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2005 ANNUAL MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

Stanford University, Stanford, CA
March 19–22, 2005

The Annual Meeting of the Association was held on the campus of Stanford University, March 19–22, 2005. The program included the Sixteenth Annual Gödel Lecture, a Retiring Presidential Address, one tutorial, and ten one-hour invited talks. The program also featured four special sessions, and seventeen contributed talks. The members of the Program Committee were John Mitchell, Michael Rathjen, Stewart Shapiro, Reed Solomon, Patrick Speisseger, and John Steel (Chair). The members of the Local Organizing Committee were Andrew Arana, Solomon Feferman, Grigori Mints, John Mitchell, and Richard Sommer (Chair). There were 125 registered participants at the meeting, including 42 graduate students. Generous financial support was provided by the Departments of Computer Science, Mathematics, and Philosophy at Stanford University, the Provost of Stanford University, and the Center for the Study of Language and Information. The ASL hosted a conference reception.

The Sixteenth Annual Gödel Lecture, *Skolem-Lowenheim theorems for generalized logics*, was delivered by Menachem Magidor (Hebrew University of Jerusalem). The tutorial, consisting of 3 one-hour lectures and titled *Automated verification = graphs, automata, and logic*, was presented by Moshe Vardi (Rice University). Richard Shore (Cornell University) gave the Retiring Presidential Address, *Degree structures: local and global investigations*. The other hour-long invited addresses were:

Steve Awodey (Carnegie Mellon University), *Algebraic set theory*.
Raf Cluckers (Katholieke Universiteit Leuven), *New directions for real integration and the Fourier transforms of subanalytic integrable functions*.
Ilijas Farah (York University), *Absoluteness and the uncountable*.
Solomon Feferman (Stanford University), *Open-ended schematic axiom systems*.
Deirdre Haskell (McMaster University), *Valued fields and elimination of imaginaries*.
Andrey Morozov (Sobolev Institute of Mathematics and Novosibirsk University), *On elementary submodels of F-parametrizable models*.
Vladimir Pestov (University of Ottawa), *On the groups of automorphisms of various structures: their geometry, dynamics, and combinatorics*.
Andre Scedrov (University of Pennsylvania), *Logic in computer security*.
Andreas Weiermann (Utrecht University), *Phase transition and universality for independence results*.
Jindrich Zapletal (University of Florida), *Potential theory and forcing*.

There were four special sessions (with the organizers given in parentheses): Computability and Randomness (Andre Nies), Model Theory (Chris Miller and Patrick Speisseger), Proof Theory and Constructivity (Solomon Feferman and Grigori Mints), and Set Theory and Borel Structure (Greg Hjorth and Slawomir Solecki).
These included a total of 38 speakers, each of whom gave a 25-minute talk.

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Abstracts of the invited talks and the contributed talks (given in person or by title) by members of the Association for Symbolic Logic follow.

For the Program Committee
JOHN STEEL

Abstract of invited Retiring Presidential Address

► RICHARD A. SHORE, *Degree structures: local and global investigations*.
Department of Mathematics, Cornell University, Malott Hall, Ithaca, NY 14853, USA.
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We survey the investigations of structures of relative computability with an emphasis on Turing reducibility.

Abstracts of invited talks

► STEVE AWODEY, *Algebraic set theory*.
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Since its introduction by Joyal and Moerdijk in [1], the algebraic approach to set theory has proven to be a flexible and powerful tool for constructing models of various elementary intuitionistic and classical set theories. The method has been further advanced through several recent collaborations (cf. [2]), the results of which are summarized in this talk. They include a model consisting of "ideals" over a logical category, which are certain sheaves that are much like schemes or manifolds whose "affine" or "euclidean" pieces are logically definable sets. A logical completeness theorem for set theory with respect to ideal completions of type theories is also given.

[1] A. JOYAL AND I. MOERDIJK, *Algebraic set theory*, ***London Mathematical Society***, Lecture Note Series 220, Cambridge University Press, 1995.

[2] www.phil.cmu.edu/projects/ast

► RAF CLUCKERS, *New directions for real integration and the Fourier transforms of subanalytic integrable functions*.
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Rolin, Comte and Lion studied parameterized integrals of real subanalytic functions. Call a function on a given space constructible if it is a product of subanalytic functions and the logarithms of subanalytic functions. Rolin, Comte, and Lion found that a parameterized integral of a constructible function is a constructible function in the remaining variables. Of course one assumes here an integrability condition, uniform in the parameters. We pursue this research further, by studying Fourier transforms of constructible functions. If we allow for products of constructible functions with a reasonable family of transcendental functions, we find a framework which is stable under taking Fourier transforms or under taking other similar parameterized integrals. Again, we take a natural integrability condition. These results are obtained by applying the o-minimal cell decomposition and the preparation theorem of Lion and Rolin for subanalytic functions to the integrand, after a suitable change of variables. There are some interesting corollaries on the decay of Fourier transforms: they go to zero, on infinity, more rapidly than given by general Fourier theory. This is interesting for mathematical physicists, who often want a quick decay.

We present joint work with M. Aschenbrenner and J.-P. Rolin.

The author is a postdoctoral fellow of the Fund for Scientific Research — Flanders (Belgium) (F.W.O.).

► ILIJAS FARAH, *Absoluteness and the uncountable*.
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Let $L_{\omega_1\omega}(Q)$ be the extension of the first-order predicate logic obtained by adding the quantifier Qx, 'there exists uncountably many' and allowing countable conjunctions with finitely many free variables. A consequence of Keisler's completeness theorem is that for every sentence ϕ of $L_{\omega_1\omega}(Q)$ the statement 'ϕ has a standard model' is absolute between transitive models of ZFC. (A model is standard if the quantifier Qx is interpreted as 'there exists uncountably many.') For a pointclass Γ let $L^{\Gamma}_{\omega_1\omega}(Q)$ be obtained from $L_{\omega_1\omega}(Q)$ by adding predicates for sets in Γ. A model for ϕ in this logic is *correct* if it correctly interprets the predicates for sets in Γ. By Keisler's theorem, if Γ=Borel then this logic retains the absoluteness property. For larger pointclasses large cardinals imply absoluteness for forcing extensions; some large cardinal assumption is necessary already when $\Gamma = \mathbf{\Pi}^1_1$.

I am also going to discuss some applications.

This is a joint work with Hrušák, Martinez Ramero, and Larson.

► SOLOMON FEFERMAN, *Open-ended schematic axiom systems*.
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An essential part of the concept of a formal system that is currently entrenched in logical work is that its language is fixed once and for all. This forces axiom schemata that may be used in such systems to be infinite bundles of all possible substitution instances by formulas from that language. By a schematic axiom system is meant one whose schemata are finitely specified by means of predicate variables (thus putting the 'form' back into 'formal systems'). In addition, the language of such a system is considered to be open-ended, in the sense that its basic vocabulary may be expanded to any wider conceptual context in which its notions and axioms may be appropriately applied. The point is that each such expansion permits new substitution instances for its formal schemata. Open-ended schematic axiom systems were originally introduced by the author to explain the concept of their unfolding. Following a review of the results obtained in collaboration with Thomas Strahm determining the unfolding of basic systems for both finitist and non-finitist arithmetic, the main part of the talk will be devoted to an exploration of the use of open-ended schematic systems for a more faithful account of mathematical practice than is provided by formal systems in the current standard sense. The background framework for these applications is a subject-neutral general theory of operations and predicates.

► DEIRDRE HASKELL, *Valued fields and elimination of imaginaries*.
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It can be said that model theory is the study of definable sets. Model theory becomes applied when the definable sets are those which are of interest in some other mathematical context. For example, the constructible sets of an algebraically closed field are precisely the sets definable in the language of fields, and the semialgebraic sets in p-adic algebraic geometry are the sets definable in the Macintyre language for p-adically closed fields. Mathematically, one naturally looks also at quotients, for example of algebraic or semialgabraic groups, but these unfortunately are less accessible to study model-theoretically. The essential difficulty is that, although any one equivalence class of a $\emptyset$-definable equivalence relation is a definable set, the quotient structure in general is only interpretable in the original structure and is not a definable object. A structure is said to *eliminate imaginaries* if the quotient by any $\emptyset$-definable equivalence relation lives as a definable set in some cartesian power of the original structure. In this talk, I will review some of the classical examples of theories of fields which eliminate imaginaries (for example, algebraically closed fields), and of some which do not (for example, algebraically closed valued fields). Then I will discuss recent results on multisorted languages in which some theories of valued fields can be proven to eliminate imaginaries.

► ANDREY MOROZOV, *On elementary submodels of F-parameterizable models*.
Sobolev Institute of Mathematics and Novosibirsk University, Koptyug prosp. 4, Novosibirsk, Russia.
E-mail: morozov@math.nsc.ru.

From time to time, I was asked the questions like these:

- Do all computable (arithmetical, hyperarithmetical, analytic, etc.) permutaions form an elementary subgroup in the group of all permutations of ω?
- Characterize subfields $\mathbb{R}' \leqslant \mathbb{R}$ such that $\mathbb{HF}(\mathbb{R}') \preccurlyeq \mathbb{HF}(\mathbb{R})$.
- Do all computable (arithmetical, hyperarithmetical, analytic, etc.) automorphisms of $\langle \mathbb{Q}; < \rangle$ form an elementary subgroup in the group $\mathrm{Aut}\langle \mathbb{Q}; < \rangle$?
- Do all computable (arithmetical, hyperarithmetical, analytical, etc.) members of ω^ω form an elementary subsemigroup in the semigroup ω^ω by composition?

In the talk, I introduce the notion of F-parameterization of models which generalizes the notion of computable models and actually is a weak version of definability in second order arithmetic. In terms of F-parameterizations, some general results on elementary submodels of F-parameterizable models will be obtained. In particular, I will show how to answer in a uniform way to the questions above. Under the assumption $\mathrm{V} = \mathrm{L}$, it is possible to obtain a more transparent characterization of elementary submodels in some classes.

► VLADIMIR PESTOV, *On the groups of automorphisms of various structures: their geometry, dynamics, and combinatorics*.
Department of Mathematics and Statistics, University of Ottawa, 585 King Edward Avenue, Ottawa, Ontario, K1N 6N5, Canada.
E-mail: vpest283@uottawa.ca.

We will survey some recent developments concerning the properties of groups of automorphisms of various structures, including many structures of importance in logic, as seen from the viewpoint of topological dynamics, metric geometry, combinatorics, representation theory, etc.

► ANDRE SCEDROV, *Logic in computer security*.
Department of Mathematics, University of Pennsylvania, 209 S. 33rd St., Philadelphia, PA, 19104-6395, USA.
E-mail: scedrov@saul.cis.upenn.edu.

While modern cryptography relies on applications of number theory, probability theory, and computational complexity, the methods of formal logic are contributing in significant ways to the wider area of computer security, including network security, access control, and policy languages. We will explore some of the applications of the methods of formal logic in these areas and we will consider several ways in which logical methods relate to mathematical applications in cryptography.

► ANDREAS WEIERMANN, *Phase transition and universality for independence results*.
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E-mail: weierman@math.uu.nl.

We will present recent results on classifying natural independent statements for first order Peano arithmetic (and related systems) and we will survey surprising connections between this area of logic and other fields in mathematics, like analytic combinatorics and Ramsey theory.

It will turn out that the fine structure of independence results shares features of phase transition and universality phenomena which are ubiquitous in statistical physics and discrete mathematics. To this end we consider combinatorial assertions, like Friedman-, Paris Harrington- or Kanamori McAloon style principles, and parameterize these with respect to a number-theoretic function. If this parameter function is bounded by a function of slow growth then the resulting assertion remains provable in the system under consideration but when the parameter function exceeds in growth a critical threshold then the resulting assertion, although still true, becomes unprovable. We will show that in a lot of examples the threshold stems from a very restricted (universal) scale.

The fine structure analysis of phase transitions yields applications to some famous open problems in mathematics and to Feferman's natural well-ordering problem. In particular we will discuss how the asymptotic of the standard Ramsey function for triples and three colors (a classical Erdoes problem) is affected by the possible independence of a certain Paris Harrington principle.

► JINDRICH ZAPLETAL, *Potential theory and forcing*.
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Let I be the ideal of sets of Newtonian capacity zero, and P(I) the poset of I-positive Borel sets ordered by inclusion. I will show that P(I) is a proper forcing, together with a number of related results about other similar capacities.

This is a joint work with Murali Rao.

Abstract of invited tutorial on computer-aided verification

► MOSHE Y. VARDI, *Automated verification = graphs, automata, and logic*.
Department of Computer Science, Rice University, Mail Stop 132, 6100 S. Main Street, Houston, TX 77005-1892, USA.
E-mail: vardi@cs.rice.edu.

In automated verification one uses algorithmic techniques to establish the correctness of the design with respect to a given property. Automated verification is based on a small number of key algorithmic ideas, tying together graph theory, automata theory, and logic. In this self-contained talk I will describe how this "holy trinity" gave rise to automated-verification tools.

Abstracts of invited talks in the Special Session on Computability and Randomness

► BARBARA F. CSIMA AND ANTONIO MONTALBÁN, *A minimal pair of K-degrees*.
Department of Mathematics, Cornell University, Ithaca, NY 14850, USA.
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We construct a minimal pair of K-degrees. We do this by showing the existence of an unbounded nondecreasing function f which forces K-triviality in the sense that $\gamma \in 2^\omega$ is K-trivial if and only if for all n, $K(\gamma \upharpoonright n) \leq K(n) + f(n) + \mathcal{O}(1)$.

► DENIS HIRSCHFELDT, *K-triviality and Bases for 1-randomness*.
Department of Mathematics, University of Chicago, 5734 S. University Ave., Chicago, IL 60637, USA.
E-mail: drh@math.uchicago.edu.

A set A is K-trivial if it has the lowest possible initial-segment prefix-free Kolmogorov complexity, up to an additive constant. That is, $K(A \upharpoonright n) \leq K(n) + O(1)$. A set A is a basis for 1-randomness if it can be computed in a set that is 1-random relative to A. I will discuss recent results of the speaker, André Nies, and Frank Stephan relating these two notions.

► BJØRN KJOS-HANSSEN, *Downward closed classes of Turing degrees and randomness*.
Department of Mathematics, University of Connecticut, 196 Auditorium Rd., Storrs, CT 06269-3009, USA.
E-mail: bjorn@math.uconn.edu.

A survey of recent work on Turing degrees of sets that are low for randomness in various senses, and degrees that do not contain diagonally non-recursive functions. We obtain equivalent conditions in terms of Kolmogorov complexity and traceability. Joint work with André Nies, Wolfgang Merkle and Frank Stephan.

► JOSEPH S. MILLER, *Contrasting plain and prefix-free complexity*.
Department of Mathematics, Indiana University, Rawles Hall, Bloomington, IN 47405, USA.
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For any c, consider the set SCR_c of strings x such that $K(x) > |x| + k(|x|) - c$. It is possible for SCR_c to contain an infinite Π^0_1 set; we show that any such set is hyperimmune. In particular, this proves that SCR_c itself is not Π^0_1, which had been an open question. The main result gives easy proofs of results due to Solovay and An. A. Muchnik that contrast plain and prefix-free complexity.

► JOSEPH S. MILLER AND ANDRE NIES, *Open questions in randomness and computability*.
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We discuss open questions in the area of randomness and computability. For each question we provide the background, motivation, and present known partial results in the direction of an answer.

► JAN REIMANN, *Kolmogorov-Loveland randomness and stochasticity*.
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One of the major open problems in the field of effective randomness is whether Martin-Löf randomness is the same as Kolmogorov-Loveland (or KL-) randomness, where an infinite binary sequence is KL-random if there is no computable non-monotonic betting strategy that succeeds on the sequence in the sense of having an unbounded gain in the limit while betting successively on bits of the sequence.

Recent work by Merkle, Miller, Nies, Reimann, and Stephan established further properties of KL-random sequences, some of which we will present here.

For instance, every KL-random sequence has arbitrarily dense, easily extractable subsequences that are Martin-Löf random. This is based on the fact that for every effective split of a KL-random sequence at least one of the halves is Martin-Löf random. However, this splitting property does not characterize KL-randomness, since one can construct a a sequence that is not even computably random such that every effective split yields subsequences that are 2-random, hence are in particular Martin-Löf random.

One can weaken the concept of KL-randomness to KL-stochasticity. A sequence X is KL-stochastic if there is no computable non-monotonic selection rule that selects from X an infinite, biased sequence. Every KL-random sequence is also KL-stochastic. We will present a proof that every KL-stochastic sequence has effective dimension 1, or equivalently, a sequence cannot be KL-stochastic if it has infinitely many prefixes that can be compressed by a factor of $\alpha < 1$ with respect to (prefix-free) Kolmogorov complexity. This improves on a result by Muchnik, who has shown a similar implication where the premise requires that such compressible prefixes can be found effectively.

► THEODORE SLAMAN, *Nonuniform measures and random reals*.
University of California, Berkeley, Department of Mathematics, Berkeley, CA 94720, USA.
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I will give a survey of recent joint work with Jan Reimann. For infinite binary sequences, we explore the relationships between having nontrivial string complexity, computing 1-random reals, and being random relative to (non-uniform) measures.

► FRANK STEPHAN, *Computing strings of maximal complexity*.
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E-mail: `fstephan@comp.nus.edu.sg`.

Let F be the class of all functions f which map every n to a string in $\{0,1\}^n$. Let A be a set of natural number. A has PA-complete degree iff one can compute a complete extension of Peano Arithmetic relative to A.

(1) The computation of incompressible strings with respect to the plain Kolmogorov complexity C requires a PA-complete degree: A has PA-complete degree iff there is $f \leq_T A$ such that $\forall n\ (C(f(n)) \geq n)$.
(2) The ability to compute maximal strings with respect to the plain Kolmogorov complexity C does not only depend on A but also on the underlying universal machine. This is illustrated by the following result: For every recursively enumerable set B there is an universal machine U_B such that for the plain Kolmogorov complexity C_B based on U_B, there is $f \leq_T A$ with $\forall n \forall x\ (C(f(n)) \geq C(x))$ iff $A \geq_T B$ and A has PA-complete degree.
(3) The computation of maximal strings with respect to the prefix-free Kolmogorov complexity H requires access to the halting problem K: There is $f \leq_T A$ with $\forall n \forall x$ $(H(f(n)) \geq H(x))$ iff $A \geq_T K$. This even holds if the condition is relaxed to being more complex than $H(n) + n - c$ for some suitable constant c: There is a constant c and $f \leq_T A$ with $\forall n\ (H(f(n)) \geq n + H(n) - c)$ iff $A \geq_T K$.

Statement (3) partially answers the following open question of Calude: Is $A \geq_T K$ whenever there is a partial A-recursive function f with infinite domain such that for all n in the domain of f, $f(n) \in \{0,1\}^n$ and $H(f(n)) \geq H(x)$ for all $x \in \{0,1\}^n$.

The talk is based on joint work with Bjørn Kjos-Hanssen and Wolfgang Merkle.

► SEBASTIAAN A. TERWIJN, *Randomness, diagonalization, and domination properties*.
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Cantor proved that there are uncountably many reals using his diagonalization method. Borel gave a proof of this result using a measure theoretic argument that avoids the explicit diagonalization. We discuss several constructions from the theory of randomness in which similar combinations of diagonalization and the probabilistic method occur. Secondly we discuss the role of domination properties in the theory of randomness. We also discuss some recent joint work with André Nies and Frank Stephan.

Abstracts of invited talks in the Special Session on Model Theory

► ALFRED DOLICH, *On structures associated to definable one forms*.
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We discuss some model theory associated to integral manifolds of a one form ω definable in an o-minimal structure $\mathcal{R}$ expanding the real field. In particular we associate a new structure M_ω with universe $\mathbb{R}^2$ to this form and study how the geometric properties of ω are reflected in the model theory of M_ω and vice versa.

This is joint with with Patrick Speissegger.

► ALEXEI KOLESNIKOV, *Generalized amalgamation properties in simple theories*.
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Our talk is about the new results in classification of simple unstable theories. The class of simple theories naturally extends the class of stable theories and contains many important examples of unstable theories such as the theories of random graphs, algebraically closed fields with a generic automorphism, and pseudo-finite fields. First introduced by Shelah [6] in 1980, simple theories became an active field of study following the work of Kim and Pillay [3], [4]. One of the properties isolated by Kim and Pillay is called the independence theorem. The property essentially asserts that a simple theory cannot interpret a generic triangle-free graph. It was observed however that a simple theory can interpret a (3-dimensional) generic tetrahedron-free hypergraph, as well as its higher-dimensional analogs.

Our research deals with the properties that preclude a simple theory from interpreting generic "tetrahedron-free" hypergraphs of dimension $n \geq 3$. One such family of properties (strong n-simplicity) was introduced and studied in our previous work [5]. Since then generalized amalgamation was used by dePiro, Kim, and Young [2] to solve a problem in geometric simplicity theory.

In our talk, we will introduce the necessary background and describe the new results obtained in this direction.

[1] ZOE CHATZIDAKIS AND EHUD HRUSHOVSKI, *The model theory of difference fields*, ***Transactions of the American Mathematical Society***, vol. 351 (1999), pp. 2997–3071.

[2] TRISTRAM DE PIRO, BYUNGHAN KIM, AND JESSICA YOUNG, *The type-definable group configuration under the generalized type amalgamation*, preprint.

[3] BYUNGHAN KIM, *Forking in simple theories*, ***Journal of the London Mathematical Society***, vol. 57 (1998), pp. 257–267.

[4] BYUNGHAN KIM AND ANAND PILLAY, *Simple theories*, ***Annals of Pure and Applied Logic***, vol. 88 (1997), pp. 149–164.

[5] ALEXEI KOLESNIKOV, *n-simple theories*, ***Annals of Pure and Applied Logic***, to appear.

[6] SAHARON SHELAH *Simple unstable theories*, ***Annals of Mathematical Logic***, vol. 19 (1980), pp. 177–203.

► DANIEL J. MILLER, *Polynomially bounded o-minimal structures with C^∞ uniformization*.
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An o-minimal expansion of the real field is said to have C^∞ uniformization if every definable compact set is the image of a definable compact C^∞ manifold under a definable C^∞ mapping. It follows from the construction of Rolin, Speissegger and Wilkie [1] that an o-minimal expansion of the real field is polynomially bounded and has C^∞ uniformization iff it is interdefinable with an expansion of the real field by a collection of restricted functions closed under differentiation and contained in a certain type of quasianalytic class, and in this language the structure is model complete. I hope to discuss the following tentative results: from the model completeness proof for this structure one can read off an AE-axiomatization of its theory which is recursive in natural oracles. When the collection of functions are Noetherian, these oracles reduce to the decidability criterion discovered by MacIntrye and Wilkie [2] for the real exponential field.

[1] J.-P. ROLIN, P. SPEISSEGGER AND A. J. WILKIE, *Quasianalytic Denjoy-Carleman classes and o-minimality*, ***Journal of the American Mathematical Society***, vol. 16 (2003), no. 4, pp. 751–777 (electronic).

[2] A. MACINTRYE AND A. J. WILKIE, *On the decidability of the real exponential field*, ***Kreiseliana***, A. K. Peters, Wellesley MA, 1996, pp. 441–467.

► DOMINIKA POLKOWSKA, *From pseudofinite fields to PAC substructures of stable structures*.
Department of Mathematics, University of Illinois at Urbana-Champaign, 1409 W. Green Street, Urbana, IL 61801, USA.
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The background for my thesis research is a 40-year interaction between field theory and model theory which began with work of James Ax on pseudofinite fields. In 1991 Hrushovski introduced the notion of a bounded PAC (bPAC) substructure of a model of a strongly minimal theory which generalized the notion of pseudofinite fields to strongly minimal models.

Influenced by Hrushovski's work on definability in pseudofinite fields and related structures, Kim and Pillay in the mid-1990's completed the foundational work on simple theories which Shelah had begun much earlier. The "bounded PAC substructures" of strongly minimal sets, studied by Hrushovski, could now be seen as supersimple structures of SU-rank 1. The basic aim of my research is to extend Hrushovski's theory from strongly minimal structures to arbitrary stable structures where possible and, as a consequence, to give a general method of building new simple theories.

This fits into a very basic theme of research over the past 10 years which creates new stable/simple structures from old ones, for example adding a generic predicate or a generic automorphism; beautiful pairs; and lovely pairs.

► DALE RADIN, *Morley degree in unidimensional compact complex spaces*.
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A compact complex space X is viewed as a first order structure by taking predicates for the analytic subsets of X. X is then a Zariski-type structure in the sense of [2]. Using properties of unidimensional Zariski-type structures, together with results in [1], we conclude that Morley degree is a definable property in $\mathrm{TH}(X)$, when $\mathrm{TH}(X)$ is unidimensional. This can be seen as a generalization of the fact that Morley degree is a definable property in the theory of an algebraically closed field.

[1] D. Radin, *A definability result for compact complex spaces*, ***The Journal of Symbolic Logic***, vol. 69 (2004), no. 1, pp. 241–254.

[2] B. Zilber, *Lecture notes on Zariski-type structures*, delivered in the *UIC logic seminar*, Fall 1991.

► CHARLES STEINHORN, *On model theory for classes of finite structures*.
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E-mail: `steinhorn@vassar.edu`.

This talk surveys joint work with D. Macpherson whose broad aim is to import into the model theory for classes of finite structures the viewpoint of "dimension theory," in its multiple senses, from model theory for infinite structures. Some results of Macpherson's students will be discussed also.

► JAMES TYNE, *An alternate approach to the Valuation Property*.
Department of Mathematics, The Ohio State University, 231 West 18th Avenue, Columbus, OH 43210, USA.
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Let $\mathcal{R}$ be an o-minimal expansion of a field with underlying set R, and let $V \subseteq R$ be a T-convex valuation ring with associated valuation v. (The ring V is *T-convex* if it is convex and satisfies $f(V) \subseteq V$ for all continuous functions $f\colon R \to R$ that are definable in $\mathcal{R}$ without parameters.)

I will examine the behavior of the (unary) functions definable in $\mathcal{R}$, under the assumption that no exponential function is definable. With this assumption, the definable functions interact well with the valuation function. For example, if $f\colon R \to R$ is definable in $\mathcal{R}$, then there is a function $\phi\colon v(R) \to v(R)$ and a finite set $B \subseteq v(R)$ such that $v \circ f(x) = \phi \circ v(x)$ for all positive $x \in R$ with $v(x) \notin B$. This and related results lead to an alternate proof of the following Valuation Property.

Suppose $\mathcal{S}$ is an extension of $\mathcal{R}$ that is generated over $\mathcal{R}$ by a single element s, and W is a T-convex valuation ring of $\mathcal{S}$, with associated valuation w, such that $W \cap R = V$. We say the *Valuation Property* holds for $(\mathcal{R}, V)$ if, for all such $\mathcal{S}$, s, and w, either $w(S) = v(R)$ or there is $r \in R$ such that $w(s - r) \notin v(R)$.

The original proof of the Valuation Property is due to Van den Dries and Speissegger [1, 9.2]. One may note that their approach requires a subtly stronger assumption than the above alternate does. Specifically, their approach requires every (unary) function definable in $\mathcal{R}$ to be ultimately bounded by x^n for some integer n.

[1] Lou van den Dries and Patrick Speissegger, *The field of reals with multisummable series and the exponential function*, ***Proceedings of the London Mathematical Society***, vol. 81 (2000), no. 3, pp. 513–565.

► MONICA VANDIEREN, *Independence, limit models and superstability in abstract elementary classes*.
Department of Mathematics, University of Michigan, Ann Arbor, MI 48109-1109, USA.
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In this talk we will describe the interaction of these three concepts in several settings which include homogeneous classes, excellent classes, local AECs as special cases. We will focus on joint work with Rami Grossberg and Andrés Villaveces.

► YOAV YAFFE, *Model companions and finite presentations in Lie differential fields*.
Department of Mathematics and Statistics, McMaster University, 1280 Main Street West, Hamilton, Ontario L8S 4K1, Canada.
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A Lie differential field is a field F given with some Lie algebra L acting on F as derivations. If we fix L we get the class of L-differential fields, which has amalgamation when the characteristic is 0.

If in addition L is finite dimensional over F the above class has a model companion (and hence a model completion). This "tameness" carries over by compactness to an L which is locally f.d., i.e., where every finitely generated sub Lie algebra of L is finite dimensional.

However if L isn't locally finitely presented, i.e., if there is a finitely generated sub Lie algebra without a finite presentation, then the class of L-differential fields does NOT have a model companion. I will describe how this result is proved by producing a non-eliminable quantifier, using a system of linear PDEs.

Abstracts of invited talks in the Special Session on Proof Theory and Constructivity

► TOSHIYAU ARAI, *Epsilon substitution method for inductive definitions*.
Graduate School of Science and Technology, Kobe University, Rokko-dai, Nada-ku, Kobe, 657-8501, Japan.
E-mail: `arai@kurt.scitec.kobe-u.ac.jp`.

Starting with a brief survey on the epsilon substitution method for the theory of Π^0_1-inductive definitions, we will discuss its possible extensions.

► MICHAEL BEESON, *Constructivity, computability, and the continuum*.
Department of Computer Science, San Jose State University, 1 Washington Square, San Jose, CA 95192, USA.
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The nature of the continuum has long been an important issue in the foundations of mathematics. It played an important role in the work of Dedekind, Weyl, and Brouwer. When recursive function theory was developed, it was immediately applied to the continuum, via Turing's "computable" numbers. But then the construction of "singular covers" showed, it seemed, that the recursive reals had measure zero! Does this mean that we have to choose between constructive reasoning consistent with Church's thesis and a unit interval of positive measure? Bishop's measure theory saved us from the horns of this dilemma—or did it just sweep the difficulty under the carpet? Nowadays, physicists tell us that space is not indefinitely subdivisible—below the Planck length space is not coordinatizable in the usual way. Does this result have philosophical implications (either ontological or epistemological) about the nature of the continuum? The talk will address these philosophical issues, and answer some related metamathematical questions.

► DOUGLAS S. BRIDGES, *Journey into apartness space*.
Department of Mathematics and Statistics, University of Canterbury, Private Bag 4800, Christchurch, New Zealand.
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The first part of this talk deals with an aspect of apartness spaces whose nearness-space analogue seems to be missing from the classical literature: namely, finding a pre-apartness structure on Y^X that provides the right setting for proximal convergence (which is normally defined in an *ad hoc* way with no reference to any apartness on Y^X). We deal with two pre-apartness structures on Y^X, where X is an inhabited set and Y a uniform space, in order to clarify the connection between proximal and uniform convergence.

In the second part of the talk we introduce two notions of connectedness for apartness spaces, and discuss various fundamental properties of connected sets.

All the work presented was carried out by the speaker and L. S. Vîţă.

► TIM CARLSON, *"Ranked β-calculi"*.
Department of Mathematics, Ohio State University, Columbus, Ohio 43210, USA.
E-mail: carlson@math.ohio-state.edu.

We present an approach to proofs for β-logic with sequents which have an explicit rank. This approach is appropriate for using patterns of resemblance in ordinal analysis.

► JOHN R. HARRISON, *Challenges for the formal approach to proof*.
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The more foundational computer theorem provers, including our own HOL Light system, will only allow theorems to be produced by very low-level logical inference rules of the kind found in traditional logic texts. Yet there are numerous algorithms used in symbolic computation that don't produce formal proofs, or indeed any other kind of certificate that the answers they give are correct. For example, formal verification often involves the analysis of finite-state systems using highly optimized state enumeration techniques. Even in pure mathematics, many major results rely on extensive computer checking of cases: the best-known example is probably the 4-color theorem, and a more recent case of some interest to us is the proof by Hales of the Kepler conjecture. In this talk we will discuss some of the interesting challenges that arise when trying to prove such results in a foundational prover.

► ROSALIE IEMHOFF, *Basic intuitionistic logic of proofs*.
Department of Discrete Mathematics and Geometry E104, Technical University Vienna, Wiedner Hauptstrasse 8–10, 1040 Vienna, Austria.
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The basic logic of proofs consists of the propositional schemes that are valid about the proof predicate $\mathrm{Prf}(x, y)$. It is a logic in the language of propositional logic extended by expressions $t : A$, where t is an atom and A a formula, that are interpreted as "t is a proof of A". We will present an arithmetical completeness theorem for the basic intuitionistic logic of proofs, that is a completeness theorem w.r.t. Heyting Arithmetic. This result disproves the conjecture that the intuitionistic logic of proofs consists of intuitionistic logic extended by the characteristic axioms of the classical logic of proofs and the law of excluded middle for expressions $t : A$. We will briefly comment on the extension of the result to the case in which functions on proof terms are allowed. This is joint work with Sergei Artemov.

► HENRY TOWSNER, *Termination of the substitution method for ID1*.
Department of Mathematical Sciences, Carnegie Mellon University, Pittsburgh, PA 15213-3890, USA.
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We present a new termination proof of Hilbert's epsilon substitution method for the system ID1 of inductive definitions. This provides witnesses for Σ^0_1 theorems and proves that the theory is 1-consistent.

The proof follows a schema from [1]. Cut-elimination for certain deductions in an ad hoc deduction system is shown by induction up to the Howard-Bachmann ordinal.

[1] G. MINTS, S. TUPAILO, AND W. BUCHHOLZ, *Epsilon substitution method for elementary analysis*, ***Archive for Mathematical Logic***, vol. 35 (1996), pp. 103–130.

► HENRY TOWSNER AND GRIGORI MINTS, *A simple substitution method for ID1*.
Department of Mathematical Sciences, Carnegie Mellon University, Pittsburgh, PA 15213-3890, USA.
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We present a new termination proof of Hilbert's epsilon substitution method for the system ID1 of iterated inductive definitions. This provides witnesses for Σ^0_1 theorems and proves that the theory is consistent.

The proof is non-effective, but is much shorter than other proofs. The definition of the process is effective, and avoids the use of history, proof-theoretic ordinals, and collapsing techniques required for effective proofs (in particular, [1]).

We show that an infinitely long process would have to include guesses at existential formulas more complicated than any the process could generate. This implies that the substitution process actually employs only finite substitutions and terminates in finitely many steps.

[1] T. Arai, *Epsilon substitution method for $ID_1(\Pi^0_1 \vee \Sigma^0_1)$*, ***Annals of Pure and Applied Logic***, vol. 121, (2003), pp. 163–208.

► SERGEI TUPAILO, *On the consistency of New Foundations*.
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New Foundations, NF, is a system of set theory named after Quine's 1937 article "New foundations for mathematical logic", where it was introduced. It was meant as a foundations of mathematics, alternative to Zermelo-Fraenkel set theory ZF and others. Obvious advantages of NF are that it's very easily formulated and many mathematical notions can be expressed in NF in a much more "natural" way than in ZF. However, in spite of efforts of many researches and many brilliant results, NF is still not known to be consistent relative to any theory in which we have reasonable confidence.

We investigate the possibility of reducing the Consis(NF) problem to consistency of various extensions of Jensen's NFU, "NF with Urelements", which is known to be consistent due to Jensen 1969. Extensions of NFU by different "large cardinal axioms" and their consistency strength have been studied by R. Jensen, S. Feferman, M. Boffa, R. Holmes, R. Solovay.

Abstracts of invited talks in the Special Session on Set Theory and Borel Structure

► JOHN D. CLEMENS, *Generating equivalence relations by homeomorphisms*.
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I will discuss ways of generating equivalence relations such as E_0 as orbits equivalence relations of continuous actions of groups. First, I will give an example of a homeomorphism of Cantor space which generates E_0, and will then consider various nice properties we might desire of such a homeomorphism and show that some of them are not possible. In particular, I will discuss actions by Lipschitz automorphisms of the Cantor space.

► UDAYAN B. DARJI, *Small sets in groups*.
University of Louisville, Louisville, KY 40292, USA.
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In this talk we introduce the notion of Haar meager sets and develop of some its basic properties. We establish relationships between Haar meager sets, meager sets and Haar null sets. We also give some concrete example of Haar meager sets.

► MATTHEW FOREMAN, *The conjugacy relation on measure preserving transformations*.
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In joint work with D. Rudolph and B. Weiss I show that the conjugacy relation on ergodic MPT's is complete Σ^1_1.

► SU GAO, *The optimality of a theorem of Becker and Kechris*.
Department of Mathematics, University of North Texas, PO Box 311430, Denton, TX 76203-1430, USA.
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We establish the following theorem:

For each Polish group that is not locally compact there exists a continuous action of the group on a coanalytic complete subset of a Polish space so that the action cannot be continuously extended to any superset. This shows that a theorem of Becker and Kechris, which is a generalization of the classical Lavrentieff's theorem, is optimal in several aspects.

This is a joint work with Ding.

► ALEXANDER KECHRIS, *De Groot's Problem*.
Mathematics, Caltech, 253-37 Sloan Hall, Pasadena, CA 91125, USA.
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I will discuss the recent solution by Trevor Wilson of De Groot's Problem, originally raised in 1958, concerning a continuous movement version of the Banach-Tarski Paradox.

► BENJAMIN MILLER, *Weakly wandering sequences*.
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In the measure-theoretic setting, the following are equivalent:

(1) There is no T-invariant probability measure.

(2) (Hopf) There is a compression in $[T]$.

(3) (Hajian-Kakutani) T admits a weakly wandering set (of full saturation).

In the descriptive setting, the first two of these remain equivalent (Nadkarni), but the third is strictly weaker (Eigen-Hajian-Nadkarni). We will discuss the circumstances under which (3) holds.

► MIKE OLIVER, *Definable cardinality of the collection of quotient Boolean algebras*.
Department of Mathematics, University of North Texas, P.O. Box 311430, Denton, TX 76203-1430, USA.
E-mail: moliver@unt.edu.

We have previously shown that there is a Borel embedding from $\aleph_1$ into the isomorphism relation on Boolean algebras that are quotients by Borel ideals, and also that there is such an embedding from E_0 into the isomorphism relation on quotients by analytic P-ideals. In joint work with Gao, we examine attempts to extend these results, with the goal of showing that *every* Borel equivalence relation may be embedded into the quotients by Borel ideals.

► CHRISTIAN ROSENDAL, *Polish groups; Automatic continuity and bounded orbits*.
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Automorphism groups of sufficiently homogeneous countable models have been known for some time to have strong properties relating their abstract and their permutation group structures. We use methods of descriptive set theory in order to obtain new stronger properties such as automatic continuity of homomorphisms (a strengthening of the small index property) and boundedness of orbits arrising from actions on metric spaces. The main tool used is the existence of comeagre orbits for the diagonal conjugation action of the group on its finite powers, the verification of which depends on the model theory of the underlying structure.

► SIMON THOMAS, *Property* (τ) *and* E_0*-ergodicity*.
Mathematics Department, Rutgers University, 110 Frelinghuysen Road, Piscataway, New Jersey 08854-8019, USA.
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I will discuss the E_0-ergodicity of the orbit equivalence relations arising from dense embeddings of finitely generated groups with Property (τ) into profinite groups.

► ASGER DAG TORNQUIST, *The Borel complexity of orbit equivalence*.
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E-mail: atornqui@math.ucla.edu.

In this talk we discuss the borel complexity orbit equivalence of free measure preserving actions of the free group F_n on n generators. Building on work of Sorin Popa and Damian Gaboriau, the main result is that there are E_0 many such actions. We also discuss the situation for certain other countable groups.

Abstract of contributed talks

► PAUL BANKSTON, *In search of categoricity in topology*.
Department of Mathematics, Statistics and Computer Science, Marquette University, Milwaukee, WI 53201-1881, USA.
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We define a Boolean (i.e., totally disconnected compact Hausdorff) space to be *categorical* if its Boolean algebra of closed open sets is categorical in the model-theoretic sense. The usual Cantor space, then, is categorical by this criterion because of the $\aleph_0$-categoricity of the theory of atomless Boolean algebras. With the aid of the topological ultracoproduct construction, this definition of categoricity naturally extends to the realm of compact Hausdorff spaces. We report on recent progress in the search for compact metric spaces of positive dimension that are categorical.

► KATALIN BIMBÓ, *Admissibility of cut in the structurally free logic* $LC_+^{\mathsf{Y},\mathsf{y}}$.
School of Informatics, Indiana University, 901 East Tenth Street, Bloomington, IN 47408–3912, USA.
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Structurally free logics incorporate combinators as constant formulas subject to certain rules into a sequent calculus. Different combinators correspond to different structural rules, and so adding or omitting combinators can result in various substructural logics. So far only proper combinators (or their duals) were considered in such logics (cf. [3], [1]). One of the interesting and sensible "improper" combinators is the fixed point combinator (usually denoted by Y). The fixed point combinator is added here to LC_+ with the rule:

$$\frac{\mathfrak{A}[\mathfrak{C};(\mathsf{Y};\mathfrak{C})] \Rightarrow \mathcal{A}}{\mathfrak{A}[\mathsf{Y};\mathfrak{C}] \Rightarrow \mathcal{A}}.$$

(The dual fixed point combinator may be added by the "dual" of this rule.) The cut rule is formulated as:

$$\frac{\mathfrak{A} \Rightarrow \mathcal{M} \quad \mathfrak{B}\langle\mathcal{M}\rangle \Rightarrow \mathcal{A}}{\mathfrak{B}\langle\mathfrak{A}'\rangle \Rightarrow \mathcal{A}}.$$

(This rule is somewhat similar to the cut rule introduced in [2], however, our reasons to use $^{\mathsf{t}}$ are different.) We demonstrate that the admissibility of the cut rule cannot be proven by a Gentzen-style double induction method alone (cf. [4]). We add an extra induction to prove the elimination theorem (that is, the admissibility of cut). The novel feature of the proof—beyond being a proof for a new extension of *LC*—is the additional induction. The latter also provides further insights into the ways the fixed point combinator can be introduced into a consecution.

[1] K. Bimbó and J. M. Dunn, *Two extensions of the structurally free logic LC*, ***Logic Journal of the Interest Group in Pure and Applied Logics***, vol. 6 (1998), pp. 403–424.

[2] J. M. Dunn, *A 'Gentzen system' for positive relevant implication*, (*abstract*), ***The Journal of Symbolic Logic***, vol. 38 (1973), pp. 356–357.

[3] J. M. Dunn and R. K. Meyer, *Combinators and structurally free logic*, ***Logic Journal of the Interest Group in Pure and Applied Logics***, vol. 5 (1997), pp. 505–537.

[4] G. Gentzen, *Untersuchungen über das logische Schließen*, ***Mathematische Zeitschrift***, vol. 39 (1935), pp. 176–210.

► OTÁVIO BUENO, *Nominalistic metalogic*.
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According to Hartry Field's nominalism, a mathematical theory need not be true to be good. It must only be conservative, that is, consistent with every internally consistent theory about the physical world (Field [1] and [2]). In order to argue for the adequacy of the notion of conservativeness, Field needs to establish that (a) conservativeness can be formulated in a nominalistically acceptable way, and that (b) mathematical theories are conservative. Field has explicitly addressed these issues (Field [2], [3] and [4]). However, in doing that he has used two metalogical results of first-order logic (the completeness and the compactness theorems). Since these results typically assume set theory, the issue arises as to whether the nominalist is entitled to use them without also providing a nominalization strategy for set theory itself. In this paper, it is argued that without such a nominalization strategy, Field's nominalism cannot be maintained.

[1] H. Field, ***Science without numbers: a defense of nominalism***, Princeton University Press, Princeton, N.J., 1980.

[2] ———, ***Realism, mathematics and modality***, Basil Blackwell, Oxford, 1989.

[3] ———, *Metalogic and modality*, ***Philosophical Studies***, vol. 62 (1991), pp. 1–22.

[4] ———, *A nominalistic proof of the conservativeness of set theory*, ***Journal of Philosophical Logic***, vol. 21 (1992), pp. 111–123.

► WILLIAM C. CALHOUN, *Monotonicity of complexity measures*.
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The monotone complexity of a string $K_m(x)$ was introduced by Levin [1] and Schnorr (who initially defined a slight variant called process complexity) [2]. Monotone complexity is defined by restricting Kolmogorov's definition of program-size complexity to monotone machines, where M is monotone if for all σ and τ in the domain of M, $\sigma \prec \tau$ implies $M(\sigma) \prec M(\tau)$. After fixing an optimal monotone machine M, $K_m(x)$ is defined to be the minimum length $|\sigma|$ where $x \preceq M(\sigma)$. Monotone complexity gives rise to a complexity measure for a real (binary sequence) α via the function $K_m(\alpha[n])$, where $\alpha[n]$ is the first n bits of α. An advantage of monotone complexity is that it is monotonic in n and the deficiency of randomness function $\mathrm{d}(n) = n - K_m(\alpha[n])$ is "nearly" monotonic. We will discuss applications of monotonicity in defining relative randomness.

[1] L. A. Levin, *On the concept of a random sequence*, ***Soviet Mathematics Doklady***, vol. 14 (1973), pp. 1413–1416.

[2] C. P. Schnorr, *Process complexity and effective random tests*, ***Journal of Computer and System Sciences***, vol. 7 (1973), pp. 376–388.

► Y. FIRAT ÇELİKLER, *Dimension theory of Denef-Pas-analytic sets*.
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Definable sets over non-Archimedean fields in the Denef-Pas Language play an important role in the theory of motivic integration. Establishing a well behaved dimension theory for such sets is essential to this role. For the definable sets in the algebraic Denef-Pas Language, such a dimension theory was established by Cluckers and Loeser in [4] using the results by van den Dries in [5]. In order to develop an analytic motivic integration theory one needs similar results in the analytic Denef-Pas languages (i.e., Denef-Pas language enriched with analytic D-functions). I will talk about results in dimension theory of D-semianalytic sets which were established in [1] and [2] and how these results can be extended to Denef-Pas-analytic sets using the quantifier elimination theorem of [3] by Cluckers, Lipshitz and Robinson.

[1] Y. F. ÇELIKLER *Dimension theory and parameterized normalization for D-semianalytic sets*, to appear, available at: arXiv:math.LO/0409109.

[2] ———, *Parameterized stratification and piece number of D-semianalytic sets*, submitted, available at: arXiv:math.LO/0409095.

[3] R. CLUCKERS, L. LIPSHITZ, AND Z. ROBINSON, *Analytic cell decomposition and analytic motivic integration*, preprint.

[4] R. CLUCKERS AND F. LOESER, *Constructible motivic functions and motivic integration*, preprint, available at: arXiv:math.AG/0410203.

[5] L. VAN DEN DRIES, *Dimension of definable sets, algebraic boundedness and Henselian fields*, ***Annals of Pure and Applied Logic***, vol. 45 (1989), pp. 189–209.

► VALERIA DE PAIVA, *Constructive* CK *for contexts*.
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This note describes possible world semantics for a constructive modal logic CK. The system CK is weaker than other constructive modal logics K as it does *not* satisfy distribution of possibility over disjunctions, neither binary $(\Diamond(A \vee B) \rightarrow \Diamond A \vee \Diamond B)$ nor nullary $(\Diamond \perp \rightarrow \perp)$. Our intuition is that $\Diamond \perp$ should not constructively imply $\perp$, thus we dispense with both forms of distribution. We are interested in this version of constructive K for its application as contexts in AI [3]. However, logicians interested in contexts prefer their semantics possible worlds style, while system CK first had only a categorical semantics [2]. Wijesekera [4] investigated possible-worlds semantics of a system similar to CK, without the binary distribution, but satisfying the nullary one. We had hoped that an adaptation of Wisejekera's results would work for us, but a flaw in his proofs meant that his results had to be reworked. We also wanted a possible-worlds semantics for CK because it can be seen as a natural generalization of our work [1] on a constructive and categorical version of modal S4.

Joint work with Michael Mendler, University of Bamberg, Germany.

[1] N. ALECHINA, M. MENDLER, V. DE PAIVA, AND E. RITTER, *Categorical and Kripke semantics for constructive S4 modal logic*, ***Proceedings of Computer Science Logic 2001*** (***CSL 2001***), (L. Fribourg, editor), Lecture Notes in Computer Science, vol. 2142, Springer Verlag, 2001, pp. 292–307.

[2] G. BELLIN, V. DE PAIVA, AND E. RITTER, *Extended Curry-Howard Correspondence for a Basic Constructive Modal Logic*, ***Methods for modalities***, 2001.

[3] V. DE PAIVA, *Natural deduction and context as (constructive) modality*, ***Proceedings of the 4th International and Interdisciplinary Conference Context 2003, Stanford, CA, USA***, (Blackburn et al., editors), Lecture Notes in Artificial Intelligence, vol. 2680, Springer Verlag, 2003.

[4] D. WIJESEKERA, *Constructive modal logic I*, ***Annals of Pure and Applied Logic***, vol. 50 (1990), pp. 271–301.

► RUMEN D. DIMITROV, *Characterization of the principal filters of quasimaximal spaces in $\mathcal{L}^*(V_\infty)$.*
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Let I_0 be a computable basis of the fully effective vector space V_∞ over a computable field F. Let $\mathcal{L}(V_\infty)$ be the lattice of c.e. subspaces of V_∞ and $\mathcal{L}^*(V_\infty)$ be $\mathcal{L}(V_\infty)$ modulo finite dimensional spaces. Let $I = \bigcap_{i=1}^{n} I_i$ be a quasimaximal subset of I_0 that is the intersection of the maximal $I_i \subset I_0$. Suppose that the maximal sets I_i $(i \leq n)$ are partitioned into k equivalence classes with respect to the relation of having the same 1-degree up to $=^*$ and suppose that the i-th equivalence class has n_i elements. We prove that the principal filter $\mathcal{L}^*(V,\uparrow)$ of $V = cl(I)$ is isomorphic to the product of the lattices $(\mathcal{L}(n_i,\overline{F}))_{i=1}^{k}$, where $\mathcal{L}(n_i,\overline{F})$ is the lattice of subspaces of an n_i-dimensional vector space over $\overline{F}$. The filed $\overline{F}$ is an extension of F if F is infinite, and is isomorphic to F if F is finite.

► LEONA F. FASS, *Agents on the Web: Some behavioral theory and applications*.
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The Semantic Web is a potential system that should adapt and learn, benefiting from human and software agent interaction. To solve problems that may be posed, web-navigating software agents are being developed to make logical decisions, choose actions, and comply with explicit and implicit instructions provided by human users. In order to effect complex problem solutions, behavioral goals may be decomposed into sub-goals to be fulfilled by suitable agents. Results may then be re-composed, to achieve behaviors as specified. Determining "suitable" agents to fulfill sub-goal tasks is, itself, a critical issue to be dealt with on the Semantic Web.

In a traditional approach as employed throughout [2], groups of agents form problem-solving coalitions by convincing themselves and each other that collective effort is beneficial. But motivated by [3], we suggest that oversight and feedback should guide agent selection and collection. Selected agents may be configured dynamically to produce a specified behavior, adapted when a preferable configuration is found, and dynamically reconfigured should a behavioral goal specification be modified. Our theoretically-designed Personal Travel Assistant illustrates these possibilities, with components that adapt as new fulfilling agents become available, and/or as behavioral goals and users may change.

[1] L. F. FASS, *An automata-theoretic view of agent coalitions*, in [2], pp. 18–21.

[2] L.-K. Soh and J. E. Anderson (editors), ***Papers from the AAAI Workshop on Forming and Maintaining Coalitions and Teams in Adaptive Multiagent Systems, San Jose, CA***, July 2004, AAAI Press WS04-06.

[3] R. WALDINGER, *Deductive chat lines for multiple agents*, Invited Symposium Talk, abstracted in ***Papers from the AAAI Spring Symposium on Logic-Based Program Synthesis: State of the Art and Future Trends, Stanford University***, March 2002, AAAI Press SS02-05, p. 5.

► HANNES LEITGEB, *Semantic dependence for type-free truth, modality, and membership*.
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In [1] we introduced a semantic dependency relation for sentences of a first-order language $\mathcal{L}_{Tr}$ with truth predicate Tr ($\mathcal{L}_{Tr}$ might, e.g., be the language $\mathcal{L}$ of arithmetic plus Tr): for $\varphi \in \mathcal{L}_{Tr}$ and $\Phi \subseteq \mathcal{L}_{Tr}$, φ is defined to depend on Φ: iff for all Ψ_1, $\Psi_2 \subseteq \mathcal{L}_{Tr}$: if $Val_{\Psi_1}(\varphi) \neq Val_{\Psi_2}(\varphi)$ then $\Psi_1 \cap \Phi \neq \Psi_2 \cap \Phi$. ($Val_\Psi(\varphi)$ is the truth value of φ as being given by an expansion of a ground model for $\mathcal{L}$ such that Ψ is used as the extension of *Tr*). Transfinite iteration of the operator $D^{-1}\colon \Phi \mapsto \{\varphi \in \mathcal{L}_{Tr} \,|\varphi \text{ depends on } \Phi\}$ leads to a least fixed point set Φ_{lf} of sentences the truth values of which may be said to *depend on non-semantic states of affairs only*. Any standard Tarskian definition of truth for $\mathcal{L}$ can be extended to a definition of truth for $\mathcal{L}_{Tr}$ in a way such that for all sentences $\phi \in \Phi_{lf}$ a T-biconditional of the form $Tr(\ulcorner\phi\urcorner) \leftrightarrow \phi$ is derivable from the truth definition (given the metatheory).

In our talk we will show that analogous theories can be introduced for cases where either (i) a necessity predicate $\Box$ of sentences or (ii) a membership or "instantiation" predicate $\in$ of open formulas is added to the arithmetic language. The main aim of the theory is to give a general account of what instances of Tarski's T-scheme, of the usual modal axiom schemes, and of the comprehension scheme are "natural" if truth, modality, and membership are expressed by type-free first-order predicates.

[1] H. Leitgeb, *What truth depends on*, ***Journal of Philosophical Logic***, (2005), forthcoming.

► LEO MARCUS, *Logical foundations of an adaptive security infrastructure*.
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Current distributed system security mechanisms cannot adapt adequately to either rapidly changing, or long-term evolution of, threat environments or system components and architectures. While substantial effort has been expended to improve the localized adaptability of individual security mechanisms (e.g., firewalls, network routers, intrusion detection systems) to identified attacks, there is little understanding of the ramifications of such adaptations on the larger system. We have developed a model-theoretic framework [1], [2] on which we are building a semantics for an "adaptive security infrastructure." We intend to use this framework and associated semantics to characterize adaptive responses to changes in the state of the larger system. Such characterizations will help security analysts specify the intended behavior of a system (even under changing conditions) and facilitate verification that the system behaves as intended.

We discuss some model-theoretic problems that must be solved to establish the desired semantics.

[1] L. Marcus, *Local and global requirements in an adaptive security infrastructure*, ***2nd International Workshop on Requirements for High Assurance Systems, September 9, 2003, Monterey Bay, California***, pp. 23–29. Carnegie Mellon Software Engineering Institute Publication.

[2] ———, *Introduction to logical foundations of an adaptive security infrastructure*, ***Proceedings of FCS'04, Workshop on Foundations of Computer Security***, Turku Centre for Computer Science Publication 31, June 2004, pp. 251–266.

► ROBERT MIGNONE, *Almost disjoint families and bounding numbers*.
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In [1] it is shown that if lambda is a singular cardinal of cofinality kappa, mu any cardinal less than lambda, then there exists a delta between mu and mu to the power kappa, inclusive, such that delta is the cardinality of a maximal almost-disjoint family within the family of all kappa-sized subsets of functions from kappa to mu.

In [2] using Shelah's pcf theory and exploiting the relationship between the size of the smallest maximal almost disjoint family and bounding numbers, questions from [1] are resolved.

Here, bounding numbers are studied further. First, it is shown that any family of functions from kappa to kappa ordered under a particular quasi-ordering, can be transformed into a kappa almost disjoint family of the same size; this family is within the family of all kappa-sized subsets of functions from kappa to kappa.

Next, in the spirit of [3], the notion of bounding number is generalized to sets of size kappa of functions from kappa plus to kappa plus and a result is given relating the comparative sizes of these bounding numbers to the bounding numbers of families of functions from kappa to kappa.

[1] P. ERDOS AND S. HECHLER, *On maximal almost-disjoint families over singular cardinals*, ***Colloquia Mathematica Societatis Janos Bolyai***, vol. 10, Infinite and finite sets, Keszthely, Hungry (1973), pp. 597–604.

[2] M. KOJMAN, W. KUBIS, AND S. SHELAH, *On two problems of Erdos and Hechler: New methods in singular madness*, ***Proceedings of the American Mathematical Society***, vol. 132 (2004), pp. 3357–3365.

[3] J. D. MONK, *The spectrum of partitions of a Boolean algebra*, ***Archives of Mathematical Logic***, vol. 40, (2001), no. 4, pp. 243–254.

► VICTOR PAMBUCCIAN, *How simple is hyperbolic geometry*?

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I will survey axiomatizations of hyperbolic geometry, and ask what the simplest possible axiomatization is (i) in a given language (here the number of quantifiers in prenex form and quantifier complexity will be the criteria for simplicity) and (ii) regardless of language (here the ideal is to minimize the number and arity of the predicate (and operation) symbols occuring in that language; the intended interpretation of the individual variables of the (one-sorted) languages in which these results are obtained will be varied: points, lines, spheres). It turns out that in certain cases it is possible to have both an absolutely minimalist language and lowest possible quantifier-complexity, in others it is not.

[1] V. PAMBUCCIAN, *Sphere tangency as single primitive notion for hyperbolic and Euclidean geometry*, ***Forum Mathematicum***, vol. 15 (2003), pp. 943–947.

[2] ———, *The simplest axiom system for plane hyperbolic geometry*, ***Studia Logica***, vol. 77 (2004), pp. 385–411.

[3] ———, *Hyperbolic geometry in terms of point-reflection and line-orthogonality*, ***Mathematica Pannonica***, vol. 15 (2004), pp. 241–258.

[4] ———, *The complexity of plane hyperbolic incidence geometry is* $\forall\exists\forall\exists$, ***Mathematical Logic Quarterly***, vol. 51 (2005), pp. 277–281.

► WIM RUITENBURG, *Very intuitionistic theories that admit quantifier elimination.*

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We present a general model-theoretic method by which to construct intuitionistic theories that admit quantifier elimination. In general these theories are very intuitionistic in that they admit sentences that are, in some strong sense, false in classical logic.

► JACK D. STECHER, *An application of intuitionistic topology to economics and accounting.*

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This talk models trade among economic agents with subjective perceptions, using the "basic picture" approach to intuitionistic topology (e.g., in [2]). Each agent i has a private collection of conceptions of the world, denoted S_i. A forcing relation $\Vdash_i$ connects the real world X to the way agent i subjectively experiences the world S_i.

In this setting, it is not immediately clear how economic activity can occur. Agents have no access to the objective world X, nor can one agent i know the private conceptions of some other agent j.

To enable agents to trade, I assume the existence of a shared language T, interpreted as an accounting system. A relation $\Vdash_i$ connects agent i's conceptions S_i to the language T. Thus, the semantics of the reporting language are subjective.

For two agents to trade, they need to know that what they might receive matches what they believe their trades mean. Following [1], I show this requirement is equivalent to a continuity condition on the forcing relations. Some additional structure enables me to discuss how economic equilibrium can be attained in this setting. In contrast to neoclassical economic theory, however, equilibrium is not guaranteed to be effcient.

[1] SILVIA GEBELLATO AND GIOVANNI SAMBIN, *The essence of continuity (the basic picture, II)*, preprint 27, University of Padua Department of Pure and Applied Mathematics, 2001.

[2] GIOVANNI SAMBIN AND SILVIA GEBELLATO, *A preview of the basic picture: A new perspective on formal topology*, ***Types and proofs for programs***, Lecture Notes in Computer Science, vol. 1657, Springer-Verlag, 1998, pp. 194–207.

► DAN E. WILLARD, *A version of the Second Incompleteness Theorem for axiom systems that recognize addition but not multiplication as a total function.*
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The purpose of this talk is to summarize the contents of a recent 32-page article that was published during September of 2004. This article was one of 17 research papers in the book ***First-Order Logic Revisited***, edited by Hendricks, Neuhaus, Pederson, Scheffler and Wansing.

Our article [6] uses the term U-Grounding function to refer to a finite set of functions symbols, one of which includes the Addition operation. It uses the terms Π_i^* and Σ_i^* to refer to the analog of Classical Arithmetic's Π_i and Σ_i formula in a context where our U-Grounding language can formalize multiplication's graph $M(x, y, z)$ only in a Π_0^* form. (Thus, our axiom systems α are *typically unable to prove* multiplication is a total function, i.e., that $\forall x\, \forall y\, \exists z\, M(x, y, z)$).

Let us use the term Tier$-k$ sentence to refer to a sentence that is a Boolean combination of such Π_k^* and Σ_k^* sentences. Also, let Consistent(α, k) denote a sentence specifying that there exists no two *Semantic Tableaux* proofs of both some Tier$-k$ sentence and its negation from the axiom system α.

Some earlier incompleteness theorems by Pudlák [1] and Solovay [2] showed that axiom systems recognizing successor as total are universally unable to prove their Hilbert consistency. Our research has sought to generalize this phenomena for semantic tableaux consistency. Our year-2002 JSL article [4] constructed a Π_1^* sentence V such that no consistent axiom system $\alpha \supset V$ could prove Consistent$(\alpha, 0)$ *whenever* α *recognized Multiplication as total*. However, [4]'s formalism did not extend to the case where α failed to recognize Multiplication as total. Indeed using a variant of [3]'s self-referencing methodology, our article [5] actually showed some systems α can prove sentences as strong as Consistent$(\alpha, 1)$.

In such a context, our just-published paper [6] closes most of the remaining gap about semantic tableaux generalizations for the Second Incompleteness Theorem. It shows there exists a Π_1^* sentence W such that no consistent axiom system $\alpha \supset W$ can prove Consistent$(\alpha, 2)$.

[1] P. PUDLÁK, *Cuts, consistency statements and interpretations*, ***The Journal of Symbolic Logic*** vol. 50 (1985), pp. 423–442.

[2] R. SOLOVAY, Private Communications during 1994; briefly summarized in 4 pages by Appendix A of the article [3].

[3] D. WILLARD, *Self-verifying systems, the Incompleteness Theorem and the Tangibility Reflection Principle*, ***The Journal of Symbolic Logic***, vol. 66 (June 2001), pp. 536–596.

[4] ———, *How to extend the Semantic Tableaux and cut-free versions of the Second Incompleteness Theorem almost to Robinson's arithmetic Q*, ***The Journal of Symbolic Logic***, vol. 67 (March 2002), pp. 465–496.

[5] ———, *Some new exceptions for the Semantic Tableaux version of the Second Incompleteness Theorem*, ***Proceedings of the Tableaux 2002 Conference***, (U. Egly and C. Fermüller, editors), Springer Verlag, Lecture Notes in Artificial Intelligence #2381, pp. 281–297.

[6] ———, *A version of the Second Incompleteness Theorem for axiom systems that recognize addition but not multiplication as a total function, first order logic revisited*, ***Proceedings of September 2003 FOL-75 Conference***, (V. Hendricks, F. Neuhaus, S. A. Pederson, U. Scheffler and H. Wansing, editors), Logos Verlag, Berlin, 2004, pp. 337–368.

► GUOHUA WU, *Isolation and nonboundings*.

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We will prove that following result:

MAIN THEOREM. Given a nonzero c.e. degree **a**, there is a nonzero c.e. degree **b** below **a** and a high d.c.e. degree $\mathbf{d} > \mathbf{b}$ such that **b** is the largest c.e. degree below **d**.

We have the following two results as corollaries:

COROLLARIES. (1) There is a high d.c.e. degree isolated by a low c.e. degree. (Ishmukhametov and Wu, 2001)

(2) There is a high d.c.e. degree bounding no minimal pairs. (Chong, Li and Yang, 2004)

It is a joint work with Angsheng Li (CAS, China) and Yue Yang (NUS, Singapore).

► BYEONG-UK YI, *Is logic axiomatizable*?

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I defend the negative answer to the question in the title by considering sentences that involve plural constructions (in short, plurals), such as the following:

[A] There are some things each of which admires one of them.
[B] There are some critics who admire only one another.

We can see that [A], for example, is logically implied by infinitely many sentences, such as the following:

[A1] c_1 admires c_2.
[A2] c_2 admires c_3.
⋮
[A$_n$] c_n admires c_{n+1}.
⋮

But [A] is not logically implied by any finite number of sentences among these. So the logic of languages that include [A] is *non-compact* and *non-axiomatizable*.

To put this argument in proper perspective, I discuss contemporary accounts of plurals and suggest that they fail to do justice to the logic of plurals because they are based on the traditional view of plurals as devices for abbreviating their singular cousins. And I give a sketch of my view of plurals as substantial devices that complement singular constructions.

[1] G. BOOLOS, *To be is to be a value of a variable (or to be some values of some variables)*, ***The Journal of Philosophy***, vol. 81 (1984), pp. 430–448.

[2] ———, *Nominalist Platonism*, ***Philosophical Review***, vol. 94 (1985), pp. 327–44.

[3] A. CHURCH, ***Introduction to Mathematical Logic***, revised and enlarged edition, Princeton University Press, Princeton, 1956.

[4] W. V. QUINE, ***Philosophy of Logic***, Prentice-Hall, Englewood Cliffs, NJ, 1970.

[5] ———, ***Methods of Logic***, 3rd edition, Routledge and Kegan Paul, London, 1972.

[6] B.-U. YI, ***Understanding the many***, Routledge, New York and London, 2002.

[7] ———, *The logic and meaning of plurals, I & II*, ***Journal of Philosophical Logic***, forthcoming.

Abstract of talks presented by title

► ADIB BEN JEBARA, *About space and time of the elementary particles*.
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In "An idea about time in cosmology" in "ASL Annual Meeting 2004", there is a remark that time need not be linearly ordered. Discrete time is also used as an approximation to time being a continuum.

Now, let us consider what could apply to the singularity of the Big Bang. But, also, let us consider that time could be not linearly ordered for elementary particles in compatibility with quantum mechanics. And, because of time being so, there would be no continuous paths and space would be itself not a continuum and not linearly ordered.

It is difficult to consider such a possibility as we are used to space or time being a continuum at our level.

An infinite space does not imply an infinite quantity of matter-energy. To the infinite set space is applied the set theory ZFU, with urelements, space being either U or $U \times U \times U$. The number of locations of the m particles in the universe is $m+m+\dots$ and is an infinite sum which may not be well defined because it could be that only a particular case of the axiom of choice is true, see web page of Andreas Blass: http://www.math.lsa.umich.edu/~ablass/dpcc.pdf.

That the set of all infinite paths of all the particles of the universe is void could explain the Big Bang. That set is $X \cup i$ with i being a count of time.

To go back to a continuum, a question of Mr. Andreas Blass, we accept the axiom of choice or that U is linearly ordered and a set of paths $X \cup i$ will be a continuum. There is no contradiction in assuming choice not to hold some time and to hold some other time. Like there is no contradiction in assuming space to be Euclidian, and non Euclidian some other time.

► JOHN CORCORAN, *Complete enumerative inductions*.
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This largely expository paper explores analogues of ω-completeness and ω-inconsistency, starting with "finite cases". For each positive integer n, let Ln be a first-order language having exactly n [individual] constants, say the digits '0', '1', etc. of a base-n arithmetic notation. An *instance of a universal sentence* is the result of deleting the quantifier and replacing every occurrence of the freed-up variable by one and the same constant. The *complete induction sentence* [for Ln] is the sentence expressing "for every object x, x is 0 or x is 1 or etc.", for L3, in symbols, $\mathrm{A}x(x = 0 \vee x = 1 \vee x = 2)$. A set of sentences is *n-complete* iff it [deductively] yields every universal sentence each of whose instances it yields. Theorem COM: In order for a set to be n-complete it is necessary and sufficient for it to yield complete induction. A set of sentences is *n-inconsistent* iff it [deductively] yields the negation of some universal sentence each of whose instances it yields. Theorem INC: in order for a set to be *n*-inconsistent it is necessary and sufficient for it to yield the negation of complete induction. In that both of these theorems reduce a condition concerning infinitely many deductions to the deducibility of a single sentence, they are syntactic, or proof-theoretic, results analogous to semantic, or model-theoretic, results about ω-completeness and ω-inconsistency in second-order languages announced in my abstract "Semantic omega properties and mathematical induction", ***The Bulletin of Symbolic Logic***, vol. 3 (1997), p. 280. When we leave either the "finite" case or the second-order case to consider intermediate cases such as sublanguages (involving constants for zero and successor) of the usual first-order languages of number theory, we find that they do not admit of ω-completeness or ω-inconsistency being reducible to the deducibility of a single sentence.

► DOLPH ULRICH, *Single axioms for a class of substructural logics contained in* **BCI**.
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C. A. Meredith's C-pure substitution/detachment system **BCI**—now re-emergent as the implicational fragment of linear logic—has, as axioms $\mathbf{B} = CCpqCCrpCrq$, $\mathbf{C} = CCpCqrCqCpr$, and $\mathbf{I} = Cpp$. Meredith also discovered [2] an equivalent *single* axiom of length nineteen with five variables (an axiom of type $\langle 19, 5\rangle$, we'll say): $CCpCqrCCCssCtqCtCpr$.

Restricted versions of **C** can be gotten by replacing one ore more of its variables with distinct elementary implications:

$\mathbf{C}^{000}$. CCpCqrCqCpr	$\mathbf{C}^{100}$. CCCpsCqrCqCCpsr
$\mathbf{C}^{001}$. CCpCqCruCqCpCru	$\mathbf{C}^{101}$. CCCpsCqCruCqCCpsCru
$\mathbf{C}^{010}$. CCpCCqtrCCqtCpr	$\mathbf{C}^{110}$. CCCpsCCqtrCCqtCCpsr
$\mathbf{C}^{011}$. CCpCCqtCruCCqtCpCru	$\mathbf{C}^{111}$. CCCpsCCqtCruCCqtCCpsCru

Kashima and Kamide [1], using Gentzen-type sequent calculi, initiated investigation of the variants of **BCI** whose simple Hilbert-style axiomatizations can be obtained by adding to **B** and **I** varying combinations of those eight formulas. Exactly five nonequivalent logics result: $\mathbf{BC^{011}I}$, it's two proper extensions $\mathbf{BC^{001}I}$ and $\mathbf{BC^{010}I}$, their common proper extension $\mathbf{BC^{001}C^{010}I}$, and its proper extension **BCI**.

Single axioms for these weaker subsystems of **BCI** exist as well. $CCpqCCCrrCqsCps$, of type $\langle 15, 4\rangle$, suffices for $\mathbf{BC^{011}I}$ and is shortest possible. $CpCCqrCCpCCssCrtCqt$, of type $\langle 19, 5\rangle$, suffices for $\mathbf{BC^{001}I}$. For $\mathbf{BC^{010}I}$, Meredith's own [2] type $\langle 19, 5\rangle$ single axiom $CCpqCCCCCrrssCqtCpt$ for the pair $\{\mathbf{B}, CCCppqq\}$ will do, that pair itself being easily shown to be a compression of $\{\mathbf{B}, \mathbf{C}^{010}, \mathbf{I}\}$. For $\mathbf{BC^{001}C^{010}I}$, the best I have found is the type $\langle 27, 7\rangle$ $CCCCpqCCrCCssCqtCptCCuuvCrv$.

[1] R. Kashima and N. Kamide, *Substructural implicational logics including the relevant logic* E, ***Studia Logica***, vol. 63 (1999), pp. 181–212.

[2] C. A. Meredith and A. N. Prior, *Notes on the axiomatics of the propositional calculus*, ***Notre Dame Journal of Formal Logic***, vol. IV (1963), pp. 171–187.

► XUNWEI ZHOU, *Second level logical calculus of mutually-inversistic logic*.
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The second level logical calculus of mutually-inversistic logic studies second level inductive composition from empirical or mathematical connection proposition to logical connection proposition and second level decomposition from logical connection proposition to empirical or mathematical connection proposition. Take $\{P \overset{-1}{\leqslant} Q\} \overset{-1}{\leqslant} \{\neg Q \overset{-1}{\leqslant} \neg P\}$ as an example, its truth table of inductive composition is shown in Table 1, its truth table of decomposition is shown in Table 2.

TABLE 1

$P \overset{-1}{\leqslant} Q$	$\neg Q \overset{-1}{\leqslant} \neg P$	$\{P \overset{-1}{\leqslant} Q\} \overset{-1}{\leqslant} \{\neg Q \overset{-1}{\leqslant} \neg P\}$
F	F	T
F	T	F / T
T	F	F
T	T	T

TABLE 2

$\{P \overset{-1}{\leqslant} Q\} \overset{-1}{\leqslant} \{\neg Q \overset{-1}{\leqslant} \neg P\}$	$P \overset{-1}{\leqslant} Q$	$\neg Q \overset{-1}{\leqslant} \neg P$
F	F	F / T
F	T	F / T
T	F	F / T
T	T	T

Implicit inductive composition goes like this: Let $P \overset{-1}{\leqslant} Q$ be true, if we can infer that $\neg Q \overset{-1}{\leqslant} \neg P$ cannot be false, so that the third row of Table 1 cannot occur, then we prove $\{P \overset{-1}{\leqslant} Q\} \overset{-1}{\leqslant} \{\neg Q \overset{-1}{\leqslant} \neg P\}$ to be true. After $\{P \overset{-1}{\leqslant} Q\} \overset{-1}{\leqslant} \{\neg Q \overset{-1}{\leqslant} \neg P\}$ has been proved, it can be used to make second level hypothetical inference, which goes like this: When the major premise $\{P \overset{-1}{\leqslant} Q\} \overset{-1}{\leqslant} \{\neg Q \overset{-1}{\leqslant} \neg P\}$ is true and the minor premise $\mathrm{man}(x) \overset{-1}{\leqslant} \mathrm{mortal}(x)$ is true, according to the fourth row of Table 2, we can infer the conclusion $\neg\mathrm{mortal}(x) \overset{-1}{\leqslant} \neg\mathrm{man}(x)$ being true.