THE BULLETIN OF SYMBOLIC LOGIC
Volume 10, Number 2, June 2004

2003–04 WINTER MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

Phoenix Civic Plaza, Phoenix, Arizona, January 9–10, 2004

A Winter Meeting of the Association for Symbolic Logic was held on January 9–10, 2004 at the Phoenix Civic Plaza, in conjunction with the annual Joint Mathematics Meetings. The members of the Program Committee were Valentina Harizanov, John Steel, and Carol Wood (Chair). The program included ten invited 50-minute talks and eight contributed talks. The ASL hosted a welcoming reception on January 9th at the Hyatt Regency Hotel. In addition, on January 7–8, prior to the ASL Meeting, there was a joint AMS–ASL Special Session, **Infinite Combinatorics and Inner Model Theory**, organized by M. D. Foreman and M. Zeman.

The 50-minute invited addresses were:

Matthew Foreman (UC Irvine), *Canonical structure in the universe of set theory*.

Steve Jackson (University of North Texas), *Supercompactness measures*.

Byunghan Kim (MIT) *The type-definable group configuration under the generalized type-amalgamation*.

Julia Knight (Notre Dame), *Computable classification*.

R. W. Knight (University of Oxford), *Vaught's Conjecture*.

Steffen Lempp (University of Wisconsin), *The proof-theoretic strength of some combinatorial principles*.

Françoise Point (FNRS-Belgium) *On theories of modules of fields with endomorphisms*.

Kobi Peterzil (Haifa and University of Illinois, Urbana) *Torsion-free groups in some o-minimal structures*.

Leonard Schulman (Caltech) *Quantum algorithms and group representation theory*.

Slawomir Solecki (University of Illinois, Urbana), *Cofinal types of topological directed orders*.

Abstracts of the invited talks and contributed talks given (in person or by title) by members of the Association for Symbolic Logic follow.

For the Program Committee
CAROL WOOD

Abstracts of invited talks

► MATTHEW FOREMAN, *Canonical structure in the universe of set theory*.
Department of Mathematics, University of California at Irvine, Irvine, CA 92697, USA.
E-mail: mforeman@math.uci.edu.

"Canonical Structure" is an informal term denoting, among other things, structure that is independent of the choices made in its definition. Classicly the only canonical structures known were cardinalities, cofinalities, behaviour of the "beth" function and fine structural inner models. Recently, following work of Shelah, a wealth of canonical structure has been

1079-8986/04/1002-0006/$1.90

discovered including PCF objects, canonical ideals such as I[lambda] and club guessing ideals and canonical stationary sets such as the "Good" ordinals.

► BYUNGHAN KIM, *The type-definable group configuration under the generalized type-amalgamation*.
Department of Mathematics 2-171, Massachusetts Institute of Technology, 77 Massachusetts Avenue, Cambridge, MA 02139, USA.
E-mail: bkim@math.mit.edu.

Under 3-amalgamation, we obtained the canonical type (hyper)-definable group from a group configuration. This is a joint work with Tristram de Piro and Jessica Young.

► JULIA F. KNIGHT, *Computable classification*.
Mathematics Department, University of Notre Dame, 255 Hurley Hall, Notre Dame, IN 46556-4618, USA.
E-mail: knight.1@nd.edu.

Classification is an important goal in many areas of mathematics. The idea is to find nice invariants for a class of mathematical objects, up to isomorphism or other equivalence, or else to say, in some concrete way, that there are no nice invariants. There are results of this kind in model theory, and in descriptive set theory. There are also some results in computable structure theory. I will give an overview of some work in computable structure theory, mainly inspired by work in descriptive set theory.

One productive approach to computable classification, pursued by Wesley Calvert, is to measure the complexity of the isomorphism problem for the computable structures in a class. The isomorphism problem is the set of pairs (a, b) such that a, b are computable indices for isomorphic structures in the class being considered. The idea is to locate the isomorphism problem at some level in the hyperarithmetical hierarchy, or to show that it is properly Σ^1_1. Calvert has a number of results of this kind (for interesting classes of fields and groups), and he will present some of them in his talk. A second approach involves computably embedding one class of structures in another in such a way that we can transfer invariants from one class to another. The notion of computable embedding is essentially uniform enumeration reducibility, respecting isomorphism types. This was the approach taken during the past summer by Wesley Calvert, Sara Miller, Desmond Cummins, and me. Miller will say more about this in her talk.

► R. W. KNIGHT, *Vaught's Conjecture*.
University of Oxford, Oxford, UK.
E-mail: knight@maths.ox.ac.uk.

We briefly describe a proposed counterexample to Vaught's Conjecture, and discuss further related questions.

► STEFFEN LEMPP, *The proof-theoretic strength of some combinatorial principles*.
Department of Mathematics, University of Wisconsin, 480 Lincoln Drive, Madison, WI 53706-1388, USA.
E-mail: lempp@math.wisc.edu.

I will discuss the proof-theoretic strength of some combinatorial principles.

► KOBI PETERZIL, *Torsion-free groups in o-minimal structures*.
Department of Mathematics, University of Haifa, Haifa, Israel and University of Illinois at Urbana-Champaign, Urbana, IL 61801, USA.
E-mail: kobi@math.uiuc.edu.

In the category of Real Lie groups, every abelian connected group is isomorphic to $\mathbb{R}_a^n + S^k$, where $\mathbb{R}_a = (\mathbb{R}, +)$ and S is the circle group.

In the category of semialgebraic sets and maps such a result fails, since R_a is not semialgebraically isomorphic to the multiplicative group $R_m = R > 0$. Moreover, there are semialgebraic abelian compact groups of dimension n which cannot be written (in the semialgebraic langauge) as a direct sum of circle groups.

We prove: Assume that a group G is definable in the expansion of $\mathbb{R}_{an}$ by power functions, or in an elementarily equivalent structure. If G is abelian, connected and torsion-free, then it is definably isomorphic to $R_a^n + R_m^k$ (where by "definably isomorphic" means in the structure $\langle R, <, +, \cdot G \rangle$).

In particular, a semialgebraic, abelian, torsion-free group is a direct product of one-dimensional semialgebraic subgroups.

This is joint work with Sergei Starchenko.

► FRANÇOISE POINT, *On theories of modules of fields with endomorphisms*.
Senior research associate at the F.N.R.S., Institut de Mathématique et Informatique, Université de Mons-Hainaut, Le Pentagone, 6, avenue du Champ de Mars, B-7000 Mons, Belgium.
E-mail: point@logique.jussieu.fr.

Given a field K with an endomorphism σ, we first view it as a vector-space over the subfield K^σ and we fix a basis C. Then, we consider a skew polynomial ring $R = K_0[t; \sigma]$ (see [2]), where K_0 is a subfield of K containing C and the set of fixed points of σ, and we consider K as a right R-module (with the action of t being induced by σ). Adding to the usual language of R-modules new unary functions (that are existentially definable in the theory of all R-modules), we write down a theory T_σ of R-modules (of which K is a model). We axiomatize the class of existentially closed models of T_σ and we get a quantifier elimination result (see [6]).

An example of such theory of modules is when K is a field of characteristic p acted upon with the Frobenius endomorphism. The unary functions in that case are the analog of the p-components functions (that one usually add to get a quantifier elimination result for separably closed fields of fixed imperfection degree (see [3])). The theory of the existentially closed models is complete and equals to the theory of modules of the separable closure of K and so is decidable (see [4]). One can describe the corresponding Ziegler spectrum, and also one obtains tighter correspondance between the pure model theoretic objects like types, forking, ranks and their algebraic counterparts than in the field case (see [5]).

Another example is when K is a non principal ultraproduct of separably closed fields K_n. We show that the corresponding theory of modules is the limit theory of the corresponding theories of modules of the K_n's. The corresponding theory of fields has been considered by Chatzidakis and Hrushovski and they showed that in some expansion by definition, they get a model-complete theory (see [1]).

[1] Z. Chatzidakis and E. Hrushovski, *Some asymptotic results on fields*, abstract ASL July 2000 in Paris.

[2] P. M. Cohn, *Skew fields*, ***Encyclopedia of mathematics and its applications***, (G.-C. Rota, editor), vol. 57, Cambridge University Press, 1995.

[3] F. Delon, *Idéaux et types sur les corps séparablement clos*, ***Supplément au Bulletin de la Société de Mathématique de France***, Mémoire 33, Tome 116, (1988).

[4] P. Dellunde, F. Delon, and F. Point, *The theory of modules of separably closed fields 2*, ***The Journal of Symbolic Logic***, vol. 67 (2002), no. 3, pp. 997–1015.

[5] ———, *The theory of modules of separably closed fields 2*, to appear in ***Annals of Pure Applied Logic***.

[6] F. Point, *Asymptotic theory of modules of separably closed fields*, submitted.

► LEONARD SCHULMAN, *Quantum algorithms and group representation theory*.
Caltech, 1200 E. California Blvd., MC 256-80, Pasadena CA 91125, USA.
E-mail: schulman@caltech.edu.

A decade ago it was found that certain computational problems, previously thought to require exponential time, can be solved in polynomial time on a computer in which quantum superpositions can be carefully controlled. I'll describe how this breakthrough and its extensions are based on Fourier transforms over large groups. I'll briefly discuss some current research challenges, and also say something about what types of computational problems we know to be beyond the reach of fast quantum algorithms.

► SLAWOMIR SOLECKI, *Cofinal types of topological directed orders*.
Department of Mathematics, University of Illinois, 1409 W. Green Street, Urbana, IL 61821, USA.
E-mail: ssolecki@math.uiuc.edu.

The talk concerns Tukey reducibility between partial orders equipped with topologies appropriately interacting with the order relation. Tukey reduction is an important way of comparing partial orders and comes up naturally, for example, in classifying partial orders (Isbell, Todorcevic), studying σ-ideals of compact sets (Christensen), or proving inequalities between cardinal coefficients of the continuum (Bartoszynski, Fremlin). Recently it has been investigated quite intensively.

The main results of the work presented in this talk concern the structure of the order induced by Tukey reduction among partial orders occurring in topology and analysis, the structure of such partial orders, and the nature of Tukey reductions among them.

This is a joint work with S. Todorcevic.

Abstracts of contributed talks

► MARK BURGIN, *Compatibility in logical varieties*.
UCLA, Los Angeles, CA 90095, USA.
E-mail: mburgin@math.ucla.edu.

An important problem of logic is to combine logics and in particular, to include a system of calculi into one calculus [3]. Logical varieties give a relevant context for solving this problem. Let **K** be a class of logical calculi and M be a deductive variety (**K**-variety) [1].

Definition 1. Components C_i of M are called (provably) compatible (**K**-compatible) if for the subvariety of M generated by these components, there is (it is possible to build) an inclusion into a consistent calculus (from **K**).

Definition 2. A logical variety is called discrete if its components are disjoint. For any deductive variety M there is a disjoint deductive variety DM that has the same components.

Proposition 1. If components C_i of M are compatible (**K**-compatible), then the same components C_i of DM are compatible (**K**-compatible).

Theorem 1. For any number $n > 2$ there is a classical connected [2] deductive logical variety M with n components such that any $n - 1$ components of M are compatible, but all components of M are not compatible.

The condition that the variety is classical is essential.

Theorem 2. For any number $n > 2$ there is a classical connected deductive logical variety M with n components such that any $n - 1$ components of M are provably compatible, all components of M are compatible, but all components of M are not provably compatible.

However, for limit ordinals similar results are not valid.

Theorem 3. For any classical deductive logical variety M with a countable number of components, if any finite subset of components of M are compatible, then all components of M are compatible.

[1] M. BURGIN, *Logical methods in artificial intelligent systems*, ***Vestnik Comp. Sci. Soc.***, vol. 2 (1991), pp. 66–78

[2] ———, *Logical varieties and covarieties*, ***Methodological and theoretical problems of mathematics and computer science***, Kiev, 1997, pp. 18–34.

[3] D. GABBAY, ***Fibring logics***, Clarendon Press, Oxford,1999.

► WESLEY CALVERT, *The isomorphism problem for classes of computable structures*.
Department of Mathematics, 255 Hurley Hall, University of Notre Dame, Notre Dame, IN 46556, USA.
E-mail: wcalvert@nd.edu.

In some classes of structures which are non-classifiable in the Shelah sense, the complexity is in some way so intrinsic to the theory that it shows up even among the countable members, in the form of high Borel complexity. Study of the recursion-theoretic complexity of the isomorphism problem demonstrates cases in which this structural complexity sometimes shows up even among the computable members of the class.

For some classes (linear orders, for instance) the isomorphism problem is Σ^1_1 complete (that is, even the class of computable models is maximally complicated), and for others it is of relatively low complexity. Classes which in this sense are not maximally complicated include the class of models of any strongly minimal theory, Archimedean real closed fields, reduced Abelian p-groups of bounded Ulm length, and finite rank torsion-free Abelian groups.

In this talk I will discuss the calculation of the complexity of the isomorphism problem. The effects of model-theoretic considerations on the isomorphism problem will be discussed, as well as recent progress toward calculating the complexity of the isomorphism problem for infinite rank torsion-free Abelian groups.

► W. CALVERT, D. CUMMINS, J. F. KNIGHT, AND S. MILLER, *Comparing classes of finite structures*.
Department of Mathematics, University of Notre Dame, 255 Hurley Hall, Notre Dame, IN 46556, USA.
E-mail: smiller9@nd.edu.

In this talk, my goal is to compare classes of structures using a notion of *computable embedding*. A computable embedding is a uniform enumeration reduction mapping the isomorphism types of one class to the isomorphism types of the other in a well-defined and one-to-one way. The computable embedding relation is a partial ordering on classes of structures. We have some "landmark" classes—finite prime fields, finite linear orderings, finite dimensional vector spaces over the rationals, and arbitrary linear orderings—forming a strictly increasing sequence. If we restrict our attention to classes of finite structures, then the class of finite linear orderings lies on top, along with the class of finite cyclic groups and the class of finite undirected graphs. There are many incomparable classes below the class of finite prime fields, and between that and the class of finite linear orderings. If we allow classes that contain infinite structures, then the class of undirected graphs lies on top. There are many incomparable classes between the class of finite linear orderings and the class of finite dimensional vector spaces, and between this class and the class of all linear orderings.

► JOHN CHISHOLM, Δ^0_n*-categorical structures need not be relatively* Δ^0_{n+1}*-categorical*.
Department of Mathematics, Western Illinois University, Macomb, IL 61455, USA.
E-mail: JA-Chisholm@wiu.edu.

Let A be a computable structure and n a positive integer. A is Δ^0_n*-categorical* iff for every computable $B \approx A$ there ia an isomorphism $f: A \to B$ with f being a Δ^0_n function. A is *relatively* Δ^0_n*-categorical* iff for every $B \approx A$ (whether computable or not) there is an isomorphism $f: A \to B$ with f being $\Delta^0_n(B)$. Similarly, a relation R on A is *intrinsically* Σ^0_n iff $f(R)$ is Σ^0_n for every isomorphism f mapping A to a computable B, and R is *relatively intrinsically*

Σ^0_n iff $f(R)$ is $\Sigma^0_n(B)$ for every isomorphism f mapping A to an arbitrary structure B.

THEOREM 1. *For every positive integer n, (1) There exists a structure that is Δ^0_n-categorical but not relatively Δ^0_{n+1}-categorical; and (2) There exists a relation that is intrinsically Σ^0_n but not relatively intrinsically Σ^0_{n+1}. [The $n = 1$ case of (2) was proved in theorem III of* [1].]

These results improve by one jump on the finite ordinal case of Theorems 6.1 and 6.2 of [2].

[1] JOHN CHISHOLM, *The complexity of intrinsically r.e. subsets of existentially decidable models*, ***The Journal of Symbolic Logic***, vol. 55 (1990), pp. 1213–1232.

[2] S. S. GONCHAROV, V. S. HARIZANOV, J. F. KNIGHT, C. MCCOY, R. MILLER, and R. SOLOMON, *Enumerations in computable structure theory*, submitted to ***Annals of Pure and Applied Logic***.

► JEFFRY L. HIRST, *Minima of initial segments of sequences of reals*.
Department of Mathematical Sciences, Appalachian State University, Boone, NC 28608, USA.
E-mail: jlh@math.appstate.edu.

Suppose that $\langle x_k \rangle_{k\in\mathbb{N}}$ is a countable sequence of real numbers. Working in the usual subsystems for reverse mathematics, RCA_0 suffices to prove the existence of a sequence of reals $\langle u_k \rangle_{k\in\mathbb{N}}$ such that for each k, u_k is the minimum of $\{x_0, x_1, \ldots, x_k\}$. However, if we wish to prove the existence of a sequence of integer indices of minima of initial segments of $\langle x_k \rangle_{k\in\mathbb{N}}$, the stronger subsystem WKL_0 is required. Computability theoretic corollaries of these results can illustrate a distinction between computable analysis and constructive analysis.

► ERMEK S. NURKHAIDAROV, *On automorphism groups of countable arithmetically saturated models of Peano Arithmetic*.
Department of Mathematics, University of Connecticut, Storrs, CT 06269, USA.
E-mail: ermek@math.uconn.edu.

A model M of Peano Arithmetic is called arithmetically saturated if and only if it is recursively saturated and the standard cut is strong. When studying the automorphism group $Aut(M)$ of a countable arithmetically saturated model M of Peano Arithmetic, one is interested to what extent M is recoverable from $Aut(M)$. We show that $Aut(M)$ can recognize if a maximal open subgroup is a stabilizer of element $a \in \Omega_\omega$, where Ω_ω is the set of all elements greater than standard cut and smaller than any non-standard definable element. We use that fact to show the following theorem:

THEOREM. *Let M_1, M_2 be countable arithmetically saturated models of Peano Arithmetic such that $Aut(M_1) \cong Aut(M_2)$. Then $(\omega, Rep(Th(M_1))) \models RT^n_2$ iff $(\omega, Rep(Th(M_2))) \models RT^n_2$.*

Here RT^n_2 is Ramsey's Theorem stating that every 2-coloring of $[\omega]^n$ has an infinite homogeneous set. From the theorem we have the following corollary:

COROLLARY. *There are countable arithmetically saturated models M_0, M_1, M_2, M_3 of Peano Arithmetic such that they have the same standard system and $Aut(M_i) \ncong Aut(M_j)$, where $i < j < 4$.*

The corollary is an improvement of a previous result which shows the existance of only 2 such models. The results presented here are part of my thesis, which is being written with James H. Schmerl.

► KERRY OJAKIAN, *The probabilistic method and Ramsey Theory in bounded arithmetic*.
Carnegie Mellon University, Department of Mathematics, 6113 Wean Hall, Pittsburgh, PA 15217, USA.
E-mail: keo@andrew.cmu.edu.

Working in bounded arithmetic (Buss's S_2), we formalize various applications of the probabilistic method. The direct counting argument does not work, but as long as the appropriate function can be defined, we can use the weak pigeonhole principle. Some of these applications are to Ramsey Theory.

► KSENIJA SIMIC, *Ergodic theorems in subsystems of second order arithmetic*.
Carnegie Mellon University, 221 Gross St., Pittsburgh, PA 15224, USA.
E-mail: ksimic@andrew.cmu.edu.

The ergodic theorems state that for an appropriately defined measure preserving transformation T on a space X, the sequence

$$S_n = \frac{1}{n}\sum_{k=0}^{n-1} f(T^k)$$

converges: in the case of the mean ergodic theorem the convergence is in L_2 norm for all $f \in L_2(X)$ and in the case of the pointwise ergodic theorem we can establish both a.e. and L_1 convergence for all $f \in L_1(X)$.

Due to the restrictions second order arithmetic imposes, difficulties arise in formalizing and proving the ergodic theorems in this framework. For one, the transformation T cannot be defined on points, or sets, but on functions. This in turn gives rise to new difficulties, for it is not immediately clear how to adapt standard proofs of the theorems.

I will talk about ways to avoid these obstacles, focus on the pointwise ergodic theorem, and show how it can be proved from the mean ergodic theorem. The main result is that both ergodic theorems are equivalent to arithmetic comprehension over the base theory RCA_0.

► REBECCA WEBER, *A substructure of the lattice of Π^0_1 classes*.
Department of Mathematics, University of Notre Dame, 255 Hurley Hall, Notre Dame, IN 46556, USA.
E-mail: rweber@nd.edu.

In [1], Cenzer and Nies showed that an end interval $[P, 1]$ of the lattice of Π^0_1 classes, $\mathcal{E}_\Pi$, is not isomorphic to $\mathcal{E}_\Pi$ if P is nonprincipal. This confirmed a conjecture of Herrmann in [2], where it was shown that there were at most two distinct types of end interval of $\mathcal{E}_\Pi$. Nies named this new structure G and defined a quotient structure $G^\diamond$, G modulo clopen Π^0_1 classes disjoint from P. G and $G^\diamond$ may also be considered as substructures of the lattice of c.e. ideals of the countable atomless Boolean algebra. From that perspective, $G^\diamond$ reveals a rich structure; in fact it is isomorphic to $\mathcal{E}^*$, the lattice of c.e. sets modulo finite difference.

[1] D. Cenzer, and A. Nies, *Global properties of the lattice of Π^0_1 classes*, ***Proceedings of the American Mathematical Society***, to appear.

[2] P. Cholak, R. Coles, R. Downey, and E. Herrmann, *Automorphisms of the lattice of Π^0_1 classes; perfect thin classes and ANC degrees*, ***Transactions of the American Mathematical Society*** vol. 353 (2001), pp. 4899–4924.

Abstracts presented by title

► JOHN CORCORAN, *Comparing Aristotle's **Prior Analytics** and Boole's **Laws of Thought***.
Philosophy, University of Buffalo, Buffalo, NY 14260-1010, USA.
E-mail: corcoran@buffalo.edu.

Aristotle and Boole are the two most original logicians before the era of modern logic. In *Prior Analytics* Aristotle presented the world's first system of logic. His system involves the standard three parts: first, a limited formalized "predicational" language; second, a formal method of step-by-step deductions for establishing validity of arguments having unlimited numbers of premisses; and third, an equally general method of countermodels for establishing invalidity. In *Laws of Thought* Boole's stated aims were to refine, systematize, and complete the project started by Aristotle and, more momentously, to demonstrate the mathematical character of logic. His two-part system involves, first, a limited formalized "equational" language capable of expressing tautologies or "laws of thought", a revolutionary breakthrough dramatically altering Aristotle's plan, and, second, a semi-formal method of derivation using equational reasoning common in analysis, but totally absent from previous systematic logic. While Aristotle's method of deduction satisfies modern standards of soundness, completeness and cogency (including "gaplessness"), Boole's method of derivation has none of these properties. Boole's primary goals did not include construction of a comprehensive method for gaplessly deducing conclusions from sets of propositions regarded as premisses as Aristotle's did, but rather construction of a method for generating solutions to sets of equations regarded as conditions on "unknowns", another unprecedented revolutionary innovation with radical implications for the future development of logic. The derivations involved in Boole's equation-solving method are far from complete, e.g., the associative laws are missing, a transparent but typical omission. As for the third part of a logic, a method of establishing invalidity, Boole's book contains no systematic discussion of independence nor does it contain anything like a method of countermodels. Despite its imperfections, Boole's book set in motion forces that would lead to the ultimate fulfillment of many of his goals including the establishment of mathematical logic.

► GRIGORI MINTS AND TING ZHANG, *Dynamical topological logic of Cantor space*.
Philosophy Department, Stanford University, Stanford, CA 94305, USA.
Computer Science Department, Stanford University, Stanford, CA 94305, USA.
E-mail: mints@csli.stanford.edu.

Dynamical topological logic [2] studies models (X, S), where X is a topological space, S a transformation on X. Propositional formulas are constructed from variables (atomic formulas) by Boolean connectives, necessity $\Box$ and a monadic operation $\circ$. Variables are interpreted by subsets of X, Boolean connectives act in a natural way, $\Box$ is the interior and $\circ$ is the pre-image under operation S. The axiom schema $\circ\Box A \rightarrow \Box \circ A$ called (C) expresses continuity of S. The propositional system S4C includes S4,(C) and axioms $\circ(A\&B) \iff \circ A\& \circ B$, $\circ\neg A \iff \neg \circ A$. J. Davoren [1] proved completeness of S4C for the class of all topological spaces, in particular for finite spaces derived from Kripke models which are not very natural mathematically. We prove completeness of S4C for Cantor space very popular in the theory of dynamical systems. P. Kremer pointed out that the real line is not complete for S4C. An elegant example was constructed by J. van Benthem. Let $\mathcal{B}$ be the Cantor space, that is the set $\{0, 1\}^\omega$ of all infinitary binary sequences $\boldsymbol{b} = b(0), b(1), \dots$. Let $\mathcal{B}_1 = \{\boldsymbol{b} \in \mathcal{B}$: $\boldsymbol{b} = b0^\omega$ for some $b \in \{0, 1\}^*\}$, $\mathcal{B}_2 = \mathcal{B} - \mathcal{B}_1$.

By [1] every formula underivable in S4C is refutable in a finite Kripke frame $\langle W, R\rangle$ with the root w_0 and operation $S\colon W \rightarrow W$ satisfying $Rww' \Rightarrow R(Sw)(Sw')$. Let $\mathcal{K}$ be the topological space with the carrier W and topology determined by R. Define a map $\mathcal{W}\colon \{0, 1\}^* \rightarrow W$ as follows. For the empty sequence Λ, $\mathcal{W}(\Lambda) = w_0$; if $\mathcal{W}(b) = w$ and w has R-successors $w, w_1, \dots, w_{m-1}$, then $\mathcal{W}(b0^i) = w$ for $i = 1, \dots, m$, while $\mathcal{W}(b0^i 1) = w_i$ for $i = 0, \dots, m - 1$. Define $w \sim_R w'$ as $Rww'\&Rw'w$. Define the stabilization point of a sequence $x \in \mathcal{B}$: $\boldsymbol{stb}(x) =$ the least finite prefix b of x such that $\mathcal{W}(b') \sim_R W(b)$ for any longer prefix b' of x. Define a stopping point by $\boldsymbol{stp}(x) = \boldsymbol{stb}(x)$ if $x \in \mathcal{B}_2$, $\boldsymbol{stp}(x) = \Lambda$, if $x = 0^\omega$, and $\boldsymbol{stp}(x) = b1$, if $x = b10^\omega$ for some $b \in \{0, 1\}^*$. It is possible to define a map $T'\colon \{0, 1\}^* \rightarrow \{0, 1\}^*$ such that $\mathcal{W}T' = S\mathcal{W}$. Moreover, T' extends to a continuous function $T\colon \mathcal{B} \rightarrow \mathcal{B}$, and every chain of the form $x, T^{-1}x, \dots, T^{-n}x$ is finite. Define $\boldsymbol{rk}(x)$ to be 0 for $x = 0^\omega$ and the least n such that $T^{-n}x$ is not in the range of T otherwise. Define a finite representative of a sequence $x \in \mathcal{B}$ by induction on $\boldsymbol{rk}(x)$: $\boldsymbol{ch}(x) = \boldsymbol{stp}(x)$, if $\boldsymbol{rk}(x) = 0$, and $\boldsymbol{ch}(x) = T'(\boldsymbol{ch}(T^{-1}(x))$ otherwise. Define $f(x) = \mathcal{W}(\boldsymbol{ch}(x))$.

THEOREM. (*a*) *The function f is a continuous open embedding of $\mathcal{B}$ onto $\mathcal{K}$ satisfying condition*: $fT = Sf$.

(*b*) *$S4C$ is complete for topological semantics in Cantor space.*

[1] J. M. DAVOREN, *Modal logics for continuous dynamics*, ***Ph.D. thesis***, Cornell University, 1998.

[2] G. MINTS and P. KREMER, *Dynamic topological logic*, ***The Bulletin of Symbolic Logic***, vol. 3, (1997), no. 3, pp. 371–372.

► XUNWEI ZHOU, *Mutual-inversism—A new school of mathematical logic*.
College of Information, Beijing Union University, 97 Beisihuandong Road, Chaoyang District, Beijing, China.
E-mail: zhouxunwei@263.net.

Mutual-inversism, constructed by the author, is a new school of mathematical logic, including zeroth, first and second level logical calculus, set theory, proof theory, model theory, recursion theory, abstract algebra, etc. In mutually-inversistic set theory, two methods are used to eliminate paradoxes: First, elements and sets are divided into three levels; secondly, empty set and universal set are regarded as distinguished sets, their uses are restricted. Mutually-inversistic set-theoretic system is both consistent and complete. Mutually-inversistic abstract algebra is based on mutually-inversistic set theory. In mutually-inversistic abstract algebra, semigroup, monoid, group, ring, field, lattice, Boolean algebra, linear algebra, etc., are inherited from classical abstract algebra; new algebraic properties are discovered such as binary bijection, complement idempotency; new algebras are established such as main-auxiliary algebra for logical theorem, Boolean sum-Boolean product algebra, NAND-NOR algebra, universal matrix. Mutually-inversistic abstract algebra is classified horizontally as term space algebra and fact space algebra, vertically as auxiliary algebra and main-auxiliary algebra. In mutually-inversistic proof theory, mutually-inversistic number-theoretic system is established which is both consistent and complete; faults in the proofs of Gödel's two incompleteness theorems are revealed. Mutually-inversistic model theory is divided into term space model theory and fact space model theory which have strong connections with term space algebra and fact space algebra respectively. Mutually-inversistic recursion theory inherits from classical recursion theory almost intactly.