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REVIEWS

The Association for Symbolic Logic publishes analytical reviews of selected books and articles in the field of symbolic logic. The reviews were published in ***The Journal of Symbolic Logic*** from the founding of the JOURNAL in 1936 until the end of 1999. The Association moved the reviews to this BULLETIN, beginning in 2000.

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In a review, a reference "JSL XLIII 148," for example, refers either to the publication reviewed on page 148 of volume 43 of the JOURNAL, or to the review itself (which contains full bibliographical information for the reviewed publication). Analogously, a reference "BSL VII 376" refers to the review beginning on page 376 in volume 7 of this BULLETIN, or to the publication there reviewed. "JSL LV 347" refers to one of the reviews or one of the publications reviewed or listed on page 347 of volume 55 of the JOURNAL, with reliance on the context to show which one is meant. The reference "JSL LIII 318(3)" is to the third item on page 318 of volume 53 of the JOURNAL, that is, to van Heijenoort's *Frege and vagueness*, and "JSL LX 684(8)" refers to the eighth item on page 684 of volume 60 of the JOURNAL, that is, to Tarski's *Truth and proof*.

References such as 49***5*** or 280***1*** are to entries so numbered in *A bibliography of symbolic logic* (the JOURNAL, vol. 1, pp. 121–218).

DOV GABBAY. ***Fibring logics,*** Oxford Logic Guides, vol. 38. Oxford University Press, 1998, xiii + 475 pp.

Studying combined logics is a growing trend in applied logic. The general question is this: Suppose language $\mathcal{L}_3$ has been formed by fusion of two languages $\mathcal{L}_1$ and $\mathcal{L}_2$. Let two logics L_1 and L_2 be defined over $\mathcal{L}_1$ and $\mathcal{L}_2$, respectively. What can be said about conservative extensions L_3 in $\mathcal{L}_3$? This is a fairly broad problem. Conservative extensions are not unique, and they may have different properties. In modal logics, several extensions have been studied in detail. For example Marcus Kracht and Frank Wolter, *Properties of independently axiomatizable bimodal logics*, ***The Journal of Symbolic Logic***, vol. 56 (1991), pp. 1469–1485, studied bimodal modal logics which are the minimal conservative extensions of their monomodal fragments (called fusions). Another example are the products of modal logics, to which the author of the present book has made important contributions. Numerous questions arise: what are the properties of the combination and can they be predicted from the properties of the fragments? Is there a general recipe by which we can obtain combined systems?

The present book promotes *fibring* as a general model construction for combined systems. The general idea is this. Let the languages $\mathcal{L}_1$ and $\mathcal{L}_2$ and two classes of models be given. Fibred models can be thought of as models for one of the languages enriched with an oracle that answers about the truth of formulae of the other language. This oracle takes the form

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of a function F, which, given a model $\mathfrak{M}$ (which typically contains a distinguished world or state), returns a model $F(\mathfrak{M})$ of the other language in which φ is evaluated. F is called the *fibring function*. If φ actually contains operators of the first language, this construction thus toggles between the two model classes until φ is totally decomposed.

The possibilities for combination are galore. We can try to combine logics whose signature is more or less similar (like two predicate logics with different but possibly nondisjoint signature), modal logics with different sets of operators, but we may also try something more radical by combining logics that are basically dissimilar. Two paradigm examples are intuitionistic modal logic (where the modal operator with its modal semantics is added on top of the intuitionistic connectives) and the temporalisation of systems. The latter is interesting insofar as it shows that it might be useful to study systems that do not use the full power of the combined language. Suppose that we have an arbitrary logical language L, say that of predicate logic. Now we study the language that arises by issuing temporal statements whose primitive clauses are constituted by members of L. Thus the connectives of L may not take temporal connectives in their scope. A similar, no less important example comes from programming. It is often desirable to import programs from a different programming language. Some implementations allow this. However, the languages may be very different (say, Prolog and C). One therefore does not wish to fuse the languages; one simply wishes to be able to import the results of a query issued in another language. At times the book actually suggests that just about any consequence relation in the joint language $\mathfrak{L}_3$ is a 'fibring' of the consequence relations induced on the fragments $\mathfrak{L}_1$ and $\mathfrak{L}_2$ (see pages 327–328).

Fibring is supposed to be a universal construction that allows to construct a semantics for combined systems. The fundamental insight on which it rests is that basically any language with operations that are either monotonic or antitonic with respect to their arguments are complete with respect to Kripke–like structures. This allows to use an idea of fibring that derives from the 'fig cactus' models, in which the fibring function is a function which assigns a model of the other language to each world of the structure of the first language. The fusion of the modal logics L and M, denoted by $L \otimes M$, is Kripke–complete if L and M are. (Actually, Frank Wolter, *Independent fusions of modal logics revisited*, in ***Advances in modal logic*** (Marcus Kracht, Maarten de Rijke, Heinrich Wansing, and Michael Zakharyaschev, editors), CSLI, 1997 pp. 361–379, has provided a construction even in case that the logics are incomplete.) Moreover, it inherits many joint properties of the fragments. Unfortunately, the intuition behind fibring suffers from a defect that is never really addressed. The fibring function effectively glues Kripke–frames for one language onto Kripke–frames of the other. This is done in parallel for each of the worlds, and each of the models added is an oracle not only for the formulae we are interested in but for all others as well. However, take the example of $\mathsf{K.B} \otimes \mathsf{K.B}$, where $\mathsf{K.B} = \mathsf{K} \oplus \Diamond\Box p \to p$, the logic of symmetric Kripke–frames. The two fragments use the same boolean connectives and a modal operator, $\Box$ for the first and $\blacksquare$ for the second fragment. Then if $w \ R_{\Box} \ x$ and $x \vDash \blacklozenge\blacksquare\Box p$ then $w \vDash p$, because $x \vDash \Box p$ and $x \ R_{\Box} \ w$, by symmetry. This is independent of the way in which the models are combined; it follows from the properties of the logics alone (namely, that $\Diamond\blacklozenge\blacksquare\Box p \to p$ is a theorem of the fusion). This can make the construction break down because we might be adding the wrong model at another world. (This is why the construction of Marcus Kracht and Frank Wolter, *Properties of independently axiomatizable bimodal logics*, ***The Journal of Symbolic Logic***, vol. 56 (1991), pp. 1469–1485, is rather involved.) The book briefly glosses over the problem without proving the innocence of the construction in general.

The book is a tour de force of applied logic. Every area is touched on: classical, nonclassical, modal, fuzzy and substructural logic, logic of programs, propositional and predicate logic. Certainly, for some readers this will be a plus because the method is amply exemplified. Unfortunately, in an attempt to show that fibring is *the* solution to the problem of combining logics, the book rarely goes beyond defining the model structures in the various cases. This however leaves little space to discuss the general theory of fibring itself. Instead, one example after the other is presented, and often it is not clear what contribution fibring actually makes. For example, in making the logic fuzzy more has to be done that just fibring the models. First, the semantics of the nonmodal language has to be fuzzified (which is not done using fibring). Second, the interpretation of the box is actually not unique despite expectation. (We could make $R_\Box$ fuzzy, too, which effectively means that we combine a fuzzy modal logic to begin with rather than *making* it fuzzy.) Also, it sometimes turns out that the logic of the fibred structures is not even conservative over its fragments. Strangely enough, in the view of the author this does not seem to disqualify them.

There are other deficiencies, too. One is the relaxed use of terminology. There is practically no distinction between 'structure' and 'model', for example. The notation has not been given much thought and occasionally changes without warning. In general, proofs are not really worked out. One example is the completeness proof of fibring for polymodal logics, another is the claim on page 61 that necessitation is admissible in $\boldsymbol{T}_I^F$ if the individual logics have the disjunction property. This is however not sufficient (a counterexample is the combination of K.T with itself, a fact that is noted on page 54).

In sum, even though the topic is certainly interesting and the technique of fibring deserves attention, this book is not the kind of book that will easily promote it. It has obviously been produced in a hurry. At numerous places the book iterates material, and sometimes it even contradicts itself. The benefits of fibring are not demonstrated beyond doubt, especially since the construction needs to be carried out with care.

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R. Amadio and P. L. Curien. ***Domains and lambda-calculi.*** Cambridge tracts in theoretical computer science, vol. 46. Cambridge University Press, 1998, 500 pp.

The book under review is a very nice introduction to the semantics of various typed versions of the λ-calculus considered as core programming languages and as meta-languages. The main goal is not the syntactic properties of such calculi (often referred as the *operational semantics*) but their *denotational semantics*. The distinction of operational vs denotational is reminiscent of the distinction between "function as extension or as a graph" and "function as a rule or as an algorithm". The denotational semantic of a program gives an interpretation that helps to prove its properties independently of its implementation. It gives a link between the mathematical structures of functional programming languages and their operational behavior. The qualities of a denotational semantics can be measured in the way it matches an independently defined operational semantics. Conversely, an operational semantics, like any formal system, can be blessed by soundness or even completeness results with respect to some denotational semantics.

The book presents in an elementary and unified way the theory of certain topological spaces presented as ordered theoretical structures that have proved to be useful in the modelling of various families of typed λ-calculi. This theory, now known as *domain theory*, has been founded as a subject by D. Scott and G. Plotkin.

This book is designated to theoretical researchers. It can motivate computer scientists to do some mathematics and may interest mathematicians in structures that are unfamiliar and far away from the traditional core of mathematics. It is really self contained, i.e., its starts from (almost) nothing and brings the reader to the level of a very good expert. Note that about one third of the sections (marked by $*$ in the table of contents) contain more advanced or less fundamental material. Many exercises help the reader to check his understanding: some are easy, some are quite difficult.

Even for a non novice reader, the material given is rather difficult but since it is very well written and pleasant to look at, despite its difficulty, it will be very useful in any library.

Here is a brief description of its 16 chapters. Chapter 1 is an introduction to *domain theory* (directed complete partial orders, Scott continuity, ...). It shows, via the Myhill-Sheperdson theorem that, in some sense, computability implies continuity. As an illustration it gives the denotational semantics of a toy language.

Chapter 2 is a complete introduction to the syntactic theory of *pure λ-calculus*. It gives its main basic results (and their proofs): confluence, standardization, finiteness of developments, sequentiality, Böhm's theorem, ... It also gives Levy's labelled calculus presented as a tool for proving many fundamental theorems.

Chapter 3 introduces the *Scott models* D_∞ as a solution to the equation $D \cong D \to D$. It is shown that this gives models of λ-calculus. Their relation with Böhm trees is given. Filter models also are considered: a term is interpreted as the set of its types. Some consequences on the relation between typability and normalization are given.

Chapter 4 is central. It shows how to model the simply typed λ-calculus in a *cartesian closed category* (CCC), introduces the notion of λ-theory and shows that every theory comes from a suitable CCC. The logical relations (a useful tool to link syntax and semantics) is also introduced. For the readers who are not familiar with categories, an appendix gives the main definitions.

Chapter 5 gives a complete presentation of the problem of classifying the largest CCC of algebraic directed complete partial orders and continuous morphisms.

Chapter 6 presents the language PCF. It is a typed λ-calculus extended with fixpoints and arithmetic operators which can be considered as a real functional programming language. The full abstraction problem i.e., the correspondence between the denotational and operational semantics is discussed and Vuillemin's sequential functions are introduced.

Chapter 7 presents general techniques for the solution of *domain equations* and the representation of domains and functors over a universal domain.

Chapter 8 considers various reduction strategies and, in particular, the *call-by-value λ-calculus* and a λ-calculus with control operators. The semantic aspects of these calculi is based on the idea, due to Moggi, of computation-as-monad. The extension of the Curry-Howard correspondence to control operators shows that the continuation passing style translations can be regarded as a way to extract an effective content from a classical proof.

Chapter 9 concentrate on *power domains*. It introduces a process calculus (Milner's CCS) and its operational semantics based on the notion of bisimulation.

Chapter 10 considers *Stone duality* applied to domains. This duality arises in topology from the consideration of points versus open sets. For the purpose of computer science, points correspond to programs and open sets to program properties.

The main goal of chapter 11 is the introduction of λ-calculi with *dependent and second order types* (in particular, Girard polymorphic types and Edinburgh LF). The related domain constructions are also given.

Chapter 12 introduces a new notion of domains where continuity is replaced by *stability*, a notion due to Berry which formalizes the property that there is always a minimum part of a given input needed to reach a given finite output.

Chapter 13 is devoted to *linear logic*. Starting from the coherent spaces, some of its models are presented: Ehrhard's hypercoherence model and Winskel's bistructures model.

Chapter 14 is devoted to the semantics of *sequentiality*. Kahn-Plotkin's concrete data structures and Berry-Curien sequential algorithms are introduced. It is shown that the model of sequential algorithms is fully abstract for the extension of PCF with a control operator catch

Chapter 15 considers the category of *partial equivalence relations* and exhibit an interpretation of system F, of a type assignment system and of a sub-typing system.

Chapter 16 introduces a simple extension of CCS known as the π-calculus, whose initial purpose was to represent the notion of reference in a *concurrent* computing setting. It allows simple encodings of various functional and concurrent models.

Finally two appendices are summaries of recursion and category theory.

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BENJAMIN C. PIERCE. ***Types and programming languages.*** The MIT Press, Cambridge, Massachusetts, 2002, xxi + 623 pp.

Types were developed in the early part of the 20th century in order to avoid inconsistencies in Frege's formulation of logic discovered by Russell. They therefore have a strong logical origin, confirmed by the development of Church's type theory, sometimes called higher-order logic. It contains the simply-typed λ-calculus which is at the core of most modern programming languages. Through the research of other pioneers such as Milner and Martin-Löf, type systems are now understood to be central in the design and analysis of programming languages. They provide the fundamental principles according to which languages are organized. A thorough textbook on type systems in programming languages had been long overdue and Pierce's book provides exactly that.

Types and programming languages is designed for an advanced undergraduate or graduate course and assumes some familiarity with functional programming. Since types and programming languages are by now a large subject, a stringent selection of topics is necessary. After providing some background on the λ-calculus and basic notions such as substitution and induction, the book starts with simple types. In this setting the pervasive idea of type safety is first introduced. Type safety is comprised of two properties relating typing with the operational semantics of a language: *preservation*, which guarantees that a term retains its type under evaluation, and *progress*, which guarantees that a well-typed term is either a value or can proceed to be evaluated further. Type safety is one of the recurring themes of the book and one of its most important lessons.

Following the thorough treatment of simple types, the book successively develops subtyping, recursive types, polymorphism, and type operators. Subtyping is motivated and presented as record subtyping; coercion semantics and coherence are covered as a short excursion. Recursive types are presented first informally, and then in both iso-recursive and equi-recursive forms. The treatment of co-induction in the study of recursive types is a nice addition that is difficult to find elsewhere in introductory form. The next part on polymorphism covers type inference, ML-style and impredicative polymorphism, existential types for data abstraction, and bounded quantification. Much of this should be core material in any advanced course concerned with programming languages. The final part treats type operators and higher-order polymorphism, a topic rather more advanced particularly in its application to the study of purely functional objects in the last chapter.

The pragmatics of type constructs are illustrated in interesting and novel ways. First, for most chapters there is an implementation available from a web site with supporting material. Several of these implementations are discussed in detail in the book, which can be quite helpful to students for gaining a deeper and more intuitive understanding of the fundamental concepts. Second, there are many exercises interleaved with the text whose solutions are given at the end of the book. This device is helpful, even to advanced readers, by giving them the opportunity to test their comprehension immediately. Third, the book contains several excursions called "*case studies*" where the type systems are exploited in sometimes unexpected ways to support object-oriented programming.

The book is very well-written, with clear motivations, cogent explanations, helpful examples, appropriate exercises, and thorough discussions. The material is well-structured; the incremental approach to developing a cumulative system quite successful. Besides serving as an excellent textbook, the detailed notes and suggestions for further reading make it an important reference and study resource. I have used the book directly for two lectures and also as general supplementary reading in my junior-level undergraduate course on *Foundations of Programming Languages*. The students and I found it quite easy to use and I will likely expand its role the next time I teach the course.

The one aspect of the book that would make me hesitate to adopt it as the sole textbook for one of my own courses is the orientation of its latter parts towards a functional interpretation of object-oriented programming at the expense of other important topics. Much of the material on subtyping, bounded quantification, and type operators is motivated and explained in terms of a functional reconstruction of objects based on structural subtyping. I do not find this approach to be particularly realistic or appealing and I think it is difficult for students to relate to their own object-oriented programming experience, which almost universally stems from languages with nominal subtyping.

In an undergraduate course, I have instead covered some further topics on implementation such as environments, abstract machines, or storage management, advanced type constructs such as monads or intersection types, or other paradigms such as concurrency or logic programming. In a graduate course I might instead expand on the meta-theory, covering topics such as parametricity or normalization for System F in more detail. Admittedly, some of these topics clearly exceed the bounds of a textbook on type systems, but others, particularly monadic types, intersection types, and dependent types would have rounded out the coverage.

In summary, ***Types and programming languages*** provides an excellent, unified introduction to type systems, suitable for an advanced undergraduate or graduate course, notwithstanding minor concerns about coverage and approach. In addition, it will serve as a standard reference in the field for many years to come. Given the central role of types in modern programming language design and analysis, ***Types and programming languages*** is probably the single most important book in the area of programming languages in recent years.

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Marcus Giaquinto. ***The search for certainty: a philosophical account of foundations of mathematics.*** Clarendon Press, Oxford, 2002, xii + 286 pp.

There was a time when mathematical knowledge was thought to be certain—indeed, mathematical knowledge was seen as an exemplar of the kind of certainty that we would like for other areas of enquiry. To use a familiar foundationalist image, mathematical knowledge was seen as the secure base on which we could construct our epistemic edifices. Unlike empirical knowledge, mathematical knowledge was not thought to be subject to the kind of error that leads to revision and replacement of scientific theories. Once proven, a mathematical theorem was thought to stand for all time. But such faith in mathematics was shaken around the end of the nineteenth century when, despite a couple of hundred years of significant advancements in the rigorization of analysis, several set-theory paradoxes came to light that shook mathematics to its core. These paradoxes and the responses to them are the subject of Marcus Giaquinto's very engaging book.

The book presents an historically informed account of the mathematical and philosophical community's struggle with the crisis brought on by the paradoxes. The reader is taken on a fascinating tour of "one of the most brilliant intellectual explorations ever" (p. vii). The book is divided into 6 sections, each typically consists of several chapters. The first section, provides some of the nineteenth century historical background. In particular, the quest for rigorization of analysis is discussed. Section II presents three of the paradoxes—the Burali-Forti paradox, Cantor's paradox and Russell's paradox. The early responses by Cantor, Frege and Russell are discussed and the inadequacies of each response is revealed. The next section presents a few of the definability paradoxes such as Berry's paradox and the well known paradoxes of truth, such as the strengthened liar paradox and cyclical liar paradoxes. It also gives a detailed account of the Russell and Whitehead response to the paradoxes in terms of the vicious circle principle and ***Principia mathematica***'s theory of types—both simple type theory and the ramified theory. Ramsey's defense of logicism as a response to the paradoxes is also discussed in this section. Section IV focusses on Zermelo's axiomatic set theory and Hilbert's formalist/finitist program. The next section presents Gödel's incompleteness theorems and how these serve to undermine Hilbert's program. The final section defends the axiomatic approach to set theory as an adequate response to the paradoxes and draws some philosophical conclusions from the preceding discussion.

The take home lesson of this book is that the crisis in the foundations of mathematics is over. There is no good reason to believe that standard set theory—ZFC—is inconsistent. Moreover, this theory can be defended against the charge of ad hocness. In particular, ZFC has shed its last vestiges of (ad hoc) limitation of size axioms. The last such vestige being the axiom of replacement, and this can be dispensed with by adopting Levy's Reflection Principle. The set theory delivered, though, is a long way from the traditional idea of a foundation. Traditionally conceived, foundations are supposed to be self evident and secure—secure enough to serve as a basis on which other beliefs may rest. But the first move away from such a foundation came with Russell who, in the light of the paradoxes, claimed that axioms may be justified by the theorems they yield: "we tend to believe the premises because we can see that their consequences are true, instead of believing the consequences because we know the premises to be true" (B. Russell, *The regressive method of discovering the premises of mathematics* reprinted in D. Lackey (editor) ***Essays in analysis***, George Braziller, 1973, pp. 272–283). This view was later endorsed by Gödel, and contemporary debates over large cardinal axioms serve to reinforce this "top-down" style of justification of the axioms (see P. Maddy, ***Naturalism in mathematics***, Oxford University Press, 1997). Such "top-down" justification suggests that set theory no longer serves as an epistemological foundation for mathematics—at least not in the traditional sense of 'foundation'. Indeed, this style of justification makes much more sense in the context of coherence epistemologies. The move from foundational epistemology to more holistic approaches strikes me as one of the legacies of the paradoxes. Traditional, foundational epistemology is just not tenable in these post-paradox times.

Although much of the material covered in this book will be familiar to philosophers of mathematics and philosophical logicians, Giaquinto is able to shed new light on many issues and he always gives a very sympathetic reading of the various views he discusses. Indeed, I think Giaquinto has written a very good book, though one that's somewhat difficult to categorize. It professes not to be a history of twentieth century developments in the foundations of mathematics—its stated goal is to "set out and engage with the main philosophical ideas and arguments" (p. vii). There is no doubt that it achieves its stated goal but it does this in an historically well-informed way. There is also a great deal of technical detail in this book. It is not, however, a book on mathematical logic (although it would make an excellent companion to a standard course on mathematical logic or philosophy of mathematics). Giaquinto prefers not to skimp on technical detail. He believes that to understand and engage with the issues in question one must be familiar, at least in broad brush strokes, with some of the key technical results. I couldn't agree more. But importantly, in providing the relevant technical details, Giaquinto does not presuppose much by way of logical and mathematical background. Indeed, one of the many admirable features of this book is that, without "dumbing down" the content, Giaquinto manages to provide an accessible account of some fairly technical material. I'd suggest that a dedicated reader with only a first course in logic or some undergraduate mathematics behind him or her would be able to benefit from this book.

It should also be clear by now that this is an ambitious book. It covers a great deal of territory and discusses many interesting issues, and, as you might expect, there are many interesting issues worthy of further discussion. Let me focus on just a couple of these here. The question of why we should pursue a uniform treatment of the paradoxes is very important and, in my view, warrants further attention. There is a strong intuition that the language paradoxes and the set theory paradoxes must be treated in much the same way. But this is so only if you think that the language paradoxes are at one with the set theory paradoxes. Opinion is divided on this latter issue. Ramsey argued that there was a significant difference between the two types of paradox, whereas Russell thought they were essentially the same. Although Giaquinto does discuss this issue (pp. 104–106), I think that there is much more to be said. In particular, it would have been good to have presented the details of some of the formal frameworks that (allegedly) demonstrate the common structure of the language and set theory paradoxes. One important contemporary framework is Graham Priest's inclosure schema (see G. Priest, ***Beyond the limits of thought***, 2nd edition, Oxford University Press, 2002, especially pp. 133–136, for details).

A closely related issue which Giaquinto does not broach is the significance of Yablo-style paradoxes. Yablo's paradox is a liar-like paradox (though Yablo-style paradoxes now come in both language and set theory versions—see R. A. Sorensen, *Yablo's paradox and kindred infinite liars*, ***Mind*** vol. 107 (1998), pp. 137–55, for a survey and discussion of some of these) that, arguably, does not involve self reference or cycles of reference. If this, is right, then simple bans on circularity will not disarm this paradox. So, in order to advance a uniform solution to the paradoxes, we need to answer the question of where Yablo's paradox fits into the picture. It would be interesting to hear Giaquinto's thoughts on this matter.

These couple of comments are not intended to signal serious shortcomings of the book; an ambitious book such as this is bound to leave some issues unresolved. These are just some of the issues that I was led to think about while reading this excellent book.

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Silvio Ghilardi and Marek Zawadowski. ***Sheaves, games, and model completions.*** Trends in Logic, vol. 14. Kluwer, 2002.

One of the most striking and useful results in the proof theory of intuitionistic logic in the 1990's is undoubtedly Andy Pitts' Theorem, which gives an interpretation of the second-order intuitionistic propositional calculus into ordinary intuitionistic propositional logic IPC: for every IPC-formula ϕ and for every propositional variable p there are formulas $\exists p\phi$ and $\forall p\phi$, involving only variables occurring in ϕ except p, such that for any other IPC-formula ψ which does not contain p, one has the following equivalences:

$$\mathrm{IPC} \vdash \phi \to \psi \quad \text{iff} \quad \mathrm{IPC} \vdash \exists p\phi \to \psi$$

$$\mathrm{IPC} \vdash \psi \to \phi \quad \text{iff} \quad \mathrm{IPC} \vdash \psi \to \forall p\phi$$

The book under review is an investigation into algebraic and model-theoretic aspects of this fact. The authors have written a number of joint papers, of which this book can be seen as the synthesis; however, there is also material that was not published before.

Their starting point is a re-interpretation of Pitts' Theorem. It is fairly easy to see that it implies that the (first-order) theory of Heyting algebras has a model completion (a fact which, nonetheless, has surprised a few reputed researchers who had thought about this). However, in a sense there is also a converse to this! That is, the fact that the theory of

Heyting algebras admits a model completion can in turn be re-interpreted by saying that $\mathrm{HA}_{\mathrm{fp}}^{\mathrm{op}}$ (the opposite of the category of finitely presented Heyting algebras) is an r-Heyting category. From this last formulation, it is almost immediate to prove Pitts' theorem! What is more, the methods developed in order to establish these re-interpretations, generalize in a straightforward fashion to other theories which occur in the study of non-classical logics, such as the various types of modal algebras in existence.

Chapters 2 and 3 lay out the general framework. Let T be an arbitrary first-order equational theory; its models are called T-algebras. Various conditions on the category $\mathrm{Alg}(T)$ of T-algebras are studied; for example amalgamation, finite model property (every finitely generated T-algebra is a subalgebra of a product of finite T-algebras), and some universal-algebraic conditions such as 'Equationally Defined Operations' (EDO).

A *Heyting category* is a category with finite limits in which for every object A the lattice $\mathrm{Sub}(A)$ of its subobjects is a Heyting algebra, for each morphism $f: A \to B$ the map $f^*: \mathrm{Sub}(B) \to \mathrm{Sub}(A)$ obtained by pulling back along f, is a Heyting map, and moreover this map has both adjoints. In short, it is a category in which one can interpret first-order (many-sorted) intuitionistic logic. If one replaces in this definition $\mathrm{Sub}(A)$ by $\mathrm{Sub}_r(A)$, the lattice of *regular* subobjects of A, one gets the notion of an r-Heyting category.

Let $\mathbf{T} = \mathrm{Alg}(T)_{\mathrm{fp}}^{\mathrm{op}}$. Then there is a theorem:

THEOREM. *If T has a model completion, $\boldsymbol{T}$ is r-Heyting; the converse holds provided T satisfies EDO.*

Chapter 4 then investigates the theory of Heyting algebras in detail; according to the theorem above, this involves developing a duality theory for finitely presented Heyting algebras.

There may be easier presentations of this duality. However, I find the one given in this book a very pleasing piece of mathematics. One needs the notions of Ehrenfeucht-Fraïssé games (bisimulations) on finite labelled posets (Kripke models), and a little bit of sheaf theory (sheaves on a site). The games define an infinite series of equivalence relations $\sim_n$ between such Kripke models. Next, one considers the jointly-epimorphic family topology J_0 on the category P_0 of finite posets with largest element, and *open* maps. J_0 is subcanonical, so representable presheaves are sheaves. For a finite poset L one has also the sheaf h_L of order-preserving maps into L. A subsheaf S of h_L is said to have *index n*, if for all $u: P \to L$ and $v: Q \to L$ it holds that if $u \in S(P)$ and $u \sim_n v$, then $v \in S(Q)$. It turns out that $\mathcal{S}(h_L)$, the collection of all subsheaves of h_L having an index, is the Heyting algebra generated by the distributive lattice of downsets of L (hence, an object of $\mathrm{HA}_{\mathrm{fp}}$). For natural transformations $\mu: h_L \to h_{L'}$ there is a similar notion of 'having an index'. It turns out that there is an equivalence of categories between $\mathrm{HA}_{\mathrm{fp}}^{\mathrm{op}}$ and the category with as objects pairs (L, C) where L is a finite poset and $C \in \mathcal{S}(h_L)$, and as morphisms natural transformations with an index. From this presentation one then shows that $\mathrm{HA}_{\mathrm{fp}}^{\mathrm{op}}$ is r-Heyting.

As a byproduct, several facts about Heyting algebras come out as corollaries; for example, there is a neat charcterization of projective finitely-presented Heyting algebras.

Chapters 5 and 6 now carry out a similar treatment for several calculi of modal logic. Many of the corresponding theories (Boolean algebras with an extra operator satisfying axioms) fail to admit model completions, but the theory of diagonalizable algebras does.

Finally, Chapter 7 on what the authors call 'prime' and 'algebraically closed' models, is more or less independent of the rest of the book but pursues related ideas. Given an equational theory T satisfying a few nice conditions (finite language, finite model property, EDO, amalgamation), let again $\mathbf{T} = \mathrm{Alg}(T)_{\mathrm{fp}}^{\mathrm{op}}$. Endowing $\mathbf{T}$ with the epimorphic family topology J, what does the topos $\mathrm{Sh}(\mathbf{T}, J)$ *classify*? The answer is: the (infinitary, geometric) theory of 'prime, locally finite, algebraically closed' T-algebras. It is a curious fact that for Heyting algebras, this theory has exactly two non-isomorphic countable models!

There is a further mini-chapter listing some open problems in this area.

I am quite enthousiastic about this book. It is a wonderful example of how different areas of research blend. The style of the proofs has just the right pace; it is excellent material for a reading seminar for advanced students, who will need to work a bit on the required background.

Of course, there is always criticism. What I find most irritating is the extensive use of footnotes for (parts of) definitions, pieces of proof etcetera; I have even seen a footnote in the statement of a theorem! The authors have not gone to great lengths in exploring existing literature that might be relevant to their work: in 1983 Blass and Scedrov had a few papers on Boolean classifying topoi of finitary theories, where model completeness also turned out to be a vital condition. It should be of interest to explore a connection here. Also, the references are sometimes a bit pedantic: for a standard fact in topos theory such as the Comparison Lemma one might refer to any of the existing textbooks (Johnstone, Barr/Wells, MacLane/Moerdijk) instead of the comparatively inaccessible (French) SGA 4. Redundant terminology should be minimized; the authors do introduce the name 'p-morphism' (which is a horror since it is absolutely non-descriptive) for surjective open map, but then hardly ever use it. Also, if you introduce an assumption which from now on will be in force, as on pp. 64–65, then stick to it and don't repeat the assumption in every theorem you state.

A few inaccuracies: the definition of 'admitting quantifier elimination' on page 25 is not entirely correct. The proof of Lemma 3.4 suggests that one needs $t\colon T \to A$ to be regular mono, which is not stated and not necessary. The height of a finite poset is *not* the 'length of a maximal chain' (p. 77, l.12). On p. 93, l.14, 'exercise 12' should have been exercise 16. But I have seen only few of these, and they just enhance the didactic value of the book, which is very good to have!

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C. Stephen Layman. ***The power of logic.*** 2nd edition. McGraw-Hill, 2002, ix + 592 pp.

General overview. This is the second edition of ***The power of logic***, a fairly standard introductory text in logic that covers most of the staples, though with more breadth than many of its competitors. Central topics include informal fallacies, categorial logic (the square of opposition, syllogisms, etc.), propositional and predicate logic, probability (including Bayes' theorem), and induction. It is a clear, well-designed book, and students should find it more accessible than many of its rivals.

What's new in the second edition. The second edition contains a number of changes from the first, most of them for the better. Exercises have been added throughout, with things simplified here, streamlined there. An unexpected final chapter of the first edition on modal sentential logic has vanished, which is for the best, given the likely audience for a book like this.

On the other hand, the first edition's treatment of syllogisms and other aspects of categorical logic has expanded from one chapter to two, growing a number of pages in the process. Devoting so much time to an arcane topic like this, which has little application in real life, seems to me a step backward. The realities of the market place, however, make it difficult to publish a text like this without some coverage of the topic.

All in all, the book is an attractive option for instructors who haven't already invested a lot of time and effort in some other text, but is it worth migrating to it if you are currently using a text you like? The thing that may justify the time and work in switching to this book is that it has an extensive and sophisticated web site written by people who know what they are doing.

The web. The book's web site contains the expected true-false and multiple-choice quizzes, but it also marks an important advance over other sites in the same niche. It consists of a seamless integration between the Power of Logic Home page (where the proof checker and the things specifically geared to the text live), and McGraw-Hill's PageOut site. The virtue of the using the two sites together is that the first site has very good software dedicated to teaching logic, while the PageOut system contains a number of things that make it much easier to manage the course.

The web site will be important to students. It has an attractive and user-friendly graphical interface, and enables them to diagram argument structure, use Venn diagrams, and construct truth tables. The real *tour de force*, though, is the proof-checker, which lets students to construct natural-deduction proofs and checks their work. Then, as is done with each module of the book, it gives them immediate feed-back on they got right, what they didn't and, often, on where and why things went wrong (if they did).

The web site can be at least as important to instructors, greatly easing the burden of grading, recording, and tracking grades. It contains a heavy-duty testing module that enables instructors to put homework, quizzes, and even detailed exams online. The students' work is then shipped to the Power of Logic server, which grades it (in accordance with the instructor's preferences), calculates their grades, and places them in an online grade book. Among other things, this makes it feasible to give frequent, indeed daily, assignments.

The system is flexible enough that instructors can fine tune things in a variety of ways that don't require them to master complicated any finicky software. I would strongly recommend that instructors considering this book take a voyage through the site.

What is the goal? The web is a new and powerful vehicle for communication and instruction. We would hope, eventually, to have smarter, more sophisticated software that can monitor each user's progress and give her problems and feedback that are geared to what she has already learned, rather than just responding with boilerplate; the current site is poised somewhere between these two locations. (One natural model for intelligent tutoring is John R. Anderson's work on cognitive tutors, e.g., Anderson, et al. *Cognitive tutors: lessons learned*, ***The journal of learning sciences***, vol. 4 (1995), pp. 167–207.)

The point of any course likely to use this book, at least for students who won't go on to do more logic, mathematics or the like, is to help students acquire cognitive tools and skills that they will use outside the classroom (on the job, in dealing with their families) once the semester is over. This involves what psychologists and educators call 'transfer of training', and it is much harder to accomplish than any of us would like.

What to do? Students need a lot of practice applying the concepts and principles (e.g., rules for calculating probabilities) in a wide range of situations to a wide range of subject matters, especially situations that matter in their own lives. The most important step toward better reasoning is learning to spontaneously recognize situations where a particular cognitive tool or skill could be applied. There are no texts on the market that focus enough on this, but the present book does have a number of exercises drawn from real life, which should help.

Different instructors and different courses have different needs. But this is a better than average textbook with a much above average web site. Moreover, with the second edition it appears quite likely that McGraw-Hill will continue supporting this project, so users aren't likely to be left with an orphaned text or superannuated web site. The book may not be the best text for a particular instructor, but she can't go far wrong using it.

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Handbook of the logic of argument and inference, edited by D. M. Gabbay, R. H. Johnson, H. J. Ohlbach, and J. Woods, Studies in Logic and Practical Reasoning, vol. 1. North-Holland, Amsterdam, 2002, 498 pp.

This book is devoted to a presentation of modern knowledge concerning Argument and Inference. The focus point of discussion is relations binding formal deductive inference in logical systems and human reasoning. The starting point being the lack of good coherence between human reasoning and logical deduction. Internal critique comes from objections to standard logic as theories of argument and inference, empirical criticisms arise from work in cognitive and experimental psychology.

The books begins by the chapter: Logic and the Practical Turn (by John Woods, Ralph H. Johnson, Dov M. Gabbay and Hans Jürgen Oldbach). This is an introductory chapter which describes the purpose of this handbook, discusses issues of problems concerning coherence and distinction of logical deduction and human reasoning.

The chapter: Standard Logics as Theories of Argument and Inference: Deduction (by John Woods) describes the notion of deductive argument and deductive inference. It begins with a historical survey. Then the author describes the deductive logic, presents an axiomatization of propositional calculus PC by axioms and inference rules, gives necessary techniques: Presentation covers elements of functional completeness, satisfiability and soundness. Next section deals with problems of formalizing of natural language, the discussion concentrates over ways to translate human reasoning to formal expressions and over approaches to trace inference within accepted formalism. Predicate calculus PredCal is given in standard capacity including syntax, first-order formulas, models, truth relation, axioms, inference rules. The chapter is concluded with pointing out problems arising to translate human sentences even into Predicate calculus and with a brief exposition of intensional logic (modal logic is taken as an example).

Next chapter: Standard Logics as Theories of Argument and Inference: Induction (by John Woods) concerns ampliative reasoning and argument. In the ampliative reasoning the conclusion has information that goes beyond the information conveyed by the premisses. As an example two king reasoning are pointed out—induction and abduction, and it is noticed that, say, induction is in a sense non-deductive argument of inference. In order to handle this kind of reasoning a conception of the inductive logic IL is offered. One variant of it is this logic in the broad sense (IL^B), another one—in the narrow sense (IL^N). First the author consider as an illustration the probability calculus. Similar to various systems of deductive logic, probability calculus—ProbCal—is a couple of a grammar and a set of interpretations. The grammar gives a vocabulary and a set of formation rules. Variables are assumed to be real valued, a functional expression 'Pr' to handle probability is fixed, the language is assumed to have vocabulary to cover the arithmetic of the real line. A series of theorems followed collects basic properties of this calculus; then the author discusses a research connected with the Dutch Book theorem concerning betting strategy. Next section covers Bayes' theorem and Bayesian inference. Then several more subjects related to previously given approaches are elucidated.

The chapter Internal Critique: A logic is not a theory of reasoning and a theory of reasoning is not a logic (by Gilbert Harman) poses a problem of distinction between human reasoning and logic. The author begins with a warning not to confuse issues of implication and issues of inference: implication is a relation between propositions rather inference and reasoning are psychological processes. The discussion followed is primary of pure philosophical style and concerns theoretical and practical rationality, legitimate practical reasons for beliefs, inference versus implication, induction and deduction, rationality in action.

The chapter Standard Logic as a Model of Reasoning: the Empirical Critique (by David N. Perkins) is devoted to discussion of coherence and distinction of human reasoning and formal logic. Some statistical observations on how people do perform reasoning are given. As an evident obstacles to reason along standard logic the following limits: Limits of Computation, Limits of Intelligence, Limits of Understanding are discussed. The concluding section concerns how to position standard logic in the human reasoning. The slogan: people should reason according to standard logic is acknowledged as doubtful and debatable, but, on another hand, "for their own sake and that of others, people generally should and often can and do" (page 218).

The chapter: A Framework for Intersubjective Accountability: Dialogical Logic (by Else M. Barth) concerns dialogue logic and its practical applications. It begins with various comments on the state of affairs in general logic and dialogue logic. The issue of dialogue logic could be concerned as an observation that any critical discussion is a subcategory of dialogue. After some discussion of relevant subjects, in section II the author proceed to the discussion whether we can construct an informal dialogue logic. The discussion involves such subjects as "what means formality", "what means pragmatize and pragmatization". Observing question how to make modern logic dialectical, the author considers semiotical roles and dialectical roles, opponent's tasks. Then a series of applications: logic in inter-cultural debate, dialogue logic in practical ethics and several others are given. Close to conclusion, in terms of offered approach the author discusses Cooperation and Real Conventions.

The chapter: Interrogative Logic as a General Theory of Reasoning (by Jaakko Hintikka, Ilpo Halonen and Arto Mutanen) is devoted to interrogative logic: logic which can use for deduction an "oracle". The formalizing of this logic contains rules in the style of "tableau" method. Rules of questioning describe the conditions provided of which we are in a position to pose our question to the oracle, the from of the reply, and how to place the answer in the tableau. The completeness theorem given states that if F is a set of all available answers and T is the set of initial premises, then a conclusion C can be established in M by means of extended interrogative logic iff $(F \cup T) \vdash C$. In the line of this approach a series of theorems on the properties of interrogative logic is given. Further authors proceed to applications, they consider a hierarchy of logics of discovery, the logic of experimental reasoning, interrogative reasoning and indefinability, semantical games and informational independence, epistemic logic for different types of knowledge. The chapter is concluded by considering new epistemic rules for interrogative logic and the completeness theorem for this logic.

The chapter Informal Logic and the Reconfiguration of Logic (by R. H. Johnson and J. A. Blair) deals with the question how informal logic emerged as an attempt to reconfigure one part of logic in the light of critiques described in proceeding chapters. It begins with a review of the origins of informal logic, then a research agenda for future work in informal logic is given. The prime attention is focused on the questions: the nature of argument and argumentation, types of argument, the elements of argument, the structure of argument, argumentation schemes, criteria of good argument. The chapter is concluded with a discussion of implications beyond informal logic.

The chapter Probability Logic (by Jon Williamson) begins with definitions of probability measure and typical properties of this measure. Probability logic described is intended to formalize laws of probability. Using this approach the author investigates how it can generalize or modify classical logic. The following subjects: probabilistic inference, Bayesian networks, a causal logic, a proof logic are discussed.

The chapter Philosophical Incidence of Logical Programming (by Louis Monad Pereira) concerns elements of logic and its connection with AI and Computing. The author discusses how the general reasoning issues explored in his paper can be approached within a uniform framework, except in the context of logic. And he states (page 444) that "logic is only conceivable venue for the core of such an enterprise". The subjects discussed as an issue of the problematic are Evolution and Reasoning, Logic and the Computer, Philosophy and AI, Defeasible Assumptions, Explicit Negation, Theory Diagnosis, Updating.

The chapter Formal Approaches to Practical reasoning: A survey (by Dov M. Gabbay and John Woods) discusses what kind of formal logical systems are suitable for modelling of actual practical reasoning behavior. Authors proceed with a discussion which components are essential for identifying of logical systems. The following components are taken to consideration: Language, Declarative Unit (like definition of well formed formulas), Databases (say, an amount of assumptions with which we reason), Consequence, Proof theory, Algorithmic Presentation, Mechanisms (abduction, revision, actions, etc.), Basic logical concepts (say, deduction theorem). In sequel authors motivate and illustrate these notions with particular logical systems. They begin by a discussion of general essence of logical systems and proof systems for them. As examples of logical systems, modal logics T, B, K4, S4, S5 are considered, axioms, inference rules, semantics are given. Then authors subsequently discuss Structured Consequence, Algorithmic Structured Consequence Relation, Mechanisms (abduction, default, update, revision), Models for Evaluation (Kripke Models), Time-Action-Revision logics.

The book is written at a level appropriate for senior undergraduates, graduate students in computer science or philosophy. It could be useful for researchers interested in practical argument and inference. The book is not uniform, 2nd and 3rd chapters form a core, they are pivotal and forms material necessary to enter the area. Some chapters are solid from technical viewpoint, some others are primary philosophical or of discussion style. But, at the same time, the book is good reading because it represents distinct viewpoints and presents a good sample of knowledge collected up to date. Some related subjects as Classical Proof Theory, Algebraizable Logics (where logical consequence operator is main subject of investigation), Admissible Inference Rules, are out of scope of this book. But it is clear that no way to describe in one book all knowledge collected, and still the mentioned above areas could look too special and technical. For readers interested primary in philosophical aspects of the subject and for the beginners the book is a very good reference. Also a number of new interesting questions are raised and this book seems to be a reasonable basis for new further investigations.

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I. Lavrov and L. Maksimova. ***Problems in set theory, mathematical logic, and the theory of algorithms.*** The University Series in Mathematics, Kluwer Academic/Plenum Publishers, New York, 2003, ix + 282 pp.

The title is an exact description of the book. These problems were originally designed to serve as a resource for Russian instructors who lectured in logic, apparently without a text. They provide a full treatment of the basics of naive set theory, propositional and predicate logic, and the rudiments of recursion theory. The emphasis is on the middle topic (85 pages while 50 and 35 pages, respectively, are devoted to the others). The authors express the hope that a student could learn the subject only from this book; such a student would be extremely capable. The book contains the definitions of the notions investigated; there is virtually no exposition. But, major theorems are broken down into quite manageable steps. For example, the Henkin proof of the completeness theorem is outlined in 7 problems in one section (for logic without equality axioms) with a $1/2$ page sketch in the solution section for the main step: defining the model. The step to properly interpret the equality symbol is given a section later. Most of the problems are routine but they cover the basic ideas extremely well. The best way to describe the scope of the book is to remark that it starts at the beginning and briefly to give the stopping point in each area. There is no treatment of independence proofs in set theory; the axioms of ZFC are given in the predicate logic section and the student is asked to formalize some simple results. There is an extensive treatment of reduced products and ultraproducts; the notion of a (model theoretic) type does not appear. There is an extensive treatment of Turing machines and of recursive and recursively enumerable sets. The incompleteness theorem is not treated; nor is there any mention of Church's thesis. This book could serve well for a reading course with an extremely strong undergraduate or beginning graduate student. In the United States, its best use may be as a study guide for students preparing for a comprehensive exam in logic.

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TERESE. ***Term rewriting systems,*** edited by Marc Bezem, Jan Willem Klop, and Roel de Vrijer, Cambridge Tracts in Theoretical Computer Science, vol. 55. Cambridge University Press, 2003, xxii + 884 pp.

Term rewriting is a fundamental concept which historically emerged out of the lambda calculus and combinatory logic. It forms the basis of functional programming languages and has applications in many areas of computer science and mathematics (e.g., algebraic specification, implementation of programming languages, automated theorem proving, recursion theory, computer algebra, etc.).

This book is a collaboration of leading members of the Dutch research community on term rewriting. This community met regularly at a "Term rewriting seminar" at the Free University of Amsterdam for 13 years, which gave rise to the pseudonym "Terese" as the author of the present book. Several researchers from this group contributed separate chapters on different aspects of term rewriting. Some chapters have the nature of a textbook and cover basic material, while others focus on very recent and highly non-trivial research results of their authors. Thus, the level of difficulty and the intended audience varies not only from chapter to chapter, but sometimes also from section to section. Hence, the book is both suitable as a textbook on term rewriting (if one only concentrates on the parts on basic material) and as a valuable reference for the specialist. Each chapter also contains a set of exercises (whose difficulty varies greatly).

The topics covered in this book are the basic subfields of term rewriting (termination, confluence, completion, etc.), several of the main current research topics in rewriting (higher-order rewriting, strategies, etc.), and also several special subfields reflecting the particular research interests of their authors (equivalence of reductions, infinitary rewriting, etc.). Of course, there are also many important topics in rewriting that are not covered or only briefly discussed (e.g., rewriting modulo equivalences, implicit induction, narrowing, Gröbner bases, explicit strategies, etc.). But this is not a drawback, since in this way, most chapters are written by specialists on the respective topic. So naturally, the selection of topics in the book mainly coincides with the research fields of its authors.

Most parts of Chapters 0–7 are an introduction to first-order term rewriting. Starting with several motivating examples, Chapter 1 introduces abstract reduction systems with their main notions (in particular, confluence and termination) and Chapter 2 focuses on the basics of term rewriting which are needed for the rest of the book. These chapters are pleasant to read and serve as a very good introduction into the topic.

Chapter 3 contains several examples of interesting term rewriting systems (such as the system of combinatory logic). In addition, two subsections briefly discuss specializations and extensions of term rewriting (such as string rewriting and conditional rewriting systems).

In Chapter 4, the important class of orthogonal term rewriting systems is introduced and it is proved that orthogonality implies confluence. In addition to the classical proof, several alternative proofs are presented, which introduce advanced proof techniques needed in later chapters. Thus, these parts clearly go beyond the material of a textbook. The chapter also presents first important reduction strategies.

Chapter 5 shows the main (un)decidability results for rewriting (e.g., undecidability of confluence and termination) and the main modularity results for disjoint unions (i.e., unions of rewriting systems over disjoint signatures). However, modularity results for other combinations where constructors or other symbols may be shared are not handled.

Chapter 6 deals with methods to prove termination of term rewriting. Classical techniques like polynomial orders, recursive path orders, and Knuth-Bendix orders are presented and different restricted notions of termination are related to each other. Moreover, powerful transformational methods for termination proofs like semantic labelling and dependency pairs are introduced, giving a good overview also on recent techniques for termination analysis.

The next chapter is on using term rewriting for deduction. It covers Knuth-Bendix completion, a short description on inductive proofs by consistency, and unification modulo equations. The chapter gives a nice overview on the classical results, but it is more in the style of a tutorial without focusing on recent research in this subfield.

Chapter 8 is a very extensive treatment (174 pages) of the concept of equivalent reductions. In contrast to most of the preceding text, this chapter mainly contains new material (which extends existing results beyond orthogonal rewriting systems). The chapter is clearly intended for specialists interested in this specific subfield. For those readers, this chapter will be a very valuable source covering, unifying, and extending the results on this topic in a very convincing way, being the most up-to-date text on this topic.

In Chapter 9, strategies to find normal forms are examined, focusing on the concept of needed reduction steps, i.e., steps that have to be performed in order to reach normal forms. Again, the chapter is on a quite advanced level, giving a very comprehensive and up-to-date treatment of this topic.

In contrast to these more research-oriented chapters, Chapter 10 is a nice tutorial on the lambda calculus which leads to Chapter 11 on higher-order term rewriting. It gives a very good overview on this topic with a focus on results on confluence and termination of higher-order systems.

While higher-order rewriting is one of the main current research areas in term rewriting, the next chapter focuses on a more specialized topic. It summarizes its authors' results on infinitary rewriting where the concept of rewriting is extended to infinite terms and infinite reduction sequences, possibly converging to limits.

Chapter 13 is a short introduction to term graph rewriting, which gives a good survey on this topic and its relation to term rewriting. More detailed treatments of term graph rewriting can be found in several other books that are exclusively devoted to this topic.

Finally, in Chapter 14, advanced specialized results on confluence of abstract reduction systems are presented and there is a short subsection on the important concept of rewriting modulo equivalences.

Chapter 15 is an overview on implementations of rewriting (in programming languages, specification languages, program transformation systems, theorem provers, etc.) which is very helpful to estimate the use and impact of term rewriting. The book concludes with an appendix on the mathematical background required.

Concerning the overall structure of the book, the order of presentation is sometimes confusing, in particular for topics which are only briefly presented, but not treated in detail. For instance, the subsections on specializations and extensions of term rewriting are in the middle of Chapter 3 on examples, instead of Chapter 2 where first-order term rewriting is introduced. As another example, rewriting modulo equivalences is introduced in Chapter 14 on advanced abstract reduction systems (although the most prominent variant of it, AC-rewriting, cannot be demonstrated in this abstract setting). In fact, this concept is already used several times before (e.g., in Chapter 6 on termination). However, the editors and authors have added many references in the text indicating in which sections concepts have been or will be introduced. Since most readers will rather concentrate on certain chapters of the book instead of reading it from the beginning to the end, such references are very helpful and even forward references may not be problematic.

The are two other recent English books on term rewriting: a textbook (F. Baader and T. Nipkow, ***Term rewriting and all that***, Cambridge University Press, 1998) and a book covering more advanced material (E. Ohlebusch, ***Advanced topics in term rewriting***, Springer, New York, 2002). Compared to them, the present book is unique in its mixture of textbook-like and research-oriented parts and by the fact that its chapters have been written by different specialists. Thus, the current book may be less suitable as a textbook than the one of Baader and Nipkow, since it goes far beyond the basics and it may not always be clear to the reader which sections of the book are the "basic" ones. Here, it would have been helpful if the editors and authors would have indicated which parts are suitable as a textbook and which parts are mainly intended for the specialized researcher. Moreover, since the chapters are written by different authors, it is unavoidable that the style of presentation is somewhat heterogeneous. On the other hand, the present book covers much more material than the one of Baader and Nipkow and treats the topics in much more depth. Hence, by selecting suitable parts of it, the book indeed serves very well both as a textbook and as an advanced text. Some of the advanced topics are similar to the ones treated in Ohlebusch's book, but since both Ohlebusch and the authors of the present book mainly report on their own research, the contents of these two books hardly overlap, but rather complement each other. To summarize, I recommend the book both as a textbook with several very nice surveys and as a comprehensive source of advanced research results which are very well presented in a unified setting.

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ALEXANDER LUBOTZKY and DAN SEGAL. ***Subgroup growth.*** Progress in Mathematics, vol. 212. Birkhäuser Verlag, Basel, 2003, xxvii + 453 pp.

The classification of infinite groups is a distant and perhaps unattainable goal. Finite simple groups have already shown us how immense such a project is likely to be. But this does not mean we cannot make inroads into understanding and sorting these groups. Given that the mathematician is essentially a finite beast at heart, the best way to get to grips with the expanse of infinite groups is to impose finiteness conditions on the group in the hope that they will become more tractable.

This was the approach taken by the Russian school of group theory at the end of the last century. For example what do finitely generated groups look like whose elements all have bounded order, the so-called Burnside Problem? The Tarski monsters created by Olshanskii and Rips in the 1970s showed that such finiteness conditions still leave room for strange creatures. To exclude such monsters, group theorists in the 1980s realized that insisting that the group be residually finite excluded such monsters and made understanding finiteness conditions on infinite groups more reasonable. For example, Zelmanov was awarded his Fields medal for the proof of the restricted Burnside Problem: every finitely generated, residually finite group of finite exponent is in fact finite.

The condition that a group is residually finite means that the intersection of all the normal subgroups of finite index is 1. This can be interpreted as saying that the group has sufficiently many finite index subgroups that one can distinguish between elements in finite quotients of the group. The central question of Lubotzky and Segal's book is how many subgroups of finite index can a residually finite group have. To quantify this question, group theorists began to explore the relationship between the structure of an infinite group and the rate of growth of the number of subgroups of each particular index in the group.

The subject took off following a talk by Segal at the Groups St. Andrews meeting in 1985. Here we find the first contribution to one of the central results of the book under review: the finitely generated, residually finite groups whose number of subgroups grow polynomially are precisely the virtually soluble groups of finite rank. Since Segal's original paper, the subject of subgroup growth has blossomed into one of the mainstream topics in group theory at present. The current book is testament to the mature and exciting state of the subject.

A major proportion of the book is dedicated to understanding the possible growth types of different groups. These range from groups whose growth rate is as fast as possible, like free groups, down to the groups of polynomial growth. The classification of groups of polynomial growth introduces the reader to a range of exciting mathematics including the classification of finite groups, the prime number theorem and properties of algebraic groups. This last theme is developed to provide estimates for the growth of congruence subgroups in arithmetic groups. This leads to Lubotzky's exciting characterization of the congruence subgroup property in terms of the growth of groups.

Groups of intermediate growth and possible gaps in growth type are considered. Interesting groups like the 'branch groups' of Grigorchuk are introduced to illustrate how to realize groups of different intermediate growth. The question of normal subgroup growth is also dealt with.

The book concludes with a more delicate analysis of the arithmetic of the subgroup lattice. The principal tool here is the idea of a non-commutative zeta function. This section of the book will be of particular interest to the model theory community since the theory of definable integrals as developed by Jan Denef is a key ingredient in analyzing these zeta functions.

One innovative feature of the book is the authors' use of 'windows' to explain key background material. Reflecting our increasing use of the Internet to access material, the windows work like hyperlinks dotted through the text. The reader is offered the chance at numerous points to jump to the back of the book where these windows are collected. The windows provide a range of background material from the theory of pro-p groups, soluble groups, permutation groups through to topics in probability, prime number theory and logic.

It is the sheer variety of techniques and ideas that are introduced throughout the book which makes this such an exciting read. One can easily appreciate why it was the winner in 2002 of the Ferran Sunyer I Balaguer prize for a mathematical monograph of expository nature. The book provides a perfect platform for a graduate seminar taking the reader from the basics to the limit of our current understanding. As the list of open problems presented at the end of the book illustrates, the subject has a long way still to go. It has reached a respectable adolescence. But the recruits that this book will surely enlist have the potential to bring the subject to full maturity in the ensuing decades. Lubotzky and Segal's book represents powerful propaganda to push for a new category in the Mathematics Subject Classification dedicated to Subgroup Growth.

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La filosofia di Gottlob Frege, edited by Nicla Vassallo, Franco Angeli, Milano, 2003, 237 pp.

The book represents the fulfilment of a dream of its editor, Nicla Vassallo, who has always been convinced of the relevance of Frege's thought also in our days and has desired to collect essays about Frege by Italian scholars, each covering and enlightening one of the many aspects of Frege's work.

The first of these essays—presented in alphabetical order, according to the surname of the author—by Claudia Bianchi, entitled "Frege on indexicals", concerns the difficulties arising from Frege's definition of sense and reference when treating expressions like "today" or "I". Bianchi recalls two different perspectives (a Neo-Russellian and a Neo-Fregean one) that tried to solve those difficulties and suggests a kind of mixture of both of them as the best solution.

The second essay ("What 'the continents are five' says about Continents"), by Andrea Bottani, concerns Frege's opinion that a number can only be attached to a concept and maintains that not in all cases Frege's opinion is justifiable (in particular, it is not justifiable in the case of the number zero) and that even in these cases the justification holds only in a strict sense, i.e., not as a negation of the fact that a quantified proposition concerns objects but as an explanation of the way in which it can concern objects not mentioned in the proposition itself by singular terms. Bottani also opposes Geach's criticism of Frege's opinion that no number can be attached to the concept "red" as Geach presupposes that identity be relative, while it was absolute for Frege.

In the third essay ("Gottlob Frege: a revolution in the concept of logic?") Carlo Cellucci considers the question whether Frege's thought represented a revolution in the way of imagining the aims of logic (Cellucci leaves aside the problem of Frege's possible revolution in conceiving logic). According to Frege, the aim of logic is protecting mathematics from scepticism. This is a stricter aim for logic with respect even to Aristotle's perspective according to which logic could be useful for finding the principles of sciences and, hence, for discovering new truths (and not only justifying those already acquired): Frege simply followed Kant in doing so and deterred the twentieth-century logic from concentrating on the question of mathematical heuristics.

With "Thinking with one's own mind. On some problems concerning philosophy of thought in Hegel and Frege" Franca D'Agostino, while honestly admitting the insuperable differences between the two authors (the historical and dialectical dimensions appear only in Hegel's thought), tries to find out some analogies between them: they are similar as far as both expressed themselves against scepticism and empirical relativism; both considered concepts the entities which instantiate objective thought; both fixed the main premises of two linguistic turns of the XX century philosophy (the hermeneutical and the analytical one).

The fifth essay ("On Frege's alleged indispensability argument"), by Pieranna Garavaso, recalls that Frege has been credited—by Penelope Maddy—with being the originator of the indispensability argument (the necessity of mathematics in scientific explanation implies the existence of mathematical entities) and criticizes this statement on the basis of three arguments: (1) Frege only refers to a general applicability of mathematics; (2) there is a structural difference between the indispensability argument and the Fregean argument considered analogous (***Grundgesetze*** sect. 91); (3) in Frege there is no use of an inductive inference towards a better explanation.

The sixth essay, by Daniele Giaretta, entitled "Concepts and arbitrary stipulations", considers Frege's conditions for a number to be attached to a concept. Giaretta shows that they amount to requiring a full determinacy of the concept and stresses that Frege did not take much care of subtle distinctions, like that between partial determinacy and indeterminacy concerning the bounds of determinacy, as he was mainly interested in the properties that arithmetical concepts have when applied to numbers.

"Frege on identity and hypostatised modes of determination" by Mauro Mariani proposes the necessity of giving a semantics also for natural languages as an explanation for Frege's change of mind concerning identity. Namely, Frege passed from a metalinguistic theory in his ***Begriffsschrift*** to the theory of identity of *Sinn und Bedeutung*, where he hypostatised the modes of determination (the senses) so that they become both intermediate entities between the signs and their referents, and what the signs refer to in *oratio obliqua*.

In "Frege's philosophy of mathematics", Dario Palladino lists Frege's main contributions to the philosophy of mathematics. He also recalls that some kinds of logicism are still alive. He stresses that, although Frege did not anticipate the later development of logic, he contributed to it just through all the criticisms put to his own opinions.

Carlo Penco and Marcello Frixione, in "Fregean senses, procedures and computational limitations", explain the different treatments that Frege gave to the problem of lack of relevant information in the case of daily statements and of logical statements: Frege admitted a problem in the first case—and tried to solve it—while he rejected the existence of a problem in the second case, presumably due to his exigency of avoiding any intrusion of psychology in logic. The two authors, on their part, recognize a problem also in the second case and try to solve it subsequently with other aspects of Frege's thought, by proposing a procedural interpretation of the sense.

Eva Picardi, in "Frege, Peano and Russell on the 'indefinables' of logic", compares the conceptions of the primitive notions of logic (variables, identity, material conditional, generality, class and membership, truth and falsehood, assertion and assertion-sign) as put forward by the three authors between 1890 and 1914.

Finally, the editor of the volume, Nicla Vassallo concludes with an own essay ("Fregean epistemology"), where she criticizes Dummett's opinion that Frege had a very limited interest in epistemology, by illustrating that Frege answered very important epistemological questions (concerning the problems of scepticism, our sources of knowledge, a theory of justification of knowledge, naturalism) in such a uniform way that a systematic epistemology can be ascribed to him, by composing all these answers together.

This is a further evidence for Vassallo's unconditioned admiration for Frege, an admiration that explains all her efforts in collecting such different voices and perspectives on Frege, but at the same time it did not prevent her from honestly reporting all of them (inside the Italian domain of research), even those that severely criticize Frege.

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Wahrheit und Beweisbarkeit. *Kurt Gödel. Dokumente und historische Analysen.* edited by Eckehart Köhler et al., Band 1. öbv et hpt, Wien, 2003, 278 pp.

Wahrheit und Beweisbarkeit. *Kurt Gödel. Kompendium zum Werk.* edited by Bernd Buldt et al., Band 2. öbv et hpt, Wien, 2003, 445 pp.

Kurt Gödel is without a doubt one of the most cited and influential logicians of all times. The amount of literature based on (or influenced by) his famous metamathematical results and his life fill whole libraries. These two volumes about Kurt Gödel's life and work under review were published almost parallel to volumes IV and V of the ***Collected Works*** (ed. by Solomon Feferman *et al.*). While volume I is dedicated to issues related to Gödel's biography, volume II is a remarkable companion to Gödel's contributions in logic, set theory, cosmology and philosophy (of mathematics). Both volumes are to be seen as helpful and enlightening additions to the ***Collected Works*** and John Dawson's biography, ***Logical Dilemmas: The Life and Work of Kurt Gödel*** (1997).

In the "Preface" of volume I Peter Weibel (one of the editors) mentions (p. 10) one central aim of this book: to understand Kurt Gödel as a part of the culture and the intellectual development during the 20th century in Austria and to document between his national and international influence. Gödel emigrated in 1940 and received U.S. citizenship in 1948. He spent more than the half of his life in the United States. The majority of the contributions to Gödel are written in English and a large part of further Gödel related achievements are due to American scientists. With respect to these facts it is clear that Gödel is mostly seen in connection with the United States and especially with the Institute for Advanced Study in Princeton (IAS). Apart from this it is important to mention that Gödel obtained almost all of his fundamental results during his time in Vienna. These two volumes published in German are dedicated to the Austrian Kurt Gödel.

Volume I seems to be a consistent continuation of a project which was inaugurated in the late 1970s and got its first highlight in the mid-80s by the movie ***Kurt Gödel: Ein mathematischer Mythos*** (ORF, Vienna 1986). Chapter (A), "Honors and letters", contains letters from Albert Einstein, Oskar Morgenstern, John v. Neumann and Gödel as well as a poem about Gödel by Hans Magnus Enzensberger and a "Gödel riddle" by Raymond Smullyan. The second chapter (B), "Biographical aspects", might be the closest one to Dawson's biography. It includes among other things a chronological sketch by Rudolf Gödel about his family (pp. 51–62), personal memories by Karl Menger (pp. 63–81) and the contribution "Gödel and the Vienna Circle" (pp. 83–108) by Eckehart Köhler. Köhler's paper can be seen as being particularly close to the former request to understand Gödel as a part of Austrian intellectual life. As a temporary corresponding member of the Vienna Circle Gödel became part of one of the most influential philosophical developments in the first half of the 20th century. Again, in "Gödel in Princeton" (pp. 149–184) Köhler focuses on the intellectual climate and on Gödel's life at the IAS. Both articles by Köhler comprise a great deal of information according Gödel's development and his personal point of view. This is supported by Maria-Elena Schimanovich-Galidescu's collection of sources on Gödel's time in Vienna (pp. 135–143) and Princeton (pp. 185–207). Sometimes the reader may get the impression that Gödel was not at all lucky in Princeton. Chapter (C), "Interviews with time witnesses", includes a lot of personal memories by friends, colleagues and students which were collected by the editors between 1978 and 1987, e.g., an abridged interview with Gödel's teacher and thesis supervisor Karl Menger (pp. 227ff). The well known "Grandjean questionnaire" already published in Hao Wang's ***Reflections on Kurt Gödel*** (1987, pp. 16ff), is presented in a German translation (pp. 223ff).

In the "Preface" of volume II Michael Stöltzner and Werner DePauli-Schimanovich-Göttig (two of the editors) mention that in contrast to the "Introductory notes" in the ***Collected Works*** the contributions to this volume are not just descriptions. Instead, the articles focus on Gödel's achievements with respect to historical and recent research aspects, whereas each chapter in this volume contains at least one prominent primary source related to the relevant issues. There is not enough space to mention all of the 21 articles. Some of the contributions are introduced by hitherto unpublished letters to and from Gödel, now available in volumes IV and V of the ***Collected Works***. Widely used are parts of the correspondence between Gödel and Paul Bernays comprising 85 items, that took 45 years to write.

The articles in chapter (D), "Formal Logic" are dedicated to Gödel's completeness result, his compactness theorem, the controversy with Zermelo and his famous "Dialectica Interpretation". Concerning the latter, in his article "The dialectics in Gödel's Dialectica Interpretation" (pp. 67–90), Jaakko Hintikka gives an account of Gödel's contributions in the light of his game-theoretical semantics. Gregory Moore's article "The controversy between Gödel and Zermelo" (pp. 55–64) deals with a topic also treated in recent investigations by Dirk van Dalen and Heinz Dieter Ebbinghaus (*Zermelo and the Skolem Paradox*, ***The Bulletin of Symbolic Logic***, vol. 6, 2000, pp. 145–161) into Zermelo's contributions against Skolem's Paradox and in support of infinitary languages. The remarkable inconsistency in their understanding of scientific developments around the year 1931 shows that Gödel and Zermelo were mathematicians of different generations.

Chapter (E), "Computer Science-Logic" focuses mainly on two classical topics and a neglected one. In contrast to the theory of recursive functions and Gödel's incompleteness results, his contributions to the complexity of proofs and the length of proofs are widely ignored. The reprinted letter to von Neumann (pp. 125f) includes the conjecture P=NP (now known as the famous conjecture P≠NP). Alexander Leitsch's correlated article, "About non-recursive shortening of proofs" (pp. 127–134) gives an illuminating account of Gödel's main idea in "*On the length of proofs*" (1936) about the shortening of proofs through the extension of theories. In "How general are general recursive functions?" (pp. 103–122) Klaus-Dieter Schulz focuses on a very exciting and influential topic: Church's thesis and Gödel's contributions in Princeton in the mid-thirties. It is well known that Gödel's extension of the concept of (primitive) recursive function to the concept of general recursive function was deeply influenced by Church (cf. Eckehart Köhler's article "Gödel in Princeton" in volume I) and—of course—in 1931 he was already prompted by a letter written by Herbrand (reprinted in this volume on pp. 99ff). Church's, Kleene's and Rosser's investigations into the λ-calculus and into the relationship (of equivalence) between Church's λ-definable functions, Kleene's μ-recursive functions and Gödel's (general) recursive functions had shown that Church's thesis is a very good approximation of the intuitive concept of computability. But Gödel was sceptical about Church's thesis until Turing's suggestion of "computability" became known as equivalent to Church's conceptual approach. Based around the interpretations and implications of Church's thesis, Schulz also focuses on the vague notion of finite computation and draws a connection (p. 111) to William Tait's thesis, "Finitism = PRA"—the cornerstone of a recent debate on the foundations of mathematics (e.g., concerning a possible finitist interpretation of Ackermann's function). Gödel had already mentioned this connective point (as Schulz notes) in his lecture "On undecidable propositions of formal mathematical systems" (1934). There he remarked that the equivalence between computation and recursiveness "cannot be proved, since the notion of finite computation is not defined, but it serves as a heuristic principle" (***Collected Works*** I, p. 348, footnote 3).

In chapter (F) about "Set Theory" Ulrich Felgner (pp. 169–185) gives a lucid account of the development of the concept of set from the beginning to Ackermann (e.g., Bolzano's as well as Cantor's views were discussed). Norbert Brunner and U. Felgner focus on "Gödels universe of constructible sets" (pp. 189–198) in the context of the consistency of the continuum hypothesis and the axiom of choice as well as of some aspects of the correlated *axiom of contructibility* $V = L$. Gödel's relative consistency proof (Con(ZF) $\Rightarrow$ Con(ZFC)) might not be his most famous result but due to its technical details it is particularly demanding. Brunner and Felgner discuss Gödel's ideas with reference to Poincaré's considerations of the concept of definability (of sets). There are some interesting 'similarities' as well as differences between Poincaré's hierarchy and Gödel's universe of constructible sets. Again, in "A letter by Gödel on the axiom of foundation" (pp. 205–213), Felgner discusses and documents Gödel's thoughts (that corresponded to Bernays' well-developed thoughts) on the axiom of foundation which was introduced independently and almost simultaneously by Fraenkel and Skolem eight years earlier in 1923 (the original version was by Mirimanoff).

Chapter (G), "Cosmology" is not on the focus of interest of Gödel research. But the five authors provide an elegant introduction to the topic and the implications of Gödel's cosmological solution of Einstein's field equations of gravitation. Gödel's solution supports a theoretical possibility of time travelling because of the time structure in the Gödel Universe. In "The Gödel Universe" (pp. 219–229) Heinz Rupertsberger gives a general and vivid overview to the topic, whereas in "The Gödel Universe and the philosophy of time" (pp. 231–250) Andreas Bartels discusses the possibility of time travelling, its implications for the philosophy of time, and reasons for the late reception of these results by philosophers. In "Gödel, Einstein, Mach: The completeness of physical theories" (pp. 251–267), Giora Hon reflects on the consequences of Gödel's solution for the so called "Mach Principle"—one of the fundamental principles for the general theory of relativity. Hon discusses how Gödel's solution refutes this principle and in what sense this result has shown the incompleteness of physics. The extraordinary role of this scientific interest in the frame of Gödel's achievements might be a consequence of his deep friendship with Einstein. The corresponding article "An example of a new type of cosmological solutions of Einstein's field equations of gravitation"

(1949) was originally written for and published in the Schilpp Volume ***Albert Einstein: Philosopher-Scientist***.

Chapter (H) on "Philosophy" is dedicated to Gödels ontological proof of the existence of God, Gödels influential Platonism and to the philosophical implications of his most famous metamathematical results. In "Sketch of the ontological argument" (pp. 309–324), after a historical introduction, Johannes Czermak presents a detailed version of Gödel's proof. This lucid presentation is imbedded into relevant aspects of modern modal logic and model theory. Petr Hájek gives a model-theoretic interpretation of Gödel's ontological proof in "The mathematician and the question of the existence of God" (pp. 325–336). In "Gödel's Platonism" (pp. 341–386) Eckehart Köhler provides a critical and elaborated justification of Gödel's epistemology and metaphysics, whereas in "Gödel's part in the discussion about Behmann's treatment of the antinomies" (pp. 387–394) Christian Thiel investigates Gödel's objection against Behmann's resolution of Russell's Paradox. Gödel's counterexample and his reservations concerning Behmann's suggestions were subject of their correspondence. The last article, "Philosophical implications of Gödel's results? A report" (pp. 395–438) by Bernd Buldt, is mainly dedicated to two aims. Firstly the article gives a critical survey of philosophical implications for some foundational accounts such as (to a large extent) Hilbert's program, Logicism and in general, the relationship between "truth" and "provability". Secondly, the paper aims at refuting some misunderstandings in Gödel's results. These misunderstandings are responsible for an unjustified myth of Gödel. In opposition to the pseudo scientific application of Gödel's results in the cognitive sciences (most prominently due to Roger Penrose), Buldt points out in short (pp. 425f) that the incompleteness results do not say anything about the mechanization of consciousness. According to Buldt's former aim, Gödel's relation to Hilbert's program still deserves interest. It was already mentioned by Gödel at the end of his famous article "On formally undecidable propositions of *Principia mathematica* and related systems" that his incompleteness results (***Collected Works*** I, pp. 194/5) are no refutation of Hilbert's program since it is conceivable that there are finite proofs that cannot be expressed in the formalism. (Of course, Gödel revised this opinion later, e.g., in his "*Dialectica* paper" (1941; published 1958).) There are, of course, a lot of philosophical implications which are particularly responsible for Gentzen's achievements in proof theory and for the possibility of reductive proof theory. A large part of Buldt's article deals with consequences, interpretations and possible ways out for Hilbert's program. Particularly remarkable are the "Concluding remarks" (p. 429) by Buldt which make—also as concluding remarks for both volumes—a responsible statement about foundational research in the post Gödelian era. Clarifying the epistemological role of arithmetic, the reduction of the actual infinite to the potential and a vast amount of other tasks can only be treated appropriately in the wider scope of epistemology, the philosophy of science and the methodology of mathematics and science. In this way, reductive proof theory, in particular, does meet the needs.

Both volumes are remarkable contributions to Gödel research. While volume I can be treated as a source book on Gödel, volume II can be seen as a very good companion to Gödel's achievements. According to the announcement in the "Preface" of volume I it is not completely clear to what extent the contributions can be seen as specific contributions to "Gödel as a part of the intellectual life in Austria". The majority of the articles, interviews and sources deals also with the time after Gödel's emigration in 1940. The reprints in volume II provide links between the biography and the scientific context. In particular, the technical topics in volume II are presented in a solid and very comprehensible way. This volume can be used without any problems by everyone who has mastered an introductory course in logic and is patient in reading. For this we have to thank the authors! They did a good job! The treatment of Gödel's achievements in the five chapters of volume II is well-balanced. With respect to the huge amount of literature on Gödel's most famous results from 1931, the vivid representation of Gödel's contributions to the complexity of proofs and his deep insights into set theory are particularly worth mentioning. Everyone who is interested in Gödel will not be disappointed. As a summary I would say: everyone should have it!

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C. Penco. *Frege: two theses, two senses*. ***History and Philosophy of Logic***, vol. 24, no. 2 (2003), pp. 87–109.

Much of the subject matter of contemporary philosophy of language was nourished by critical discussion and elaboration of Frege's contribution to the philosophy of language, logic and mathematics. In this regard, perhaps no one contributed more than M. Dummett to the subject and to the interpretation of Frege's philosophy of logic. It has been argued, against Dummett's interpretation, that there is a hidden tension underlying Frege's philosophy of logic. This tension concerns Frege's conception of thought and is brought to light by the fact that Frege held two contrasting views, two theses, concerning the relationship between thought and language. In some contexts, Frege assumes that the structure of language reflects the structure of thought, while in other contexts he recognizes that structurally different sentences can express the same thought. It is claimed that this ambivalence goes hand in hand with an ambivalent definition of thought as the 'sense' of a sentence. More specifically, it is Frege's identification of the thought with the sense of a sentence that is hiding the tension in question. In some contexts, 'sense' is taken as 'cognitive content', while in other contexts, 'sense' is taken as stating the truth condition of the sentence. For the past decades, many contributions to contemporary philosophy of language were indeed motivated by the effort to clarify the distinction between what came to be described as 'epistemic' and 'semantic' aspects concerning Frege's notion of 'thought'. The paper under review brings to light this continuity in the recent history of the field. In his article "Frege: two theses, two senses" Penco agrees with the view that there is a tension between two different notions of 'sense' implicit in Frege's conception of thought. Yet siding with Dummett, he disagrees with the specific solutions thus far proposed to resolve the tension. Some of the solutions the author briefly reviews rely on the introduction of a sharp distinction between epistemological and non-epistemological points of view concerning 'Fregean thought'. Pointing out that the sharp divide introduced to take into account the ambivalence at issue leads to consequences that are difficult to reconcile with the textual evidence, Penco suggests a way out which he claims is more coherent with Frege's explicit views. He thinks that we must look for a unifying answer within the ambivalence underlying Frege's uses of 'sense'. In other words, the differentiation between epistemic and semantic aspects does not need to be identified with the sharp divide between epistemological and ontological conceptions of thought. What is required is a unified theory which can clarify "the connection between these two levels of sense" (p. 99). Accordingly, the problem for us to solve is to understand the 'internal link' between sense as epistemic (cognitive content) and sense as truth condition. We may summarize the paper's strategy in three basic steps. Firstly, the author looks at the textual evidence focusing on Frege's writings. Secondly, he tries to show that Frege was unaware of two contrasting uses of thought found in his work. Finally, the author offers some tentative conclusions. In this paper we find much emphasis on getting right the textual evidence. A substantial part of the paper is therefore devoted to contextualizing Frege's views. Yet despite the said emphasis, Penco's way out of the problem aims at what he calls a 'rational reconstruction' of Frege's views. Under the heading "Why Frege did not get it right", Penco discusses Frege's oscillation between different concerns at the basis of his understanding of thought. Here the author rightly points to the fact that Frege's views were put forward in different contexts following different concerns and, we should emphasize, at different times in his career. Penco accounts for some motivations for what he calls ‘Frege’s inability’ to resolve the tension, such as his antipsychologism and the 19th century dual use of the German expression ‘Bedeutung’. According to Penco, there is a way in which Frege himself could develop a clear distinction of the claim of what a thought is. Notwithstanding textual evidence, Penco’s tentative proposal thus requires some revision of Frege’s original position. He generalizes the tripartite division of sense, reference and extension made by Frege for predicates (Letter to Husserl, 1891) extending it to sentences and singular terms. His ‘Fregean solution’ proposes to understand epistemic sense as given by computational procedures. These are structurally related to reference, or semantic sense, given by truth conditions (Carnapian ‘intensions’, i.e., functions from possible worlds to truth values), while the extension of the sentence gives the truth-value. The problem could accordingly be solved in a ‘Fregean setting’ by conceiving thought “as a multi-level structure: both as intension and as procedure, i.e., both as semantic and objective truth condition and, procedurally, as objective ways to determine truth conditions—so that a given intension (truth condition) may be associated with different procedures” (p. 106). A computational theory of thought may indeed take advantage of such ‘Fregean’ framework, but at this stage of the debate, it may just sound misleading to present such proposal as a ‘rational reconstruction’ of Frege’s views.

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