theory is first order, in the sense that type and predicate variables are only allowed as parameters (as placeholders for types and formulas). The fragment without inductive definitions is conservative over Heyting arithmetic, hence follows Kreisel's plea for theories with a strong language but weak existence axioms.

► MICHAEL SHEARD, *A transactional approach to the logic of truth*.
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Formal systems which axiomatize the logic of a truth predicate can differ as they try to capture differing aspects of our conception of truth. This talk explores a "transactional" approach: one which seeks to embody the principles governing the use of truth talk as a tool of communication. It examines the question of which axioms and rules of inference are necessary to encode, decode, and evaluate information conveyed by sentences which contain a truth predicate.

► SERGEI TUPAILO, *Monotone inductive definitions and consistency of New Foundations*.
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New Foundations, NF, is a system of set theory named after Quine's 1937 article "New foundations for mathematical logic", where it was introduced. It was meant as a foundations of mathematics, alternative to Zermelo-Fraenkel set theory ZF and others. Obvious advantages of NF are that it's very easily formulated and many mathematical notions can be expressed in NF in a much more "natural" way than in ZF. However, in spite of efforts by many researches and many brilliant results, NF is still not known to be consistent relative to any theory in which we have reasonable confidence.

We investigate a possibility of reducing the Consis(NF) problem to consistency of various extensions of Jensen's NFU, "NF with Urelements", which is known to be consistent due to Jensen 1969. Extensions of NFU by different "large cardinal axioms" and their consistency strength have been studied by R. Jensen, S. Feferman, M. Boffa, R. Holmes, R. Solovay. Specifically, we describe a surprising connection between the Monotone Inductive Definitions principle and consistency of New Foundations.

► KLAUS WEIHRAUCH, *Computable Analysis*.
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Computable Analysis studies those functions on the real numbers and related sets which can be computed by machines such as digital computers. It connects two traditionally disjoint fields, namely Analysis/Numerical Analysis on the one hand and Computability/Computational Complexity on the other hand, combining concepts of approximation and of computation. In particular, Computable Analysis supplies an algorithmic foundation of numerical computation.

While essentially all computablity models for number functions are equivalent (Church's Thesis) several non-equivalent mathematical models for real number computation are still being discussed. In this lecture we concentrate mainly on the representation model of computation (TTE) which is based on a definition of computable real functions introduced by A. Grzegorczyk and D. Lacombe in 1955. In TTE, infinite sequences of symbols (more generally and informally, of rational numbers, rational balls, finite rational step functions etc.) are used as "names" of the points of sets like real numbers, continuous real functions, open subsets of Euclidean space or square-integrable functions. Computability is defined by Turing machines which read from infinite files and write to infinite files. We introduce and explain basic concepts, discuss examples, and present applications.

Despite remarkable progress during recent years Computable Analysis is still in its infancy, still only the surface of what can be done has been scratched. Numerous secrets are waiting for becoming detected.

► JINDRICH ZAPLETAL, *Forcing idealized*.
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I will give an outline of the theory of definable proper forcing as developed in the last five years. I will describe connections to fields outside of logic—such as measure theory, potential theory, or dynamical systems—as well as connections with more traditional areas of logic—such as descriptive theory or the methodology of forcing.

Abstract of invited talk in the short course on Computer Science Logic

► PHOKION G. KOLAITIS, *Constraint satisfaction, complexity, and logic*.
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Constraint satisfaction problems constitute a broad class of algorithmic problems that arise naturally in several different areas of computer science and artificial intelligence. The aim of this tutorial is to present an overview of results in constraint satisfaction with emphasis on the computational complexity of constraint satisfaction problems and on the connections of this area of research with logic, universal algebra, and database theory.

Abstract of invited talk in the short course on Model Theory

► ITAY BEN-YAACOV, *Model theory in positive and continuous logics*.
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Classically, model theory studies elementary classes using full first order logic. I will discuss two alternatives to this setting which have been studied in recent years, allowing to apply model-theoretic methods to a larger family of classes of structures:

— Positive model theory consists of restricting the language to a "positive fragment" of the full first order logic, which needs only be closed for positive boolean combinations.

Motivated originally by the inability of first order logic to accommodate hyperimaginary elements, positive model theory turned out to be a very general setting which nevertheless satisfies the Compactness Theorem (for sets of positive formulas). It is a particularly natural setting for the study of simple theories.

— Continuous first order logic [joint project with Alexander Berenstein, C. Ward Henson and Alex Usvyatsov] consists of replacing the discrete set of truth values $\{0, 1\}$ with the compact interval $[0, 1]$. It is particularly well-adapted for the study of classes of complete metric structures, such as classes of Banach spaces and probability algebras. It has many advantages over previous logics used for the study of such classes: its proximity to first order logic allows to generalise many classical results such as various definability theorems (e.g., Beth's Theorem) and local (i.e., formula-by-formula) stability theory.

I will discuss these two frameworks, the relations between them, and some applications and open problems concerning them.

Abstract of invited talk in the short course on Philosophical Logic

► GREG RESTALL, *The shape of proofs: inference and meaning in classical and non-classical logics*.
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In this course, we will cover central techniques in proof theory, such as normalisation for natural deduction and cut elimination for sequent systems. We will look at the corollaries of these results for technical properties like the subformula property, conservative extensions and decidability, and their implications for theories of meaning and the nature of logical consequence. While it has been claimed that these considerations favour intuitionistic logic over its rivals, the application to classical logic of techniques from proofnets allow a proof-first account of even classical logic and modal logic.

Abstract of invited talk in the short course on Proof Theory and Constructivity

► PETER ACZEL, *Constructive Set Theory*.
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Constructive Set Theory (CST) was initiated in [9] as an extensional set-theoretical system to represent the brand of constructive mathematics introduced in [5]. A closely related system, CZF, was introduced and given an interpretation in Martin-Löf's Intuitionistic Type Theory, [8], in [1] and, in [2, 3], extensions of CZF obtained by adding various axioms such as choice principles or the Regular Extension Axiom, were also shown to be interpreted. A generalisation of that interpretation may be found in [6].

CZF is a proof theoretically weak subsystem of ZF using intuitionistic logic which has the same theorems as ZF when classical logic is used, but has the same proof theoretic strength as the classical system KP, [7].

Our aim is to introduce CST as a setting in which constructive mathematics can be developed and presented in essentially the same extensional style as classical mathematics is usually presented while only using principles that are generally accepted as constructively correct.

Some notes on CST may be found in [4].

[1] P. Aczel, *The type theoretic interpretation of constructive set theory*, ***Logic Colloquium '77***, pp. 55–66, North-Holland, 1978.

[2] ———, *The type theoretic interpretation of constructive set theory: choice principles*,

The L. E. J. Brouwer Centenary Symposium, pp. 1–40, North-Holland, 1982.

[3] ———, *The type theoretic interpretation of constructive set theory: inductive definitions*, ***Logic, Methodology and Philosophy of Science VII***, pp. 17–49, North-Holland, 1986.

[4] P. ACZEL AND M. RATHJEN, *Notes on Constructive Set Theory*, Technical Report 40, Mittag-Leffler Institute, 2001. Available at `http://www.cs.man.ac.uk/~petera/papers.html`.

[5] E. BISHOP AND D. BRIDGES, ***Constructive Analysis***, Springer, 1985.

[6] N. GAMBINO AND P. ACZEL, *The generalised type-theoretic interpretation of constructive set theory*, To appear in ***The Journal of Symbolic Logic***, March, 2006. Available at `http://www.cs.man.ac.uk/~petera/papers.html`.

[7] E. R. GRIFFOR AND M. RATHJEN, *The strength of some Martin-Löf type theories*, ***Archive for Mathematical Logic***, vol. 33 (1994), pp. 347–385.

[8] P. MARTIN-LÖF, *Intuitionistic Type Theory — Notes by G. Sambin of a series of lectures given in Padua, June 1980*, Bibliopolis, 1984.

[9] J. R. MYHILL, *Constructive Set Theory*, ***The Journal of Symbolic Logic***, vol. 40 (1975), pp. 347–382.

Abstracts of invited talks in the Special Session on Computability in Analysis

► VASCO BRATTKA, *Computability on non-separable Banach spaces*.
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In computable analysis computability on the real numbers and other topological spaces is studied. In order to represent these spaces effectively (e.g., on Turing machines), it is usually important to assume separability of the spaces. However, in some branches of analysis non-separable spaces occur naturally, for instance as dual spaces of separable spaces. For a computational treatment of duality it is desirable to obtain a way to handle non-separable spaces effectively. In this talk we will discuss Landau's Theorem as an example of a theorem which expresses a duality result for sequence spaces. It turns out that it depends on the underlying axiomatic setting and the version of the Axiom of Choice whether certain non-separable sequence spaces admit an effective representation or not.

► DIETER SPREEN, *On some problems in computable topology*.
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Computations in spaces like the real numbers are not done on the objects of the space itself but on some representation. If one considers only computable elements, finite objects as the natural numbers can be used for this. In the case of the real numbers such a numbering can e.g., be obtained by taking the Gödel numbers of those total computable functions that enumerate a fast Cauchy sequence of rational numbers. Obviously, this numbering is only a partial map. It can be shown that this is not the consequence of a bad choice, but is so by necessity. We will discuss some consequences.

Abstracts of invited talks in the Special Session on Computer Science Logic

► STEPHAN KREUTZER, *Gaifman's theorem and approximation schemes*.
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In 1982, Gaifman proved his locality theorem stating that every first-order formula over a relational signature is equivalent to a local sentence, i.e., a sentence that only makes statements about the neighbourhood of elements which are pairwise far apart in the structure. Gaifman's and similar theorems exhibiting the "local" nature of first-order logic have found numerous applications in finite model theory and related areas.

In this talk we will present a variant of Gaifman's theorem for formulae with free set-variables. In particular we will show that any first-order formula which is positive in a set-variable X is equivalent to a local formula which is also positive in X.

Formulae like this can be used to define minimization problems on graphs. For instance, we can easily construct a first-order formula $\phi(X)$ which defines X to be a vertex cover. The *optimization* problem is then to find a minimal set X satisfying the formula. We will use Gaifman's theorem to show that every optimization problem on graphs defined by such a formula has a polynomial-time approximation scheme on classes of graphs excluding a minor.

► WIEBE VAN DER HOEK, *Dynamic epistemic logic*.
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Since the work of Hintikka on Logic for Knowledge, researchers from Computer Science, Artificial Intelligence and Multi-Agent Systems have appreciated and added to this formal theory, in which one can reason about how agents (participants in a communication protocol, or in a secret exchange, or in a game like scenario) are guaranteed to either obtain knowledge or remain ignorant. We will briefly present the basics of this logic, in which in particular the Multi-Agent notions of knowledge are intriguing, like common knowledge as illustrated in the muddy children example.

An important issue of such scenarios that is only recently formally analyzed, however, is that of the dynamics. How do the agents update their knowledge, and how does their newly acquired knowledge influence their decision making? We argue that in a true Dynamic Epistemic Logic, in which one wants to represent both higher order (knowledge about knowledge) epistemic notions, and the change of knowledge, raises some problems not present in traditional Belief Revision. We then give some logics in which the change of information is central: Public Announcement Logic and variuos logics in which one can model 'private communication'.

This is based on work with Hans van Ditmarsch and Barteld Kooi.

Abstracts of invited talks in the Special Session on Model Theory

► MARIO EDMUNDO, *O-minimal cohomology, definable groups and Lie groups*.
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We will explain a conjecture by Anand Pillay connecting definable groups and Lie groups and how o-minimal cohomology theories are used in this context.

► PIOTR KOWALSKI, *Projective iterative δ-varieties*.
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Let (K,δ) be a differentially closed field of characteristic O. Finite Morley rank sets definable in (K,δ) are closely related to Buium's δ-varieties. A δ-variety is an algebraic variety V definable over K together with a section $s\colon V \to \tau(V)$ of its prolongation bundle (being a torsor of the tangent bundle). To a given δ-variety (V, s) we associate a (K, δ)-definable set $(V, s)^\sharp = \{x \in V(K) : s(x) = (x, \delta(x))\}$. Then $(V, s)^\sharp$ has a finite Morley rank and any definable finite Morley rank set X is essentially of the form $(V, s)^\sharp$ for a δ-variety (V, s).

Model-theoretical properties of $(V, s)^\sharp$ are related to geometric properties of (V, s). Buium proved that if V is projective, then any δ-variety (V, s) is δ-isotrivial.

We extend the definition of a δ-variety to the context of a Hasse field (K, δ). In this case the set $(V, s)^\sharp$ is type-definable and of finite U-rank. However there many type-definable sets of finite U-rank not being of the form $(V, s)^\sharp$. We prove a part of Buium's isotriviality theorem in the Hasse case.

This is joint work with Anand Pillay.

Abstracts of invited talks in the Special Session on Philosophical Logic

► HANNES LEITGEB, *A type-free theory of modality and truth*.
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We are interested in developing a type-free theory of modality and truth in which modalities are treated syntactically in the same way as truth, i.e., as predicates of sentences. As Tarski [1935] has shown, given a first-order language that both contains the truth predicate and the arithmetic language, the unrestricted truth scheme is inconsistent with arithmetic. Accordingly, Montague [1963] proved that if necessity is expressed by a predicate of sentences, the unrestricted predicate counterparts of the axioms and rules of the standard modal system T turn out to be inconsistent with a sufficiently strong background theory of syntax. Similar theorems have been derived for the predicate versions of other modal systems, including the minimal system of temporal logic (Horsten and Leitgeb [2001]). On the semantic side, Halbach, et al. [2003] show that certain modal frames do not allow for possible worlds evaluations if the standard satisfaction clause for modal operators is translated to the predicate case unrestrictedly. In contrast to Tarski and Montague, we do not conclude from these results that truth predicates have to be typed or that modal predicates are to be replaced by sentential operators. Just as ZF combines the standard first-order language of set theory with adequate restrictions on the comprehension scheme, we suggest to hold on to an unrestricted syntax but rather to restrict the admissible instances of the truth scheme and the modal axiom schemes and rules. We are going to present a possible worlds semantics for modal predicates in which these restrictions are taken care of: the semantics does not exclude any modal frames; the standard modal axioms and rules hold for a restricted but plausible class of instances; the language is type-free and much more expressive than the usual operator languages of modal logic. The possible worlds semantics for the reflexive frame that contains exactly one world coincides with the theory of truth in Leitgeb [2005].

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► SARA NEGRI, *Proof analysis in non-classical logics*.
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A method is presented by which it is possible to formulate calculi with good proof-theoretical properties for modal logic and related logics. The method starts from the G3-class of cut- and contraction-free sequent calculi for pure logic. Frame properties characterizing systems of non-classical logics are converted into rules that extend the purely logical calculi. By a general theorem, all frame properties expressible as universal formulas for the accessibility relation give cut- and contraction-free sequent calculi. Standard systems of modal logic fall within this category. By a stronger result, frame properties that correspond to what are known as geometric implications, as those characterizing deontic, intermediate, and substructural logics, lead to similar sequent calculi. Finally, cases of frame properties are treated that cannot be expressed as first-order formulas, the most important example being the Noetherian condition of Gödel-Lob provability logic, for which a cut- and contraction-free sequent calculus is given.

Abstracts of contributed talks

► ZOFIA ADAMOWICZ AND LESZEK ALEKSANDER KOŁODZIEJCZYK, *Partial collapses of the Σ_1 formula hierarchy in models for fragments of bounded arithmetic*.
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We show that for any $n \geq 1$, there exists a model of S_2^n (or T_2^n) satisfying $\neg\exp$ in which every $\exists s\Pi_{n+1}^b$ formula is equivalent to an $\exists\Pi_n^b$ formula.

We first use a standard iterative construction to obtain a model of $S_2^n + \neg\exp$ which is maximal with respect to $s\Sigma_{n+1}^b$ formulae (i.e., all Σ_n^b-elementary extensions to a model of S_2^n are also $s\Sigma_{n+1}^b$-elementary). It then turns out that in any such model, a $s\Sigma_{n+1}^b$ formula is equivalent to an infinite conjunction of Π_{n+1}^b "consistency statements", which can be expressed as a single $\forall\Sigma_n^b$ formula. This also gives $\forall s\Sigma_{n+1}^b \subseteq \forall\Sigma_n^b$, from which the main result follows by transposition.

► SEMIHA AKINCI, *Actualism in possible world realism*.
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There are many different versions of Actualism. All versions hold the following five characteristics. 1. All the possible worlds, the actual world included, exist, are real in a sense. 2. However, there is an ontological distinction between the actual and the other non-actual possible worlds. The actual world is the instantiated, obtained world. 3. The contents of the other possible worlds are derived from the actual world. 4. The content of possible worlds are modal entities such as, 'obtainable states of affairs'. 5. Transworld Identity is possible because in the other worlds although an individual posseses its essential characteristics, some of its accidental characteristics change and therefore what exist in different possible worlds are not identical individuals.

In this paper I divide Actualism first into two main group: Modal Actualism and Non-Modal Actualism (Linguistic Instrumentalism). I divide also Modal Actualism into two:

De-Re Modal Actualism and De-Dicto Modal Actualism. I divide De-Re Modal Actualism into two: Plantinga's Version and Stalnaker's Version. After I clarified and criticised the characteristics of all the versions, I give the modal notions which they explain and couldn't explain and which have theoretical benefits over other views.

► S. ALEKSANYAN AND A. HOVAKIMYAN, *Some characteristics of Frege proofs*.
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It is well-known, that the investigations of the propositional proof complexity are very important because of its tight relation to the problem $P \stackrel{?}{=} NP$. In particular, it is interesting to obtain the good lower and upper bounds in Frege system—the most natural calculi for propositional logic.

In [1] it was shown that (1) proof complexity of every tautology φ in Frege system is no more than $c \cdot s(\varphi) \cdot |\varphi|$, where $s(\varphi)$ is some parameter, depending on φ, by $|\varphi|$ is denoted the size of φ and c is constant; (2) for sufficiently large n and every i $\left(1 \leqslant i \leqslant \left[\frac{n}{2}\log_n 2\right]\right)$ there is a sequence of such tautologies φ_i, that $|\varphi_i| = \theta(n)$ and $s(\varphi_i) = n^i$. In particular, for the tautology

$$\varphi_k = \bigvee_{(\varepsilon_1,\dots,\varepsilon_k)\in E^k} \bigwedge_{j=1}^{2^k-1} \bigvee_{i=1}^{k} P_{i\,j}^{\varepsilon_i}$$

$$|\varphi_k| = \theta\left(k \cdot 2^{2k}\right) \text{ and } s(\varphi_k) = 2^{\theta(k\cdot 2^k)}.$$

It is interesting to investigate how long must be (can be) the size of $\mathcal{F}$-proof, in order to obtain sufficiently large values of $s(\varphi)$ for provable φ (note that the parameter s is small for each Frege axiom). Let $\mathcal{F}$ be a Frege system and $t_\varphi^{\mathcal{F}}$ is the number of the lines in the shortest $\mathcal{F}$-proof of formula φ.

We show that

1. For every formula φ, provable in $\mathcal{F}$, if $t_\varphi^{\mathcal{F}} \leqslant t$, then $s(\varphi) \leqslant 2^{c_1\cdot t}$ for some constant c_1.
2. For every sufficiently large k there is such formula φ_k (for example the above tautology φ_k), that $s(\varphi_k) = 2^{\theta(|\varphi_k|)}$ and $t_{\varphi_k}^{\mathcal{F}} \leqslant c_2 \cdot |\varphi_k|^3$ for some constant c_2. Thus the great value of $s(\varphi)$ can be "quickly" obtained.

An open problem: whether every tautology φ with $s(\varphi) = 2^{\theta(|\varphi|)}$ has polimonially bounded $\mathcal{F}$-proof? If yes, then $NP = coNP$.

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► THEODORE ANDRONIKOS, GEORGE KOLETSOS, AND GEORGE STAVRINOS, *An alternative tableaux method for propositional linear temporal logic*.
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This article presents a sound and complete tableau decision algorithm for propositional linear temporal logic (PLTL). The first tableau method for PLTL was proposed by Wolper in [4], [5]. His method, which actually builds a graph whose nodes are *sets* of PLTL formulae, performs various checks during the construction of the tableau and is, as a result, prohibitively expensive in practice. A different approach was advocated in [2], where formulae are augmented with integer constraints, and in [3], where the tableau nodes also contain time intervals. Time intervals are also used in [1], where a tableau system for finite time structure is presented.

In our algorithm each tableau node contains one PLTL formula augmented with one integer. To decide the validity of φ we construct the tableau for $\neg\varphi$, which is a tree with root node $\langle 0\colon \neg\varphi\rangle$. All nodes of this tree are ordered couples of the form $\langle n\colon \psi\rangle$, where $n \in \mathbb{N}$ and ψ is a PLTL formula. The intuition here is that formula ψ is true at moment n. The novelty of our method is the following: a tableau branch is either *open* or *closed*; here we introduce a third type of branch that we call "*don't care*" branch. A don't care branch is not eligible for expansion, but is not closed (a closed branch is known to be unsatisfiable). If upon completion of the algorithm there is an open branch, then the tableau is open, otherwise the tableau is closed. Don't care branches play no role at all in deciding whether the tableau is open or closed. The concept of don't care branches is a tool that accelerates the tableau construction because it stops the expansion of branches that would, if expanded, be copies of existing branches. Upon completion, every open branch corresponds to a model of the root formula and the information incorporated in the tableau nodes (viz. n in $\langle n\colon \psi\rangle$) greatly facilitates the construction of the model. It is a well-known fact that the satisfiability problem for PLTL is PSPACE-complete [6]. We expect however our procedure to handle relatively efficiently many PLTL formulae that appear in many practical situations.

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[5] ———, *The tableau method for temporal logic: An overview*, ***Logique et Analyse***, vols. 110–111 (1985), pp. 119–136.

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► EVANGELIA ANTONAKOS, *Comparing justified and common knowledge*.
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What is public information in a multi-agent logic of knowledge such as T_n, $\mathsf{S4}_n$ or $\mathsf{S5}_n$? Traditionally this notion has been captured by *common knowledge*, which is an epistemic operator $C\varphi$ given roughly as $\bigwedge_{n=0}^{\infty} E^n\varphi$ where $E\varphi = K_1\varphi \wedge K_2\varphi \wedge \cdots \wedge K_n\varphi$ (*everyone knows* φ) and K_is are individual agent's knowledge operators. In common knowledge systems T_n^C, $\mathsf{S4}_n^C$, and $\mathsf{S5}_n^C$, C depends directly on the agents' logic ([2]).

A stronger notion of public information is introduced in [1] with the new epistemic operator J. This provides *evidence-based common knowledge* for which $J\varphi$ (φ *is justified*) asserts that there is access to a proof of φ. Evidence-based systems are more flexible than their C counterparts as the logic for J can be chosen independently from that of the agents. The paper [1] considers evidence-based systems T_n^J, $\mathsf{S4}_n^J$, and $\mathsf{S5}_n^J$ obtained from the base logics T_n, $\mathsf{S4}_n$, and $\mathsf{S5}_n$ by adding a copy of $\mathsf{S4}$ for J and a connection principle $J\varphi \to K_i\varphi$, $i = 1, \dots, n$.

In this paper we compare common knowledge and justified knowledge in a canonical situation of the base logic S4 and specify an exact logic principle that distinguishes them. Let I.A. stand for the "Induction Axiom"

$$\varphi \wedge J\,(\varphi \to E\varphi) \to J\varphi.$$

THEOREM. $\mathsf{S4}_n^C \equiv \left(\mathsf{S4}_n^J + \mathsf{I.A.}\right)^*$, *where* **is replacing* J *by* C.

This theorem provides an alternative formulation for $\mathsf{S4}_n^C$.

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► O. ANTONOVA AND S. SOLOVIEV, *On historical roots of type theory*.
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It is largely accepted that the origins of modern Type Theory can be traced as far as the work of Aristotle. He clearly stipulates that certain assertions make sense only when their subjects are of right types (for example, in [1]). Historically, modern Type Theory was developed as a restraint to abstractions of Cantor's Set Theory: a major motivation for Russel was the necessity to avoid paradoxes, cf. [2] and [3]. At the same time, the philosophical intuition of set as a more or less arbitrary collection of objects remained for him essentially Cantorian. The same intuition is often "projected" to the past, when the origins of Type Theory are studied. In fact in Aristotle's work much more restrictive intuition of set can be discerned, for example, if we consider his theory of prime- and secondary substance. This more restrictive intuition (taking, sometimes, radical forms) continued to exist and was only temporarily shadowed by the success of Cantorian set theory. In this respect modern Type Theories are very heterogenous. As a manifestation of restrictive vision of a set we may mention the theories of inductive types (the sets there require some common generative principles underlying their elements). In our work we trace in history of philosophy and logic various forms of basic intuition of a set in connection with modern Type Theory.

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► L. Y. BELKOVA, *About sufficient conditions for the existence of finite generative system in closed classes*.
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E. Post has described all classes of Boolean functions and has shown that all closed classes in P_2 are finitely generated. In P_k there are some classes that are not finitely generated.

In this paper we dealt with the closed classes that contain some set of unary functions and are finitely generated. We have obtained that the closed classes that consist of monotone functions and contain the set of all monotone unary functions are finitely generated. In addition the order diagram is given.

Also we have examples of closed classes that are finitely generated but do not satisfy classical sufficient conditions for the existence of finite generative system. These classes don't contain the set of all unary functions or any pre-complete class of unary functions, also they have not any near unanimity function.

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► IZABELA BONDECKA-KRZYKOWSKA AND ROMAN MURAWSKI, *Structuralistic epistemology of mathematics*.

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Mathematical structuralism can be briefly characterized as a view that objects studied by mathematics are structures. The chief motivation and aim of structuralism is to avoid some ontological and epistemological problems of the latter without the necessity of rejecting realism.

Structuralism attempts to avoid these problems by maintaining that mathematical objects, such as numbers or points, are only positions in appropriate (mathematical) structures and that we cannot possess knowledge about such isolated objects outside the structures. We can cognit only structures or their parts and not single numbers or points.

The paper is devoted to structuralistic solutions to principal problems of the epistemology of mathematics, in particular to the problem: how can one get knowledge of abstract entities and what are the methods of developing mathematical knowledge. Various answers proposed by structuralistic doctrines will be presented and discussed.

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S. SHAPIRO [1997], ***Philosophy of Mathematics. Structure and Ontology***, Oxford University Press, New York.

► PIOTR BORODULIN-NADZIEJA, *Measures on minimally generated Boolean algebras*.

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We examine the notion of minimally generated Boolean algebras and additive set functions defined on such algebras. We present few results connected with long standing question if every infinite compact space contains either a nontrivial converging sequence or a copy of $\beta\omega$.

► ELENA I. BUNINA AND ALEXANDER V. MIKHALEV, *Elementary theories of Abelian p-groups and second-order theories of their endomorphism rings*.
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Two algebraic structures U and U' are called *elementary equivalent* (denoted by $U \equiv U'$) if for every first order sentence φ this sentence holds in U iff it holds in U'. The theory of a model $\mathcal{U}$ in the second order language is denoted by $Th_2(\mathcal{U})$. For every cardinal number $\varkappa$ the set of all those sentences φ of the second order language that are true for arbitrary sequence $\langle a_1, \dots, a_q, b_1^{l_1}, \dots, b_s^{l_s}\rangle$, where $a_1, \dots, a_q \in \mathcal{U}$, $b_i^{l_i} \subset \mathcal{U}^{l_i}$, and $|b_i^{l_i}| \leqslant \varkappa$, is denoted by $Th_2^{\varkappa}(\mathcal{U})$.

We study elementary equivalence of endomorphism rings of Abelian p-groups.

If for an Abelian p-group A we have $A = D \oplus G$, where the group D is divisible and the group G is reduced, then the *expressible rank of the group A* is the cardinal number $r_{\exp} = \max(\mu_D, \mu_G)$, where μ_D is the rank of D and μ_G is the rank of the basic subgroup of G.

Note that $r_{\exp}(A) = |A|$ in all cases except the case where $|D| < |G|$, any basic subgroup of A is countable, and the group G itself is uncountable. In this case $r_{\exp}(A) = \omega$.

Theorem 1. *If A and A' are p-groups, $r = r_{\exp}(A)$, $r' = r_{\exp}(A')$, then $End(A) \equiv End(A')$ implies $Th_2^r(A) = Th_2^{r'}(A')$.*

Theorem 2. *For any Abelian groups A and A', if $Th_2(A) = Th_2(A')$, then $End(A) \equiv End(A')$.*

Theorem 3. *If Abelian groups A and A' are reduced and their basic subgroups are countable, then $Th_2^{\omega}(A) = Th_2^{\omega}(A')$ implies $End(A) \equiv End(A')$.*

Therefore for all Abelian groups, except the case where $A = D \oplus G$, $D \neq 0$, $|D| < |G|$, and $|G| > \omega$, a basic subgroup in A is countable, $r = r_{\exp}(A)$, $r' = r_{\exp}(A')$, we have $End(A) \equiv End(A') \Leftrightarrow Th_2^r(A) = Th_2^{r'}(A')$.

► JEFF BURDGES, *Sylow theory for "$p = 0$" in groups of finite Morley rank*.
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The Cherlin-Zilber Algebraicity Conjecture proposes that: All simple groups of finite Morley rank are algebraic groups over algebraically closed fields. One of the longstanding difficulties in the work on this conjecture has been the behavior of torsion free subgroups of a solvable group. At face value, the problem is that a torsion-free abelian group may behave as a semisimple group in one context, and as a unipotent group in another. An abstract definition of torsion-free unipotent radical resolved several major problems, but remained silent on several matters where such a unipotent radical was expected to help. Considering the failure of this notion of unipotence leads us to a notion of (mostly) torsion-free Sylow subgroup, which has several important applications.

► DUMITRU BUŞNEAG, FLORENTINA CHIRTEŞ, AND DANA PICIU, *The prime and maximal spectra of some algebras of fuzzy-logic*.
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By A we denote a BL (pseudo BL)- algebra and by $F(A)$ the lattice of all filters of A. If A is a pseudo BL- algebra by $F_n(A)$ we denote the lattice of normal filters of A. The aim of this contributed paper is to put in evidence new characterization for prime and maximal elements of $F(A)$ (if A is a BL- algebra) and for $F_n(A)$ (if A is a pseudo BL- algebra).

Finally, the same problems if A is a n-valued Lukasiewicz-Moisil algebra (LM_n for short).

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► ERICA CALARDO AND VLADIMIR V. RYBAKOV, *Combining time and knowledge, a semantic approach*.
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The language of multi-modal logic is a good tool to study the interaction of agents' knowledge in the flow of time. Many different axiomatic systems for multi-modal propositional logics involving time and knowledge modalities have been introduced. We study a logic of this kind, but generated in a semantic way by introducing a certain class $\mathbf{K}$ of multi-modal frames, called $\mathcal{T}_{\mathcal{K}}$-frames. In these frames the flow of time is linear and agents are operating synchronously. One agent—the *wise agent*—plays a special role: the *wise agent* knowledge is the universal modality on time clusters in these frames. The multi-modal logic $L(\mathbf{K})$ is the set of all $\mathcal{T}_{\mathcal{K}}$-valid formulae. It has been already proved by us that $L(\mathbf{K})$ has the *fmp*, and hence $L(\mathbf{K})$ is decidable.

Our present research is devoted to extending this result to admissible inference rules. We consider the logic $L(\mathbf{K}_1)$, a weaker variant of $L(\mathbf{K})$, with only one agent besides the *wise agent*. Our approach involves the construction of n-characterising models $Ch_{L(\mathbf{K}_1)}(n)$ for $L(\mathbf{K}_1)$ and the description of free temporal algebras $\mathcal{F}_w(L(\mathbf{K}_1))$ by means of these models. A rule r is admissible in $L(\mathbf{K}_1)$ if and only if r is valid in the models $Ch_{L(\mathbf{K}_1)}(n)$ with respect to all definable valuations. Using this approach we prove

Theorem 1. *The logic $L(\mathbf{K}_1)$ is decidable w.r.t. admissible inference rules.*

Corollary 2. *The quasi-equational theory of free temporal algebra $\mathcal{F}_w(L(\mathbf{K}_1))$ is decidable.*

► STEFANO CAVAGNETTO, *Speed-up theorem and Kreisel's conjecture*.
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Gödel [3] announced a speed-up of proofs that occurs when passing from first-order arithmetic to second-order (more generally, a stronger theory may speed-up proofs of a weaker theory). He did not prove the claim but subsequently proofs were given by Parikh [5], Krajíček [4], and the most general one by Buss [1].

Kreisel's Conjecture (KC) is an open problem (cf. [2]) in the study of lengths of proofs. KC states in the following: If there exists k such that Peano Arithmetic PA proves $A(s^{(n)}(0))$

in k steps for every n then, in fact, PA proves $\forall x\, A(x)$. We briefly recapitulate some basic fact about KC and the speed-up phenomenon, and present the following statement.

PROPOSITION. *There exists a formal system T such that T has an unbounded speed-up over PA but is still a conservative extension of PA.*

We show that the proposition is a corollary of KC but we also give a proof not using any unproven assumption.

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► AN. CHUBARYAN, ARM. CHUBARYAN, AND H. BOLIBEKYAN, *On the most generalized speed-up in first order logic*.
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There are many results about speed-up phenomena, caused by having a stronger theory. Some of them relate to *size*, the others to *steps* of proofs. Earlier An. Chubaryan introduced the common conception of proof complexity and notions of *ordinary theory* and its special extension, for which the most generalized speed-up of proofs (analogy of Rabin's speed-up for calculations) was proved. Now we show, that for many pairs of the different systems of pure predicate calculus the above speed-up holds.

DEFINITION. *The theory Φ is called ordinary, if there are a pair recursively enumerable, effectively unseparable sets M_+^Φ and M_-^Φ of the formulas of Φ and two algorithm $\mathfrak{A}_1$ and $\mathfrak{A}_2$, which produce for each formula β of Φ accordingly such formulae $\mathfrak{A}_1(\beta)$ and $\mathfrak{A}_2(\beta)$, that the following holds*:
(a) $\beta \in M_+^\Phi \longleftrightarrow \vdash_\Phi \mathfrak{A}_1(\beta)$ *and* $\beta \in M_-^\Phi \longleftrightarrow \vdash_\Phi \mathfrak{A}_2(\beta)$;
(b) *for each formulae γ of Φ if $\vdash_\Phi \gamma \vee \mathfrak{A}_1(\beta)$ and $\vdash_\Phi \mathfrak{A}_2(\beta)$, then $\gamma \in M_+^\Phi$.*

By C_α^Φ we denote the minimal proof-complexity of the formula α in Φ, where C is such recursive function, that for each n the equation $C(x) = n$ has only limited number of solutions and there is an algorithm, which for every n produces the list of all solutions of this equation.

MAIN THEOREM. *Let Φ_1 be an ordinary theory, α be such formula of Φ_1, that $\alpha \notin M_+^{\Phi_1}$, let, further, Φ_2 be such extension of Φ_1, that $\vdash_{\Phi_2} \alpha$, then for every recursive function f*

$$\exists n_0\, \forall n > n_0\, \exists \alpha_n \quad \left(C_{\alpha_n}^{\Phi_2} \leqslant n \wedge C_{\alpha_n}^{\Phi_1} > f(n) \right).$$

Let HP_C, HP_I, HP_M, SP_C, SP_I, SP_M, NP_C, NP_I, NP_M, SP_C^-, SP_I^-, SP_M^-, RP_C, RP_I, RP_M are accordingly Hilbert-style (H), sequent (S), natural (N), cut-free sequent (S^-) and resolution (R) systems of pure predicate calculus, based on classical $({}_C)$, intuitionistic $({}_I)$ and minimal $({}_M)$ logic. We show, that all of these systems are ordinary, therefore the statement of Main Theorem is valid for every pair $\Phi_1 \subset \Phi_2$ of above systems.

► GABRIEL CIOBANU, *Rational and irrational divergence of partial recursive functions*.
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Since the partial recursive function are while-computable, we work with the class WP_{array} of the nondeterministic while programs with array assignments. We use the version DL_{array} of Dynamic Logic built over WP_{array}. For every first-order language L, the expressive power of DL_{array} is between the first-order predicate calculus with equality over L (i.e., L_{ww}) and the infinitary language allowing countable disjunction (i.e., L_{w_1w}). Rational divergence is a divergence in which a sequence is repeated periodically, while irrational divergence is a divergence in which such a looping sequence does not exist. First, these divergence predicates are expressed as formulas of DL using arithmetics. Moreover, we express in DL_{array}, step by step, the predicates *Div* corresponding to the divergence, *rationalDiv* corresponding to the rational divergence, and *irrationalDiv* corresponding to the non-periodic divergence. Finally, for each program p in WP_{array}, there is a DL_{array} formula equivalent to $Div(p) = rationalDiv(p) \vee irrationalDiv(p)$.

► STAVROS COSMADAKIS AND EUGÉNIE FOUSTOUCOS, *Datalog complexity tools applied to the problem of MSO evaluation on trees*.
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The *evaluation problem for MSO formulae in coloured trees* is the problem of computing, for each MSO (Monadic Second Order) formula $\phi(X_1, \dots, X_k)$ and each tree $\mathcal{T}$, the set $\mathcal{A}_{\phi,\mathcal{T}}$ of k-tuples of subsets of the domain of $\mathcal{T}$ which satisfy ϕ, i.e., $\mathcal{A}_{\phi,\mathcal{T}} = \{(B_1, \dots, B_k) \mid \mathcal{T} \models \phi(B_1, \dots, B_k)\}$. The aforementioned problem is decidable ([3], Thatcher and Wright 1968) and it has been solved using model-theoretic methods like compositionality (Courcelle 1990), or automata-theoretic methods (Flum, Frick, Grohe 2002).

It has been proved in [4], [5] that (1) the evaluation problem SAT-ASSIGN(t) for MSO formulae in coloured trees of size at most t is Datalog-definable i.e., it can be formulated as a Datalog query SAT-ASSIGN(t)-QUERY (via the program $\Pi(t)$ of maximal arity $2t$), (2) SAT-ASSIGN(t) is Monadic Datalog-definable (via the monadic program $\Pi_{mon}(t)$), (3) the complexity of evaluating SAT-ASSIGN(t)-QUERY on an input tree $\mathcal{T}$ of size t, is the same as the complexity obtained via automata-theoretic tools (i.e., linear in t, with a hyperexponential constant).

Here we investigate the complexity of SAT-ASSIGN(t)-QUERY by means of recently developed tools: a. the persistency numbers [1] and b. the linear formulae [2], both elaborated specifically for the analysis of datalog programs/queries. We show that the persistency number that characterizes the query SAT-ASSIGN(t)-QUERY is 0 and that the width of the linear formulae of $\Pi(t)$ is a small constant independent of t; the same holds for $\Pi_{mon}(t)$ which, moreover, presents syntactic similarities with program *ACCESS* expressing the (prototypical P-complete) *path accessibility query*.

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► GIOVANNA D'AGOSTINO AND GIACOMO LENZI, *On modal mu-calculus over transitive frames*.
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In this contribution we will discuss two extensions of modal logic, one with fixpoints, and one with bisimulation quantifiers.

The first extension is widely used in computer science in the area of verification of computer systems, whereas the second has a more logical flavor, in fact it has been introduced as a tool for the study of uniform interpolation.

It is known that over arbitrary frames (that is, over the logic K) fixpoints are much more expressive than bisimulation quantifiers: the first gives the modal mu-calculus, the second gives nothing more than K itself.

On the other hand, over transitive frames (that is, over the logic K4), every fixpoint is expressible by bisimulation quantifiers.

So, if a logic above K4 is closed under bisimulation quantifiers, then it is closed under fixpoints as well, and the two extensions trivially coincide. This is the case of logics such us GL, Grz, S4Grz, S5. On the other hand, we will see the existence of logics over K4 where the two extensions are nontrivial but nonetheless coincide, such as K4 and S4.

If time permits, we will also discuss the problem of whether, and to which extent, the fixpoint hierarchy of the modal mu-calculus above K4 collapses.

► ANUJ DAWAR, DAVID RICHERBY, AND BENJAMIN ROSSMAN*, *Choiceless polynomial time, counting and the Cai–Fürer–Immerman graphs*.
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In the quest for a logical characterization of polynomial-time computability, several authors have considered extensions of fixed-point logics with choice mechanisms. Choiceless polynomial time ($\tilde{C}$PT) is a language introduced by Blass, Gurevich and Shelah [1] that can be seen as an attempt to characterize what can be done without choice. It is not a traditional logic but a machine model that operates directly on finite relational structures via their hereditarily finite sets.

Blass et al. show that $\tilde{C}$PT is strictly more expressive than inflationary fixed-point logic (IFP) but still cannot define simple cardinality properties. In [2], they show that the natural extension of $\tilde{C}$PT with counting is strictly more expressive than the corresponding extension of IFP, by using a padded version of the query originally defined by Cai, Fürer and Immerman [3] to separate IFP with counting from **P**. Blass et al. leave open the questions of whether $\tilde{C}$PT with counting captures **P** and whether it can express the unpadded version of the CFI query.

We show that the unpadded version is definable in $\tilde{\mathrm{C}}$PT. Our program does not use counting but depends crucially on using sets of unbounded finite rank. We demonstrate that the query cannot be defined by any program, even in $\tilde{\mathrm{C}}$PT with counting, that has a constant bound on the rank of sets used [4].

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► IOANNA MATILDE DIMITRIOU, *Fragments of choice and determinacy*.
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It is known that the axioms of determinacy and choice are inconsistent together but weaker forms of choice, for example countable choice for reals, are consequences of determinacy. Fragments of choice referring to the real line have been investigated in detail in papers by Keremidis, Tachtsis, Howard, Rubin and Stanley [1], [2], [3] as part of the "consequences of the axiom of choice project" that gathers results from over 100 years of "choice studies".

We investigate these fragments of the axiom of choice and classify them according to their status under the axiom of determinacy. The paper is no original research but should be understood as a survey of the connections between the areas of determinacy theory and fragments of the axiom of choice.

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► NATASHA DOBRINEN, *κ-stationary subsets of $\mathcal{P}_{\kappa^+}\lambda$*.
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Consider the following when $\kappa^{<\kappa} = \kappa$. A set $C \subseteq \mathcal{P}_{\kappa^+}\lambda$ is *κ-club* if it is cofinal and closed under increasing sequences of length exactly κ. $S \subseteq \mathcal{P}_{\kappa^+}\lambda$ is *κ-stationary* if $S \cap C \neq \emptyset$ for all κ-club sets C. We present the general theory of κ-club and κ-stationary sets. What is particularly nice about this class of stationary sets is that there is a Kueker Theorem for them on $< \kappa$-ary functions. This allows us to push through to κ-stationary sets general results about stationary sets which hinge on Kueker's Theorem for stationary sets. We give sufficient conditions for preserving some or all κ-stationary sets. Included in this discussion is a game generalizing the proper game Γ^κ. If Player II has a winning strategy for Γ^κ played in P and P is $(<\kappa,\kappa)$-distributive, then P preserves all κ-stationary sets. We will also show that κ-stationarity is exactly the notion needed to completely characterize the relationship between the $(\kappa,\lambda,<\mu)$-distributive law in complete Boolean algebras and a related game $\mathcal{G}^{\kappa}_{<\mu}(\lambda)$, when $\mu\leq\kappa\leq\lambda$. We show how to construct Boolean algebras in which many games are undetermined, using only GCH.

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In [2] and [3] we suggested that whereas the real number system should merely be regarded as constituting an arithmetic continuum modulo the Archimedean axiom, Conway's ordered field *No* of surreal numbers [1] may be regarded as a sort of absolute arithmetic continuum (modulo NBG). In the present paper we introduce a formal replacement for the intriguing linear continuum sketched by Charles Sanders Peirce (cf. [5], [6]) at the turn of the twentieth century, and point out that by limiting *No* to the real-closed integral domain consisting of its finite and infinitesimal members one obtains a well-suited arithmetization of the formal replacement whose properties mimic those of *No* established in [3] and [4].

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Let $\mathcal{M}=\langle M,+,<,0,\{d\}_{d\in D}\rangle$ be an ordered vector space over an ordered division ring D, and $G=\langle G,\oplus,e\rangle$ an n-dimensional group definable in $\mathcal{M}$, equipped with the t-topology ([1]). We show that if G is abelian, definably compact, and definably connected, then there is a convex subgroup U of M^n, such that G is definably isomorphic to a definable quotient U/L with topological group structure induced by M^n, for some lattice L of rank n. This constitutes an o-minimal analogue of the classical theorem that every compact, connected, abelian topological group in the Euclidean space is isomorphic to a real torus.

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► A. A. ESIN, *Precomplete subclasses of functions in P_3 with regard to a certain strengthened closure operator*.
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Classical Post's theorem describes the five precomplete classes of the Boolean algebra. In 1958 S. V. Yablonskij proved that there are 18 precomplete classes in P_3. Those papers regarded a closure of set of functions with regard to substitution operator. It is easily seen that if the functions differing by dummy variables only are not considered to be equivalent it will result in the same precomplete classes.

This paper considers the closure operators that are the strengthening of classical substitution operator on the set of 3-valued logic functions. Let M be a subset of P_3. $\mathcal{R}(M)$ denotes the following set: the closure of set of functions M with regards to the substitution operation, and the addition of a function $f \sim g$ for every g in M, where

$$f \sim g \Leftrightarrow \forall \vec{x} \left[\left(f(\vec{x}) = g(\vec{x}) \right) \vee \left(f(\vec{x}), g(\vec{x}) \in \{0, 1\} \right) \right].$$

We will consider two types of such closure operator:

(1) $\mathcal{R}_1$—the functions are considered equivalent if they differ from each other by dummy variables only.

(2) $\mathcal{R}_2$—the functions differing by dummy variables only are not considered equivalent

In this paper it is proved that for $\mathcal{R}_1$ only three following classes are precomplete: T_2—the set of all functions in P_3 that preserve 2, T_{01}—the set of all functions in P_3 that preserve the set $\{0, 1\}$, $T_\sim$—the set of all functions in P_3 that preserve the $\sim$ relation.

For the T_{01} and $T_\sim$ classes the structures of closed subclasses (which are countable) are fully described. We set up a hypothesis that T_2 has denumerable lattices of closed subclasses too.

For the $\mathcal{R}_2$ operator only two additional precomplete subclasses exist: $P_3(x)$—the set of all unary functions, C_2—the set of all constants and functions $f(x_1, \dots, x_i, \dots, x_n)$, that for every i in $\{1, \dots, n\}$ the equation $f(x_1, \dots, x_{i-1}, 2, x_{i+1}, \dots, x_n) = 2$ holds.

For this operator it is also proved, that the number of closed subclasses is infinite.

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► HORACIO FAAS, *Heterogeneous inference from the standpoint of information flux*.
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Since long ago, validity of inferences is the main subject of logic, and whenever a new logical system appears it immediately attempts to fit to it. Heterogeneous inference intends such validity and if reaching that target it would be able to explain some of the mistakes of several erroneous mathematical conjectures led by misleading intuitions. As is well known, heterogeneous inference relates distinct representation systems and there are computer programs such as Barwise & Etchemendy's "Hyperproof" that make and control inferences from and to linguistic and diagrammatic representations. Visualization plays a fundamental role in diagrammatic representations, but in mathematics it has been only accepted as helpful for, but not as a rigorous component of proofs. Notwithstanding, things are slowly changing during the last twenty years or more, and there is a renewed interest in visualization based on three different sources: the development of visualization techniques in computer science that produced results not attainable without them; the role of 'mental imagery' in neurosciences; and the promising level of rigor reached by diagrammatic inference. One may conjecture that in all these three cases there is valid inference when information is extracted that already was in the premises, i.e., information flux holds validity. In this paper the notion of logical consequence is considered as making explicit information already implicit, and it is so applied to heterogeneous inference to show how several mathematical problems—some of the so called 'mathematical monsters'—could be so explained, by means of the notion of 'infon'. It is also commented how these conjectures would influence standpoints in philosophy of mathematics.

► NIKOLAOS GALATOS, *Algebraic methods in Substructural Logics*.
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Logics that when formulated in Gentzen systems lack one or more of the structural rules of *exchange*, *weakening* and *contraction* are called *substructural logics*. We focus on systems related to the full Lambek calculus, a sequent calculus system that encompasses intuitionistic, relevant and multi-valued propositional logic, as well as the multiplicative-additive fragment of propositional linear logic.

Many substructural logics are *algebraizable* and their algebraic semantics are varieties of residuated lattices. Recall that a *residuated lattice* is an algebra $\mathbf{L} = \langle L, \wedge, \vee, \cdot, \backslash, /, 1\rangle$, such that $\langle L, \wedge, \vee\rangle$ is a lattice, $\langle L, \cdot, 1\rangle$ is a monoid, and for all elements $a, b, c \in L$,

$$ab \leq c \Leftrightarrow a \leq c/b \Leftrightarrow b \leq a\backslash c.$$

Because of the lack of the rule of exchange, the logic contains two conditionals $\backslash$ and $/$, instead of the usual $\to$. Moreover, unlike in intuitionistic logic where the conditional is the residual of conjunction, the conditionals are the residuals of *fusion*, because of the lack of the rules of contraction and weakening.

We discuss the algebraization and the decidability of substructural logics. In particular, we provide an algebraic proof of the cut elimination theorem for the full Lambek calculus, obtaining in this way the decidability of the logic, as well as of the equational theory of the corresponding variety of residuated lattices. Also, we prove that the full Lambek calculus with exchange and distribution has a decidable set of theorems, but an undecidable provability relation.

Furthermore, we establish a weak form of the deduction theorem for substructural logics, called *parametrized local deduction theorem*. A stronger from of a *local deduction theorem* holds for logics with the exchange rule, since the corresponding varieties have the *congruence extension property*.

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There are many approaches to development of analytic tableaux for temporal logics nowadays. Among them tableaux with prefixed formulae appeared to be relatively easy for use and flexible in the sense of providing general framework for logics, formalizing different properties of time. Prefixes themselves are intended to name possible worlds or states in a Kripke frame. An accessibility relation on a frame can be encoded into a prefix structure or shown explicitly. We use the later case in our constraction.

An elementary building block of tableau construction is so called $\mathcal{R}$–Γ pair, where $\mathcal{R}$ is a set of pairs of natural numbers (used for prefices) and Γ is a set of prefixed formulae. Explanations of rules: $\Gamma[\psi]$ means that the set Γ contains distinguished formula ψ; Γ^* under the horizontal line of rule is the set Γ above the line without distinguished element. These rules are for the system $\mathbf{K}_t$. Extensions of this system can be obtained by adding some family of rules for the set $\mathcal{R}$, simulating first order properties of Kripke frames, corresponding to particular logic.

$$[\neg\neg]\quad \frac{\langle \mathcal{R} \mid \Gamma[n\neg\neg\varphi]\rangle}{\langle \mathcal{R} \mid \Gamma^* \cup \{n\varphi\}\rangle} \qquad [\wedge]\quad \frac{\langle \mathcal{R} \mid \Gamma[n(\varphi \wedge \psi)]\rangle}{\langle \mathcal{R} \mid \Gamma^* \cup \{n\varphi, n\psi\}\rangle}$$

$$[\neg\wedge]\quad \frac{\langle \mathcal{R} \mid \Gamma[n\neg(\varphi \wedge \psi)]\rangle}{\langle \mathcal{R} \mid \Gamma^* \cup \{n\neg\varphi\}\rangle\ \langle \mathcal{R} \mid \Gamma^* \cup \{n\neg\psi\}\rangle}$$

$$[G]\quad \frac{\langle \mathcal{R}[(m,n)] \mid \Gamma[mG\varphi]\rangle}{\langle \mathcal{R} \mid \Gamma \cup \{n\varphi\}\rangle} \qquad [H]\quad \frac{\langle \mathcal{R}[(m,n)] \mid \Gamma[nH\varphi]\rangle}{\langle \mathcal{R} \mid \Gamma \cup \{m\varphi\}\rangle}$$

$$[\neg G]\quad \frac{\langle \mathcal{R} \mid \Gamma[m(\neg G\varphi)]\rangle}{\langle \mathcal{R} \cup \{(m,k)\} \mid \Gamma^* \cup \{k\neg\varphi\}\rangle} \qquad \text{where } k \text{ is a fresh prefix}$$

$$[\neg H]\quad \frac{\langle \mathcal{R} \mid \Gamma[m(\neg H\varphi)]\rangle}{\langle \mathcal{R} \cup \{(k,m)\} \mid \Gamma^* \cup \{k\neg\varphi\}\rangle} \qquad \text{where } k \text{ is a fresh prefix}$$

► PAVEL HRUBEŠ, *Two hierarchies of universal predicates*.
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For every countable structure $\mathcal{M}$ on natural numbers there exist subsets of N undefinable in $\mathcal{M}$. However, there are subsets of N which, though not definable in $\mathcal{M}$, can be obtained by reflection on the semantics of $\mathcal{M}$. The truth and the universal predicate for $\mathcal{M}$ are prominent examples of such sets. The structure $\mathcal{M}$ thus points to a richer structure $\mathcal{M}'$ and we can consider a transfinite sequence of such structures and we can try to determine which subsets of N can be reached in such fashion.

We investigate two hierarchies of arithmetical structures, denoted as *Tarski* and *proper Tarski hierarchy*. Both the hierarchies are defined in terms of transfinite addition of a certain minimum universal predicate (a *canonic universal* and a *proper canonic universal predicate* respectively) to a given initial structure. It is shown that the Tarski hierarchy reaches only the first non-recursive ordinal, ω_1, and is equivalent to the class of hyperarithmetical sets (see [2]). The proper Tarski hierarchy, on the other hand, goes far beyond ω_1 and HYP. The sets obtained in the proper Tarski hierarchy are characterised as a particular model of a second-order arithmetic (see [1]).

[1] R. Boyd, G. Hensel, and H. Putnam, *A recursion-theoretic characterisation of the ramified analytic hierarchy*, ***Transactions of the American Mathematical Society***, vol. 141 (1969), pp. 37–62.

[2] Pavel Hrubeš, ***On hierarchies of universal predicates***, unpublished

► KOICHIRO IKEDA, *The stability spectrum of generic graphs*.
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In [2], [3], Hrushovski used an adaptation of the Ehrenfeucht-Fraïssé homogeneous universal relational structures to refute two well-known conjectures. The way of his construction is called a generic construction. To construct new structures, many model theorists are still applying Hrushovski's techniques. However all known generic stable examples are strictly stable or ω-stable. In [1], Baldwin asked whether or not there exists a 'generic' structure which is superstable but not ω-stable. In [4], I have proved that there is no generic pseudoplane which is strictly stable or ω-stable. In this talk, I will sketch the proof of the following result:

Any saturated generic graph is strictly stable or ω-stable.

[1] , J. T. BALDWIN, *Problems on pathological structures*, ***Proceedings of 10th Easter Conference in Model Theory***, (Helmut Wolter and Martin Weese, editors), pp. 1–9, 1993.

[2] , E. HRUSHOVSKI, *A stable $\aleph_0$-categorical pseudoplane*, preprint, 1988.

[3] , ———, *A new strongly minimal set*, ***Annals of Pure and Applied Logic***, vol. 62 (1993), no. 2, pp. 147–166.

[4] , K. IKEDA, *A remark on the stability of saturated generic graphs*, ***Journal of the Mathematical Society of Japan***, to appear.

► VLADIMIR KANOVEI AND VASSILY LYUBETSKY, *A cofinal family of equivalence relations generated by Borel ideals*.

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Following Rosendal's construction in [1], we present an explicit definition of an increasing ω_1-sequence of Borel equivalence relations on a Polish space, cofinal in the family of all Borel equivalence relations, in the sense of Borel reducibility. We prove that equivalence relations in this sequence are generated by explicitly defined Borel ideals.

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Support of Minpromnauka grant RF 37-053-11-0061 acknowledged.

[1] C. ROSENDAL, *Cofinal families of borel equivalence relations and quasiorders*, January 2003, Preprint.

► VINCENT KIEFTENBELD, *Notions of strong compactness without the Axiom of Choice*.

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For large cardinal axioms, there are often different definitions that are equivalent in ZFC. For instance, 'there is a κ-complete nonprincipal ultrafilter on κ' is ZFC-equivalent with 'there is an elementary embedding from V to an inner model with critical point κ'.

Without the Axiom of Choice, these definitions may not be equivalent anymore. Depending on which definition you choose, the consistency strength of the large cardinal axiom may change with the ambient set theory. An example is the axiom X: 'there is a κ with a normal measure on $\wp_\kappa(\kappa^+)$'. The consistency strength of $\mathsf{ZFC} + \mathsf{X}$ is much larger than that of $\mathsf{ZF} + \mathsf{AD}$, but $\mathsf{ZF} + \mathsf{AD}$ proves X. Therefore, the consistency strength of $\mathsf{ZF} + \mathsf{X}$ must be strictly less than the consistency strength of $\mathsf{ZFC} + \mathsf{X}$. For details, cf. [1, Kapitel 6].

In this talk, we discuss similar questions for notions of strong compactness without the Axiom of Choice. Very little is known so far, and we point out mostly basic answers and open questions whose solution would require the development of new techniques.

[1] STEFAN BOLD, *AD und Superkompaktheit*, Diplomarbeit, Rheinische Friedrich-Wilhelms-Universität Bonn, April 2002.

► JONATHAN KIRBY, *Exponential and Weierstrass differential equations*.

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The theory of exponential and Weierstrass differential equations in a differential field can be explained in terms of an amalgamation construction in the style of Hrushovski. This gives a simple means of determining when a system of such equations has a solution in some differential field, simply by computing the "predimension" of the system.

Part of this work is contained in J. Kirby, *A Schanuel condition for Weierstrass equations*, ***The Journal of Symbolic Logic***, vol. 70, (2005), pp. 631–638.

► MIROSŁAWA KOŁOWSKA-GAWIEJNOWICZ, *Some connections between bilinear logics and Grishin algebras*.
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We consider bilinear logics, being non-commutative versions of Girard's linear logics and examples of a substructural logics [1], [2].

First, we introduce definition, dualities and an equivalent axiomatization of Grishin algebra [3], [5]. Next, we give the selected connections between bilinear logics and Grishin algebras. In particular, we discuss a weak version of bilinear logic and Grishin's laws, a sequent calculus for weak bilinear logic and Grishin's mixed associative laws [4].

Finally, we present some results concerning a new view on Grishin algebra.

[1] V. M. Abrusci, *Phase semantics and sequent calculus for pure noncommutative classical linear propositional logic*, ***The Journal of Symbolic Logic***, vol. 56 (1991), pp. 1403–1451.

[2] Y. Girard, *Linear logic*, ***Theoretical Computer Science***, vol. 50 (1987), pp. 1–102.

[3] V. N. Grishin, *On a generalization of the Ajdukiewicz-Lambek system*, ***Studies in non-commutative logics and formal systems***, Nauka, Moscow, 1983, pp. 315–343.

[4] J. Lambek, *From categorial grammar to bilinear logic*, ***Substructural logics***, (K. Došen and P. Schroeder-Heister, editors), Oxford University Press, 1993, pp. 207–237.

[5] ———, *The metamathematics of sentence structure*, ***American Mathematical Monthly***, vol. 65 (1958), pp. 154–169.

► ROMAN KONTCHAKOV, AGI KURUCZ, AND MICHAEL ZAKHARYASCHEV, *Undecidability of first-order intuitionistic and modal logics with two variables*.
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It is well known that the two-variable fragment of classical first-order logic is decidable, while the fragment with three variables is undecidable, even without constants and equality. Our main result is the following:

Theorem 1. *The two-variable fragment of first-order intuitionistic logic* (*without constants and equality*) *is undecidable.*

The undecidability of the two-variable intuitionistic logic extended with the constant domain principle was first proved in [1]. For first-order modal logics we have an even stronger result:

Theorem 2. *The two-variable monadic fragments of almost all standard first-order modal logics* (*e.g.*, **K**, **K4**, **GL**, **S4**, **S5**, **K4.1**, **S4.2**, **GL.3**, *etc.*) *interpreted in models with expanding* (*or constant*) *domains are undecidable.*

[1] D. Gabbay and V. Shehtman, *Undecidability of modal and intermediate first-order logics with two individual variables*, ***The Journal of Symbolic Logic***, vol. 58 (1993), pp. 800–823.

► KRZYSZTOF KRUPINSKI AND FRANK WAGNER, *Small profinite groups and rings*.
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We investigate profinite groups and rings as profinite structures in the sense of Newelski [1], [2], i.e., we consider a profinite group [or ring] together with a distinguished group of automorphisms (called the structural group) respecting the appropriate inverse system. We say that a profinite group [or ring] is small if there are only countably many orbits over any finite set under the action of the structural group. The main conjecture concerning small profinite groups, which was stated by Newelski in [1], is the following:

CONJECTURE (*). Each small profinite group contains an open abelian subgroup.

To show this conjecture it is reasonable to investigate small profinite rings. More precisely, we have formulated the following conjecture:

CONJECTURE (**). Each small commutative profinite ring has an open nilpotent subring.

If this conjecture was true, we would get an important step in the proof of Conjecture (*). We show a partial result that each small commutative profinite ring has an open subring which is nil of finite nilexponent. We also show some other results describing the structure of small profinite rings.

[1] L. NEWELSKI, *Small profinite groups*, ***The Journal of Symbolic Logic***, vol. 66 (2001), pp. 859–872.

[2] ———, *Small profinite structures*, ***Transactions of the American Mathematical Society***, vol. 354 (2002), pp. 925–943.

► OLIVER KULLMANN, *Generalised clause-sets and hypergraph colouring*.
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In [4] it was shown that a minimally non-2-colourable hypergraph has at least as many hyperedges as vertices. In [3] this result has been shown to be closely related via autarky theory to the well-known property of minimally unsatisfiable boolean clause-sets of having more clauses than variables.

I want to discuss the framework for these investigations, and how hypergraph theory and the theory of (generalised) clause-sets can enrich each other via these new links. An important notion here is the notion of matching autarkies (see [2]), and the related questions of polynomial time satisfiability decision for clause-sets with bounded maximal deficiency as shown in [1]; generalising these notions to the non-boolean framework poses some challenging problems.

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[1] HERBERT FLEISCHNER, OLIVER KULLMANN, AND STEFAN SZEIDER, *Polynomial–time recognition of minimal unsatisfiable formulas with fixed clause–variable difference*, ***Theoretical Computer Science***, vol. 289 (November 2002), no. 1, pp. 503–516,

[2] OLIVER KULLMANN, *Lean clause-sets: Generalizations of minimally unsatisfiable clause-sets*, ***Discrete Applied Mathematics***, vol. 130 (2003), pp. 209–249.

[3] ———, *Conjunctive normal forms with non-boolean variables, autarkies and hypergraph colouring*, Technical Report CSR 5-2005, University of Wales Swansea, Computer Science Report Series, March 2005.

[4] P. D. SEYMOUR, *On the two-colouring of hypergraphs*, ***The Quarterly Journal of Mathematics***, vol. 25 (1974), pp. 303–312.

► MILOŠ S. KURILIĆ, *Splitting families and forcing*.
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A family $\mathcal{S}$ of infinite subsets of ω is called a *splitting family* iff for each infinite set $X \subseteq \omega$ there is $S \in \mathcal{S}$ satisfying $|X \cap S| = |X \setminus S| = \aleph_0$. If $\mathbb{P}$ is a forcing notion, then we say $\mathcal{S}$ is $\mathbb{P}$-stable iff $\Vdash_{\mathbb{P}}$ "$\mathcal{S}$ is a splitting family". In [2], for $\mathbb{P} =$ the Cohen forcing, the following characterization is given:

THEOREM 1. *A splitting family $\mathcal{S} \subseteq [\omega]^{\omega}$ is not Cohen-stable iff there is a bijection $f : \omega \to \mathbb{Q}$ such that for all $S \in \mathcal{S}$ the boundary of the set $f[S]$ is a nowhere dense set in the space $\mathbb{Q}$ of rational numbers.*

A characterization of Cohen-stable maximal almost disjoint families on ω, given in [2], was generalized in [1] for a wide class of forcings including the Sacks, Miller, Laver, Solovay, and the Hechler forcing. Using a similar method we obtain characterizations of stable splitting families for the forcings mentioned above. Also we consider the existence of stable and unstable splitting families and, regarding the forcing notions mentioned above, we investigate the hierarchy of forcing stability of splitting families.

[1] J. BRENDLE AND S. YATABE, *Forcing indestructibility of MAD families*, ***Annals of Pure and Applied Logic***, vol. 132 (2005), nos. 2–3, pp. 271–312.
[2] M. S. KURILIĆ, *Cohen-stable families of subsets of the integers*, ***The Journal of Symbolic Logic***, vol. 66 (2001), no. 1, pp. 257–270.

► THIERRY LIBERT, *Class abstraction as functional abstraction*.
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It has often been suggested that there is some relation between class abstraction '$\{x \mid \varphi(x)\}$' and functional abstraction '$\lambda x.f(x)$'. Besides, historically, the former was already conceived as a particular case of the latter in Frege's work. The purpose of this talk is to review and compare some attempts in that direction, as in [2] or more recently in [1].

[1] T. LIBERT, *Positive abstraction and extensionality revisited*, submitted.
[2] D. SCOTT, *Combinators and classes*, ***λ-Calculus and Computer Science***, vol. 37 of Lecture Notes in Computer Science, Springer, Berlin, 1975, pp. 1–26.

► ROBERT S. LUBARSKY, *Reflection in constructive set theory*.
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It is a classical result that, under the other axioms of ZF, Replacement, Collection, and Reflection are equivalent. The proofs from right to left are all soft proofs, and carry over to (any sufficiently strong) intuitionistic set theory. However, the proofs from left to right are not and do not. That Replacement does not imply Collection has already been investigated, most notably by Friedman and Scedrov [1], and, for classical systems weaker than ZF, by Zarach [2]. Here we consider the case of Reflection, and show that various forms of Reflection do not follow from various axiomatizations of intuitionistic set theory. There are some obvious but difficult open questions remaining, which will be mentioned.

[1] HARVEY FRIEDMAN AND ANDREJ SCEDROV, *The lack of definable witnesses and provably recursive functions in intuitionistic set theories*, ***Advances in Mathematics***, (1985), pp. 1–13.
[2] ANDRZEJ ZARACH, *Replacement $\not\to$ Collection*, ***Gödel '96*** (Petr Hájek, editor), Lecture Notes in Logic, Springer, no. 6, 1996, pp. 307–322.

► LARISA MAKSIMOVA, *On a form of interpolation in modal logics*.
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We define a new variant WIP of interpolation that is equivalent to some form WRP of Robinson's consistency property and find its algebraic equivalent.

Let $A(\mathbf{p})$ denote a formula whose all non-logical symbols are in $\mathbf{p}$, L a normal modal logic. In addition to the known forms CIP, IPD and IPR [2] of interpolation, we define the *weak interpolation property* WIP: If $A(\mathbf{p},\mathbf{q}), B(\mathbf{p},\mathbf{r}) \vdash_L \bot$, then there exists a formula $A'(\mathbf{p})$ such that $A(\mathbf{p},\mathbf{q}) \vdash_L A'(\mathbf{p})$ and $A'(\mathbf{p}), B(\mathbf{p},\mathbf{r}) \vdash_L \bot$.

It is easily proved that *CIP* $\Rightarrow$ *IPD* $\Rightarrow$ *IPR* $\Rightarrow$ *WIP*. The converse arrows are, in general, not valid in normal modal logics, although CIP is derivable from WIP in the classical predicate logic. In modal predicate logics CIP is equivalent to Robinson's consistency property [1]. We prove that WIP is equivalent to *Weak Robinson's Property* WRP:

Let T_1, T_2 be two open L-theories in the languages $\mathcal{L}_1, \mathcal{L}_2$ respectively, $\mathcal{L}_0 = \mathcal{L}_1 \cap \mathcal{L}_2$, $T_{i0} = T_i \cap \mathcal{L}_0$. If $T_{i1} \cup T_{i2}$ is strongly L-consistent, then $T_1 \cup T_2$ is strongly L-consistent.

An *open L-theory of the language* $\mathcal{L}$ is a set $\Gamma \subseteq \mathcal{L}$ closed with respect to $\vdash_L$; Γ is *strongly L-consistent* if $\Gamma \nvdash_L \bot$.

Now we find an algebraic equivalent of WIP. It is well known that there exists a duality between propositional normal modal logics and varieties of modal algebras. A logic L has CIP iff its corresponding variety $V(L)$ is super-amalgamable, IPD is equivalent to the amalgamation property AP, and IPR to the restricted amalgamation property RAP [2]. We define a property

PRAP. For each $\mathbf{A},\mathbf{B},\mathbf{C} \in V$ such that $\mathbf{A}$ is a common subalgebra of $\mathbf{B}$ and $\mathbf{C}$ there exist an algebra $\mathbf{D}$ in V and homomorphisms $\delta\colon \mathbf{B} \to \mathbf{D}$, $\varepsilon\colon \mathbf{C} \to \mathbf{D}$ such that $\delta(x) = \varepsilon(x)$ for all $x \in \mathbf{A}$, where $\mathbf{D}$ is non-degenerate whenever $\mathbf{A}$ is non-degenerate.

Theorem 1. *For any normal modal logic L the following are equivalent*:
(1) *L has WIP*, (2) *L has WRP*, (3) $V(L)$ *has PRAP*, (4) *the class of simple algebras in* $V(L)$ *has AP.*

Proposition 1. *Any normal modal logic containing* $S4.1$ *or GL has WIP.*

Proposition 2. *There are only three propositional modal logics over* $S5$ *with WIP. For any logic containing* $S5$, *CIP* $\iff$ *IPD* $\iff$ *IPR* $\iff$ *WIP.*

[1] D. M. Gabbay, ***Investigations to modal and tense logics with applications to problems in philosophy and linguistics***, D. Reidel Publishing Company, Dordrecht, 1976.

[2] L. Maksimova, *Restricted interpolation in modal logics*, ***Advances in modal logics***, vol. 4, King's College London Publications, London, 2003, pp. 297–312.

► JUAN CARLOS MARTÍNEZ, *On cardinal sequences of scattered Boolean spaces*.
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If X is a scattered Boolean space and α is an ordinal, we denote by $I_\alpha(X)$ the α^{th}-Cantor-Bendixson level of X. The *height of* X is defined by $\mathrm{ht}(X) =$ the least ordinal α such that $I_\alpha(X)$ is finite. Then, if $\tau = \langle \kappa_\alpha\colon \alpha < \eta \rangle$ is a sequence of infinite cardinals, we say that τ is the *cardinal sequence of* X, if $\mathrm{ht}(X) = \eta$ and $|I_\alpha(X)| = \kappa_\alpha$ for any $\alpha < \eta$.

It has been proved in [1] that if $\tau = \langle \kappa_\alpha\colon \alpha < \omega_1 \rangle$ is a sequence of infinite cardinals such that $\kappa_\alpha \leq 2^\omega$ for each $\alpha < \omega_1$, then there is a scattered Boolean space whose cardinal sequence is τ. However, no general consistency result is known for cardinal sequences of length greater than ω_1. In [2], it was shown that for every specific regular cardinal κ, it is consistent with ZFC that for every non-zero ordinal $\eta < \kappa^{++}$ there is a scattered Boolean space X such that $\mathrm{ht}(X) = \eta$ and $|I_\alpha(X)| = \kappa$ for any $\alpha < \eta$. Nevertheless, no consistency result is known for cardinal sequences $\langle \kappa_\alpha : \alpha < \eta \rangle$ where $|\eta|$ is a singular cardinal or the successor of a singular cardinal.

Then, we shall show in this talk that if GCH holds and $\tau = \langle \kappa_\alpha : \alpha < \eta \rangle$ is a sequence of infinite cardinals of any length such that $\kappa_\alpha \geq |\eta|$ for each $\alpha < \eta$, then there is a cardinal-preserving partial order that forces the existence of a scattered Boolean space whose cardinal sequence is τ.

[1] I. Juhász and W. Weiss, *Cardinal sequences*, ***Annals of Pure and Applied Logic***, to appear.

[2] J. C. Martínez, *A forcing construction of thin-tall Boolean algebras*, ***Fundamenta Mathematica***, vol. 159 (1999), pp. 99–113.

▶ HEIKE MILDENBERGER AND SAHARON SHELAH, *Specializing Aronszajn trees and preserving some weak diamonds*.

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This is a talk about [2]. We show that $(\mathbb{R}; \mathcal{N}; \in)$ together with "Aronszajn trees are special" is consistent relative to ZFC. The weak diamond for the covering relation of Lebesgue null sets was the only weak diamond in the Cichoń diagramme for relations whose consistency together with "Aronszajn trees are special" was not yet settled. Our forcing proof gives also new proofs to the known consistencies of several other weak diamonds stemming from the Cichoń diagramme together with "Aronszajn trees are special" and CH. The main part of our work is a strengthening of [3, Chapter V, §§1–7], by using a genericity game. We extend with the forcings from [1], for which we show new preservation properties.

[1] Uri Abraham and Saharon Shelah, *A Δ^2_2 well-order of the reals and incompactness of $L(Q^{MM})$*, ***Annals of Pure and Applied Logic***, vol. 59 (1993), pp. 1–32.

[2] Heike Mildenberger and Saharon Shelah, *Specializing Aronszajn trees and preserving some weak diamonds*, [MdSh:848], submitted, 2005.

[3] Saharon Shelah, ***Proper and improper forcing***, 2nd edition, Springer, 1998.

▶ GEORG MOSER AND RICHARD ZACH, *Complexity of elimination procedures in the epsilon calculus*.

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Hilbert's ε-calculus [1] is a powerful but complex logical formalism based on an operator allowing the formation of terms $\varepsilon_x\, A(x)$ ($A(x)$ any formula). The ε-operator is axiomatized by *critical formulas* $A(t) \rightarrow \varepsilon_x\, A(x)$. It extends the first-order predicate calculus in the sense that formulas and proofs with quantifiers can be translated into the ε-calculus by mapping $\exists x\, A(x)$ to $A(\varepsilon_x\, A(x))$. The First Epsilon Theorem, due to Paul Bernays [3, 4], establishes a fundamental result about the ε-calculus, viz., that ε-terms can be eliminated from proofs of ε-free formulas. The Extended First Epsilon Theorem also establishes a version of Herbrand's Theorem for existential formulas. We give an analysis of the ε-elimination procedure and establish bounds on the complexity of the resulting Herbrand disjunctions. It has been claimed [2] that the ε-calculus with equality is unsuitable for automated deduction since the length of Herbrand disjunctions in that case is not bounded by any function which depends on the size of the proof. We give an alternative formulation of the ε-calculus with equality for which the length of the Herbrand disjunction extracted from by the ε-elimination procedure can be so bounded.

[1] JEREMY AVIGAD AND RICHARD ZACH, *The epsilon calculus*, ***Stanford Encyclopedia of Philosophy***, `http://plato.stanford.edu/entries/epsilon-calculus/`, 2002.

[2] MATTHIAS BAAZ AND AALEXANDER LEITSCH, *Methods of function extensions*, ***Collegium Logicum—Annals of the Kurt Gödel Society***, vol. 1 (1995), pp. 87–122.

[3] PAUL BERNAYS, *Logical calculus*, unpublished typescript, Institute for Advanced Study, Princeton, 1935–36.

[4] DAVID HILBERT AND PAUL BERNAYS, ***Grundlagen der Mathematik***, vol. 2, Springer, Berlin, 1939.

► ANDREY NOVIKOV, *On decidability of a class of program invariants*.

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Consider a program in an object-oriented language, for distinctness C#. Let us assume that the methods of all classes defined and used by the program contain only the following constructs:

- `while, if, break,` {};
- assignments of variables with additive expressions on class members (and members of members, etc.) in the right part;
- method calls.

Let us consider any first-order formulas φ_1, φ_2 in the signature $\{+, =, >\}$ on class integer-typed members or members of object-typed class members, etc.

Denote by $\varphi_1 \to \bigcirc\varphi_2$ the fact that "for any initial state S if $S \models \varphi_1$ then in the next state S' holds $S' \models \varphi_2$.

By adapting the model from [3] and building the language semantics in terms of the adapted model (see [4] for details), we prove the following.

THEOREM 1. *For any φ_1, φ_2 as described above the formula $\varphi_1 \to \bigcirc\varphi_2$ is decidable.*

[1] Abstract State Machines academic web page, `http://www.eecs.umich.edu/gasm/`

[2] E. BOERGER, N. FRUHA, V. GERVASI, AND R. STAERK, *A high-level modular definition of the semantics of C#*, ***Theoretical Computer Science***, (2004), pp. 1–35.

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► MARCIN PETRYKOWSKI, *Examples and applications of weak generic types*.

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Assume G is a definable group in some first order structure M. We call a definable set $A \subseteq G$ weak generic if there exist a generic set B and a non-generic definable set C such that $A = B \setminus C$. A complete type p is weak generic if every formula from p defines a weak generic subset of G.

We give some interesting examples of weak generic types and show that their structure can be very complicated.

We also consider the possible ways of using weak generic types to explore model-theoretic properties of groups.

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The prototypical Lie-ring is $(\mathrm{End}(E), \circ - \circ')$, where E is an abelian group, $\mathrm{End}(E)$ is the abelian group of its endomorphisms, $\circ$ is composition, and $x \circ' y = y \circ x$. Then an arbitrary ring $(E, \cdot)$ is a Lie-ring if

$$x \cdot y + y \cdot x = 0,$$
$$(x \cdot y) \cdot z = x \cdot (y \cdot z) - y \cdot (x \cdot z).$$

The latter identity means that the map $u \mapsto (z \mapsto u \cdot z)$ is a homomorphism from $(E, \cdot)$ to $(\mathrm{End}(E), \circ - \circ')$. Commutative rings then have a parallel definition: $x \cdot y - y \cdot x = 0$, and $u \mapsto (z \mapsto u \cdot z)$ is a homomorphism from $(E, \cdot)$ to $(\mathrm{End}(E), \circ)$. Other rings can be defined in terms of different linear combinations $*$ of $\circ$ and $\circ'$, but they don't behave so nicely: we do not in general have that $u \mapsto (z \mapsto u * z)$ is a homomorphism from $(\mathrm{End}(E), *)$ to $(\mathrm{End}(\mathrm{End}(E)), *)$ unless $*$ is $\circ$ or $\circ - \circ'$ or trivial.

To what extent will the model-theory of Lie-rings parallel that of commutative rings (integral domains, fields)? The set of derivations of a commutative ring is naturally a Lie-ring. Hence, for example, the model-theory of differential fields gives rise to model-complete and ω-stable theories of (expansions of) Lie-rings.

► TOMASZ POŁACIK, *Quantified intuitionistic propositional logic on pseudo-trees and trees*.
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We investigate intuitionistic logic in the language of the standard propositional connectives—negation, disjunction, conjunction, implication—augmented with the propositional quantifiers. Our approach is semantical: we focus on the topological and Kripke-style semantics. In particular, we consider the so-called principal Kripke semantics for propositional quantification and identify the propositions with the upward-closed (or open in the natural topology of a Kripke frame) subsets of the frame and then let the quantifiers range over the set of *all* propositions. It is known that the set of all propositional second-order formulae which are valid in the class of all Kripke frames, is recursively isomorphic to full classical second-order predicate logic. So, looking for a more plausible interpretation of intuitionistic propositional quantification, we focus on the class of pseudo-trees, the class of trees, and the class of finite trees and consider the sets of all formulae which are valid in the classes in question; we also consider the full binary tree (Cantor space). In our investigation we tackle such issues as Explicit Definability, decidability, and axiomatizability. We show, among others, that the second-order propositional intuitionistic logic of the class of trees enjoys the Explicit Definability Property. We also provide a natural axiomatization of the class of rooted Kripke frames whose logic enjoys this property. Finally, we point to different properties of propositional quantification in the cases of the class of pseudo-trees, trees and finite trees and, as an important case, we study the logic of the full binary tree.

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We deal with the decidability of monadic second-order theory of the natural numbers expanded by unary predicates. Elgot and Rabin [2] found many interesting unary predicates P such that the monadic theory of $\langle \mathbb{N}, <, P \rangle$ is decidable. Their results and techniques have been extended over the years. In [1] a class of effectively profinitely ultimately periodic (e.p.u.p.) predicates was introduced. Many examples of e.p.u.p. predicates were provided, and it was shown that if P is an e.p.u.p. predicate then the monadic theory of $\langle \mathbb{N}, <, P \rangle$ is decidable. We show that this is a necessary condition.

THEOREM. *The monadic second-order theory of* $\langle \mathbb{N}, <, P \rangle$ *is decidable if and only if* P *is effectively profinitely ultimately periodic.*

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► GEMMA ROBLES, FRANCISCO SALTO, AND JOSÉ M. MÉNDEZ, *A constructive negation defined with a negation connective for logics including* Bp_+.
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The concept of constructive negation we refer to in this paper is (minimally) intuitionistic in character (see [1]). The idea is to understand the negation of a proposition A as equivalent to A implying a falsity constant of some sort. Then, negation is introduced either by means of this falsity constant or, as in this paper, by means of a propositional connective defined with the constant. But, unlike intuitionisitc logic, the type of negation we develop here is, of course, devoid of paradoxes of relevance.

The logic Bp_+ is the result of adding the prefixing axiom to the well-known Basic positive logic B_+ of Routley and Meyer. We have claimed in [2] and [3] that B_+ is too weak a positive logic to introduce the full reductio axioms. The aim of this paper is to prove that any logic including Bp_+ can be endowed with a (strong) constructive negation including the full reductio axioms.

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► N. M. RUBTSOVA, *Evidence-based knowledge for* $S5$.
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Logics of evidence-based knowledge was proposed by S. Artemov (see S. Artemov. Evidence-Based Common Knowledge. Technical Report TR-2004018, CUNY Ph.D. Program in Computer Science, 2005). It is multi-agent logic of knowledge enriched by evidence assertions of the form $t\colon \varphi$ where t is an *evidence term*. Knowledge of every agent is axiomatized by a modal logic L (L is T or $S4$ or $S5$), evidence-based knowledge operator is described by the logic of proofs LP (see S. Artemov. Explicit provability and constructive semantics. ***The Bulletin of Symbolic Logic***, vol. 7 (2001), no. 1, pp. 1–36). The resulting systems L_nLP are provided with Kripke-style semantics. The corresponding completeness theorems are proven. For $L = T, S4$ it is shown that forgetful projection which replace all evidences by a uniform modality J coincides with L_nS4. We introduce the multi-agent logic of evidence-based knowledge $S4_nLP(S5)$ in which the evidence component corresponds to $S5$-modality. In order to do this we extend the language of $S4_nLP$ by new unary operation of *negative checker* "?".

Axioms and rules of $S4_nLP(S5)$ are those of $S4_nLP$ plus a new axiom describing negative checker:

$$\neg[\![t]\!]\varphi \rightarrow [\![?t]\!]\neg[\![t]\!]\varphi.$$

For this logic we define Kripke-style models and prove the completeness theorem. A *model* is a structure $M = (W, R_1, \dots, R_n, R, \varepsilon, \Vdash)$ where $W \neq \emptyset$ is a set of states, R_i, R are reflexive transitive accessibility relations on W. An *evidence* function ε is a mapping from states and evidence terms to sets of formulas. It is monotonous (uRv implies $\varepsilon(u,t) \subseteq \varepsilon(v,t)$) and respects operations on evidence terms. $\Vdash$ is a validity relation between states and sentence variables. The validity of formulas is defined in the usual way with the only new clause: $u \Vdash [\![t]\!]\varphi$ iff $\varphi \in \varepsilon(u,t)$. We require that $\varphi \in \varepsilon(u,t)$ implies $v \Vdash \varphi$ for all $v \in W$ with uRv.

THEOREM. *$S4_nLP(S5) \vdash \varphi$ iff φ holds in all $S4_nLP(S5)$-models.*

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It is well-known that there is some hierarchy of the proof systems under the p-simulation relation, which determines the relative efficiency of proof systems for Classical Propositional Logic (CPL). But the natural, real conclusions in most cases have constructive character, therefore the investigation of the proofs complexities is important for the systems of Intuitionistic Propositional Logic (IPL) and in some cases also for Minimal (Johansson's) Propositional Logic (MPL). Note, that the analogies of some interesting systems of CPL are described for IPL recently, but they haven't analogies for MPL.

We constructed a hierarchy for the well-known systems of IPL (Frege, sequent, cut-free sequent, natural, resolution, cut-free sequent tree, resolution tree, Frege with substitution), as well as for analogous systems of MPL, some of which are described by us. It is natural, that the technique for obtaining of the upper and lower bounds of proofs complexities in CPL is not always applicable for the systems of IPL and MPL, but some modifications of them allow us to obtain the same hierarchy for the systems of non-classical logic as the hierarchy for the systems of CPL.

It is shown also, that some of the systems of CPL have the superpolinomial speed-up over the analogous systems of IPL and MPL.

For some "weak" systems of IPL and MPL the tautologies of the size n are described, the proofs complexities of which in these systems are by order $n, n^2, n^3, \dots, n^{[\frac{n}{2}\log_n 2]}$ (the similar result for CPL was obtained by An. Chubaryan earlier).

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Larisa Maksimova in [1], [2], [3] and [4] proved that exist only finitely many modal logics with Craig Interpolation Property (CIP) and Deducibility Interpolation Property (IPD) which contain the logic $S4$. In particular, there are exactly nine extensions of $S4.3$ with CIP and there are precisely thirteen extensions of $S4.3$ with IPD.

Due to these results in order to check up when modal logic which contains S4.3 have CIP it is enough to find out whether it will coincide with one of above-mentioned nine logics.

Similarly in order to check up when modal logic have IPD it is enough for us to find out whether it will coincide with one of above-mentioned thirteen logics.

Denote by $L + \{A_1, \dots, A_n\}$ the extension of logic L by extra axiom schemes $A_1, \dots, A_n$.

The algorithms of automatized recognition of the CIP (IPD) for logic in the form $S4.3 + \{A_1, \dots, A_n\}$ is obtained by modifying description from above-mentioned papers.

The author creates the program realizing above described algorithms.

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Σ^1_3-absoluteness for *ccc* forcing means that for any *ccc* forcing P, ${H_{\omega_1}}^V \prec_{\Sigma_2} {H_{\omega_1}}^{V^P}$. "$\omega_1$ inaccessible to reals" means that for any real r, ${\omega_1}^{L[r]} < \omega_1$. To measure the exact consistency strength of "Σ^1_3-absoluteness for *ccc* forcing and ω_1 is inaccessible to reals", we introduce a weak version of a weakly compact cardinal, namely, a (lightface) Σ^1_2-indescribable cardinal; It turns out that κ has this property exactly if it is inaccessible and $H_\kappa \prec_{\Sigma_2} H_{\kappa^+}$.

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It is well known that the Wadge game characterizes the set of continuous functions, in the sense that a function $f : {}^\omega\omega \to {}^\omega\omega$ is continuous if and only if there is a strategy for Player II in the Wadge game that induces f.

It was shown by Alessandro Andretta in 2003 that the set of first level Borel functions is characterized in the same way by the backtrack game. (A function is first level Borel if the preimage of every $\mathbf{\Sigma}^0_2$ set is $\mathbf{\Sigma}^0_2$).

We present the multitape game, which is a generalization of the Wadge and backtrack games. We show that the multitape game characterizes the set of second level Borel functions that have a countable decomposition. (A function is second level Borel if the preimage of every $\mathbf{\Sigma}^0_3$ set is $\mathbf{\Sigma}^0_3$. A second level Borel function f has a countable decomposition if there is a $\mathbf{\Pi}^0_2$ partition $\langle A_n : n \in \omega \rangle$ of ${}^\omega\omega$ such that $f \restriction A_n$ is first level Borel).

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The Degree spectrum $\mathrm{DS}(\mathfrak{A})$ of a countable structure $\mathfrak{A}$ is the set of all enumeration degrees generated by all enumerations of $\mathfrak{A}$. The Co-spectrum of the structure $\mathfrak{A}$ is the set of all enumeration degrees which are lower bounds of the $\mathrm{DS}(\mathfrak{A})$.

We present a generalization of the notions of Degree spectra and Co-spectra for countable many (finitely or infinitely) structures [2], [3]. Let ζ be a constructive ordinal, and $\mathfrak{A}_0, \mathfrak{A}_1, \dots \mathfrak{A}_\xi \dots,\ \xi \leq \zeta$ be a sequence of countble abstract structures. The Joint spectrum of the sequence $\{\mathfrak{A}_\xi\}_{\xi\leq\zeta}$ is the set $\mathrm{DS}(\{\mathfrak{A}_\xi\}_{\xi\leq\zeta})$ of all elements of $\mathrm{DS}(\mathfrak{A}_0)$, such that $\mathbf{a}^{(\xi)} \in \mathrm{DS}(\mathfrak{A}_\xi)$, for each $\xi \leq \zeta$.

The properties of the Degree spectra for a structure $\mathfrak{A}$ are presented in [1]. We prove an analogue of the Minimal Pair Theorem and an existence of Quasi-Minimal degree from [1] for the Joint spectrum of sequence of structures.

THEOREM 1. *For any sequence of structures* $\{\mathfrak{A}_\xi\}_{\xi\leq\zeta}$, *there exist enumeration degrees* $\mathbf{f}$ *and* $\mathbf{g}$ *in* $\mathrm{DS}(\{\mathfrak{A}_\xi\}_{\xi\leq\zeta})$, *such that for any enumeration degree* $\mathbf{a}$ *and* $\alpha < \zeta$:

$$\mathbf{a} \leq \mathbf{f}^{(\alpha)} \ \ \& \ \ \mathbf{a} \leq \mathbf{g}^{(\alpha)} \Rightarrow \mathbf{a} \in \mathrm{CS}^{\alpha}(\{\mathfrak{A}_\xi\}_{\xi\leq\zeta}).$$

THEOREM 2. *For any sequence of structures* $\{\mathfrak{A}_\xi\}_{\xi\leq\zeta}$ *there exists an enumeration degree* $\mathbf{q}$ *such that*:

1. $\mathbf{q}^{(\alpha)} \in \mathrm{DS}(\mathfrak{A}_\alpha)$, *for* $0 < \alpha < \zeta$ *and* $\mathbf{q} \notin \mathrm{CS}(\{\mathfrak{A}_\xi\}_{\xi\leq\zeta})$;
2. *If* $\mathbf{a}$ *is a total degree and* $\mathbf{a} \geq \mathbf{q}$, *then* $\mathbf{a} \in \mathrm{DS}(\{\mathfrak{A}_\xi\}_{\xi\leq\zeta})$;
3. *If* $\mathbf{a}$ *is a total degree and* $\mathbf{a} \leq \mathbf{q}$, *then* $\mathbf{a} \in \mathrm{CS}(\{\mathfrak{A}_\xi\}_{\xi\leq\zeta})$.

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► CLAES STRANNEGÅRD, *A proof system for modeling reasoning processes in propositional logic*.

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Logical problem-solving can be investigated in timed psychological experiments in which subjects are asked questions such as the following:

- Is the propositional sentence A valid?
- Is the propositional sentence A consistent?
- Is the propositional sentence A true in the model M?

In [1] and [2], experiments on the logical consequence relation were performed and then analyzed in terms of algorithms for reasoning in natural deduction systems.

Cognitive modeling using natural deduction is sometimes problematic, however, since the working memory content at each stage of the reasoning process is not being represented explicitly. The same holds, e.g., for sequent calculi and Hilbert systems.

A proof system for propositional logic is presented in the form of a transition system, whose states represent working memory content. These states are sets containing information of two kinds, called facts and goals. Proofs are modeled as computations in this transition system. This makes it relatively easy to model reasoning with limited cognitive resources. The proofs may be used for showing validity, consistency or truth/falsity and they may contain both data-driven (forward) and goal-driven (backward) reasoning steps. The main focus here is not on the particular system, but rather on the format for modeling logical reasoning processes.

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Zakharyaschev [1] introduced canonical formulas for intermediate logics and proved the Refutability Criterion and Completeness Theorem for intuitionistic canonical formulas.

Denote by *THN* the class of all proper paraconsistent extensions of minimal logic.

We define canonical formulas for logics in *THN*, give a necessary and sufficient condition for the refutability of such canonical formulas in general frames and prove the following

THEOREM. *There is an algorithm which, for any formula* Φ, *constructs canonical formulas* $J_1, \dots, J_n$ *such that*

$$L_j + \Phi = L_j + J_1 + \cdots + J_n.$$

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An algorithm based on a lemma of Ackermann for computing first-order equivalents of modal formulas is presented in [1]. A refined and implemented version of this algorithm is known under the name of DLS-algorithm. DLS works successfully for the classical Sahlqvist formulas (SF), but there are generalized SF, called complex SF (see [2]), for which DLS does not work successfully. An example of such a formula is the following one:

$$\Diamond\Box(p_1 \vee \neg p_2) \wedge \Diamond\Box(\neg p_1 \vee p_2) \wedge \Diamond\Box(p_1 \vee p_2) \Rightarrow \Box(p_1 \vee \neg p_2) \vee \Box^2(p_1 \Leftrightarrow p_2) \vee \Box^3(p_1 \wedge p_2)$$

In this work we consider a generalization of the Ackermann lemma from [1] in order to be applicable for complex Sahlqvist formulas. A special version of this generalization is for instance the following one.

Generalized Modal Ackermann Lemma. *Let* **B** *be a modal algebra,* $a_i \in B$, $i = 1, 2, 3, 4$ *and* $A(q_1, q_2, q_3)$ *be a modal formula considered as an expression over* **B** *with only positive occurrences of* q_1, q_2, q_3. *Consider the following system of equations with respect to* x_1, x_2:

$$(*) \quad \left\| \begin{array}{lll} a_1 & \leq & x_1 \vee x_2 \ = D_1 \\ a_2 & \leq & x_1 \vee \neg x_2 = D_2 \\ a_3 & \leq & \neg x_1 \vee x_2 = D_3 \\ a_4 & \leq & A(\neg D_1, \neg(D_1 \wedge D_2), \neg(D_1 \wedge D_2 \wedge D_3)). \end{array} \right.$$

Then $(*)$ *has a solution in* **B** *iff* $a_4 \leq A(\neg a_1, \neg(a_1 \wedge a_2), \neg(a_1 \wedge a_2 \wedge a_3))$.

The full version of the lemma is for arbitrary finite number of 2^n equations on the variables $x_1, \dots, x_n$, where D_i, $i = 1, \dots, 2^n - 1$ is a fixed sequence of different elementary disjunctions. The algebraic modal generalization of the original Ackermann lemma is for $n = 1$. Such an algebraic form of the Ackermann Lemma makes possible to obtain first-order equivalents of ordinary and complex Sahlqvist formulas without translating them into the second-order logic, and working directly in the modal algebras over the corresponding Kripke frames.

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Partial functions and "undefinedness" have been around in mathematics for a long time, without causing any trouble. It was only when mathematics and computer science met in projects on "automatization" of formal reasoning that some problems came up [3]. Where humans are able to avoid the application of a partial function on an argument outside "the domain" of the function, formalizing the rules for this activity seems to be less trivial. In [1] Farmer states that there does not exist a consensus on how partial functions should be mechanized and the developer of a mechanized mathematics system must choose among many different possible ways of representing and reasoning about partial functions. We want to add one more possibility by introducing "partially defined iota terms" of the form $\iota x_\psi(\varphi)$ which represents the unique x satisfying φ whenever the condition ψ is fulfilled. We present an extension of a two-valued first order sequent calculus for predicate logic with identity [2], where we are able to reason correctly about partially defined iota terms. A sequent is sound only if all terms appearing in it are defined. In this sense the sequent $\Gamma \vdash \iota x_\psi(\varphi) = z$ is sound only if $\Gamma \vdash \psi$.

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The chief purpose of this paper is to show that the theory of positive implication algebras (Rasiowa) alias Hilbert algebras (Diego) alias (in dual form) implicative models (Henkin) alias finitely deductive implication algebras (Vukomanovic) provides a complete characterization of intuitionistic implication, as defined on Heyting algebras. This is accomplished by showing that every positive implication algebra can be embedded into a dual symmetrical Heyting algebra (of implicative filters of the given algebra) dually preserving intuitionistic implication, thus yielding completeness. Moreover, such a symmetrical Heyting algebra is dually isomorphic to itself. The same applies, mutatis mutandis, to the theory of S4 implication algebras which provides a complete characterization of S4 implication as defined on finitely deductive lattices. That is, every S4 implication algebra can be embedded into a dual finitely deductive lattice dually preserving S4 implication.

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[6] ———, *Representation theorems for strict and non-strict deductive implicative algebras (Abstract)*, ***The Journal of Symbolic Logic***, vol. 52 (1987), pp. 347–348.

► MLADEN VUKOVIĆ, *Some notes on correspondence theory of the system IL*.
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The provability logic GL (Gödel-Löb) is a simple modal description of provability. The language of the provability logic contains propositional letters p_0, $p_1, \dots$, the logical connectives $\neg$, $\wedge$, $\vee$, $\rightarrow$ and $\leftrightarrow$, and the unary modal operator $\Box$. The axioms of system GL are all tautologies, $\Box(A \rightarrow B) \rightarrow (\Box A \rightarrow \Box B)$, $\Box(\Box A \rightarrow A) \rightarrow \Box A$. The system GL complete expresses provability predicate of Peano arithmetic. The interpretability logic IL is an extension of provability logic with a new modal binary operator $\rhd$. This operator stands for interpretability, considered as a relation between extensions of a fixed theory. The new axioms of the system IL are $\Box(A \rightarrow B) \rightarrow (A \rhd B)$, $((A \rhd B) \wedge (B \rhd C)) \rightarrow (A \rhd C)$, $((A \rhd C) \wedge (B \rhd C)) \rightarrow ((A \vee B) \rhd C)$, $(A \rhd B) \rightarrow (\Diamond A \rightarrow \Diamond B)$ and $\Diamond A \rhd A$. The paper [2] provides the necessary definitions and detailed explanation and gives several examples of interpretations. We are only interested in IL as a system of modal logic. The operator $\rhd$ is not an ordinary binary modal operator because its semantic is defined by:

$$w \Vdash A \rhd B \Leftrightarrow \forall v\,(wRv \ \ \& \ \ v \Vdash A \Rightarrow \exists V\,(vS_w V \ \ \& \ \ V \models B)).$$

We consider basic operations on generalized Veltman models as disjoint union, generated submodel, bounded morphism, and bisimulation. We show that the operations leave modal satisfaction unaffected, and that all of them are bisimulations. We prove Hennessy–Milner theorem for generalized Veltman semantic. We define standard translation of interpretability logic. Our aim is to consider correspondence theory for IL.

[1] P. BLACKBURN, M. DE RIJKE, AND Y. VENEMA, ***Modal Logic***, Cambridge University Press, 2001.

[2] A. VISSER, *An overview of interpretability logic*, ***Advances in modal logic, vol. 1. Selected papers from the 1st international workshop (AiML'96), Berlin, Germany, October 1996***, (Marcus Kracht et al., editors), CSLI Lecture Notes, CSLI Publications, Stanford, CA, vol. 87, 1998, pp. 307–359.

► VOJKAN VUKSANOVIC, *Canonical equivalence relations on* $[\mathbb{R}]^n$.
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We determine the set of canonical equivalence relations on $[2^{\mathbb{N}}]^n$ for the class of Baire equivalence relations and prove that the list of equivalences given in the paper "Canonical partition behaviour of Cantor spaces" is not complete. More precisely, we determine a finite list $\{\mathbb{E}_1, \dots, \mathbb{E}_{L_n}\}$ of equivalence relations on $[2^{\mathbb{N}}]^n$, such that for every Baire equivalence relation $\mathbb{E}$ on $[2^{\mathbb{N}}]^n$ there is a perfect set $X \subseteq 2^{\mathbb{N}}$ and there is an equivalence $\mathbb{E}_i$ from the list such that:

$$\mathbb{E} \restriction [X]^n = \mathbb{E}_i \restriction [X]^n.$$

► DAN E. WILLARD, *On two partial (and not full) respects where an axiom system can recognize its own consistency and multiplication as a total function*.
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Let $A(x, y, z)$ and $M(x, y, z)$ denote 3-way predicates indicating $x + y = z$ and $x * y = z$. An axiom system α will be said to **recognize** addition and multiplication as **Total Functions** iff it can prove:

$$\forall x\, \forall y\, \exists z\; A(x, y, z) \quad \text{AND} \quad \forall x\, \forall y\, \exists z\; M(x, y, z). \tag{1}$$

Pudlák [2] has strengthened the earlier work of Bezbourah-Shepherdson [1] to prove that there is no reasonable axiom system α that can simultaneously prove Equation (1) and prove its own Hilbert consistency. The approximate generalization of this incompleteness result for cut-free definitions of consistency, concerning semantic tableaux or Herbrand deduction, was derived by us in [4]. At the same time we showed in [3], [5] that there was a rich variety of axiom systems, called IS(A), IS-1(A) and IS-1*(A), that could verify their own cut-free consistency—where the said systems are able to recognize integer addition **BUT NOT** "integer multiplication" as a total function.

The chief question left unexplored by [3], [5] *was what pragmatic applications could the system IS(A) and its more general analogs plausibly possess?* We have found a useful application of these techniques in the context of a computer's floating point instruction set—where an ordered pair of integers (M, E), called the "mantissa" and "exponent", formally represents the real number $M \cdot 2^{E - \log M}$, and where addition, subtraction, multiplication and division among two real-numbers with mantissa-lengths of b_1 and b_2 will produce an output mantissa whose bit-length $= Max(b_1, b_2)$ (after the usual computerized rounding operations are performed). In a context where IS(A) and its cousins can prove conventional integer addition is a total function, they can each prove all these functions are total, as well as the operation Expand(M) that increases the mantissa's bit-length by one.

There is also a second sense in which axiom systems recognizing their cut-free consistency can *partially* (*and not fully*) conceptualize multiplication. Let Arith(s) indicate that s is a string formally encoding a term with addition and multiplication symbols applied to digital-style numbers, as illustrated by say "15 × (9 + (2 × 7))", Numb(t) specify t represents a digital number without any function symbols, such as say "345", and NumbArith(s, t) denote both these conditions hold and t represents the numeric value for s. Then IS(A) can prove $\forall s\, \exists t\, \{\text{Arith}(s) \rightarrow \text{NumbArith}(s, t)\}$, *although it cannot prove* Equation (1)'s statement: $\forall x\, \forall y\, \exists z\; M(x, y, z)$. This counter-intuitive result holds because t's string-length is bounded by the length of *s while z's length* can potentially equal *twice* Max(x, y)'s string-length.

[1] A. Bezboruah and J. Shepherdson, *Gödel's second incompleteness theorem for Q*, ***The Journal of Symbolic Logic***, vol. 41 (1976), pp. 503–512.

[2] P. Pudlák, *Cuts, consistency statements and interpretations*, ***The Journal of Symbolic Logic***, vol. 50 (1985), pp. 423–442.

[3] D. Willard, *Self-verifying systems, the incompleteness theorem and the tangibility reflection principle*, ***The Journal of Symbolic Logic***, vol. 66 (2001), pp. 536–596.

[4] ———, *How to extend the semantic tableaux and cut-free versions of the second incompleteness theorem almost to Robinson's Arithmetic Q*, ***The Journal of Symbolic Logic***, vol. 67 (2002), pp. 465–496.

[5] ———, *Some new exceptions for the semantic tableaux version of the Second Incompleteness Theorem*, ***Proceedings of the Tableaux 2002 Conference***, vol. 2381 of Lecture Notes in Artificial Intelligence, Springer-Verlag, pp. 281–297.

Abstracts presented by title

► SERGEI ARTEMOV AND NIKOLAI KRUPSKI, *On Reflexive Combinatory Logic*.
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The Reflective Combinatory Logic RCL [1] is an extension of the Typed Combinatory Logic $\mathsf{CL}_{\to}$ which admits typing judgments of the form "t is a term of type F" inside types via an additional type constructor $t\colon F$. Unlike the case of the Intuitionistic Type Theory [2] where a similar type constructor is also available, the type $t\colon F$ in RCL is inhabited by nontrivial terms that store a complete information about the typing judgment.

In the definition of RCL the following two judgments have been introduced by simultaneous induction: "F is a well-formed formula (or type)" and "F is derivable from $F_1, \dots, F_n$". The intended meaning of the latter is "type F is inhabited provided all types F_i are". In this talk we give an equivalent cut-free sequent formulation for RCL and establish complexity bounds for the decision problems concerning these judgements.

THEOREM 1. *The well-formedness property for* RCL *is decidable in polynomial time. The derivability in* RCL *is decidable and PSPACE-complete.*

[1] S. ARTEMOV, *Kolmogorov and Gödel's approach to intuitionistic logic: current developments*, ***Russian Mathematical Surveys***, vol. 59 (2004), no. 2, pp. 203–229.

[2] P. MARTIN-LÖF, *An intuitionistic theory of types*, ***Twenty-five years of Constructive Type Theory***, (G. Sambin and J. Smith, editors), Oxford Logic Guides, vol. 36, 1998, pp. 127–172.

► ANATOLY P. BELTIUKOV, *A polynomial programming language as a formal theory*.
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An arithmetic programming language (similar to Pascal) is proposed in which only all polynomial time computable functions can be programmed. All program objects in the language have strong types. The language can operate with arbitrary natural numbers with addition, non-negative subtraction, multiplication, integral division, natural numbers from bounded ranges, arrays, records, unions (records with variants), functions (defined by polynomial time algorithms), including functionals of higher orders, variables with pre bounded value ranges, assignments, branching, loops with pre computed number of steps, error processing. Function definitions do not allow recursions.

From the other side, this language can be considered as a formal ultraintuitionistic theory of the above mentioned objects. As it is usually happens in similar situations, the formulas of this theory are the value types in the programming language, and the proofs are the programs themselves. Negations are simulated by functions, that extract "error messages" from attempts to realize the negated formula. Equality formulas correspond to special data types that define a set of "permissible" modules that supply the desired number congruence. All proved formulas have polynomial time computable realizations.

In contrast with usual intuitionistic arithmetic this theory does not contain "double negated" classical Peano arithmetic statements as theorems. For example, it does not contradict to the hypothesis that exponent function (inverse to the easily defined logarithm) does not exist. Moreover, a realizational model with partial exponent function can be easily built for this theory.

► ADIB BEN JEBARA, *About space being not a continuum*.
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In "an idea about time in cosmology" in ASL Annual Meeting 2004 and in "About space and time of elementary particles" in ASL Annual Meeting 2005, we assumed that time and then space at the level of the elementary particles can be considered being U, applying the set theory ZFU, with urelements, Ui being a set of locations in the case of space with i a count of time. So, time and also space would be not linearly ordered.

About what is assumed as a particular case of the axiom of choice, see web page of Andreas Blass:
`http://www.math.lsa.umich.edu/~ablass/dpcc.pdf`

If no distance can be defined in $U \times U \times U$, that might lead to explain why things appear so strange with elementary particles and with the Big Bang. For instance, a particle would not need to go to a location in between two others it would go to.

As a set of paths, a cartesian product of sets of locations XU_i could be the void set, then, may be, the universe comes from no-space. For $U_1 \times U_2 \times U_3 \times \cdots$ would be equal to the void set with the number of elements of U_i being m, the number of particles in the universe, in case countable choice holds only for sets of 2 through $(m-1)$ elements. And no-space would be a void without even space itself and not a vacuum.

So, may be we can consider that the particles in the Big Bang come from no-space, instead of from a point, to deal with a question from Mr Andreas Blass about from where they come.

It could be that the involvement of no-space is possible only in the Big Bang. Or it could be that teleportation of elementary particles as well involves no-space.

► AYŞE BERKMAN, *Identification of simple generic L^*-groups of finite Morley rank*.
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A group of finite Morley rank is said to be of even type if its Sylow 2-subgroups are infinite and of bounded exponent. If every infinite simple proper definable section of such a group is either a simple algebraic group over an algebraically closed field, or a group with finite Sylow 2-subgroups, then we call it an L^*-group. Let p be an odd prime.

THEOREM 1. *Let G be a simple L^*-group of finite Morley rank and even type, S a 2-Sylow° subgroup of G, and D a maximal p-torus of Prüfer rank at least 3 in G which normalizes S. Then $N_G^\circ(S)$ is solvable, $O_2(C_G(x)) = 1$ for every element $x \in D$ of order p, and $G = \langle B(C_G(x)) \mid x \in D, |x| = p\rangle$.*

This technical result yields the following identification theorems.

THEOREM 2. *Let G be a simple L^*-group of finite Morley rank and even type, and S a 2-Sylow° subgroup of G. Assume at least one of the following holds.*

a. *There is a p-torus of Prüfer rank at least 3 in G normalizing S.*

b. *The size of $\mathcal{M}$ is at least 3, and for every $P_1, P_2 \in \mathcal{M}$, $O_2(\langle P_1, P_2\rangle) \neq 1$.*

Then G is isomorphic to a Chevalley group over an algebraically closed field of characteristic 2.

Above, $\mathcal{M}$ stands for the set of minimal 2-local° subgroups which contain $N_G^\circ(S)$ as a proper subgroup. A group P is called a 2-local° subgroup if $P = N_G^\circ(U)$ for some non-trivial definable connected 2-subgroup U.

This is joint work with Alexandre Borovik.

► JOHN CORCORAN, *Meanings of inference, deduction, and derivation*.
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The verbs 'infer', 'deduce', and 'derive' used in logic are ambiguous; logicians use each with multiple normal meanings. Several of their meanings are vague in that they admit borderline cases. This paper juxtaposes, distinguishes, and analyzes several senses of these three-place action verbs, focusing on a constellation of recommended senses. In the sense to be recommended, the verb 'infer' is used for the epistemic activity of [a person] judging a proposition to be true by determining that it is a consequence of given propositions known to be true. The verb 'deduce' is used for the epistemic activity of determining that a proposition is a consequence of given propositions. Aristotle discovered that the same process of deduction used inferentially to draw a conclusion from premises known to be true is also used non-inferentially to draw a conclusion from premises whose truth or falsity is unknown, or even from premises known to be false. Applying his grasp of this point in the first few pages of Prior Analytics, he distinguished demonstrative from non-demonstrative deductions. He wrote: "Every demonstration is a deduction but not every deduction is a demonstration." Inference, or demonstration, producing knowledge of the truth of its conclusion, and deduction, producing knowledge that its conclusion is logically implied by its premises, must both be distinguished from derivation, which consists in arriving at a string of characters by means of rule-governed manipulations starting with given strings of characters. Derivation by itself does not and cannot produce knowledge. No proposition can be inferred from a contradiction because no contradiction is known to be true. Every proposition can be deduced from a contradiction. And what can be derived from what depends upon which character-manipulating rules are allowed. This paper treats the relevant views of Aristotle, Boole, Frege, Lewis, Hilbert, Tarski, Church, and others.

► PIERRE JORAY, *Higher order quantifiers without ontological commitment*.
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Since Quine, quantification has been widely thought by philosophers to be the central logical tool for the access of a theory to its references. Restricting logicians practice, Quine was even able to arouse suspicion about any development of quantification theory outside the framework of a first order referential interpretation. Nevertheless, at its very begining, in Frege and also in Peirce, the use of quantifiers was seen as a very general tool, able to bind variables of any order.

In this talk, I will show that a referential interpretation is not incompatible with the idea to use in logic quantifiers of higher orders. I will present a general framework for the interpretation of quantifiers which permits to distinguish radicaly ontological questions from semantical or model-theoretical preoccupations. While preserving all classical deductive properties of quantifiers, this framework I call *categorial quantification* is neither referential, nor substitutional. It is essentially grounded on the notion of semantic category, which was introduced in logic by S. Leśniewski and developed by K. Ajdukiewicz. My central aim is to show that it is perfectly possible to formalise a theory on the ground of a higher order formal language, without being commited by the use of quantifiers to any ontological position concerning the nature of the valuable domains of interpretation.

► REINHARD KAHLE, *David Hilbert and the paradoxes*.
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100 years ago, in the summer term 1905, Hilbert gave a course on *Logical principles of mathematical thinking*. In this course he gave, for the first time, a detailed discussion of certain paradoxes, namely, (a variant of) *Richard's paradox*, his own—*Hilbert's paradox*, [2]—and *Zermelo-Russell's paradox*. Up to 1920 he repeated and refined the discussion of the paradoxes in several lectures on the foundations of mathematics.

In this talk, we summarize the development of Hilbert's treatment of the paradoxes based on the (unpublished) lecture notes of his lectures in Göttingen. Together with the early correspondence with Cantor and Frege, and some few traces in his publications, one gets a complete picture of the rôle of the (set-theoretic) paradoxes in Hilbert's interest in the foundations of mathematics.

[1] R. KAHLE, *David Hilbert über Paradoxien*, Pré-Publicações do Departamento de Matemática, Universidade de Coimbra, Preprint Number 06-17, 2006.

[2] V. PECKHAUS AND R. KAHLE, *Hilbert's paradox*, ***Historia Mathematica***, vol. 29 (2002), pp. 157–175.

► ROMAN KUZNETS, *Logic of Proofs as a measure of Hilbert-style proof complexity*.
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Logic of Proofs LP introduced by Artemov gave an exact intended semantics for Gödel's logic of provability S4 (see [1]). This Logic of Proofs considers statements of the form $t{:}F$, where a proof term t (called proof polynomial) denotes a proof for F. Logic of Proofs has a natural arithmetical semantics where $t{:}F$ is interpreted as a formal arithmetical statement "t is a proof of F in PA." We show that the smallest proof polynomial that proves F can serve as a measure of how long is the shortest Hilbert-style proof of $t{:}F$.

THEOREM 1. *Let t be the smallest term such that $\mathsf{LP} \vdash t{:}F$. Let $\mathcal{L}_1$ be the shortest Hilbert-style derivation of $t{:}F$ in* LP. *Then*

$$dag(t) \leq |\mathcal{L}_1| \leq 3|t|,$$

where $|t|$ is the size of the syntactic tree of term t, $dag(t)$ is the size of the syntactic direct acyclic graph (dag) of term t, and $|\mathcal{L}_1|$ is the number of formulas in derivation $\mathcal{L}_1$.

We further extend this result to Hilbert-style proofs of propositional tautologies:

THEOREM 2. *Let F be a propositional tautology. Let t be the smallest term such that $\mathsf{LP} \vdash t{:}F$. Let t' be the dag-smallest term such that $\mathsf{LP} \vdash t'{:}F$. Let $\mathcal{L}_2$ be the shortest Hilbert-style propositional derivation of F. Then*

$$dag(t') \leq |\mathcal{L}_2| = O(|t|).$$

[1] SERGEI N. ARTEMOV, *Explicit provability and constructive semantics*, ***The Bulletin of Symbolic Logic***, vol. 7 (2001), no. 1, pp. 1–36.

► PAOLO LIPPARINI, *m-permutable varieties satisfy $\alpha\beta_h = \alpha\gamma_h$*.
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Juxtaposition denotes intersection and * denotes transitive closure. If α, β, γ, δ are congruences on some algebra, define

$$\beta_0 = 0 \quad \gamma_0 = 0 \quad B_0(\delta) = \delta \quad G_0(\delta) = \delta$$
$$\beta_{n+1} = \beta + \alpha\gamma_n \quad \gamma_{n+1} = \gamma + \alpha\beta_n$$
$$B_{n+1}(\delta) = \beta \circ \alpha G_n(\delta) \circ \beta \quad G_{n+1}(\delta) = \gamma \circ \alpha B_n(\delta) \circ \gamma$$

THEOREM 1. (i) *Every m-permutable variety satisfies both $\alpha B_m(\delta) = \alpha G_m(\delta)$ and $\alpha\beta_h = \alpha\gamma_h$, with $h = m[\frac{m+1}{2}]$, where [ ] denotes integer part.*

(ii) *If a variety $\mathcal{V}$ satisfies $\alpha B_m \subseteq (\alpha G_m)^*$ for some m then $\mathcal{V}$ satisfies $\alpha\beta_h = \alpha\gamma_h$ for some h.*

[1] P. LIPPARINI, *n-permutable varieties satisfy non-trivial congruence identities*, ***Algebra Universalis***, vol. 33 (1995), pp. 159–168.

► JOÃO MARCOS, *Generalizing truth-functionality*.
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A common standard for the interpretation of classical propositional logic is set by the functionally complete collection of 2-valued truth-tables. The structure of the free algebra of formulas is faithfully mirrored in the semantics: In each admissible model, each sentential letter freely ranges over the set of truth-values, and to each n-ary logical constant there corresponds a convenient n-ary operator over those same truth-values. The whole approach is easily generalizable so as to define the class of *truth-functional logics*, i.e., *many-valued* logics with *matrix semantics*, where operators are characterized by truth-tables over some fixed set of truth-values of an arbitrary cardinality.

A further interesting generalization of the above idea is produced by the so-called *non-deterministic matrix semantics* (Nmatrices), where each n-ary operator is allowed to choose in between a number of possible outputs for each given n-uple of inputs. The set of admissible models may thus be enlarged, allowing for the natural adequate interpretation of more generous classes of non-classical logics. Yet another generalization of the same idea is given by the so-called *possible-translations semantics with many-valued ingredients* (Many-valued PTS), where each model of a logic $\mathcal{L}$ is given by an admissible translation of $\mathcal{L}$ into an appropriate many-valued logic $\mathcal{L}_k$ coupled with a standard many-valued valuation from $\mathcal{L}_k$.

For the finite-valued case, matrices and Nmatrices share interesting meta-properties such as *compacity* and *decidability*. Finite-valued PTS are also guaranteed to share those properties as soon as all the corresponding admissible translations are recursively defined. However, several logics known to be uncharacterizable by finite-valued matrices can be adequately characterized by finite-valued Nmatrices, and several logics that have no finite-valued Nmatrix characterization can be characterized by finite-valued PTS.

The present contribution will examine the multiple relations between the three above mentioned alternative styles of semantics.

► ELENA NOGINA, *On logic of proofs and provability*.
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Joint logics of proofs and provability were studied in [1] and [5]. The latter found an arithmetically complete logic which contained both the provability logic GL ([4]) and the logic of proofs LP ([2]) in their joint language augmented by some additional operations. A recent application in the logic of knowledge [3] prompted a question of a complete axiomatization of the logic of proofs and provability in the original language of GL and LP.

In the current note we find such an axiomatization, GLA, consisting of axioms and rules of GL and LP together with axioms $t\colon P \rightarrow \Box P$, $\neg t\colon P \rightarrow \Box\neg t\colon P$, $t\colon \Box P \rightarrow P$, and the reflection rule $\vdash \Box P \Rightarrow \vdash P$. GLA has been supplied with a corresponding Kripke-style semantics and shown to be complete with respect to the intended arithmetical provability interpretation.

[1] S. Artemov, *Logic of proofs*, ***Annals of Pure and Applied Logic***, vol. 67 (1994), no. 1, pp. 29–59.

[2] ———, *Explicit provability and constructive semantics*, ***The Bulletin of Symbolic Logic***, vol. 7 (2001), no. 1, pp. 1–36.

[3] S. Artemov and E. Nogina, *Basic epistemic logics with justifications*, CUNY Ph.D. Program in Computer Science, 2005, TR-2005004.

[4] G. Boolos, ***The logic of provability***, Cambridge University Press, Cambridge, 1993.

[5] T. Yavorskaya (Sidon), *Logic of proofs and provability*, ***Annals of Pure and Applied Logic***, vol. 113 (2002), nos. 1–3, pp. 345–372.

► ZORAN OGNJANOVIĆ, ZORAN MARKOVIĆ, AND MIODRAG RAŠKOVIĆ, *A logic with imprecise and conditional probabilities*.
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In this paper we present a propositional probability logic which allows making formulas that speak about imprecise and conditional probabilities. A class of Kripke-like probabilistic models is defined to give semantics to probabilistic formulas. Every possible world of such a model is equipped with a probability space. The corresponding probabilities may have nonstandard values. The proposition "the probability is close to r" means that there is an infinitesimal ϵ, such that the probability is equal to $r - \epsilon$ (or $r + \epsilon$). We provide an infinitary axiomatization and prove the corresponding extended completeness theorem.

► ISABEL OITAVEM, *Tree recursion and Pspace*.
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In this talk we discuss recursion schemes over free algebras and we give an implicit characterization of Pspace—the class of functions computable in polynomial space, by deterministic Turing machines. The presence of pointers in the recursion on notation scheme is the main difference between this and the well-known Bellantoni-Cook characterization of the polytime functions—Ptime.

[1] , S. Bellantoni and S. Cook, *A new recursion-theoretic characterization of Polytime functions*, ***Computational Complexity***, vol. 2 (1992), pp. 97–110.

[2] , D. Leivant and J. Marion, *Ramified recurrence and computational complexity II: substitution and poly-space*, ***8th Proceedings of CSL***, Lecture Notes in Computer Science, vol. 993, Springer, 1994, pp. 486–500,

► MAURICIO OSORIO, *Computing safe beliefs*.
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The following notion is very important in Non-monotonic reasoning. We assume the 3-valued Gödel logic G3 in our presentation. A safe belief complete theory of a finite propositional theory T is a complete theory T' obtained from T after adding only negated formulas. From T, we are interested in the set of literals that can be proved in some such extension T' from T. Cal such sets, safe beliefs of T. We present two related algorithms to compute set beliefs from a given theory. The first one is based on a kind of the well known Davis-Putnam (that computes models in classical proposional logic). It however, adds either $-a$ or $--a$ in every recursive call. When the theory becomes positive, it must prove the conjunction off all the atoms that occur in it. Here, we can however switch to classical logic to verify the desired result. We prove the correcteness of our algorithm. Our second algorithm works as follows: First translated our theory to a normal form and then we apply the first algorithm. The advantage of our second is that many pruning strategies are known to this restricted class of theories. However the size of the resulting theory is much greater than the original one. Our normal forms are based on what we call free formulas. A free formula is a formula of the form: $X \to Y$, where X is a conjunction of literals and Y is a disjunction of literals. The translation is supported by a set of tautologies of G3 Logic, some of them are the following:

$$[((a \to b) \ \& \ y) \to x] \leftrightarrow [(((-a \mid b) \ \& \ y) \to x) \ \& \ (y \to (x \mid a \vee -b)]$$

$$[(a \to b) \to x] \leftrightarrow [((-a \mid b) \to x) \ \& \ (x \mid a \mid -b)]$$

$$[y \to (a \mid (c \to x))] \leftrightarrow [((c \ \& \ y) \to (x \mid a)) \ \& \ (y \to (-c \mid -x \mid a))]$$
$$[z \leftrightarrow (a \to b)] \leftrightarrow [((z \ \& \ a) \to b) \ \& \ ((-a \mid b) \to z) \ \& \ (z \mid a \mid -b)].$$

[1] MAURICIO OSORIO, JUAN ANTONIO NAVARRO PÉREZ, AND JOSÉ ARRAZOLA, *Safe beliefs for propositional theories*, accepted in ***Annals of Pure and Applied Logic***.

► RAFFAELE PISANO, *Mathematics of logics or Logic of mathematics. Critical problems in the History of Science*.
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In the past Logic had glorious years, even in medieval times, during which it was part of the famous Quadrivium of teaching. It historically developed its solidity and identity by *shouldering* the development of those Sciences such as Mathematics, Physics and Chemistry; even though its methodological process has been above all influenced by the first of these. However the strong historical and cultural reasons which confirm the importance of this Science are contrasted by the difficulty of its fundaments, above all, Mathematics of Logic. *How can we overcome something that seems to be an aporia?* In this paper, which begins with a reflection on the ambiguity found both in natural and scientific language, I investigate a particular aspect of Logics: negative and the double negative sentences in ancient scientific books. Such an aspect would produce in fundaments of Science, still more than the validity of the *Tertium non datur* law, the valid distinction between classical Logic and the non-classical forms. Henceforth, I shall attempt to show that non-classical Logic may assume, besides its intellectually based role on which a theory is built, also a sharp category of historical interpretation (i.e., Sadi Carnot's Thermodynamics), from which it is possible to understand what the author thought even implicitly, so it is possible to elevate his fundamental hypotheses beyond his written work.

[1] A. DRAGO AND R. PISANO, *S. Carnot's* Réflexions: *a theory based on non-classical Logic*, ***The Bulletin of Symbolic Logic***, vol. 8 (2002), pp. 130–131.
[2] ———, *Interpretation and reconstruction of S. Carnot's* Réflexions *trough original sentences belonging to non-classical logic*, ***Fond. Ronchi Journal***, LIX, vol. 5 (2004), pp. 615–644.

► WIM RUITENBURG, *Models of basic arithmetic*.
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Basic Predicate Calculus BQC is the predicate logic parallel to Albert Visser's Basic Propositional Calculus BPC, a sublogic of Intuitionistic Predicate Calculus IQC. Basic Arithmetic BA is the BQC equivalent of Heyting Arithmetic over IQC, and of Peano Arithmetic over Classical Predicate Calculus CQC. We study some elementary Kripke model theory of BA.

[1] W. RUITENBURG, *Basic predicate calculus*, ***Notre Dame Journal of Formal Logic***, vol. 39 (1998), no. 1, pp. 18–46.
[2] A. VISSER, *A propositional logic with explicit fixed points*, ***Studia Logica***, vol. 40 (1981), pp. 155–175.

► SAEED SALEHI, *Variants of Kleene's Recursive Realizability*.
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The notion of Recursive Realizability introduced by Kleene is a useful tool for measuring the strength of intuitionistic arithmetics. Various generalizations of this realizability has been proposed by restricting the class of recursive functions to a proper sub-class. As such examples one could mention Damnjanovic's or López-Escobar's primitive recursive realizabilities or Plisko's Σ_n–realizability.

In this talk, we present definitions and basic properties of two realizabilities introduced by the author, namely that of realizability by primitive recursive functions [2] and realizability by polynomially bounded recursive functions [3]. Applying these to Ruitenburg's Basic Arithmetic (a sub-system of Heyting Arithmetic [1]) yields interesting results about its provably total functions. Applicability of these realizabilities to Basic Arithmetic results from the observations that

(1) the Gödel codes of primitive recursive and polynomially bounded recursive functions are arithmetically definable, and

(2) the S-m-n functions in Recursion Theory can be chosen from the class of primitive recursive or polynomially bounded recursive functions.

[1] W. Ruitenburg, *Basic predicate calculus*, ***Notre Dame Journal of Formal Logic***, vol. 39 (1998), no. 1, pp. 18–46.

[2] S. Salehi, *Provably total functions of basic arithmetic*, ***Mathematical Logic Quarterly***, vol. 49 (2003), no. 3, pp. 316–322.

[3] ———, *Polynomially bounded recursive realizability*, to appear in ***Notre Dame Journal of Formal Logic***.

► DARIUSZ SUROWIK, *Towards independency of axioms of some intuitionistic tense logic system*.

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In the paper [1] W. B. Ewald constructs the minimal intuitionistic tense logic system and call it $IK_{t.}$. He adds the temporal operators G, H, F and P to the connectives of propositional intuitionistic calculus and offers a Hilbert axiom system for the $IK_{t.}$

The Ewald's axiomatization of $IK_{t.}$ is the following:

(A1) All axioms of the intuitionistic propositional calculus

(A2)	$G(\alpha \Rightarrow \beta) \Rightarrow (G\alpha \Rightarrow G\beta)$	(A2′)	$H(\alpha \Rightarrow \beta) \Rightarrow (H\alpha \Rightarrow H\beta)$
(A3)	$G(\alpha \wedge \beta) \Leftrightarrow (G\alpha \wedge G\beta)$	(A3′)	$H(\alpha \wedge \beta) \Leftrightarrow (H\alpha \wedge H\beta)$
(A4)	$F(\alpha \vee \beta) \Leftrightarrow (F\alpha \vee F\beta)$	(A4′)	$P(\alpha \vee \beta) \Leftrightarrow (P\alpha \vee P\beta)$
(A5)	$G(\alpha \Rightarrow \beta) \Rightarrow (F\alpha \Rightarrow F\beta)$	(A5′)	$H(\alpha \Rightarrow \beta) \Rightarrow (P\alpha \Rightarrow P\beta)$
(A6)	$(G\alpha \wedge F\beta) \Rightarrow F(\alpha \wedge \beta)$	(A6′)	$(H\alpha \wedge P\beta) \Rightarrow P(\alpha \wedge \beta)$
(A7)	$G\neg\alpha \Rightarrow \neg F\alpha$	(A7′)	$H\neg\alpha \Rightarrow \neg P\alpha$
(A8)	$FH\alpha \Rightarrow \alpha$	(A8′)	$PG\alpha \Rightarrow \alpha$
(A9)	$\alpha \Rightarrow GP\alpha$	(A9′)	$\alpha \Rightarrow HF\alpha$
(A10)	$(F\alpha \Rightarrow G\beta) \Rightarrow G(\alpha \Rightarrow \beta)$	(A10′)	$(P\alpha \Rightarrow H\beta) \Rightarrow H(\alpha \Rightarrow \beta)$
(A11)	$F(\alpha \Rightarrow \beta) \Rightarrow (G\alpha \Rightarrow F\beta)$	(A11′)	$P(\alpha \Rightarrow \beta) \Rightarrow (H\alpha \Rightarrow P\beta)$

Rules of inference:

$$MP: \frac{\alpha \Rightarrow \beta, \alpha}{\beta} \qquad RH: \frac{\alpha}{H\alpha} \qquad RG: \frac{\alpha}{G\alpha}$$

In the talk I will proove, that there are several dependent axioms in the set of axioms proposed by W. B. Ewald. Moreover, I will propose a new axiomatization of the IK_t system. The new axiomatization is equivalent to the Ewald's axiomatization.

[1] W. B. Ewald, *Intuitionistic tense and modal logic*, ***The Journal of Symbolic Logic***, vol. 51 (1986), no. 1.

► ALDO URSINI, *Linear reasoning inside classical logic*.
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Linear Logic (LL) (see for instance [2]) is interpretable inside classical logic enriched with a constant 1 and a binary (modal) operator $\langle A, B\rangle$. We call MK the (sequent calculus presentation) of this binary modal logic. The idea behind the interpretation I is a follows: atomic formulas are the same in both languages, linear negation $A^{\perp}$ is defined as

$$\neg\langle A, 1\rangle$$

and linear implication $A \multimap B$ is defined as

$$\neg\langle A, B^{\perp}\rangle.$$

Thus formulas of LL are also formulas of MK. Intuitively, $\langle A, B\rangle$ can be read as "A, B are linearly compatible".

THEOREM 1. *There is a procedure which from any proof* π *in* LL *of a sequent* $\Gamma \vdash \Delta$ *of* LL *produces a proof* π^* *in* MK *of the sequent* $\mathbf{1} \vdash (\otimes\Gamma) \multimap (\wp\Delta)$.

THEOREM 2. *A sequent* $\Gamma \vdash \Delta$ *of* LL *is provable in* LL *iff* $\mathbf{1} \vdash (\otimes\Gamma) \multimap (\wp\Delta)$ *is provable in* MK.

There is also a complete Kripke-style semantics for MK. This logic has some relations to a fragment of the so-called Ambient Logic introduced in theoretical computer science by L. Cardelli and others (see for instance [1]).

[1] L. CARDELLI AND A. D. GORDON, *Mobile ambients*, ***Foundations of Software Science and Computational Structure***, (M. Nivat, editor), Lecture Notes in Computer Science, Springer, vol. 1378, 1998, pp. 140–145.

[2] J.-Y. GIRARD, *Linear Logic: its syntax and semantics*, ***Advances in Linear Logic***, (J-Y. Girard et al., editors), London Mathematical Society, Lecture Notes, vol. 222, Cambridge, 1995.

► LUIS A. URTUBEY, *Reasoning about contextual knowledge in multi-agent systems*.
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I investigate an extension of the formalism of interpreted systems by Halpern et al. to model the notion of contextual knowledge in a multi-agent system. Borrowing some ideas of a paper by Lomuscio and Sergot, I apply their semantical deontic model, which allows for the representation and reasoning about states of correct and incorrect functioning behavior of agents, to get a formalization of contextual reasoning. This formal framework introduces an operator representing the knowledge that an agent would have if it operates under the assumption that a group of agents share a context c.

The basic idea of interpreted systems is to describe a distributed computing system by specifying the states in which every agent and the environment can find itself. In this setting, the level of abstraction at which one chooses to operate is left open. If needed, logics similar to the ones used in philosophical logic can be defined on these semantics. One obvious advantage is the possibility of studying the logical characterization of systems defined semantically.

In this contribution I dare the following exercise. Starting from the basic notion of interpreted system and its adaptation to cope with deontic systems, I show how it can be arranged to provide a basic grounded formalism for contextual reasoning.

In particular, the allowed/disallowed labeling of states could be used to model a distinction that renders possible to pick out the normal states from the abnormal ones, which do not fit into the assumed context.

Some issues on the axiomatization of the resulting system and relations with alternative notions of common knowledge are explored.

[1] R. FAGIN, J. HALPERN, Y. MOSES, AND M. VARDI, ***Reasoning about knowledge***, MIT Press, 1995.

[2] A. LOMUSCIO AND M. SERGOT, *Deontic interpreted systems*, ***Studia Logica***, vol. 75, (2003).

► XUNWEI ZHOU AND HONG BAO, *Second level hypothetical inference*.
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The mutually-inversistic logic has been constructed by the authors. In the second level calculus of mutually-inversistic logic there is second level hypothetical inference. First level hypothetical inference is proposed by Socrates: From the major premise "all men are mortal" and the minor premise "Socrates is a man" to infer the conclusion "Socrates is mortal". An example of second level hypothetical inference is from the major premise "if all P are Q, then all not Q are not P (contrapositive law)" and the minor premise "all men are mortal" to infer the conclusion "those who are not mortal are not men". In the first level hypothetical inference "all men are mortal" is the major premise, while in the second level hypothetical inference it becomes the minor premise, the major premise is a logical theorem. Taking "all integers are rationals, all rationals are real numbers, therefore all integers are real numbers" as another example. The inference of this example made by mutually-inversistic logic is different from that made by classical logic. In mutually-inversistic logic, the inference goes like this (the second level calculus of mutually-inversistic logic is quantifier-free and $\leq^{-1}$ denotes mutually inverse implication thereinafter):

(1) $\mathrm{int}(x) \leq^{-1} \mathrm{rat}(x)$ P

(2) $\mathrm{rat}(x) \leq^{-1} \mathrm{real}(x)$ P

(3) $\{\mathrm{int}(x) \leq^{-1} \mathrm{rat}(x)\} \wedge \{\mathrm{rat}(x) \leq^{-1} \mathrm{real}(x)\}$ T, (1), (2), conjunction rule

(4) $\{P \leq^{-1} Q\} \wedge \{Q \leq^{-1} R\} \leq^{-1} \{P \leq^{-1} R\}$ P

(5) $\mathrm{int}(x) \leq^{-1} \mathrm{real}(x)$ T, (3), (4), second level hypothetical inference

The inference of the same example made in classical logic is well known, so omitted here. The biggest advantage of second level hypothetical inference over classical logic is that with second level hypothetical inference theorem proving becomes fully automatic. Second level hypothetical inference is very useful in computer science and artificial intelligence. Based on it, the authors have established second level Prolog, second level expert systems, second level relational database, theory of automatic proving of partial correctness of structured programs, formal semantics of second level Prolog, second level machine learning, second level semantic network, mutually-inversistic robot planning, and mutually-inversistic formal language.