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HETEROGENEOUS REASONING AND ITS LOGIC

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CONTENTS

Let me start by saying that I had the privilege of witnessing the birth of Jon Barwise's new research on heterogeneous logic and its subsequent developments. I entered the Stanford philosophy graduate program in the Fall of 1987, became Barwise and Etchemendy's first research assistant on the project of diagrammatic/heterogeneous reasoning during summer of 1989, and under their guidance completed my thesis, "Valid reasoning and visual representation," in August, 1991. With this experience I would like to focus on the more personal and informal aspects of Jon's research on heterogeneous logic which may not be conveyed by his articles. (Accordingly, I have written this paper without footnotes or other references except to Jon's work.) The present article can only hint at the depth and the influence of Jon's work in this area.

In the first section, I single out an important feature of the project on heterogeneous logic Jon founded together with John Etchemendy about 15 years ago. I title it "resolving conflicts" since the research, I strongly believe, grew out of Jon's personal attitude toward how to resolve a tension between opposite extremes.

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The second section focuses on how teaching logic itself was shaped as part of Barwise and Etchemendy's research agenda. It is worthwhile noting that their textbook *Language, Proof, and Logic* constitutes part of their research and, hence, the success of the book vindicates the goal of the overall project.

In the third section, I describe what was done by Barwise and Etchemendy in the initial stages of their research on heterogeneous logic in three different subsections: (i) They wrote introductory articles and gave numerous talks about the new project. (ii) They applied the main idea of the research and laid out the architecture for a computer program as part of the project. (iii) They provided a mathematical framework for heterogenous reasoning so that heterogenous logic research could get off the ground as an important research area.

These aspects of their research led Barwise and Etchemendy to write up a mathematical framework for their computer program *Hyperproof*. The main ideas behind this work are explored in the fourth section. In the last section, I emphasize the influence of Barwise and Etchemendy's heterogenous logic project in various areas and briefly discuss the current interdisciplinary research on this subject.

§1. Resolving conflicts. The project on heterogeneous reasoning and logic is a product of Jon's special way of approaching the conflicts between ordinary versus formal reasoning, teaching versus research, and theoretical versus applied work.

It is an undeniable fact that we ordinary reasoners rely on many different forms of media (e.g., sentences, maps, diagrams, smells, etc.) and at the same time it is also true that mathematicians and logicians, one of whose primary goals is to formalize valid reasoning, have traditionally focused on sentential forms of representation only. Jon himself, as one of the top logicians in the 20th century, has made a great contribution to this tradition in set theory and model theory. However, Jon was not hesitant in questioning the long-standing preference traditional logicians, including himself, have had for symbolic logic. Recognizing a seeming conflict between what is going on with our ordinary reasoning and what has been done in logic and mathematics, he combined the merits of two opposite poles and made sense out of them. The merit of modern logic is its formalization and rigor, and the merit of the multi-modal reasoning is its practical power. Jon's idea was very straightforward: There is no reason why one cannot have both, that is, to develop formal logic out of our daily valid reasoning. Where others might see an irreconcilable conflict, Jon saw a good opportunity to invent something new and better.

Many of us live or do not mind living a split existence between teaching and research. That is what Jon never accepted. His insistence on the unity of teaching and research was one of the main forces which led him to the discovery of heterogeneous reasoning and logic. Jon's concern for students was extraordinary. As I describe below, one might consider the establishment of the new research on heterogeneous logic to be a natural outgrowth of Jon's intensive efforts to examine logic teaching and his determination to make logic teaching work.

Also deserving of our attention is the way Jon introduced the development of computers into his research. One might say that it is a lucky coincidence that rapid advances in the computer industry, especially in computer graphics, took place when he and Etchemendy realized the power of visualization in our reasoning, but without Jon's on-going interest in current technology the connection would not have been made. While his heart never left one of the most traditional academic subjects, mathematical logic, Jon successfully introduced a cutting-edge technology to this time-honored area. To him, there was no strife between them, but rather complementation.

§2. Logic with visualization. I took my first logic course From Jon Barwise at Stanford during my first quarter as a Stanford graduate student in philosophy. Phil. 159, entitled "Basic concepts in mathematical logic," was rather unique to Stanford, and in retrospect the nature of the course was one of the main contributing factors to the development of Jon's new research on diagrammatic reasoning.

Phil. 159 was designed to prepare students for taking a course on the metatheory for first-order logic. Among logic teachers and students, it is well known that there is a gap between what a student learns in an introduction to logic class (Phil. 57 at Stanford) and what is supposed to be taught in the next level logic class (Phil. 160A at Stanford). What I mean by 'gap' is that students do not see that what they learned in Phil. 57 is at all helpful for Phil. 160A. Many students who understood the material in Phil. 57 felt at a complete loss in 160A. Considering that the main goal of Phil. 160A is to show that the system learned in Phil. 57 is sound and complete, the existence of this gap has been frustrating to both students and teachers. The main idea behind Phil. 159 was to reduce the gap, by teaching the mathematical tools needed for 160A. Since there is no fixed list of the necessary and sufficient tools for the purpose, what and how the course is taught vary depending on the instructor. Jon took a full advantage of this flexible curriculum and taught important concepts and tools of mathematical logic (e.g., models, logical consequence, non-logical consequence, consistency, mathematical induction, recursion, etc.) by using Barwise and Etchemendy's then new and experimental software, *Tarski's World* and *Turing's World*.

Being a first-year graduate student who did not have much background in logic and computers, I did not fully appreciate how revolutionary his teaching methods were. Moreover, I had no idea about Jon's ambitious plan behind the introduction of visual software to the teaching of logic. However, I was more than impressed with Jon's genuine care for his students. He was always available in the office and ready to help us with any questions about the course material or about logic in general. He did not dismiss our questions but took them seriously.

The final project of the course was to write a "book" which discussed every topic covered in class. Jon believed that understanding the material involved more than just solving relevant problems but knowing the motivations behind each concept and being able to explain them. Towards the completion of the project, we realized how much we had learned, mainly thanks to Jon's patience, his enthusiasm for the subject, and his innovative teaching methods. To our surprise, Jon read everybody's "book" and commented on them at great length. This means that he read more than 50 "books" during the short Christmas break while writing "reviews" for each of them!

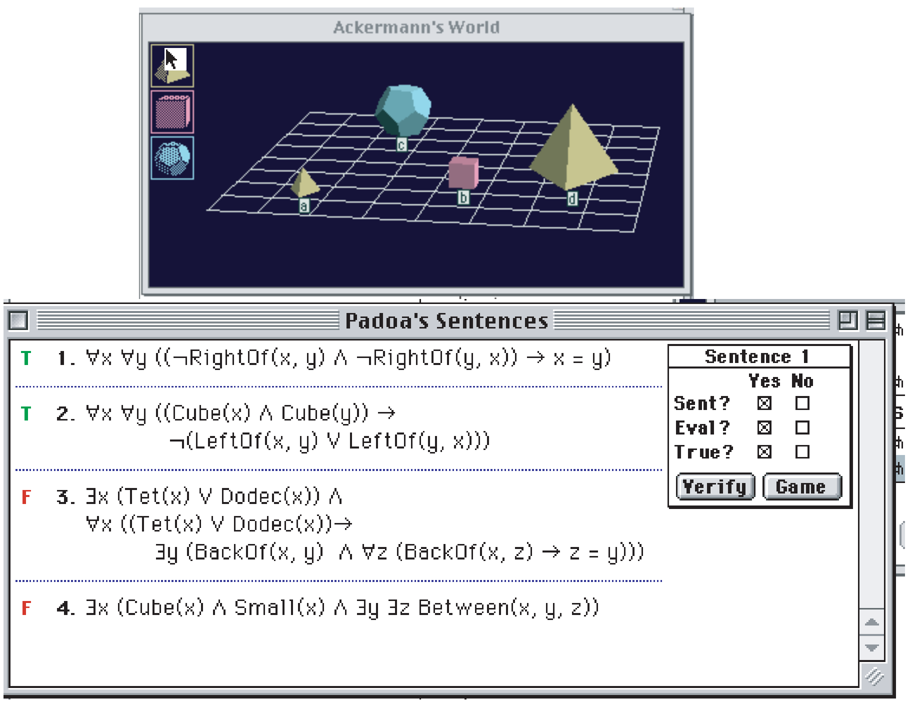
Above all, what we learned through Barwise and Etchemendy's new software, *Tarski's World* and *Turing's World*, was the most unforgettable. In retrospect, the use of these programs made a crucial contribution not only to students' learning but also to Jon's research. While we were learning a first-order language and important logical concepts in a very interesting way, both Jon and John [Etchemendy] were, apparently, learning(!) how easily students could learn the material as compared to when they were taught in a traditional way. This experience led Jon and John to rethink not only how logic has been taught in a classroom but also how logic has been studied among logicians. Let me explain briefly how these computer programs made a change in teaching logic, before I move on to how they shaped the direction of Jon's research.

In most American universities, there are two levels of logic courses available to undergraduates. One is aimed at improving our analytic reasoning (let me call it "Informal Logic"), and the other is designed to teach a formal system of logic (let me call it "Formal Logic"). While the former course has been taught in very different ways from school to school, the latter one is quite uniform in that deductive systems are always taught. Unlike Informal Logic, Formal Logic has more specific goals, for example, to introduce the concept of validity with respect to certain deductive systems, to make students familiar with notation used in philosophy papers, to help students prepare for further meta-logic courses, and so on. Accordingly, Formal Logic has usually been considered an important requirement for the philosophy major in most universities.

As a logic teacher, I have found it the most challenging to face common complaints that neither of these two kinds of undergraduate logic courses have fulfilled their own expectations. The Informal Logic courses have been frustrating both to teachers and students, since these courses have not been successful in improving our reasoning skill outside class. The Formal Logic course left an impression that after spending the entire term in learning an artificial and inflexible language and its inference rules, students do not feel that they have learned any interesting and important material. As a result, almost only philosophy majors take the course, just to satisfy a requirement. Any logic teacher who takes teaching undergraduates seriously would admit a grain of truth in these complaints.

Barwise and Etchemendy correctly identified the main source of the problems with the traditional way of teaching formal logic, that is, that so much emphasis has been given to syntactic manipulations only. *Tarski's World* was designed mainly to counter this traditional narrowness, by illustrating the semantic conception of logic in a very successful way. After using the program as supplementary material in a course for several years, they wrote an introductory logic textbook based on the program, *Language, Proof, and Logic*. In the following, based upon my experience, I will show how this program has provided a breakthrough in teaching formal logic to undergraduates.

While traditional logic textbooks and several existing computer programs for the material handle only first-order sentences, *Tarski's World* consists of three parts: first-order sentences, a toy world, and an evaluation window.



The two extra features (i.e., the toy world and the evaluation window), I find, have been immensely helpful to teach students basic concepts of logic and to achieve the goals of the formal logic class. For example, after answering one or two questions, students learn the concepts of non-consequence, consistency, and satisfiability very naturally. They figure out by themselves how to prove that one of these relations or properties holds of a given set of sentences. I was quite surprised to witness how helpful understanding semantic concepts is for carrying out formal proofs in a deductive system. The evaluation window checks not only the syntax of a sentence (i.e., well-formedness) but also its semantics by yielding its truth-value relative to a given toy world. If a user does not understand how this truth-value is produced, he or she can play a game with the program. This game, I find, is the best way to teach students multiple-quantifier sentences. As all logic teachers will admit, teaching first-order sentences with multiple quantifiers on a blackboard in a classroom is one of the most frustrating teaching experiences.

The first time I adopted their textbook in my formal logic course was when I was teaching a summer course at Stanford University in 1992. Since it was a summer quarter, the class consisted of students who had different backgrounds—engineering, mathematics, philosophy, physics, and political science students. There were several students who had failed a formal logic course during the year or who wanted to finish this not-so-fun requirement during the summer. However, literally all of the students remained enthusiastic about the subject until the end of the summer quarter, saying that this program did not let them forget important concepts in the course. I still remember the phrase from a teaching evaluation: "I would like to let the department know that there is a different way to teach this course!"

Since then, I have been using their textbook, and the results have been as good as what I experienced the first time. As clear evidence that they learned important concepts from this course well enough to be interested in the subject, some of my students have been taking an advanced graduate metalogic course after this formal logic course. It hardly happens that philosophy majors volunteer to take more logic courses during college years. Also, at the request from students who got interested in the basic concepts of logic, I had to open a new undergraduate Philosophy of Logic course. It is almost amusing to see the course that has been notorious among philosophy majors transformed into a course that generates other courses. I know that without Barwise and Etchemendy's innovative *Tarski's World* this change would not have taken place. As a logic teacher, I strongly believe that they have effected a revolution in the teaching of formal logic to undergraduates.

Turing's World is another innovative teaching tool. It was designed to teach computability in an easier and more natural way. The program consists of two windows: A flow graph for a Turing machine may be drawn in one window and a Turing tape is in the other window so that a Turing machine may be simulated for the given flow graph. Compared with other descriptions of Turing machines, a flow graph is much easier to read off. More importantly, how a given flow graph is supposed to work is easily seen by running the program. As a neat and useful feature, the program allows us to build a submachine so that it may be used for another program.

Logic teachers know that writing up a Turing machine program is one of the best ways to have a good sense of what an algorithm is, but admit that writing up a long description of a machine or reading off a flow graph is time-consuming as well as difficult. When one follows Barwise and Etchemendy's booklet containing *Turing's World*, one will be surprised to see how one can understand a great amount of computability theory with such ease.

§**3. Birth of heterogeneous logic.** The innovations that both *Tarski's World* and *Turing's World* provided did not end in the teaching of logic. Barwise and Etchemendy drew a valuable lesson from the way these programs have been received by students and teachers, and opened a new area of research in logic. They gained important insights when they asked the following questions: Why are these programs more effective than traditional teaching methods? Does it mainly have something to do with ease or convenience thanks to the use of a computer?

As Barwise and Etchemendy have written, "we already knew that it was much easier to use the diagrammatic representation of a Turing machine: that is why we designed Turing's World with its graphical capabilities in the first place." ([9], p. 7) Being convinced of the special cognitive aspects of a non-symbolic representation, Barwise and Etchemendy became interested in the general topic of visual programming languages.

However, observing a preference for diagrammatic representation is one thing and explaining its cognitive power is another. While exploring the latter, Barwise and Etchemendy discovered an interesting and important relation between visualization and reasoning: Graphical representation makes certain kinds of reasoning easier, but in many cases visual reasoning (i.e., the reasoning one adopts involving visual representation) is not easy to capture in a traditional symbolic logical system. This realization became the driving force behind Jon's new research on diagrammatic reasoning. Before moving on to how he pursued the subject in more detail, let me further explain his main motivations for the research in this area.

All of us engage in and make use of valid reasoning, and in the process of reasoning human beings obtain information through many different kinds of media, including diagrams, maps, smells, sounds, as well as written or spoken statements. An everyday example would be reasoning about the path of a trip, using directions not only in English or some other language but also in the form of a map. While nobody would deny that we more than often adopt diagrams or pictures for ordinary reasoning, formal logic has thus far been primarily concerned with valid reasoning based on information in one form only, i.e., in the form of sentences. The intensive study of reasoning by means of artificial symbolic languages has been the major direction in logic.

On the other hand, considering that logic is the study of reasoning, the discrepancy between how our ordinary reasoning is performed and what has been studied by working logicians deserves our special attention. The research on diagrammatic reasoning took off in the late 80's when Barwise and Etchemendy began an inquiry emerging from their puzzles over teaching a logic class involving diagrams. As mentioned above, while modern logicians have concentrated on sentences only, actual reasoning as performed by human beings typically involves information obtained through more than one medium, that is, multi-modal reasoning. Such reasoning is called "heterogeneous reasoning" and Barwise and Etchemendy aimed to come up with a formal framework for our heterogeneous reasoning. The project is truly revolutionary and ambitious both at a theoretical and at a practical level, as I will explain below.

First of all, when the proposal is accomplished, the scope of logic will be enlarged. Some might find it strange to talk about a change in the scope of logic, since logic sounds more or less like a fixed subject. However, as we know, what Aristotle's logic covers is only a proper subset of what modern logic is about. Many Kantian scholars believe that Kant's distinction between logic and mathematics was based on Kant's narrow concept of logic, that is, Aristotelian monadic logic. When Frege and Peirce independently presented quantified logic, the scope of logic changed from being monadic to relational. What (relational) predicate logic could do became so extensive that Frege even thought mathematics was built on logic. Since the turn of the 20th century, it is only first-order predicate logic which has been identified as logic without controversy. Whether second order logic is a logic has been debated without resolution.

What the heterogeneous logic project proposes is much more ambitious than encompassing second order logic under logic. From this project's point of view, the second order logic debate is limited to one and the same form of representation, i.e., symbolic systems. On the other hand, the heterogeneous logic project aims to expand the territory of logic by freeing logic from a mode of representation. Logic is not inherently tied up with sentential representation only. On the contrary, as long as reasoning can be represented in a system and we can determine its validity, we may call it a logical system. Nothing prevents logicians from working on such systems.

At a practical level, the heterogeneous logic project could solve one of the puzzles about logic teaching. The complaint most often made against logic courses is that studying logic does not help students improve their reasoning ability. So far, there have been two views on the relation between ordinary reasoning and formal logic. Some have concluded that we cannot teach reasoning skill in a classroom. Everyday reasoning involves such an indefinite number of different situations that it is almost impossible to formalize how we reason in various cases. Believing that the valid reasoning a logic course usually teaches is different from the natural valid reasoning human beings perform in everyday life, others advocate a different kind of course for these two different kinds of reasoning. Either way, a formal logic course has been considered as a course which has nothing to do with ordinary reasoning.

However, both views were challenged when Barwise and Etchemendy presented the thesis that a main reason for the failure of traditional formal logic courses with respect to improving reasoning skill is that formal logic classes have taught only one of many ways to handle information, that is, to manipulate symbols. Based on this conjecture, they have raised fundamental questions: What prevents us from teaching more than symbolic manipulations in a logic class? Is there any legitimate reason why only symbolic systems have been welcomed in a formal logic class? If we find a negative answer for the justification of the longstanding bias for symbolic systems, it is time to expand the territory of formal logic by investigating different forms of logical systems other than symbolic ones.

Thus, Barwise and Etchemendy's project on heterogeneous reasoning took off sometime during 1988. During the first stage of the research, Barwise accomplished three different tasks together with Etchemendy: One was to motivate the reader with the goal of the project, another was to develop computer software based on a heterogeneous system, and the third was to lay out a more theoretical and general framework for heterogeneous inference. As I explain below, all of these three efforts turned out to be tremendously successful.

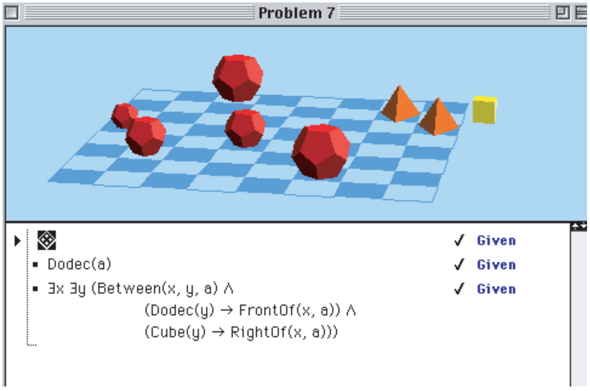
3.1. Enlarging the audience. Barwise and Etchemendy's fist paper on the subject, entitled "Visual information and valid reasoning," instantly attracted attention from many researchers among various areas—logic, computer science, mathematics, cognitive science, and philosophy. In the paper, they illustrated the role of visual representation in our ordinary reasoning through examples and claimed that logic, whose main goal is to investigate valid reasoning, should encompass non-symbolic forms of representation. This paper was presented at numerous conferences, and I vividly recall the enthusiasm the audience showed for the paper at every conference I attended.

Not surprisingly, the paper has been reprinted in many volumes, becoming one of the classics in the field, and it has inspired many people interested in logic, computer science, and reasoning. I am one of those who participated in the project since reading a draft of their paper, and my first book *The Logical Status of Diagrams* practically grew out of the following sentence in the paper: "We think it is possible to give an information-theoretic analysis of this system, one that shows, among other things, that the demonstration above [the derivation by Venn circles for the proof that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$] is in fact a valid proof." ([3], p. 9) As a case study of a *non*-linguistic form of representation as a standard formal representation system, my work showed not only that their hunch is correct, but also that a widespread worry of mathematicians and logicians about diagrams is not justified. The traditional worry is that the use of diagrams tends to yield fallacies compared with the use of symbolic systems. By proving that a Venn diagrammatic system is sound and complete, I showed that the fallacious inferences we often get in the use of diagrams do not originate in the nature of diagrams but in a lack of clarity about the legitimate manipulation of diagrams.

3.2. Applying the ideas. At the same time, Barwise and Etchemendy were designing the computer program *Hyperproof*. The outline of the program was presented at the Asilomar meeting in the Fall of 1988, but it was a mock demonstration, far from being a final product. During the Summer of 1989, there was a meeting from 1pm to 4pm every day among those involved in the program. The energy and the enthusiasm I experienced in those meetings are beyond description. It was exciting to Jon and John since the success of the program would strongly support their main thesis that we can formalize valid inference based on multi-modal information. The rest of us were inspired not only by the project itself but also by their enthusiasm. They never lost the overall picture of the project, but at the same time, they were thorough about its details. That summer I witnessed Jon's impressive work ethic and his brilliant mind, and found the rest of us being voluntarily driven by them.

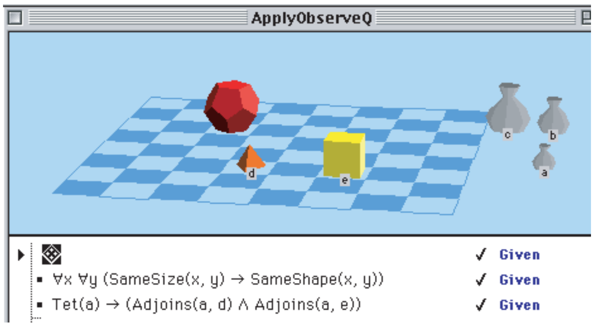
Hyperproof presents us with information about a block world in two forms, in a first-order language and in a diagram. A screen contains two windows, one for each form of representation. By combining these two forms of information, the user is supposed to reason about a certain piece of information, for example, the name, the size, the shape of a figure, or the relation among figures. Sometimes the piece of information the user aims to obtain can be expressed in a diagram, for example, by locating a block, by assigning a name to a block, or by giving a definite size or a definite shape to a block. Sometimes the user expresses the conclusion in a sentence, for example, how many blocks are to the left of a given block, whether a large cube is at the back of a given block. As shown in the examples below, the information is represented in two forms, and the goal is to determine the location of the block off the board.

In the first question of the next page, the conclusion will be expressed in a diagram, by locating the block in a specific place in the board. When the user faces a question about a given block world, in most cases he knows how to reason between two different forms of information even though no rule is given to him. Then, why cannot the natural reasoning adopted here be formalized? A main challenge is that the inference the user takes on cannot be completely captured by the inference rules of first-order logic. In order to manipulate different forms of information, we need more rules to allow us not only to reason in visual representation but also to reason

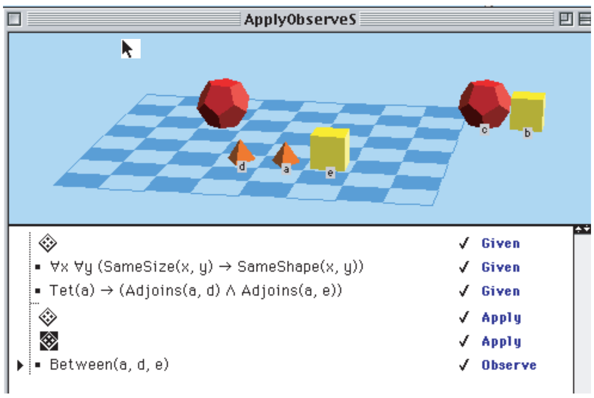


between linguistic and visual representation. For example, the user might want to make a statement by simply observing a given diagram. (*Hyperproof* has the rule Observe.) Conversely, sometimes one needs to apply sentential information to a diagram. (The rule Apply is available.)

From the following information, we would like to know whether object **a** is between objects **d** and **e**:



What we need is to observe the conclusion after applying the given two premises to the diagram:



In this way, the software teaches multi-modal reasoning—reasoning involving information in more than linguistic mode, e.g., sentences and diagrams—with inference rules which are not part of standard natural deductive systems but which are used for our ordinary reasoning.

Hence, Barwise and Etchemendy's *Hyperproof* serves as a nice case study to confirm their new view of logic, and at the same time the program itself will have a significant practical effect on logic teaching. This is an excellent example of how teaching and research do not have to be separated from each other. On the contrary, their great concern about logic teaching has been one of the driving forces for the project, and at the same time their research goal has been one of the major premises behind the development of the new teaching method.

3.3. Laying out a mathematical framework. In addition (i) to motivating researchers with a new view of logic and reasoning, and (ii) to launching a blueprint for a computer program based on the enlarged view of logic, Barwise and Etchemendy completed another important initial step of the heterogeneous logic project, that is, to provide a general mathematical framework for heterogeneous inference. This they did in the paper "Information, Infons, and Inference." Traditional model theory allows us to justify the inference within a sentential context. Hence, one could develop a theory tailor-made for another form of representation, for example, for visual representation. Next, in order to handle cross-multi-modal information, an inter-mechanism could be developed among the different theories developed for each form or representation. Instead of taking this alternative, Barwise and Etchemendy decided to develop a theory that transcends any specific form of representation from the beginning: "When we turn our attention to heterogeneous inference, we are naturally drawn to look for a model of information that is independent of the particular form in which the information is represented." ([1], p. 34)

First of all, the theory they were looking for should not be tied up with syntactic features of any particular representation system. The semantic aspect of representation, rather than the syntactic, needs to play a key role in the theory. What a system represents, i.e., the information content, should become the basis for building a model for heterogeneous inference. What they proposed is an information-based theory of inference.

In this scheme, admittedly, what is needed first is a theory of information content. Their information theory adopts situation semantics, which was developed by Barwise and Perry in *Situations and Attitudes* (1983), mainly because situation semantics, unlike many other formal semantics, puts an emphasis on partial information. Since the main goal of the heterogeneous reasoning project is to formalize our ordinary reasoning and working on limited information is one of the main features of the daily reasoning process, Barwise and Etchemendy's information theory should be able to handle limited parts of the world, not always the entire complete world.

Starting with basic units of information, they developed an algebra of pieces of information. (Refer to §2 of [1].) Their final algebraic product has two important features: One is to accommodate heterogeneous inference, i.e., inference made between different forms of representation. The other is to respect partial information, which is quite natural to our ordinary reasoning process.

A major role Barwise and Etchemendy assigned to the information theory in the heterogeneous reasoning project is that their inference rules are written at the level of information, not at the level of representation. They allow us to manipulate pieces of information, not to manipulate units of representation. This meta-level approach, naturally, raises the following question: The manipulation of information is accomplished by the manipulation of syntactic units of representation. If inference rules do not tell us how to work with syntactic objects, how can we ever get to manipulate pieces of information?

It is true that each manipulation is actually performed at the level of representation. Traditional inference rules are formulated at this level, that is, at the level of symbolic representation. This is why existing transformation rules are system-bounded and hence, fail to be applicable to heterogeneous reasoning. Barwise and Etchemendy pushed the overall picture up to a meta-level, identifying five valid inference principles (Given, Assume, Subsume, Merge, and Recognize-as-Possible) among pieces of information. Now, all one needs to do is to check whether each manipulation either in a system or between systems conforms to one of the five valid rules. This is a way their semantic theory transcends any specific form of representation, so to speak.

The next question, then, is how to justify their basic five principles. Using a water metaphor, Barwise and Etchemendy adopted the expression "information flow" for valid steps among pieces of information and developed a model of information flow. More detailed explanations of their information flow graphs require further discussion of the theory of information, and this is why Jon's two areas of research—information theory and heterogeneous logic—are closely related to each other and it is not a pure coincidence that Jon became interested in the two topics around the same time.

How their theory can be used for reasoning is illustrated by a simple example involving the use of diagrams. (p. 67, [1]. Numbers are mine.)

> You are to seat four people, A, B, C, and D in a row of five chairs.
> (p-i) A and C are to flank the empty chair.
> (p-ii) C must be closer to the center than D,
> (p-iii) who is to sit next to B.

On the basis of this information,

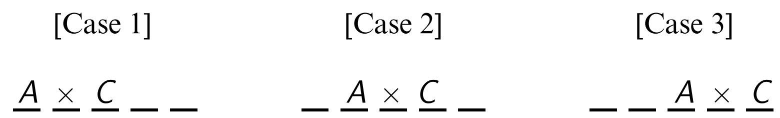
(c-i) show that the empty chair is not in the middle or on either side.

(c-ii) Can you tell who must be seated in the center?

(c-iii) Can you tell who is to be seated on the two ends?

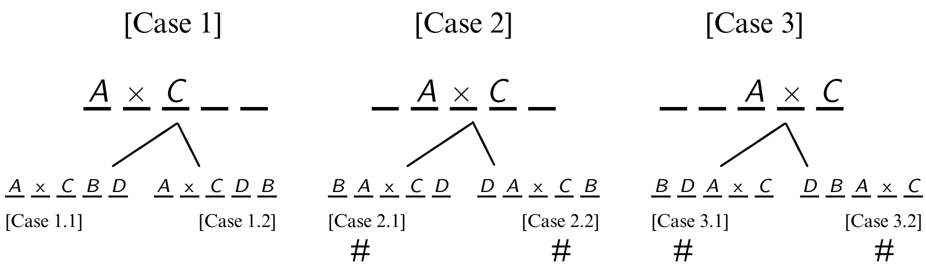
Most probably, we would like to draw diagrams to solve the question instead of translating this information into first-order sentences. First, to represent the information (p-i), we realize that there are six possible cases, of which we draw these three (noting that the other three are their mirror images).

FIGURE 1



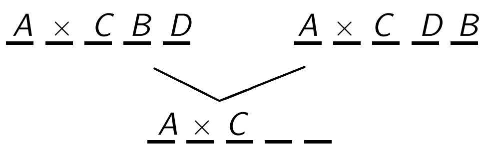
Since we have two more people to be seated and two seats are available, there are two subcases under each case. But, cases 2.1, 2.2, 3.1, and 3.2 are all closed, since they conflict the information of (p-ii) or (p-iii):

FIGURE 2



Since both cases 1.1 and 1.2 are possible, we extract the information common to these two diagrams to get the following diagram:

FIGURE 3



From the final diagram, we can read off the three pieces of information we would like to find out, (c-i), (c-ii), and (c-iii).

This is a very natural way to solve the simple puzzle, but a question is how we know this reasoning process is valid. No traditional inference rule or semantics can be adopted here, since we are representing information in a non-symbolic form and manipulating diagrams, not first-order sentences. The information-theoretic analysis Barwise and Etchemendy developed in the paper provides a mathematical scheme to justify this natural process. After the five inference principles, that is, Given, Assume, Subsume, Merge, and Recognize-as-Possible, are proven to be valid, we know the reasoning is a valid performance as long as each step applies one of the five principles. In the above, the step processed in Figure 1 belongs to the Assume rule, the step in Figure 2 to the principle of Subsume, and the step illustrated in Figure 3 belongs to the Merge principle. In order to justify our natural reasoning process, we do not have to go back to one of the existing symbolic systems whose soundness is proven.

I find the main framework presented in "Information, Infons, and Inference" intuitive and convincing as well as clever and revolutionary for the following reasons. At a naïve level, the reason why we judge certain steps in our reasoning valid and others not valid has nothing to do with any form of representation. In many cases, we are not aware of what kind of representation has been used in the reasoning process. (Imagine that one follows a road instruction one read an hour ago. One might be remembering that piece of information visually or linguistically depending on the person's tendency. Also, when one applies the given information while driving, it is not clear how different pieces of information are combined together to get one to the right location.) Instead, we intuitively think of valid reasoning in terms of information, i.e., a valid inference is a process of extracting information from given information. Barwise and Etchemendy's proposal to capture validity at the level of information, transcending forms of representation, fits with our pre-theoretic notion of inference.

At the same time, their theory suggests a major shift in the study of inference. Traditionally deductive logic has concentrated on inference rules within one and only one system and the justification of its inference rules is done at a semantic level. But each semantics is, again, tied up with a form of representation and is developed differently from one system to another. Barwise and Etchemendy's theory of logic shifts our attention to inference *from* at a syntactic and system-related semantic level *to* at an informational and system-free semantic level. Let me elaborate by making a distinction between inference rules within a system (call them 'transformation' rules) and inference rules among piece of information regardless of a representation form (call them 'inference' rules). Since the former belong to a syntactic level and the latter to a semantic level, the distinction might sound similar to the traditional distinction between the syntax and semantics of a system. However, this would miss two important differences:

First, the traditional way of developing semantics is so local that each semantics can be applied to only one corresponding representation system and, therefore, the justification of transformation rules is done independently from system to system. By contrast, Barwise and Etchemendy's theory proposes a semantic scheme underlying any form of inference, and they correctly believe that the powerful semantics they are looking for should be free from any particular form of representational system. Hence, their semantics is done at a higher level than semantics in the traditional sense.

Second, traditionally there are only transformation rules, but there is no rule at a semantic level. Semantics checks whether a given set of transformation rule is legitimate and/or powerful enough to be interesting. Barwise and Etchemendy are not satisfied with justifying transformation rules of a system case by case, but propose rules governing valid inference at the semantic level (in their enlarged sense) and a more comprehensive method to justify a transformation rule of any system. The way they enlarged the concept and the role of semantics is truly revolutionary in logic, and I do not doubt that their proposal will make a significant contribution to the theory of reasoning and inference in the near future.

§4. Case study for heterogeneity. Pushing their thesis that valid inference is justified ultimately at the level of information, not at the level of representation, Barwise and Etchemendy launched another important and groundbreaking claim about the theory of representation in their paper "Heterogeneous Logic." Also in this paper they "put some logical meat on the bones of the informal argument" for heterogenous logic they had been making.

Many have thought that there is a universal representation system in which every logician should be interested. This ideal system would allow us to represent any domain of our reasoning and, hence, the reasoner would count on the system to make an inference. The logician's traditional faith in first-order logic is a good example to illustrate the longstanding belief in the existence of a universal language. However, more recently, computer scientists, who have been frustrated with the limit of first-order logic's expressive power and its inefficiency, have been searching for a diagrammatic (not a linguistic), but universal, system. Barwise and Etchemendy are skeptical about any attempt to invent a universal representation system, whether linguistic or diagrammatic, and they urge us to embrace heterogeneity itself. That is, the user should be able to adopt different kinds of representation case by case as long as each system is legitimate.

In "Heterogeneous Logic," Barwise and Etchemendy take up the case for non-uniform representation in the area of hardware design and its verification. For example, in order to represent three different aspects of an electronic device, that is, timing, control, and logic gate, engineers have designed three separate diagrammatic systems. (Refer to Figures 7.1, 7.2, and 7.3 on p. 217 in [8].) While each diagram serves its own purpose very well, the three aspects are of such different nature that it is almost impossible to come up with one effective diagrammatic system which is able to represent all of them at the same time. They recommend us that we should respect the practice of engineers' adopting different diagrams for different purposes and heterogeneous reasoning should be accepted instead of searching for one homogeneous system.

The next question, then, is how to justify heterogeneous reasoning at a theoretical level. Is there any danger that one system might be inconsistent with another? In the case of one and the same system, we may develop its semantics and establish its legitimacy. However, when we adopt various systems to talk about different aspects of one and the same domain, the justification could be more at issue. In the above example, a line represents a different thing from one diagram to another, but whatever may be represented is about one and the same unit of electronic device, and hence, they should not be inconsistent with each other.

Some have supposed that we need eventually an "interlingua" in which various representation systems are imbedded. They did not see how heterogeneous logic obtains its rigor without the justification of an interlingua. Barwise and Etchemendy through a mathematical analysis of their own computer program *Hyperproof* present a case study for the legitimacy of a heterogeneous system without supposing an interlingua. As briefly outlined before, the program is designed to resolve questions about a block world in terms of both linguistic (i.e., first-order language) and non-linguistic (i.e., diagrammatic) representations.

In most cases, the user must commute between a diagram and a list of first-order sentences to solve a question. The user sometimes infers a piece of information only from sentences, but sometimes reads off information from a given diagram and writes it up in a first-order sentence, sometimes adds the information (a sentence contains) to the diagram, etc. It is obvious that inference rules in the program should allow us to commute between two forms of representation. Then, how can we show these rules are sound without an interlingua?

Their answer is based on the theory of valid reasoning developed in their earlier paper "Information, Infons, and Inference." Valid inference is justified ultimately at the level of information, not at the level of representation. Hence, the legitimacy of linguistic and diagrammatic representations used in *Hyperproof* can be proven against the semantic domain which both aim to represent:

> It is important to remember that *Hyperproof* maintains two distinct forms of representing information about blocks worlds. There is no sense in which the program has a single underlying representation into which both diagrams and sentences are translated. What holds

> the two together is not an underlying representation scheme, or interlingua, but simply the fact that they are both about *the same worlds*. It is this fact that underwrites the rules of inference which govern how we can reason with both at the same time. ([8], p. 220. My italics.)

The simple fact (i.e., that there is a common semantic domain we reason about) establishes a theoretical background for the validity of the heterogeneous inference rules, inference rules which allow us to traffic between distinct forms of representation. This is the most fundamental reason why ordinary reasoning requires heterogeneous reasoning, and at the same time provides us with the most convincing justification for heterogeneous logic. A rigorous analysis of *Hyperproof* rules is presented at great length in the paper, which I outline briefly below.

The syntax and the semantics of a first-order language is as usual, but the syntax of diagrams requires some thought since traditional logic has been devoted to linguistic systems only. For example, a concept of token is not that important in the case of sentential languages: Thanks to the conventions of a writing system, we know that neither the size nor the font of a symbol makes a difference to telling whether one string is the same as another. This convention is so ingrained that we almost do not see a visual difference among different tokens of the same sentence-type, and, even if we do, we consider the difference in font and size to be trivial. Therefore, an ambiguity between sentence-token and sentence-type, if any, is negligible. This is why the syntax of symbolic logic is always a type-syntax, not a token-syntax. However, a diagrammatic system is much more sensitive to tokens.

In the case of a single *Hyperproof* diagram, different icon-tokens (even of the same icon-type) in the diagram represent distinct objects in a block world. This is why the syntax of diagrams focuses on tokens of icons, while the syntax of sentences is type-oriented. Since belonging to the same type of icon does not guarantee the representation of the same block, the "sameness" among tokens needs to be defined separately, which is why the syntax of diagrams has one more mechanism, "counterpart relations."([7], p. 226.)

While we needed two kinds of syntax for two distinct languages, one for sentences and the other for diagrams, the basic mathematical framework for the semantics is more or less shared between the two systems. Both the semantics of sentences and the semantics of diagrams are defined against the same modeling of a block world, but, as we may expect, there are interesting differences in mathematical details. Tarski's definition of satisfaction is adopted for the semantics of sentences so that we may talk about the truth of a sentence with respect to a given world. However, as they point out, in the case of a diagram, it is more natural to say whether the diagram depicts a certain world, as opposed to being true in the world. Hence, for both kinds of representation, they introduce the notion "representation holding in a world."

With a uniform relation "holding" between representation and world, all the important logical notions are defined and the soundness of inference rules, especially heterogeneous inference rules, are proven. Nothing prevents us from adopting distinct forms of representation and from moving one form to another by using trafficking inference rules.

What they showed in this case study is a milestone in the establishment of heterogeneous logic. The development of logic out of different forms of representation requires neither the existence of a universal representation system nor the existence of an interlingua underlying various modes of representation. Now, we have a very good piece of evidence to be optimistic about the main goal of the project, since the practice of our ordinary reasoning in terms of multi-modal representation can be rigorously justified in a mathematical framework.

§5. **Interdisciplinary research.** In the Spring of 1992 when the first interdisciplinary forum entitled "Reasoning with Diagrammatic Representation" took place under the auspices of the American Association for Artificial Intelligence, there was the pleasant surprise of finding common ground among a large and heterogeneous group, that is, philosophers, psychologists, logicians and mathematicians working on reasoning and logic. All of us were agreed about the importance of heterogeneous reasoning and, moreover, we realized that much research has been undertaken in the area of non-symbolic, especially diagrammatic, representation systems. At this conference, Barwise and Etchemendy's presentation of their on-going project left a strong positive impression on the audience and inspired many who come from various disciplines. To challenge a long-standing prejudice against non-symbolic representation, those working on heterogeneous reasoning have taken different kinds of approaches, which we may roughly categorize into three distinct groups. I witnessed Jon's influence over all of these three areas of the research.

One branch of research can be found in recent philosophy of mind and psychology. Since the limits of linguistic forms are clear to those who have been working on mental representation and our reasoning process, some philosophers and cognitive scientists have embraced this new direction of heterogenous reasoning with enthusiasm and have explored human reasoning and mental representation involving non-linguistic forms. Historically, especially in psychology, the importance of non-linguistic forms for human thought has raised heated discussions, e.g., the "imagery debate" between analogists and propositionalists. Instead of debating over the ontological status of internal visual images, some cognitive scientists have concentrated on the functions of mental imagers of diagrams in our various cognitive activities, for example, memory, imagination, perception, navigation, inference, problem-solving, etc. Hence, it is not surprising at all that Keith Stenning, a cognitive scientist at Edinburgh, worked together with Barwise and Etchemendy in order to explore how different forms of representation vary in their cognitive effects on human inference. In this context, Atsushi Shimojima, one of Jon's students, coined the term 'free ride' to refer to an inference which is characteristic of visual representation; the conclusion seems to be read off almost automatically from the representation of premises.

Another strand of work on heterogeneous reasoning shows that there is no intrinsic difference between symbolic and diagrammatic systems as far as their logical status is concerned. Some logicians have presented case studies to prove that a diagrammatic system can be as sound and complete as a symbolic system. Jon and John guided several of their graduate students (including the present author) in this direction. These results directly refuted a widely-held assumption that diagrams are inherently misleading, and abolished a theoretical ground to prevent diagrams from being used in proofs. In spite of the limited expressive power of diagrammatic systems, their effort is significant in that they support one of the main theses of the heterogeneous reasoning project: Logic should not be system-bounded. As long as we have a way to formalize a non-symbolic system, we are very close to formalizing a heterogenous system, from which not only theorists but also practitioners working on human reasoning will greatly benefit.

A third direction of research on heterogeneous reasoning has been taken by computer scientists, whose interest is much more practical than those of the other groups. Not surprisingly, those working in many areas in computer science—for example, knowledge representation, design of efficient systems, and development of computer programs—found new and exciting opportunities in this novel concept of 'heterogeneous system' and have implemented non-symbolic representation in their research areas. I know Jon's presence at Indiana University since the early 90's excited many in the Computer Science department.

However, since things have been changing rapidly, my descriptions must be too coarse and will become soon outdated. While I am writing this article, new work must be coming out. A new journal on the topic is being launched and many conferences are being planned. Hence, I will not even pretend to know the exact scope of Jon's influence on this new research topic. For more updated information about books, articles, conferences, journals, and research sites on the topic, I would like to refer to the website `http://plato.stanford.edu/entries/diagrams/`.

Needless to say, it is truly heart-breaking to all of us to see Jon leaving in the middle of his life and his research. In this great sadness, however, we all know that Jon's marvelous achievements in logic, mathematics, philosophy, and computer science during his short life will keep being explored by many scholars and will never stop inspiring the rest of us. The research on "heterogenous reasoning and its logic" will be a source of common ground to those in many disciplines, as they all remember Jon Barwise's great mind.

Articles/Books on Logic with Diagrams by Jon Barwise

[1] Jon Barwise, *Information, infons, and inference* (with John Etchemendy), ***Situation theory and its applications I*** (R. Cooper, K. Mukai, and J. Perry, editors), Center for the Study of Language and Information, Stanford, CA, 1989, pp. 33–78.

[2] ———, *Hyperproof: an exercise in situated logic*, ***Proceedings of the North American logic programming conference***, 1990.

[3] ———, *Visual information and valid reasoning* (with John Etchemendy), ***Visualization in teaching and learning mathematics*** (W. Zimmerman and S. Cunningham, editors), Mathematical Association of America, Washington, DC, 1991, pp. 9–24. Reprinted in ***Philosophy and the Computer*** (L. Burkholder, editor) Westview Press, Boulder (1992) pp. 160–182. Reprinted in ***Logical Reasoning with Diagrams*** (G. Allwein and J. Barwise, editors) Oxford University Press, New York (1996) pp. 3–25 [In the text, I refer to the page numbers of this reprint.].

[4] ———, *Toward the rigorous use of diagrams in reasoning about hardware* (with Steve Johnson and Gerry Allwein), ***Working papers on diagrams and logic*** (Gerard Allwein and Jon Barwise, editors), Indiana University Logic Group Preprint Series, May 1993, pp. 169–212.

[5] ———, *Diagrams and the concept of logical system* (with Eric Hammer), ***What is a logical system?*** (D. M. Gabbay, editor), Oxford University Press, 1994, pp. 73–106.

[6] ———, Heterogeneous reasoning, pp. 64–74, Springer-Verlag, Berlin, 1993, pp. 64–74.

[7] ———, ***Hyperproof*** (with John Etchemendy), Center for the Study of Language and Information, Stanford, CA, 1994.

[8] ———, *Heterogeneous logic* (with John Etchemendy), ***Diagrammatic reasoning: cognitive and computational perspective*** (B. Chandrasekaran, J. Glasgow, and N. H. Narayanan, editors), AAAI Press, Menlo Park, Calif., 1995, MIT Press, Cambridge, Mass., pp. 209–232.

[9] ———, *Computers, visualization, and the nature of reasoning* (with John Etchemendy), ***The digital phoenix: How computers are changing philosophy*** (T. W. Bynum and James H. Moor, editors), Blackwell, London, 1998, pp. 93–116.

[10] ———, ***Language, proof and logic*** (with John Etchemendy), Center for the Study of Language and Information, Stanford, CA, 1999.

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