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REVIEWS

The Association for Symbolic Logic publishes analytical reviews of selected books and articles in the field of symbolic logic. The reviews were published in ***The Journal of Symbolic Logic*** from the founding of the JOURNAL in 1936 until the end of 1999. The Association moved the reviews to this BULLETIN, beginning in 2000.

The Reviews Section is edited by Alasdair Urquhart (Managing Editor), Lev Beklemishev, Mirna Džamonja, David M. Evans, Erich Grädel, Geoffrey P. Hellman, Denis Hirschfeldt, Julia Knight, Michael C. Laskowski, Roger Maddux, Volker Peckhaus, Wolfram Pohlers, and Sławomir Solecki. Authors and publishers are requested to send, for review, copies of books to *ASL, Box 742, Vassar College, 124 Raymond Avenue, Poughkeepsie, NY 12604, USA.*

In a review, a reference "JSL XLIII 148," for example, refers either to the publication reviewed on page 148 of volume 43 of the JOURNAL, or to the review itself (which contains full bibliographical information for the reviewed publication). Analogously, a reference "BSL VII 376" refers to the review beginning on page 376 in volume 7 of this BULLETIN, or to the publication there reviewed. "JSL LV 347" refers to one of the reviews or one of the publications reviewed or listed on page 347 of volume 55 of the JOURNAL, with reliance on the context to show which one is meant. The reference "JSL LIII 318(3)" is to the third item on page 318 of volume 53 of the JOURNAL, that is, to van Heijenoort's *Frege and vagueness*, and "JSL LX 684(8)" refers to the eighth item on page 684 of volume 60 of the JOURNAL, that is, to Tarski's *Truth and proof*.

References such as 49***5*** or 280***1*** are to entries so numbered in *A bibliography of symbolic logic* (the JOURNAL, vol. 1, pp. 121–218).

Reflections on the Foundations of Mathematics: Essays in Honor of Solomon Feferman. edited by Wilfred Sieg, Richard Sommer, and Carolyn Talcott, Lecture Notes in Logic, vol. 15. A. K. Peters, Ltd., Natick, MA, 2002, viii + 440 pp.

This review of a festschrift marking the 70th birthday of distinguished logician Solomon Feferman I have composed from my own perspective, mathematical intuitionism. Alas, intuitionism is a view upon which Professor Feferman does not smile, despite its historical proximity to a position he has long maintained. That position is the predicativism bequeathed to us variously by Henri Poincaré, Bertrand Russell and Hermann Weyl. As an intuitionist, I will therefore be gazing over my garden wall onto an adjacent property, trying to descry new building projects under way there and any recent improvements.

The topics addressed in the volume mirror the breadth of its dedicatee's interests and recall the range of his accomplishments. One finds papers of quality and import on proof theory (Part I), logic and computation (Part II), applicative and self-applicative theories (Part III), and philosophy of modern mathematics and logic (Part IV). Part I makes a promising start with Jeremy Avigad's *Ordinal analysis without proofs*, a model of the clear, direct exposition much to be desired in mathematical logic, but too often neglected or even disparaged. Am I utterly foolish in supposing that more of this kind of writing—open and welcoming to the neophyte, sensitive to history—would have retained for mathematical logic and philosophy of mathematics the respect of mathematicians and philosophers generally?

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Avigad's facility in both word and metamathematics is reminiscent of his mentor's celebrated *Systems of Predicative Analysis I* (***The Journal of Symbolic Logic***, vol. 29, 1964, pp. 1–30). In other quarters of Part I, the troubled waters of technicality rise alarmingly high, as the merest glance at the tide of symbols washing over pages 54 through 56 of Wilfried Buchholz's ironically-entitled contribution, *Relating ordinals to proofs in a perspicuous way*, so gloriously will confirm. Harvey Friedman marches on stage to sing a few more bars of an old song about Kruskal's Theorem, independence results and reverse mathematics he started composing over twenty years ago. Reverse mathematics is also the theme of papers by Ulrich Kohlenbach and by Stephen Simpson. At six pages in length, Simpson's has the megaphonic character of an advertising spot. Prospective readers of either piece are strongly advised to have memorized huge chunks of Simpson's ***Subsystems of Second Order Arithmetic*** (Perspectives in Mathematical Logic, Berlin: Springer-Verlag, 1999) before proceeding. Logicians who are not avid proof-theorists living on the Stanford–Munich fault line are warned that, unless they read Avigad, they will not gain from this section of the book easy entrée to the latest developments in ordinal analysis or subsystems of second-order arithmetic.

The articles of Part II are a mixed lot, but here the going is easier for novices, with Jens Erik Fenstad writing on the present state and future prospects of computability theory, J. Tucker and J. Zucker on algebraic specifications with infinitary logic, Tim Fernando on type theory and natural language, Robert Constable and Karl Crary on a theory of types supporting intensional reasoning and appropriate to an automated analysis of complexity. The majority of these papers bear the earmarks of Feferman's approaches to proof and type theory. Fenstad's survey, *Computability theory: structure or algorithm*, is especially attractive, manifesting admirable tolerance for alternative viewpoints, breadth of vision, and historical sense. That said, the piece suffers from a sad hermeneutic disequilibrium. In his introductory and concluding sections, Fenstad cries in the wilderness for a meeting of minds between recursion theorists and theoretical computer scientists within a single, unified study of both abstract and quotidian computability. However, apart from citing a few promising (now middle-aged) results on computability over the real numbers, e.g., those inspired by M. Pour-El and J. Caldwell's *On a simple definition of computable functions of a real variable—with applications to functions of a complex variable* (***Zeitschrift für Logik und Grundlagen der Mathematik***, vol. 21, 1975, pp. 1–19) the author devotes little of the survey itself to the confluence he desires. There is only a single mention of computable analysis and no mention of the advantages, widely recognized, that intuitionistic mathematics holds for the proper care and management of computability theories. One advantage is the metamathematical discipline imposed on theorists by the use of realizability interpretations that neatly isolate the distracting clutter of coding and machines, which are taken up in proving the soundness of the realizability in question, from the clean and often machine-free business of pursuing computable mathematics. Under realizability, that mathematical business appears in the welcome guise of a pure constructive mathematics governed by an extension of intuitionistic logic. Given the liaison between intuitionism and continuity, one regrets that Fenstad did not expand on his cursory remark concerning discontinuity.

I pass rapidly over the papers comprising Part III, Applicative and Self-Applicative Theories, only one of which is not devoted to an examination of Feferman's ideas, or further developments of his ideas, on explicit mathematics and the clutch of systems he devised to capture Errett Bishop's constructivism. Part IV features contributions of a more reflective or philosophical character, those of Charles Parsons, *Realism and the debate on impredicativity 1917-1944*, and William Tait, *Remarks on Finitism*, standing out. Parsons wishes to assay the prospects for defending impredicative definitions, assumptions and principles in mathematics. This seems an odd project, there being little call to defend our hugely successful and thoroughly impredicative mathematics from the charges Russell and Poincaré brought against it. Their charges were neither cogently stated nor proven. They were so packed with fallacies, e.g., illicit congress between use and mention, that the worries they once fostered now seem more than mildly neurotic. Parsons looks hard and long to see if a defense of impredicativity requires platonism in the philosophy of mathematics. I would have thought that a quick glance at intuitionistic mathematics—defensible, highly impredicative, nonplatonistic—would suffice to show that there is no such requirement. Tait labors to shore up the contention, first advanced in his *Constructive reasoning* (***Logic, Methodology and Philosophy of Science III***, Van Rootselaar and Staal (editors), Amsterdam: North-Holland, 1967, pp. 185–199) that Hilbert's (or anybody's) finitistic arithmetic has to be primitive recursive arithmetic (PRA). Tait is willing to allow that, on occasion, Hilbert and Bernays may have mooted the possibility that finitism extends beyond the limits of PRA. Even so, he insists that this possibility is no counterexample to his original claim.

Predicativism was conceived in the same overheated chamber as intuitionism; their common parent was the marvelous suggestion that conventional mathematicians, especially set theorists, were committing gross errors of sorts hitherto undetected. Intuitionists took the error to lie in a thoughtless construction of higher mathematics on a framework of rotten materials: invalid contentual inferences and principles preventing a proper analysis of the phenomenal continuum. Early predicativists touted the supposed discovery of a vicious-circle fallacy, the blithe assumption that such higher-order entities as sets, functions and real numbers can properly be defined by referring in the *definiens* to collections and systems that contain the *definiendum*. Poincaré, Russell and, later, Weyl demanded that impredicativity be excised and mathematics trimmed to fit new predicativistic patterns. In those days, some accomplished mathematicians seemed willing to pick up the scissors and cut.

But that was long ago. In the volume under review, one searches in vain for even a brief expert assessment of the scientific prospects for predicative trimming. Predicativism, again like Brouwerian intuitionism, seems to have suffered a most ignominious intellectual extinction: it was ignored to death. After handing over to mainstream projects in logic, computability and theoretical computer science intellectual legacies of great value, its potent philosophical manifesto was seldom rationally evaluated. 'Working mathematicians,' snug in departmental offices, publish or perish in happy ignorance of predicativism or intuitionism, and couldn't give a damn. Yet, no one in a book devoted to praising and appraising the continuing achievements of a scholar who wears the predicativist mantle of Weyl, wishes to pause and ask, "What went wrong and why?"

Often, the volume's editing, typesetting and printing, if such terms still apply in the computer age, are less than happy: evidence the mangled appearance of the section title to Part III on page 249. A. K. Peters's editors and proofreaders were understandably too worn out by marching narrow tracks through forests of Greek letters to stalk such grade school solecisms as failures in subject-verb agreement and misspellings of common English words.

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George Tourlakis. ***Lectures in Logic and Set Theory, volumes 1 and 2.*** Cambridge studies in advanced mathematics, vol. 83. Cambridge University Press, Cambridge, UK, 2003. xi + 328 and xv + 575 pp.

This is a detailed two-volume development of mathematical logic and set theory, written from a formalist point of view, aimed at a spectrum of students from the third-year undergraduate to junior graduate level. Volume 1 presents the heart of mathematical logic, including the Completeness and Incompleteness theorems along with a bit of computability theory and accompanying ideas. Tourlakis aspires to include "the absolutely essential topics in *proof*, *model* and *recursion* theory" (vol. 1, p. ix). In addition, for the final third of the volume, Tourlakis provides a proof of the Second Incompleteness Theorem "right from Peano's axioms, … gory details and all," which he conjectures "is the only complete proof in print [from *just* Peano arithmetic] other than the one that was given in Hilbert and Bernays (1968)" (vol. 1, p. x). In the opening page of Chapter II, Tourlakis provides a lucid explanation of the proof in plain language, before diving into the details and emerging a hundred pages later with the provability predicate, the derivability conditions and a complete proof. Tempering his formalist tendencies, Tourlakis speaks "the formal language with a heavy 'accent' and using many 'idioms' borrowed from 'real' (meta)mathematics and English," in a mathematical argot (vol. 1, p. 39). In his theorems and proofs, therefore, he stays close to the formal language without remaining inside it.

But let me focus on volume 2, a stand-alone development of axiomatic set theory, containing within it a condensed version of volume 1. The book emphasizes the formal foundations of set theory and, like the first volume, gives considerable attention to the details of the elementary theory. Tourlakis is admirably meticulous in maintaining the theory/metatheory distinction, with a careful explanation of the role of inductive arguments and constructions in the metatheory (vol. 2, p. 20) and a correspondingly precise treatment of axioms, theorems and their respective schemes throughout. What is more, he sprinkles the text with philosophical explanations of the theory/metatheory interaction, giving a clear account, for example, of how it is that we may use apparently set theoretic arguments in the metatheory without circularity (vol. 1, p. 10–12). After developing the logical background, he paints the motivating picture of the cumulative hierarchy, the process by which we imagine sets to be built, with Russell's paradox as a cautionary tale. In Chapter III, the axioms of set theory march forward in succession. He presents them gradually, motivating them from the cumulative hierarchy and deriving consequences as they appear. This treatment includes the Axiom of Choice, which he motivates, impressively, by developing Gödel's constructible universe L sufficiently to see that the Axiom of Choice holds there. Later, he revisits the constructible universe more formally, and by the end of the book his formal set theoretic development encompasses even the sophisticated topic of forcing. The book culminates in Cohen's relative consistency proof, via forcing, of the failure of the Continuum Hypothesis.

Interestingly, Tourlakis' version of ZFC set theory, like Zermelo's, allows for (without insisting on) the existence of urelements, atomic objects that are not sets, but which can be elements of sets. His reason for this is philosophical and pedagogical: he finds "it extremely counterintuitive, especially when addressing undergraduate audiences, to tell them that all their familiar mathematical objects—the 'stuff of mathematics' in Barwise's words—*are* just perverse 'box-in-a-box-in-a-box ... ' formulations built from an infinite supply of empty boxes" (vol. 2, p. xiii). The enrichment of the theory to allow urelements requires only minor modifications of the usual ZFC axioms, such as the restriction of Extensionality to the sets and not the urelements. The application of the definition $a \subseteq b \iff \forall z(z \in a \Longrightarrow z \in b)$ even when a or b are urelements, however, causes some peculiarities, such as the consequence that urelements are subsets of every object, including each other. Consequently, the axiom asserting that the urelements form a set (Axiom III.3.1), is actually deducible via the Comprehension Axiom from Tourlakis' version of the Power Set Axiom, which asserts that for every object a there is a set b such that $\forall x(x \subseteq a \Longrightarrow x \in b)$, since any such b must contain all urelements.

At times, the author employs what some might take as an exaggerated formal style. For example, after introducing the Pairing Axiom, stating that for any a and b there is c with $a \in c$ and $b \in c$, he considers Proposition III.5.3, the trivial consequence that $\{a, b\}$ is a set. His first proof of this is set out in eleven numbered steps, with duly noted uses of the Leibniz axiom and modus ponens. To be sure, he later adopts what he calls a "relaxed" proof style, but even so, in the "Informal" Example III.9.2, he fills a page with tight reasoning and explicit appeals to the deduction theorem, the principle of auxiliary constants and more, to show merely that if x is a set and $x \subseteq \{\emptyset\}$, then $x = \emptyset$ or $x = \{\emptyset\}$. Similar examples of formality can be found on pages 118, 120, 183–184 and elsewhere in volume 2, as well as volume 1.

The preface of volume 2 explains that the book weaves a middle path between those set theory books that merely build set-theoretic tools for use elsewhere and those that aim at research in set theory. But I question this assessment. Many of the topics constituting what I take to be the beginnings of the subject appear only very late in the book. For example, the von Neumann ordinals appear first on page 331; Cantor's theorem on the uncountability of $P(\omega)$ occurs on page 455; the Cantor-Bernstein theorem appears on page 463; the definitions of cardinal successor and $\aleph_\alpha$ wait until page 465; and the definition of cofinality does not appear until page 478, with regular and singular cardinals on page 479. Perhaps it was the elaborate formal development of the early theory that has pushed this basic part of set theory to the end of the book. This may not be a problem, but I worry that students may wrongly understand these topics to constitute "advanced" set theory, when surely the opposite is true. Furthermore, many other elementary topics, which one might expect to find in a set theory text aimed in part at graduate students, do not appear in the text at all. This includes closed unbounded sets, stationary sets, ω_1-trees (such as Souslin trees or Kurepa trees), Borel sets, regressive functions, Martin's axiom, the diamond principle and even ultrafilters. Large cardinals are not mentioned beyond the inaccessible cardinals. The omission of ultrafilters is particularly puzzling, given the author's claim to have included "all the fundamental tools of set theory as needed elsewhere in the mathematical sciences" (vol. 2, p. xii). Certainly ultrapowers are one of the most powerful and successful such tools, whose fundamental properties remain deeply connected with logic.

In the final chapter, the author provides a formal account of the foundations of forcing, with useful explanations again of the important theory/metatheory interaction arising in connection with it. Because his account of forcing is based on countable transitive models, some set theorists may find it old-fashioned. This way of forcing tends to push much of the technique into the metatheory, which Tourlakis adopts explicitly (vol. 2, p. 519), and can sometimes limit forcing to its role in independence results. A more contemporary view of forcing makes sense within ZFC of forcing over V, for example via the Boolean-valued models $V^{\mathbb{B}}$, and allows one sensibly to discuss the possibilities achievable by forcing over any given model of set theory.

Despite my reservations, I welcome Tourlakis' addition to the body of logic texts. Readers with a formalist bent especially will gain from it.

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T. JECH. ***Set theory.*** The third millennium edition, revised and expanded. Springer-Verlag, Berlin, 2003, viii + 769 pp.

It is a pleasure to sit down to write a review of the new and expanded version of Jech's magnificent work.

While there is a great mass of new material, and this book is a fundamental rewrite and reorganization of the earlier text, it is also shares the strengths of the previous edition. Set theory is an unusually diverse field, overlayed with strange connections, and peculiar subfields, and obscure historical precedents, and Jech does an upstanding job trying to give us a picture of this breadth.

An illustration can be made by picking one example out of many.

The book begins with the usual discussion of the axioms of ZFC, which is to say the commonly accepted postulates of modern set theory, and by implication the foundations of mathematics; after that he introduces the ordinals, and then the formal definition of the real number line. At the appropriate point, page 39 of the new edition, the author sneaks in an aside about Suslin's question—is the real number line characterized by its completeness, first order properties (dense linear ordering without endpoints), along with the non-existence of uncountably many disjoint non-empty open sets. Previously Cantor had shown a positive answer if we assume the stronger condition that there is a countable dense subset, and Suslin was naturally drawn to ask whether we can replace that assumption by the more modest proposition of non-existence of uncountably many disjoint open sets. This timely aside lies in waiting, until the best part of a hundred pages later when the author indicates, at exactly the right time, the connection with combinatorial set theory: There is a counterexample if and only if there is a *Suslin tree* on $\aleph_1$. On page 191 we see Jensen's *diamond* principle in the context of a general discussion of Gödel's *constructible universe* L. Later in the sections on Cohen's method of showing independence results by *forcing* we see at page 239 how to force a Suslin tree and then how diamond outright implies one; the author sensibly relegates to the exercises a proof that forcing can be used to obtain a homogeneous Suslin tree. At this stage the reader only knows that the existence of a Suslin tree is relatively consistent with ZFC, but then the next section deals with iterated forcing and Martin's axiom, and we are thereby handed the tools to see that the non-existence of a Suslin tree is equally consistent with ZFC. In conclusion, nearly a century after the first glimmer of this problem, it is established that the existence of a Suslin tree is formally independent of ZFC.

Much, much later there is a charming footnote to the whole saga: Shelah's result that if we use forcing to add a *Cohen real* to the universe of sets, then in the resulting model there exists a Suslin tree.

Of course there are many other similar examples in Jech's book, and I only picked Suslin's question because it was so near at hand. From the perspective of modern set theory one might find a better example in the problem of the existence of a two valued, countably complete, non-atomic, measure—a question which has its roots in measure theory, and has given flower to much of the post 60's work on large cardinals, as well as Solovay's results on *real-valued measurable* cardinals. Or one could trace through how the theory of *stationary sets* has been developed over the decades and discuss the disparate applications of these ideas. Or issues related to measure and category. And so on.

It is not possible to give a complete summary of this book. Perhaps it will suffice to say that Jech covers all the major topics one would expect, as well as a good deal more in the way of advanced topics.

Part I consists primarily of the material which can be discussed without metamathematics. Part II is more sophisticated, using forcing and constructibility to examine various kinds of independence results; it goes on to discuss large cardinal assumptions, both those small enough to reside in Gödel's L, such as weakly compact, as well as those incompatible with the axiom of constructibility, such as measurable cardinals and the ultrapower constructions they allow. There is a pithy and well organized introduction to descriptive set theory and the projective hierarchy. In the final part, part III, there is an eclectic presentation of several modern topics, including some more advanced techniques in forcing as well as applications of large cardinals to descriptive set theory.

Apart from the book's great scope, I am struck by the perfect level of detail in both this book and the original. The organization is outstanding. The proofs are short and elegant. The author rarely gets caught in the kinds of extended discussion of technicalities which befall so many expository works. At the end of each chapter there is a helpful summary of the history. The numerous exercises give color and flesh to the topics.

In passing I should concede that some readers find Jech's style overly terse. Indeed it may well be that the book is not suitable as a textbook for a class of students who are not already committed to studying the subject, or for a casual researcher who simply wants to browse. This is a work for the devotee.

There should also be some discussion regarding what is specifically new to the "third millennium" edition. This appears both in the form of new material added to preexisting sections as well as entirely new chapters. The new book is around 25% longer.

One very important addition is the discussion of the subject of *extenders*. These are a crucial tool in the theory of large cardinals, and the overwhelming majority of work in inner model theory from the last 20 years relies on these in one way or another. The original edition of Jech appeared in 1978, just before their advent, and was as such almost outdated from the moment of birth. This addition alone justifies the cost of purchase.

The new edition also has much more on descriptive set theory. The former did well what it set itself to do—which is to say, develop set theory up to the point needed to understand Solovay's model—but had nothing in the way of modern uniformization theorems which have been so central to the "Cabal" school of descriptive set theorists. The latest edition introduces Moschovakis' notion of scale, a basic tool in this field, and provides a definition of the Martin-Solovay tree. There is also a whole section on Polish spaces.

Naturally the author presents new material on Shelah's PCF theory. There is new discussion on cardinal invariants of the continuum. Later there is a subsection on Namba forcing. After there is a section on Laver's model. A bit later Silver's dichotomy for $\mathbf{\Pi}^1_1$ equivalence relations. Discussion of universally Baire sets. There are many small, specific, but important, topics which cannot be found easily in other sources.

Part III provides entirely new discussion of several subfields of set theory. There is a chapter on Ramsey theory and combinatorial principles. There is a chapter on proper forcing. He makes some effort to summarize recent work on determinacy and large cardinals past a Woodin cardinal—though the topic does not lend itself to being properly treated in a more general text of this nature, and here there is little in the way of proofs. There is a chapter on using large cardinals to obtain relative consistency proofs in the cardinal arithmetic at singulars and measurables; there are some proofs, though a more extensive treatment would require core model theory to ascertain the precise consistency strengths, and thereby, again, require a more specialized treatment. Many of the totally new chapters have this flavor. There is a summary of the results, but a relatively little explanation of the mechanics. Still, this is not the author's fault; much of the most important work in modern set theory require book long proofs, and even the simplest proofs in core model theory have several hundred pages of implicit prerequisites.

An entirely novel feature of the latest edition is Jech's promise in the introduction to present topics which "reflect the state of the art of set theory at the end of the millenium." This remark could have been omitted, since a choice of topics is so unvoidably a personal matter. One topic which might have found its way into a book of this nature Harrington-Kechris-Louveau dichotomy theorem; and after that there is tapestry of other results on Borel equivalence relations under Borel reducibility. Alternatively, one might have discussed the very, very large, "brink of inconsistency", cardinals which have proven important in work of Foreman, Laver, Woodin, and others. Or invariant descriptive set theory in the sense of Becker, Kechris, and Vaught, along with connections to the topological Vaught conjecture. Or the applications to Banach spaces of Ramsey type theorems, and how it has influenced the work for which Gowers won a fields medal; or how Todorcevic has applied similar kinds of partition theorems to set theoretical topology. Jech does mention Martin's proof of Borel determinacy, but not Friedman's proof that it necessarily required $\aleph_1$ many iterations of the power set. In discussing currently topical areas, there could have been some description of iteration stragies and fine structure, as it appears in the work of Mitchell, Neeman, Schindler, Steel, Zeman, and others.

There is one other entirely new feature of the new edition over which the author probably had no control. The new cover is a ghastly blend of bright yellow and distressing green. Presumably some executive in the marketing division of Springer felt that this would be more "contemporary", more "hip", more "MTV". The earlier Academic Press edition was an appropriately serious dark green. The bible of set theory deserves a cover which is dark and timeless.

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Dominique Perrin and Jean-Eric Pin. ***Infinite words: automata, semigroups, logic and games.*** Pure and Applied Mathematics Series, vol. 141. Elsevier, Amsterdam, 2004, xi + 538 pp.

This book has quite a long history; I remember first versions of first chapters being published as a technical report in 1991. Since the field the book covers—infinite words—is such an active field, the authors had to incorporate more and more new material over the years, for instance, new work on varieties and ordered varieties, now an entire chapter, and new insights in infinite games and finite automata on infinite trees, a topic which now accounts for two chapters. But, finally, this long awaited book, a fundamental reference for infinite words, which covers almost all aspects of infinite words, has appeared.— All good things come to those who wait.

The monograph is divided in 10 chapters and two appendices, it has 538 pages, a table of contents, an extensive bibliography with 370 entries, a short index, and a list of figures, but does not have a list of symbols, which would often come in handy. Each chapter has a section with exercises, which give the reader the opportunity to verify his or her understanding of the material covered and also present new material, and a section with detailed historical notes and references.

The core of the book covers regular languages of infinite words and infinite trees: seven of the ten chapters deal directly with this. The other three chapters take a broader perspective; they cover infinite games and topological aspects of sets of infinite words.

In general, the authors look at regular languages from a specific—a mathematical—angle: they are interested in closure properties (the authors prove, for instance, Rabin's complementation lemma, which says that the class of regular languages of infinite trees is closed under complement), classification (the authors give, for instance, a very nice and clear account of Wagner's hierarchy of the regular Wadge degrees), expressiveness (the authors show, for instance, that the star-free languages of infinite words are exactly the ones with an aperiodic syntactic semigroup), and applications in mathematics or mathematical logic (the authors prove, for instance, that the monadic theory of the infinite tree is decidable, using tree automata). What Perrin and Pin do not cover in depth are computational aspects, applications in branches of computer science such as verification, and alternating automata.

The first chapter, Automata and Infinite Words, lays the ground for the entire book. It contains basic definitions and notation, definitions of finite automata and regular expressions for infinite words, translations between different types of automata and regular expressions, and proofs of fundamental closure properties. As part of this, Perrin and Pin provide a nice account of Safra's construction for determinizing Büchi automata (McNaughton's theorem) and lower and upper bounds for the various translations. (There is a flaw though in Proposition I.10.2: the authors claim that a Muller automaton can be turned into an equivalent ω-regular expression of polynomial size, which is not true, for an old result of Ehrenfeucht and Zeiger provides an exponential lower bound.) The only thing that is missing (from my point of view) is a description of Klarlund's complementation construction.

The second chapter, Automata and Semigroups, presents the algebraic approach for studying regular languages of infinite words, which extends the semigroup-theoretic approach for studying ordinary regular languages, that is, regular languages of finite words. Sets of finite and infinite words are viewed as subsets of so-called free ω-semigroups, which are two-sorted structures extending semigroups in an appropriate fashion. The authors even use the algebraic approach to give an alternative proof of McNaughton's theorem. The chapter concludes with a thorough treatment of prophetic automata (co-deterministic and co-complete Büchi automata) from an algebraic point of view.

The third chapter is one of the three chapters that takes a broader perspective; it covers material from descriptive set theory relevant for a deeper understanding of the next chapter, which treats games, and the one after that, on Wagner's hierarchy. It studies the set of infinite words over a fixed (countable) alphabet as a topological space by providing it with the product topology. Perrin and Pin prove the strictness of the Borel hierarchy, introduce countable and Borel automata, and use the latter to show the separation theorem for Polish spaces.

The fourth chapter, Games and Strategies, deals with infinite games in general, Martin's theorem on the determinacy of Borel games, which is proved for Σ_3-games, games on infinite graphs with regular (rational) winning conditions, and Wadge games. In particular, Perrin and Pin give proofs of the forgetful determinacy theorem and Wadge's lemma.

Chapter V, Wagner Hierarchy, is devoted to the classification of all regular languages of infinite words with respect to the Wadge preorder. More precisely, the lattice of all regular Wadge degrees—also known as Wagner's hierarchy—is described (along with an appropriate naming scheme) and procedures are presented for determining the Wadge degree of a given language. Again, just as for McNaughton's theorem, Perrin and Pin take two approaches, an automata-theoretic one and an algebraic one, based on ω-semigroups.

Chapter VI, Varieties, explains how the algebraic approach of Chapter II can be used to classify regular languages, based on the notions of (pseudo-)variety of ω-semigroups and (pseudo-)variety of ordered ω-semigroups. The strength of this approach is demonstrated by effective characterizations of some topological classes of regular languages of infinite words using identities and inequalities, effective characterizations of concatenation hierarchies modelled after Brzozowski's hierarchy for finite words, and variations of McNaughton's theorem.

Chapter VII, Local Properties, can be seen as an extension of Chapter VI. It treats seperately different classes of regular languages of infinite words that are characterized by local properties, that is, where membership of a word to the language is determined by the (multi-)set of its infixes of a fixed size. Just as in the previous chapter, effective characterizations in terms of varieties of (ordered) ω-semigroups are given.

Chapter VIII, An Excursion into Logic, covers much of the connection between regular languages of infinite words and predicate and temporal logic. It is demonstrated how Büchi's theorem, the decidability of the monadic theory of the successor, can be proved using regular ω-languages, it is shown that star-free ω-regular languages correspond to first-order logic with order and to temporal logic. Moreover, first-order logic with successor only and restricted temporal logic are characterized.

The two final chapters, Chapter IX and Chapter X, deal with two extensions of infinite words: bi-infinite words and infinite trees. In each of the two cases, Perrin and Pin show how the theory developed for regular languages of infinite words can be extended or adapted to the different models. For instance, using finite tree automata it is proved that the monadic theory of the binary tree (the monadic theory of two successors) is decidable.

The material is presented in an easily accessible fashion. The authors managed to make the individual chapters as independent of each other as possible, and they provide additional background on finite semigroups and varieties of semigroups in two appendices.

This book is an indespensable reference for researchers working in the field. It is the first monograph that tries to cover the topic of infinite words in all aspects. I would have liked to see some open problems stated.

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RICHARD ZACH. *Hilbert's* 'Verunglückter Beweis'*, the first epsilon theorem, and consistency proofs*. ***History and Philosophy of Logic***, vol. 25 (2004), pp. 79–94.

Hilbert's ε-calculus has received a good deal of attention in the past, due to its fruitful applications in proof-theory and other fields, and due to the fact that it played a crucial role in Hilbert's programme of establishing reliable foundations of mathematics. It consists in the quantifier-free fragment of the predicate calculus together with an ε-operator and the "transfinite axiom" for the introduction of ε-terms. The latter can be employed to define unbounded quantifiers, so that propositions of infinitary mathematics can be represented. For eliminating such terms from proofs of finitary propositions Hilbert developed the ε-substitution method in the early 1920's, which was subsequently extended by Ackermann and von Neumann. The ε-theorems (first published in ***Grundlagen der Mathematik***, vol. 2, 1939) state that the ε-calculus is a conservative extension of the quantifier-free fragment of predicate logic.

In the paper under review Richard Zach discusses an interesting historical detail in the development of the ε-calculus, namely an alternative attempt by Hilbert to prove the eliminability of the ε-symbol from proofs of ε-free formulas, which has hitherto gone almost completely unnoticed in the literature. The latter should not be too great of a surprise, given that no published record of this approach exists and that it is nowhere mentioned in Hilbert's lectures notes. There is, however, a 6-page sketch of a proof in Bernays' handwriting attached to a copy of lecture notes by Hilbert, and a discussion of this proof—in which it is referred to as Hilbert's "Verunglückter Beweis," his failed proof—in a letter by Bernays to Ackermann dated 16 October 1929. These two sources form the basis of Zach's discussion. His analysis of this proof reveals that it can be seen as a direct precursor to the first ε-theorem (of which it attacks the simplest case) and also to a 'general consistency result' (formulated by Bernays in the second volume of ***Grundlagen der Mathematik***), which exhibits conditions that a formal system must satisfy in order to be amenable to a consistency proof of the kind under consideration. The approach itself, however, was abandoned because of difficulties in generalizing it beyond the simple case.

Since this second approach to consistency proofs reveals some aspects of Hilbert's practice, the author uses it to inform his discussion of Hilbert's philosophy of mathematics. With respect to the question of the original aim of Hilbert's programme, whether it was to prove consistency of infinitary mathematics or conservativity of "ideal" extensions of finitary mathematics (which has become a popular interpretation), Zach convincingly argues for the former. Furthermore, based on the 'failed proof' and on Hilbert's published writings he makes the case that Hilbert's "real" formulas should be understood to be decidable variable-free formulas, rather than all quantifier-free formulas, as seems to be a common view among Hilbert commentators.

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