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2005 SPRING MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

The Westin St. Francis Hotel, San Francisco, CA, March 25–26, 2005

The 2005 Spring Meeting of the Association for Symbolic Logic was held at the Westin St. Francis Hotel, March 25–26, 2005, in conjunction with the annual meeting of the Pacific Division of the American Philosophical Association. The members of the program committee were Andrew Arana, Anthony S. Gillies, Shaughan Lavine (Chair), and Richard Zach. The ASL hosted a reception on the evening of March 25th.

The program included two three-hour invited symposia: the first, Revising Beliefs about Game Theory: Strategies for Rational Choice, which was organized by Anthony S. Gillies, consisted of three one-hour talks; and the second, Logic Instruction and Philosophy Graduate Training, which was organized by Richard Zach and Andy Arana, consisted of an introduction by the Chairs, followed by a panel discussion. The symposiasts were as follows:

Revising Beliefs about Game Theory: Strategies for Rational Choice.

Rohit Parikh (City University of New York), *Going beyond first order logic.*

Robert Stalnaker (Massachusetts Institute of Technology), *Counterfactuals and dispositions in games.*

Johan van Benthem (University of Amsterdam and Stanford University), *Update and revision in the course of a game.*

Logic Instruction and Philosophy Graduate Training.

Michael Glanzberg (University of California—Davis).

Ted Sider (Rutgers University).

Brian Weatherson (Cornell University).

The program also included ten contributed talks and three abstracts presented by title. Richard Zach and Shaughan Lavine chaired the two sessions of contributed talks.

Abstracts of the panel discussion held, and the invited talks and contributed talks given (in person or by title) by members of the Association for Symbolic Logic follow.

For the Program Committee
Shaughan Lavine

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Abstracts of invited talks in the Symposium on Revising Beliefs about Game Theory: Strategies for Rational Choice

► ROHIT PARIKH, *Going beyond first order logic*.
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In Robert Stoll's 1961 book ***Set Theory and Logic***, which I have used to teach Logic, there is a section entitled *The Predicate Calculus: Symbolizing Everyday Language*. However, the work of Stalnaker, Lewis and others, including modal logicians, shows quite clearly that the predicate calculus does not fully express everyday logical notions. An analysis of counterfactual conditionals for instance requires at least some notion of "possible world", and dealing with Gettier-like puzzles seems to require some sort of non-monotonic reasoning. There is also a fresh connection between games and logic. Game theoretic notions can be used, via Eloise-Abelard games, to understand semantics, both for first order logic and for the independence-friendly logic of Hintikka and Sandu. Conversely, logical notions, especially from the logic of knowledge and belief, can be used to understand how conventional games are played. Here again there are interesting and influential contributions by Stalnaker and others.

► ROBERT STALNAKER, *Counterfactuals and dispositions in games*.
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There is no question that counterfactuals—propositions about what would have happened if certain non-actual circumstances had been realized—are deeply involved in the kind of strategic reasoning studied in game theory, but such propositions usually remain implicit in the informal intuitive commentary on the theory. This paper sketches a way of bringing a formal semantic theory of conditional propositions together with an epistemic model for games, and show how the resulting theory can shed light on some patterns of strategic reasoning, on the representation of the causal structure of a game, on the concept of a strategy, and on the analysis of rationality in a dynamic context.

► JOHAN VAN BENTHEM, *Update and revision in the course of a game*.
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Understanding agents' behaviour in the course of a game involves both information update about their current situation and belief revision of their expectations about the future. Finding an elegant logical framework merging these two phenomena is still a topic of ongoing debate. My talk gives a temporal-doxastic model where the two live in harmony, and shows what sort of logical issues arise. This setting also allows for comparison with other logical frameworks.

Abstracts of invited talks in the Panel Discussion on Logic Instruction and Philosophy Graduate Training

► Panelists: ANDREW ARANA (CO-CHAIR), MICHAEL GLANZBERG, TED SIDER, BRIAN WEATHERSON, AND RICHARD ZACH (CHAIR), *Panel Discussion: Logic instruction and philosophy graduate training*.
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Formal logic in the philosophy curriculum. Over more than half a century, formal logic has held an important position in analytic philosophy and consequently in the philosophy curriculum at English-speaking philosophy programs, both at the undergraduate and graduate level. Typically, undergraduates are required to complete a first course in formal logic covering semantics and proof theory of classical first-order logic. A graduate course on metalogic dealing with completeness and Löwenheim-Skolem theorems, undecidability and often also Gödel's Incompleteness Theorems is a standard requirement in Ph.D. programs.

The Special Session on Philosophy and Logic Education provides a forum for reflection on and evaluation of the form and content of such courses, and the place and role formal logic courses play or should play in training in philosophy, especially at the graduate level.

Logic and philosophy. One issue explored at the session is the question of how formal logic relates to other areas of philosophy, and how logic courses and requirements should relate to other courses and requirements in Ph.D. programs. On the one hand, working in formal logic is certainly a different kind of enterprise than, say, working in metaphysics or ethics. At the introductory level, the main motivation to require courses in logic is simply that it trains students in reasoning and assessing arguments. Logic provides the tools (formalization, deductive proofs, truth tables and interpretations) to do this. At this level, logic courses are more in the business of imparting skills than of a body of knowledge. The situation is somewhat different regarding requirements at the graduate level, where logic training presupposes those skills to a large extent, and students are taught *results*, such as the completeness theorem, and their proofs. And as these are in the first instance mathematical results, the question might be raised, "Why burden philosophy Ph.D. students by requiring such courses?"

Methodology. There are undeniably similarities in the methodology of formal logic and philosophical methodology: often, the limitative results of metalogic have served as examples for how questions could be made precise so that they are amenable to a (often negative) solution, and how to give such solutions with mathematical rigor. One motivation for requiring metalogic in philosophy graduate training then is that it imparts to students an appreciation for limitative results, and how they can be proved.

Content. There are also connections between formal logic and other areas of philosophy in terms of content. This is most obviously so in the philosophy of logic (e.g., theories of truth), the philosophy of mathematics (e.g., Gödel's theorems as refutation of logicism and formalism), and the philosophy of language (formal semantics), but also in other areas. One might think of Lucas' argument against mechanism, Putnam's model-theoretic argument, or the contributions of formal logic in illuminating modality, the logic of knowledge, mereology, etc. Philosophy Ph.D.'s arguably should be able to understand, appreciate, and apply such results, and acquire the foundation necessary for further training enabling them to contribute to this literature. A more specific question for the panel then is how a graduate logic course would best accomplish this. Which results should be taught? How should they be taught? What is the relative importance of the topics now standard in graduate logic courses and more recent developments such as formal theories of truth or intensional logics and possible worlds semantics? Which recent developments should be taught, in what form, and where (required courses, supplementary courses, or incorporated into subject-specific courses, e.g., possible worlds semantics in metaphysics courses)?

History of analytic philosophy. The history of philosophy is rightfully considered a central part of the philosophy curriculum. As the history of *analytic* philosophy matures as a recognizable field of study (and teaching), a background in logic and metalogic becomes increasingly important. For the major figures in early (Frege, Russell, Wittgenstein) and more recent (Carnap, Quine, Lewis) analytic philosophy, logic was a central tool in philosophy and an area to which they themselves contributed. On the one hand, this raises similar questions as above: What to include or emphasize in graduate logic courses so to enable graduate students to understand, e.g., Russell's theory of types, or what Frege's Axiom V says? On the other hand, perhaps graduate logic courses should incorporate the philosophical aspects of the development of logic in the 19th and 20th century?

Additional questions. (1) *Textbooks*: To some extent, the form and content of courses is influenced by the available textbooks. In graduate level logic courses, the two most popular texts are probably Boolos, Burgess, and Jeffrey's ***Computability and Logic*** and Enderton's ***A Mathematical Introduction to Logic***. In light of the issues outlined above, how do they and other texts serve the purpose? What would an ideal graduate level logic text for philosophy look like? (2) *Logic in the Profession*: One important aspect of graduate training is, of course, preparation to teach. How important is it to have received advanced training in logic in order to effectively teach an introductory course? How can graduate training in logic enhance the effectiveness of introductory logic teachers?

Materials from this session are available online at
`http://www.phil.ucalgary.ca/asl-cle/sf05/`

Abstracts of contributed talks

► MAJID AMINI, *Justification of deduction*.
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Rudolf Carnap once remarked that the epistemological situation in inductive logic is not worse than that in deductive logic, but indeed quite analogous to it. The purpose of this paper is to pursue parallelisms between induction and deduction in terms of the issue of justification. David Hume famously argued that induction faces a crisis of rationality as it cannot be justified either deductively or inductively. Apparently, a direct analogue could be constructed for deduction where it will be shown that deduction suffers from the same problem of justification: that is, it cannot be justified either inductively or deductively. The discussion then focuses on the limitations and shortcomings of claims made by, for example, Michael Dummett and Dag Prawitz that, unlike induction, at least some fragments of deductive logic can be justified. However, an attempt will be made to show that the parallelism could still be extended to an additional and more interesting analogy between deduction and induction in terms of Nelson Goodman's 'the new riddle of induction', *i.e.*, the problem of “grue” predicate. It will be contended that likewise a “new riddle” could be formulated for deduction in terms of Arthur Prior’s “tonk” connective. Yet, it may be objected, especially in the case of the latter parallelism, that there is a sharp disparity between deduction and induction, and the similarity is only superficial. For, again unlike induction, in deduction there are, for example, proof-theoretic principles such as Emil Post’s “conservative extension”, or some model-theoretic variants of it, that proscribe the introduction of such rogue logical connectives. The remainder of the paper will, therefore, be devoted in a Carnapian spirit to an examination of the status of those constraints.

► OTÁVIO BUENO, *Pluralism about logical pluralism*.
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According to logical pluralism, there is more than one right logic; that is, more than one consequence relation adequate for a given domain. J. C. Beall and Greg Restall [1] have articulated this view in terms of cases: A is a consequence of B if every case in which B is true is also a case in which A is true, and depending on the cases that are considered, different logics emerge. For example: for consistent and complete cases, we obtain classical logic. For consistent and incomplete cases, we get intuitionistic logic. And for inconsistent and complete cases, paraconsistent logic is obtained. Despite the fact that this account provides a systematic formulation of logical pluralism, I argue that Beall and Restall’s proposal faces significant difficulties. (i) The proposal over-generates since it takes as logical truths (and hence as logical consequences of the empty set) statements that are not usually taken to be so, such as the continuum hypothesis. (ii) The proposal also under-generates since it fails to take as logic perfectly reasonable instances of logic, such as quantum logic. After supporting these claims, I provide an alternative conceptualization of logical pluralism that, by highlighting the context dependence of applied logic, overcomes the difficulties raised.

[1] J. C. Beall and G. Restall, *Logical pluralism*, ***Australasian Journal of Philosophy***, vol. 78 (2000), pp. 475–493.

► PHILIP EHRLICH, *The Peircean linear continuum: a surreal model*.
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In [1], J. H. Conway introduced a real-closed field containing the reals and the ordinals as well as a great many other numbers including $-\omega$, $\omega/2$, $1/\omega$, $\sqrt{\omega}$ and $\omega - \pi$ to name only a few. Indeed, this particular real-closed field, which Conway calls *No*, is so remarkably inclusive that, subject to the proviso that numbers—construed here as members of ordered “number” fields—be individually definable in terms of sets of von Neumann-Bernays-Gödel set theory with Global Choice (NBG), it may be said to contain “All Numbers Great and Small.” In [2] and [3] we suggested that whereas the real number system should merely be regarded as constituting an arithmetic continuum modulo the Archimedean axiom, *No* may be regarded as a sort of absolute arithmetic continuum (modulo NBG). In the present paper we introduce a formal replacement, C_P , for the intriguing linear continuum sketched by Charles Sanders Peirce at the turn of the twentieth century, and point out that by limiting *No* to the real-closed integral domain consisting of its finite and infinitesimal members one obtains a well-suited arithmetization of C_P whose properties mimic those of *No* established in [3] and [4].

[1] J. H. Conway, ***On numbers and games***, Academic Press, 1976.
[2] P. Ehrlich, *Universally extended continua*, ***Abstracts of Papers Presented to the American Mathematical Society***, vol. 10 (January, 1989), p. 15.
[3] ———, *Universally extending arithmetic continua*, ***Le Continu Mathematique, Colloque***

de Cerisy, (H. Sinaceur and J. M. Salanskis, editors), Springer-Verlag France, Paris, 1992, pp. 168–178.

[4] ———, *Number systems with simplicity hierarchies: a generalization of Conway's theory of surreal numbers*, ***The Journal of Symbolic Logic***, vol. 66 (2001), no. 3, pp. 1231–1258.

► JAAKKO HINTIKKA, *How to prove the consistency of elementary arithmetic*.
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When a system of elementary arithmetic is based on independence-friendly (IF) logic instead of received first-order logic, its consistency can be proved in the same system by formulating its own syntax by means of Gödel numbering and then forming the sentence Dis(h) with the Gödel number h where h is the numeral representing h and where Dis(x) says that the sentence with the Gödel number x is disprovable. This predicate can be formulated because there exists a complete disproof procedure in IF first-order logic. Moreover, the completeness of this procedure can be proved by means of König's lemma, which is a valid principle of IF first-order logic. Dis(h) cannot be true unless the system is inconsistent. Hence there is a sentence in the system that is not disprovable and hence is true on the no-counterexample interpretation. Moreover, since the principle of excluded middle does not hold in IF logic, this argument can be considered elementary and hence a solution to Hilbert's second problem.

► DOMINIC J. D. HUGHES, *Proofs without syntax*.
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"Mathematicians care no more for logic than logicians for mathematics." Augustus De Morgan, 1868.

Proofs are traditionally syntactic, inductively generated objects. This talk presents an abstract mathematical formulation of propositional logic in which proofs are combinatorial (graph-theoretic), rather than syntactic. It defines a *combinatorial proof* of a proposition P as a graph homomorphism $h\colon G \to G(P)$, where $G(P)$ is a graph associated with P, and G is a colored graph. The main theorem is soundness and completeness: P is true iff there exists a combinatorial proof $h\colon G \to G(P)$.

A six-page technical report detailing this work is available at the Mathematics arXiv [1].

[1] Dominic J. D. Hughes, *Proofs without syntax*, Technical report, Mathematics arXiv e-print archive, August 20, 2004. Available at http://arxiv.org/abs/math/040828.

► JUN MIYOSHI, *Conversations and situations*.
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The purpose of this paper is to show that conversations are games and to explain how conversations develop using the ideas of equilibrium, utility, and situation. Conversations are games in the game theoretic sense. The speakers are the players and their speech acts are the actions in the game. A conversation has the subgame perfect equilibrium. Suppose that A says "Could you open the window?" and B says "Sure" and opens the window. In this conversation, roughly, A's asking B to open the window, and B's accepting it and opening it make up the subgame perfect equilibrium (asking is better than not asking for A, accepting than rejecting for B, and opening than not opening for B). Ideally, a conversation develops along the subgame perfect equilibrium path. Simple conversations may proceed along these paths. But complicated conversations need not, because their players do not see the total game trees and can not choose the global best strategies.

In complicated conversations, a player is merely able to choose the local best strategy in his or her horizon at each turn. He or she has the static evaluation function(s) and can evaluate the situations corresponding to the terminal nodes relative to horizons. These evaluations are expressed as the utilities of the situations. Thus, a player can find the local best strategy. That can also explain why long conversations might be winding and unforeseeable.

[1] J. L. Austin, ***How to do things with words***, 2nd edition. Harvard University Press, 1975.

[2] J. Barwise and J. Perry, ***Situations and attitudes***, MIT Press, 1983.

[3] P. Parikh, *The use of language*, ***Stanford University Center for the Study, J. R. Searle, 1969***, Speech Acts, Cambridge University Press, 2001.

[4] J. von Neumann and O. Morgenstern, ***Theory of games and economic behavior***, 3rd edition, Princeton University Press, 1953.

► MATTHEW E. MOORE, *Naturalism, truth and beauty in mathematics*.
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In this paper I explore the relationship between mathematical realism (MR: mathematics is a form of knowledge) and scientific naturalism (SN: all our knowledge is scientific). In the first half of the paper I consider an argument for mathematial *ir*realism (i.e., not-MR) which proceeds by way of mathematical naturalism (MN: mathematics is methodologically immune from criticism on scientific grounds). Clearly MN, taken together with SN, implies not-MR. If SN implies MN, then a scientific naturalist should be a mathematical irrealist.

The most natural derivation of MN from SN relies on the Benefit Thesis (BT: the methodological autonomy of mathematics is in the best interests of science) and the Autonomy Thesis (AT: mathematical methods are not scientific methods). Since AT and SN already imply mathematical irrealism, anyone who accepts this derivation of MN from SN already has sufficient grounds for mathematical irrealism. But can AT itself be derived from SN? Here again a natural strategy suggests itself to the scientific naturalist: show that no reasonable naturalized epistemology can exhibit mathematical methods as scientific ones. This is easily done if one's naturalized epistemology is an empiricist one, and due attention is paid to branches of mathematics like the Gödel Program in set theory, whose methods are hard to square with empiricism.

Of course the realist can accept all this and still maintain that we would do better to modify the epistemology than to discard MR. I will argue in the second half of the paper that a mathematical naturalist should go along with the realist on this. For it proves to be exceedingly difficult for an irrealist to justify BT if MR is not true; and without BT our earlier derivation of MN will not go through.

► ROHIT PARIKH, ERIC PACUIT, AND EVA COGAN, *The logic of knowledge based obligation*.
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When we consider the obligation of an agent to do something or to refrain from doing something, then the question of what the agent knows can become relevant. Suppose for instance that Jill is a physician whose neighbour Sam is ill. If she does not know that he is ill, she is under no obligation to act, but if she does know, then she has an obligation to treat him. We can deal with such cases using a history based model where the set of possible global histories (including all events) is determined (for an agent) by the sequence of local events which the agent herself has seen. What is known is what is true in all such global histories. The logic of knowledge here is S5.

A more interesting case arises if the agent only has default knowledge, (as in the Gettier puzzle) where the knowledge is obtained not by quantifying over all global histories possible for the agent, but only the most likely or default ones. The logic now is KD45.

In such a situation an agent may lose an obligation arising from its default or defeasible 'knowledge' if such knowledge is negated by further information.

The Kitty Genovese murder case (1964) can be dealt with in our framework which explains the unfortunate behavior of the neighbours who watched the murder but did not act to help.

► MICHAEL RESCORLA, *Church's Thesis and the conceptual analysis of computability*.
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Church's Thesis asserts that a number-theoretic function is intuitively computable if and only if it is Turing-computable. A related thesis asserts that Turing's discussion provides a conceptual analysis of the intuitive notion of computability. I endorse Church's thesis, but I argue against the related thesis. I argue that purported conceptual analyses based upon Turing's work involve a subtle but persistent circularity. Turing machines manipulate syntactic entities. To specify which number-theoretic function a Turing machine computes, we must correlate these syntactic entities with numbers. I argue that, in providing this correlation, we must demand that the correlation itself be computable. Otherwise, the Turing machine will compute uncomputable functions. But if we presuppose the intuitive notion of a computable relation between syntactic entities and numbers, then our analysis of computability is circular. We fail to illuminate computability in more primitive terms.

► NEIL W. TENNANT, *On minimal mutilation*.
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Suppose K is a consistent theory and A is a contingent K-theorem.

A contraction of K with respect to A is a consistent subtheory of K that does not imply A, and which "contains as much of K as possible". That is, contractions should be minimally mutilating.

A revision of K with respect to $\sim A$ is a consistent theory implying $\sim A$ and which "contains as much of K as possible". So revisions, too, should be minimally mutilating. But a revision ought also to contain "as little as possible outside K"; that is, revisions should be minimally bloating.

AGM-theory offers an explication of what it is for a contraction J of K with respect to A to be minimally mutilating: for every p in K, $J, A \vdash p$. This is the Postulate of Recovery. For well-known reasons, Recovery does not capture the intuitive idea of minimal mutilation. Surprisingly, AGM-theory does not offer any account of what it is for a revision to be minimally bloating.

In this paper I shall offer new formal explications of both minimal mutilation and minimal bloating.

Abstracts presented by title

► JOHN CORCORAN, *Logically equivalent false universal propositions with different counter-example sets*.
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This largely expository and pedagogical article discusses the phenomenon indicated by the title and other related phenomena. Two propositions are *logically equivalent* if each is a logical consequence of the other. Our examples use standard first-order logic with the class of numbers [non-negative integers] as universe of discourse. A number x is a counterexample for a universal proposition "Ax P(x)" iff it is not the case that P(x). In some familiar cases logically equivalent false propositions have the same counterexamples. "Every number that is not even is prime" and "Every number that is not prime is even" are both *counterexemplified* by the non-prime odd numbers. Moving along a spectrum, we find cases which share some but not all counterexamples. "Every number divides every other number" is counterexemplified by every number except one, whereas its equivalent "Every number is divided by every other number" is counterexemplified by every number except zero. On the other end of the spectrum there are cases having no counterexamples in common: "Every even number precedes every odd number" is counterexemplified only by even numbers, whereas its equivalent "Every odd number is preceded by every even number" is counterexemplified only by odd numbers. One easy result is that given any non-empty finite set of numbers every false universal proposition is logically equivalent to another counterexemplified exclusively by numbers in the given set. If n is a numeral, then "P(n)" and "Q(n)" are corresponding instances of universal propositions "Ax P(x)" and "Ax Q(x)". As seen above, it is not necessary for corresponding instances of logically equivalent universal propositions to be logically equivalent. The above phenomenon is quite common; indeed every non- tautological universal proposition is in the same logical form as a false universal proposition that is logically equivalent to another false universal having a different set of counterexamples.

► DOLPH ULRICH, *Single axioms for the finitely axiomatizable extensions of strict-implicational* **S3** *and of some of its weaker subsystems*.
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For any theorems $\alpha_1, \alpha_2, \ldots, \alpha_n$ of a logic **L** whose connectives include C, let $\mathbf{T_1} = CC\alpha_1 C\alpha_2 x_1 x_1$, $\mathbf{T_2} = CC\mathbf{T_1}C\alpha_3 x_2 x_2, \ldots$, and $\mathbf{T_{n-1}} = CC\mathbf{T_{n-2}}C\alpha_n x_{n-1} x_{n-1}$. Generalizing the results of [1] (cf. [2, p. 44, Theorem 8]), we have:

THEOREM. $\mathbf{T_n} = CCCCpqCCrsCpqC\mathbf{T_{n-1}}CCyzx$ *is provable in any substitution/detachment system* **L** *in which* $\alpha_1, \alpha_2, \ldots, \alpha_n$, $CCpqCCrsCpq$, *and* $CCpqCCCpqrr$ *are theorems and* $C\alpha\beta \Rightarrow CC\gamma\alpha C\gamma\beta$ *and* $C\alpha CC\beta\gamma\delta \Rightarrow CC\beta\gamma C\alpha\delta$ *are derived rules. In turn,* $\mathbf{T_n}$ *gives, with substitution and detachment,* $CCpqCCrsCpq$ *and each of* $\alpha_1, \alpha_2, \ldots \alpha_n$.

One of the weaker logics (though certainly not the weakest) covered by the Theorem is the system **C3** with axioms **Syl** $= CCpqCCqrCpr$, **Hilbert** $= CCpCpqCpq$, and **Weak Irrelevance** $= CCpqCrr$, which is the strict-implicational fragment of the modal logic **S3**.

COROLLARY. *Every finitely axiomatizable extension of* **C3** *can be axiomatized by a single axiom*.

In individual cases, the single axioms provided by the Theorem can typically be shortened. The one provided for **C3** itself, for example, is 43 symbols long and contains 14 variables, but can be improved at least to the 37-symbol 11-variable $CCCCpCpqCCrsCpqCCCtuCCvwC$-$CCxxCwyCvyzz$. Taking the latter formula as our axiom, 1, and letting Dx.y be the most general result of detaching an instance of formula y from an instance of formula x, D1.1 = 2. $CCpqCCCrrCqsCps$ and D2.2 = 3. $CCCppCCCCqqCrsCtsuCCtru$. Thereafter, DD3D2.3D3D2.3 = **Syl**, DDD1D1.2.1D3.2 = **Hilbert**, and DD1D3.1DD2.3D3.3 = **Weak Irrelevance**.

[1] S. TANAKA, *On variants of axiom systems of propositional calculus. I.*, ***Proceedings of the Japan Academy***, vol. 42 (1966), pp. 108–110.

[2] A. TARSKI, ***Logic, semantics, metamathematics***, Clarendon Press, Oxford, 1956.

► XUNWEI ZHOU, *First level explicit multiple inductive composition*.
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First level explicit multiple inductive composition is from zeroth order fact propositions to establish first order multiple empirical or mathematical connection proposition. Suppose we want to establish $\mathrm{nat}(x)/ \overset{-1}{\wedge} \{\mathrm{nat}(y) \overset{-1}{\leqslant} x \leq y\}$ (there exists natural number x such that for all natural number y, x is less than or equal to y, that is, there exists the least natural number) from nat(0), nat(1), nat(2), nat(3), $1 \leq 2$, $2 \leq 2$, etc. *Then*, we make the relation matrix of $x \leq y$ shown below.

$$\begin{array}{c|c} & y \quad 0 \quad 1 \quad 2 \quad 3 \\ x & \\ \begin{array}{c}0\\1\\2\\3\end{array} & \begin{bmatrix} \mathrm{T} & \mathrm{T} & \mathrm{T} & \mathrm{T} \\ \mathrm{F} & \mathrm{T} & \mathrm{T} & \mathrm{T} \\ \mathrm{F} & \mathrm{F} & \mathrm{T} & \mathrm{T} \\ \mathrm{F} & \mathrm{F} & \mathrm{F} & \mathrm{T} \end{bmatrix} \end{array}$$

Notice that the first row of the relation matrix are all T, that is to say, there exists natural number x, that is 0, such that for all number y, x is less than or equal to y. Therefore $\mathrm{nat}(x)/ \overset{-1}{\wedge} \{\mathrm{nat}(y) \overset{-1}{\leqslant} x \leq y\}$ holds.