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2004 SPRING MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

Palmer House Hilton Hotel, Chicago, Illinois
April 23–24, 2004

The 2004 Spring Meeting of the Association for Symbolic Logic was held at the Palmer House Hilton Hotel, April 23–24, 2004, in conjunction with the annual meeting of the Central Division of the American Philosophical Association. The members of the program committee were Steve Awodey, Timothy Bays, and Michael Kremer (Chair). The ASL hosted a reception on the evening of April 23rd.

The program included three three-hour invited symposia: the first, on Frege's Contribution to Logic, was organized by Michael Kremer; the second, on The Philosophy of Proof Theory, was organized by Timothy Bays; and the third, on Category Theory and Structuralism, was organized by Steve Awodey. Each of the symposia consisted of two hour-long talks and commentary as follows.

Frege's Contribution to Logic.

Warren Goldfarb (Harvard University), *On Frege's conception of logic.*
Erich Reck (University of California, Riverside), *The logic in Frege's 1910–14 lectures.*
Commentator: Peter Sullivan (University of Stirling).

The Philosophy of Proof Theory.

Jeremy Avigad (Carnegie-Mellon University) *Mathematical method and proof.*
Andrew Arana (Stanford University), *The epistemic value of purity.*
Commentator: William Tait (University of Chicago).

Category theory and structuralism.

Geoffrey Hellman (University of Minnesota), *Axioms, categories, and structure.*
Colin McLarty (Case Western Reserve University), *Misgivings on categorial foundations.*
Commentators: Stewart Shapiro (Ohio State University and St. Andrew's University) and Dana Scott (Carnegie Mellon University).

The program also included nine contributed talks and two abstracts presented by title.

Abstracts of the invited talks and contributed talks given (in person or by title) by members of the Association for Symbolic Logic follow.

For the Program Committee
MICHAEL KREMER

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Abstracts of invited talks in the Session on Category Theory and Structuralism

▶ GEOFFREY HELLMAN, *Axioms, categories, and structure*.
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In previous work [1], we raised questions about category theory as an autonomous framework for understanding mathematics as a study of abstract structure, arguing that, notwithstanding its insights, it is insufficiently articulate about mathematical existence and about a background (external) logic of relations. A modal-structural theory of "large domains" was sketched to illustrate one way of filling the gaps, compatible with a "many worlds for mathematics" view, promoted by Bell. Here we consider two lines of response that have emerged, one defending CT as "non-foundational" and "schematic" [2], the other espousing a more traditional platonist reading of certain topos axiom systems [3]. To "take this fork in the road", we probe the idea of "schematic mathematics" further, while also suggesting that the platonist reading of topos axioms reinstates problems, especially regarding set theory, that categorical foundations held out hope of avoiding.

[1] G. Hellman, *Does category theory provide a framework for mathematical structuralism?*, ***Philosophia Mathematica***, vol. 11 (2003) no. 3, pp. 129–157.

[2] S. Awodey, *An answer to Hellman's question: 'Does category theory provide a framework for mathematical structuralism?'* ***Philosophia Mathematical***, forthcoming.

[3] C. McLarty, *Exploring categorical structuralism*, ***Philosophia Mathematica***, forthcoming.

▶ COLIN MCLARTY, *Misgivings on categorical foundations*.
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Various people have expressed misgivings on categorical foundations for mathematics. Notably Solomon Feferman has suggested they may be "logically impossible", and they have no "external theory of relations". In [1] Geoffrey Hellman suggests they have not answered the "home address problem". Each of these questions illuminates an aspect of foundations for mathematics, in general, but also specifically categorical foundations. The talk will look at them in relation to several different programs for categorical foundations.

[1] G. Hellman, *Does category theory provide a framework for mathematical structuralism?*, ***Philosophia Mathematica*** vol. 11 (2003) no. 3, pp. 129–157.

Abstracts of invited talks in the Session on Frege's Contribution to Logic

▶ WARREN GOLDFARB, *On Frege's conception of logic*.
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Frege's notions of inference and proof, and hence his conception of logic, have features that look odd from a contemporary perspective. Many of these features are clearly exhibited in his correspondence with Hilbert and the polemic he subsequently published on Hilbert's Foundations of Geometry. I chart the philosophical views about logic from which these apparent oddities flow, and discuss how these shape the whole notion of "foundations of mathematics" that Frege develops.

► ERICH H. RECK, *The logic in Frege's 1910–14 lectures*.
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Gottlob Frege is generally acknowledged to be one of the founders of modern logic. He first presented his logic in ***Begriffsschrift*** (1879), then an expanded, updated version in ***Basic Laws of Arithmetic*** (vol. I: 1893; vol. II: 1903). Recently a third, even later presentation has become available, from his 1910–1914 Jena lectures [1]. In this talk, I first summarize the logical system contained in Frege's lectures. Then I compare it to his earlier systems in ***Begriffsschrift*** and ***Basic Laws***, thus revealing some significant developments in Frege's views. Finally, I discuss the relationship of Frege's logic to its current successors, as well as the specific influence of the Jena lectures on the history of logic.

This talk is based on joint work with Steve Awodey, Carnegie Mellon University.

[1] E. Reck and S. Awodey, editors and translators, ***Frege's Lectures on Logic: Carnap's Student Notes, 1910–1914***, with an introduction by G. Gabriel, Open Court, Chicago, forthcoming.

Abstracts of invited talks in the Session on The Philosophy of Proof Theory

► ANDREW ARANA, *The epistemic value of purity*.
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A proof of a proposition P is pure, roughly speaking, if it is comprised of propositions that are definitions of terms in P or are deductive consequences of those definitions. We consider two specific proposals for purist projects in mathematics. We then examine and evaluate each project's epistemic value.

This is a joint work with Mic Detlefsen.

► JEREMY AVIGAD, *Mathematical method and proof*.
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On the traditional view, the primary role of a mathematical proof is to warrant the truth of the resulting theorem. This view fails to explain why it is very often the case that a new proof of a theorem is deemed important. Three case studies from elementary arithmetic show, informally, that there are many criteria by which ordinary proofs are valued. I argue that at least some of these criteria depend on the methods of inference the proofs employ, and that standard models of formal deduction are not well-equipped to support such evaluations. I discuss a model of proof that is used in the automated deduction community, and show that this model does better in that respect.

Abstracts of contributed talks

► A. W. CARUS, *Carnap's dream*.
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In January 1931 Rudolf Carnap had a "vision" during a sleepless night. Years later, in his autobiography, he reports that "the whole theory of language structure and its possible applications in philosophy" became clear to him, in a flash, and he wrote it all down in shorthand the next day. He gave these notes the title "Attempt at a Metalogic". They became the germ of *Logical Syntax of Language*, he tells us. But wherein did this momentous insight actually consist? What was that "germ"? Using Carnap's shorthand manuscript of the "Attempt at a Metalogic", and other notes from this period, we try to answer this question. The punch line, somewhat oversimplified, is this: Carnap uses Gödel's then-recent logical discoveries to overcome the conception of meaning he had derived from Wittgenstein's *Tractatus*, and which had turned out to be incompatible with other Vienna Circle goals. The "vision" was Carnap's liberation from the notion of meaning.

This is joint work with Steve Awodey.

▶ DENNIS F. CUDIA, *Conditional inductive logic*.
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The conditional form of Bayes Theorem is stated as follows: for any wffs k, r, s of propositional logic, $P(r \,\&\, k)P(s \,\&\, k)P(k) \neq 0$ implies that $P(s \mid r \,\&\, k)P(r \mid K) = P(r \mid s \,\&\, k)P(s \mid k)$. The conditional form of Bayes Theorem is proven. Note if k is a tautology then the result is Bayes Theorem. For notation see [1]. With the wffs $k, p, q_1, \ldots, q_n$ as *condition* k, *effect* p and possible *causes* $q_1, \ldots, q_n$, the *conditional likelihood tables* are denoted by $LT(k)$ and $LT(\sim q, k)$ and defined by

$LT(k)$	q_1	$\ldots$	q_n
p	$P(p \mid q_1 \,\&\, k)$	$\ldots$	$P(p \mid q_n \,\&\, k)$
$\sim p$	$P(\sim p \mid q_1 \,\&\, k)$	$\ldots$	$P(\sim p \mid q_n \,\&\, k)$

$LT(\sim q, k)$	$\sim q_1$	$\ldots$	$\sim q_n$
p	$P(p \mid \sim q_1 \,\&\, k)$	$\ldots$	$P(p \mid \sim q_n \,\&\, k)$
$\sim p$	$P(\sim p \mid \sim q_1 \,\&\, k)$	$\ldots$	$P(\sim p \mid \sim q_n \,\&\, k)$

Note that $LT(k)$ is defined iff $P(q_i \,\&\, k) > 0$ for $1 \leq i \leq n$. If $P(k) > 0$ let $C(k)$ denote the *prior conditional probability* matrix and define $C(k)$ as the $n \times n$ diagonal matrix $\operatorname{diag}(P(q_1 \mid k), \ldots P(q_n \mid k))$. Let $JP(k)$ and $JP(\sim q, k)$ denote the *conditional joint probability matrices* and define $JP(k) = LT(k) \cdot C(k)$ and $JP(\sim q, k) = LT(\sim q, k) \cdot (I - C(k))$ where I is the identity matrix. Denote the *conditional probablility of the effect* p by $D(k)$ and define $D(k)$ as the 2×2 diagonal matrix $\operatorname{diag}(P(p \mid k), P(\sim p \mid k))$ provided $P(k) > 0$. Denote the *conditional predictive value tables* by $PV(k)$ and $PV(\sim q, k)$ and define $PV(k)$ and $PV(\sim q, k)$, respectively, by interchanging p and q_i and p and $\sim q_i$, respectively, in the first row of $LT(k)$ and $LT(\sim q, k)$, respectively and interchanging $\sim p$ and q_i and $\sim p$ and $\sim q_i$, respectively, in the second row of $LT(k)$ and $LT(\sim q, k)$, respectively. It is proven that the conditional form of Bayes Theorem for $r = p$ or $r = \sim p$ and $s = q_i$, $1 \leq i \leq n$, or $s = \sim q_i$, $1 \leq i \leq n$, implies the matirix equations $D(k) \cdot PV(k) = JP(k)$ and $D(k) \cdot PV(\sim q, k) = JP(\sim q, k)$.

[1] D. Cudia, *Probability and inductive logic*, ***The Bulletin of Symbolic Logic***, vol. 9 (2003), p. 246.

▶ SEAN A. FULOP, *Substructural logic as Universal Grammar*.
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A *type-logical grammar* is a triple $G = \langle V_G, I_G, R_G \rangle$ consisting of a vocabulary V_G of words, a *type logic* R_G having formulae (types) F_R, and a lexical assignment function $I_G \colon V_G \to \wp(F_R)$. The formulae of R_G are now defined: $\mathcal{F} ::= \bigcup_{i,j} \mathcal{A} \mid \mathcal{F}/_i\mathcal{F} \mid \mathcal{F}\backslash_i\mathcal{F} \mid \Diamond_j\mathcal{F} \mid \Box^{\downarrow}{}_j\mathcal{F}$, $1 \leq i \leq n$, $1 \leq j \leq m$, over primitive types $\mathcal{A}$ consisting of a distinguished s and a denumerable set of variables. R_G deduces Gentzen *sequents* $\Gamma \Rightarrow C$ with Γ a linearly ordered tree (*G-term*) of formulae and C a formula. G-terms are so defined: $\Gamma ::= \bigcup_{i,j} \mathcal{F} \mid \Gamma \circ_i \Gamma \mid \langle \Gamma \rangle^j$, $1 \leq i \leq n$, $1 \leq j \leq m$.

R_G must have the rules below, governing all slashes and modals regardless of subscript

($\Gamma[\Delta]$ indicates a G-term with a position occupied by an occurrence of G-term Δ). $A \Rightarrow A$ (Axiom); $\Delta \Rightarrow A$, $\Gamma[A] \Rightarrow C \vdash \Gamma[\Delta] \Rightarrow C$ (Cut); $\Delta \Rightarrow B$, $\Gamma[A] \Rightarrow C \vdash \Gamma[(A/B \diamond \Delta)] \Rightarrow C$ (/L); $\Delta \Rightarrow B$, $\Gamma[A] \Rightarrow C \vdash \Gamma[(\Delta \diamond B\backslash A)] \Rightarrow C$ ($\backslash$L); $(\Gamma \diamond B) \Rightarrow A \vdash \Gamma \Rightarrow A/B$ (/R); $(B\diamond\Gamma) \Rightarrow A \vdash \Gamma \Rightarrow B\backslash A$ ($\backslash$R); $\Gamma[\langle A\rangle] \Rightarrow B \vdash \Gamma[\Diamond A] \Rightarrow B$ ($\Diamond$L); $\Gamma[A] \Rightarrow B \vdash \Gamma[\langle\Box^{\downarrow}A\rangle] \Rightarrow B$ ($\Box^{\downarrow}$L); $\Gamma \Rightarrow A \vdash \langle\Gamma\rangle \Rightarrow \Diamond A$ ($\Diamond$R); $\langle\Gamma\rangle \Rightarrow A \vdash \Gamma \Rightarrow \Box^{\downarrow}A$ ($\Box^{\downarrow}$R). There may also be structural rules.

G generates a sentence structure over leaves $c_1, \dots, c_n \in V_G$ iff there is a structurally equivalent tree Γ over types $A_1, \dots, A_n$ such that $I_G : c_i \mapsto \mathcal{X}_i \ni A_i$ for all $1 \leq i \leq n$ and $\Gamma \Rightarrow s$ is provable in R_G.

A type logic R_G will be *permitted* under the following conditions: 1. R_G is finitary and enjoys cut-elimination, the subformula property, and decidability. 2. All structural rules are restricted to the presence of certain G-term modalities $\langle\Delta\rangle^j$ and cannot apply circularly. 3. Every provable sequent $\Gamma \Rightarrow C$ is determined by the multiset of formula occurrences in Γ.

For linguistic theory, some permitted logic R_G is to be selected as the Universal Grammar, supposed to be able to generate any natural language with a properly chosen lexicon. For a specific language, V_G and I_G are posited to be learned by the human during language acquisition.

► JAAKKO HINTIKKA, *On the interpretation of negation*.
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In independence-friendly (IF) first-order logic, the negation that results from the usual "classical" rules of game-theoretical semantics is not the contradictory negation but a dual negation not obeying tertium non datur. (See e.g., [1].) By adding a sentence-initial contradictory negation, we obtain an extended IF logic with two negations. Algebraically, it is a Boolean algebra with an additional operator in the sense of Jónsson and Tarski [2], wherefore their results apply, including the possibility of a set-theoretical interpretation. (Also see, [3].) In such an interpretation, the strong negation can be taken to be a generalization of the notion of orthogonality. This means that by its means we can define such notions as dimensions and coordinate representation on a purely logical level. The resulting structure also turns out to be a topology closely related to intuitionistic logic.

[1] Jaakko Hintikka, *Hyperclassical logic (aka independence-friendly logic) and its general significance*, ***The Bulletin of Symbolic Logic***, vol. 8 (2002), pp. 404–423.

[2] Bjarni Jónsson and Alfred Tarski, *Boolean algebras with operators I–II*, ***American Journal of Mathematics***, vol. 73 (1951), pp. 891–939 and vol. 74 (1952), pp. 127–162.

[3] Alfred Tarski, **Collected Papers** vol. 3 (1986), edited by Steven R. Givant and Ralph N. McKenzie, Birkhäuser, Basel, pp. 369–419 and pp. 421–458.

► THOMAS HOFWEBER, *Consistency, existence, and the philosophical ambitions of proof theory*.
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In his early work Hilbert suggests a thesis that could be summarized as "consistency implies existence". Hilbert's thesis, if correct, would give proof theory a central role in answering large scale philosophical questions about mathematical truth, objects, and objectivity. But Hilbert's thesis is not correct. There are consistent theories that aren't about anything, and there are consistent mathematical theories that are false in their intended domain. In light of this the philosophical ambitions of proof theory have shifted away from the large scale philosophical questions and moved towards the foundational study of various systems.

And proof theoretic investigations have largely abandoned the goal of achieving (relative) consistency proofs as an end in itself. I will argue that a modified version of Hilbert's thesis is defensible, and that, together with work in other domains, proof theory can play a positive role in the answer of large scale philosophical questions.

► MATTHEW MCKEON, *On the Kripkean account of logical truth in first-order modal logic*.
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The concern of this talk is the criticism of the Kripkean representation of modal, logical truth as truth at the actual-world element of every subset of the set of possible worlds. The point of contention is the variability of the collection of worlds from one model structure to another in fixing the extension of logical truth for a modal language. For the logical reading of the modal operators, this feature has been called curious, completely lacking in motivation, and the cause of Kripke semantics missing some logical truths. The criticism, largely ignored in the literature, has been made by Field and Hintikka, among others. In earlier work (1996), I have provided a rationale for Kripke-type semantics. Here I test a defense of its correctness, which runs roughly as follows. In first-order logic a quantified sentence is a logical truth only if it remains true no matter how the quantifiers are restricted. I consider three accounts of why logical truth for first-order quantifiers should take into account different quantifier ranges and argue that the natural extension of the best account to the semantics for the modal operators—which on many views are analogous to the quantifiers—is to say that a modal sentence is a logical truth only if it remains true no matter how these "quantifiers" are restricted. But then Kripke-type semantics is justified. The challenge posed to the critics is to explain why the cardinality of the ranges of the modal quantifiers may be a logical element of modal sentences, but the cardinality of the ranges of first-order quantifiers is a non-logical element of quantificational sentences.

Logical truth in modal logic, ***Pacific Philosophical Quarterly***, vol. 77 (1996) no. 4, December, pp. 351–361.

► FRED SEYMOUR MICHAEL, *The logic of sequent tableaux*.
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The many tableau systems, such as those of Beth, Smullyan, Rasiowa and Sikorski, all derive directly or indirectly from Gentzen's sequent calculus. The tableau system to be examined here is the most directly derived from Gentzen's calculus. Proofs in the sequent calculus are commonly constructed by reasoning backwards. The system of sequent tableaux simply institutionalizes this common practice. Every formula in a sequent tableau is a sequent. The rules are basically just sequent calculus rules inverted. Apart from the rules for negation, the tableau rules are all just decomposition rules. The rules for negation can be characterized as movement rules. Since only sequent calculus rules are being used, it is evident that the subformula principle holds. In a sequent tableau, a branch is closed iff it terminates in a sequent calculus axiom, a formula of the form $A \vdash A$. A tableau is closed iff all its branches are closed. It is a mechanical matter to convert a closed sequent tableau into a sequent calculus proof. So sequent tableaux provide a simple means of constructing cut-free sequent calculus proofs. Since its rules are all proof theoretic, the system is syntactic, not semantic; but as in any other tableau system, it is a straightforward matter to read off a counterexample when we are confronted with a tableau with a branch that does not terminate in an axiom. But the principal advantage of sequent tableaux over other tableau systems is that it can be readily adapted so as to apply to non-classical as well as classical logics.

► GREGORY R. WHEELER, *A resource bounded default logic*.
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Standard statistical inference is a genuine example of non-monotonic inference. The connection between classical (i.e., Reiter) defaults and standard inferential statistics has been explored recently from a knowledge representation perspective by Henry Kyburg and Choh Man Teng [1], but classical default logic proves insufficient to compute extensions under this interpretation. Statistical Default Logic (SDL) is a resource bounded default logic, an expansion of classical default logic that allows us to represent the structure of basic techniques found in inferential statistics, including hypothesis testing and the estimation of a population's mean, variance and proportions. A statistical default may be thought of as an ordered pair consisting of a Reiter default in the first coordinate and an error-bound parameter denoting a real number within the unit interval in the second coordinate. The error-bound parameter is an upper-bound limit on the probability of accepting the consequent of an applied statistical default and yet that consequent being false, what is known as a Type-I error within inferential statistics. A method for constructing extensions is then defined that preserves this upper bound on the probability of error under a (skeptical) non-monotonic consequence relation. It is noted that skeptical default logic admits of a consequence relation that sanctions a single conclusion set bounded in error by a pre-assigned threshold of error and that this consequence relation is non-monotonic. Thus statistical default logic features a means for preserving the property of a monotonically decreasing measure of the upper limit of the probability of error associated with a sequence of inference steps in a statistical argument and yet preserves the non-monotonic behavior of statistical inference.

[1] HENRY E. KYBURG, JR. AND CHOH-MAN TENG, *Statistical inference as default reasoning*, ***International Journal of Pattern Recognition and Artificial Intelligence***, vol. 13, no. 2, (1999), pp. 267–283.

► FRANK WROBLEWSKI, *The definition of class resolving Russell's paradox and the empirical account of mathematics*.
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The definition of class is obtained by individuating Quine's contextual definition of class. In ***Set Theory and its Logic***, Quine defines the subject-predicate class-in-use notation $\beta \in \{\alpha \mid \varphi(\alpha)\}$ to be $\exists\gamma(\beta = \gamma.\exists\alpha(\alpha = \gamma.\varphi(\alpha)))$, at bottom a subject-predicate proposition equivalent to class condition $\varphi(\beta)$, where variable β is the subject term. To isolate the individuating definition of class notation $\{\alpha \mid \varphi(\alpha)\}$ from Quine's contextual definition, it remains to isolate the predicate term of the defining proposition where '$\in$' is notation for the 'is' of predication. Isolating this predicate term hinges on the equivalence of the mathematician's and Leibniz' definitions of equality. Mathematicians unwittingly assume this equivalence in using substitutivity of equality, whereas logicians axiomatize it with extensionality. To validate it, to avoid vitiating the definition, we bring to bear an empirical account of mathematics.

That mathematics is found to be true of the world is compelling evidence that mathematics is true under some empirical interpretation. Accordingly, we identify nature's bearers of mathematical terms, the variables and classes, and identify nature's truthmaker articulated by comprehension, the sole irreducible axiom of mathematics. Contrary to axiomatic set theory, the irreducibility of the AC is unsettled since confirming instances prove it not false. Thus warranted, mathematics translates to natural if regimented language mediating nature's truthmakers which makes its truth empirical whose certainty is the temporal stability of nature's truthmakers. Benacerraf's epistemic challenge is met and Quine's indispensability argument is confirmed.

Russell's paradox resolves by proving Russell's set cannot be an element of itself.

Familiarity with Quine's ***Mathematical Logic*** and ***Set Theory and its Logic*** is assumed.

Abstracts presented by title

▶ JOHN CORCORAN, *First-order logical form*.
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Let L be a first-order language. A one-one function from the entire vocabulary of L onto itself is *form-preserving*, or *formal*, if it carries every non-logical constant to one in the same syntactic category and leaves every other character fixed. Any formal function f extends naturally and uniquely to the set of sentences: if s is a sentence of L, then the n-th character-occurrence in the sentence fs is fk where k is the character occurring in the n-th place in s. Any [extended] formal function extends naturally and uniquely to sets P of sentences, to [premise-conclusion] arguments (P, c) where P is a set [premise-set] and c a sentence [conclusion], and to "discourses" which include formal deductions whether "axiomatic" or "natural". A given sentence p is said to be *formally similar to* (or *formally identical to* or to have *the same logical form as*) a sentence q if there is a formal function f such that fp is q itself or an alphabetic variant of q. This equivalence relation, *formal similarity*, also extends to sets, arguments, discourses, etc. Among the several obvious metatheorems are analogues of many traditional "principles of form": every two sentences having the same form are both tautological or both informative (non-tautological), every two sets having the same form are both satisfiable or both unsatisfiable, every two arguments in the same form are both valid or both invalid, every two discourses in the same form are both formal deductions (in a given system) or neither is. This framework enables restatement of the Hilbert-Bernays-Quine theorem that, in a sufficiently rich interpreted language without identity, in order for a sentence to be consistent (respectively, tautological) it is necessary and sufficient for it to be in the same logical form as some true sentence (respectively, no false sentence).

▶ XUNWEI ZHOU, *Sufficient condition cannot be defined, what can be defined is the establishment and employment of sufficient condition*.
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Sufficient condition plays a central role in mathematics. Various implications, the abstractions of sufficient condition, plays a central role in logic. Sufficient condition is a cognitive operator, reflecting two unary cognitive processes: establishment and employment. For example, first, a mathematical theorem must be established (proved), then, it can be employed (used) to make hypothetical inference. Sufficient condition itself cannot be defined, what can be defined is the establishment of sufficient condition and the employment of sufficient condition. Take $x > 0 \Rightarrow x^2 > 0$, where $\Rightarrow$ denote sufficient condition, as an example. The establishment of $x > 0 \Rightarrow x^2 > 0$ goes like this: First, it is required that the antecedent $x > 0$ not be permanently false, the consequent $x^2 > 0$ not be permanently true; when $x = 1$, the antecedent $1 > 0$ is true, so $x > 0$ is not permanently false; when $x = 0$, the consequent $0^2 > 0$ is false, so $x^2 > 0$ is not permanently true, thus the first requirement is met; secondly, it is required that the antecedent $x > 0$ and the consequent $x^2 > 0$ share the same variable; $x > 0$ and $x^2 > 0$ share the same variable x; thus, the second requirement is also met; thirdly, suppose $x > 0$ is true, if we infer that $x^2 > 0$ cannot be false, then we establish (prove) $x > 0 \Rightarrow x^2 > 0$. The employment of $x > 0 \Rightarrow x^2 > 0$ goes like this: From $x > 0 \Rightarrow x^2 > 0$ being true and $1 > 0$ being true we can infer $1^2 > 0$ being true; or from $x > 0 \Rightarrow x^2 > 0$ being true and $0^2 > 0$ being false we can infer $0 > 0$ being false. The establishment and employment of sufficient condition can be depicted by truth tables. The establishment of $\Rightarrow$ is shown in Table 1, the employments of $\Rightarrow$ are shown in Tables 2 and 3. F/T in Table 1 means "needn't determine", F/T in Tables 2 and 3 mean "unable to determine".

Table 1

$x > 0$	$x^2 > 0$	$x > 0 \Rightarrow x^2 > 0$
F	F	T
F	T	F/T
T	F	F
T	T	T

Table 2

$x > 0 \Rightarrow x^2 > 0$	$x > 0$	$x^2 > 0$
F	F	F/T
F	T	F/T
T	F	F/T
T	T	T

Table 3

$x > 0 \Rightarrow x^2 > 0$	$x^2 > 0$	$x > 0$
F	F	F/T
F	T	F/T
T	F	F
T	T	F/T