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REVIEWS

The Association for Symbolic Logic publishes analytical reviews of selected books and articles in the field of symbolic logic. The reviews were published in ***The Journal of Symbolic Logic*** from the founding of the JOURNAL in 1936 until the end of 1999. The Association moved the reviews to this BULLETIN, beginning in 2000.

The Reviews Section is edited by Alasdair Urquhart (Managing Editor), Lev Beklemishev, David M. Evans, Erich Grädel, Geoffrey P. Hellman, Denis Hirschfeldt, Thomas J. Jech, Julia Knight, Michael C. Laskowski, Volker Peckhaus, Wolfram Pohlers, and Sławomir Solecki. Authors and publishers are requested to send, for review, copies of books to *ASL, Box 742, Vassar College, 124 Raymond Avenue, Poughkeepsie, NY 12604, USA.*

In a review, a reference "JSL XLIII 148," for example, refers either to the publication reviewed on page 148 of volume 43 of the JOURNAL, or to the review itself (which contains full bibliographical information for the reviewed publication). Analogously, a reference "BSL VII 376" refers to the review beginning on page 376 in volume 7 of this BULLETIN, or to the publication there reviewed. "JSL LV 347" refers to one of the reviews or one of the publications reviewed or listed on page 347 of volume 55 of the JOURNAL, with reliance on the context to show which one is meant. The reference "JSL LIII 318(3)" is to the third item on page 318 of volume 53 of the JOURNAL, that is, to van Heijenoort's *Frege and vagueness*, and "JSL LX 684(8)" refers to the eighth item on page 684 of volume 60 of the JOURNAL, that is, to Tarski's *Truth and proof*.

References such as 49***5*** or 280***1*** are to entries so numbered in *A bibliography of symbolic logic* (the JOURNAL, vol. 1, pp. 121–218).

JAMES ROBERT BROWN. ***Philosophy of mathematics, an introduction to the world of proofs and pictures.*** Routledge, 1999, vii + 215 pp.

This book is a breath of fresh air for undergraduate philosophy of mathematics. Very accessible and even entertaining, Brown explains most of the issues without technicalities. The wide variety of topics, the nice examples, and the division of chapters into sub-headed sections makes it fun and easy to read.

The book includes both traditional/historical-based issues and more current topics. On the whole it works well, though there is no obvious organization of chapters in terms of these two themes. The introductory chapter, "The Mathematical Image," sketches what Brown takes to be ingredients in the common conception of mathematics such as the objectivity of mathematics and the certainty of its results. Chapter 2, "Platonism," includes an explication and a partial defense. Chapter 3, "Picture-proofs and Platonism," argues that pictures can be proofs, and that Platonism best accommodates this fact. "What is Applied Mathematics?" is the important question of Chapter 4, which also goes on to argue that Platonism has no trouble answering it. Chapter 5, "Hilbert and Gödel," explains Hilbert's program and the impact of the Gödel results on it, concluding with a brief endorsement of Penrose's interpretation (which Brown takes as another piece of evidence for Platonism). The role of notation in mathematics is the interesting topic of Chapter 6, "Knots and Notation," which compares mathematics with poetry (as two areas where innovation is closely tied to notation). Chapter 7 considers several answers to the question, "What is a Definition?"; and it

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raises some doubts about the current "official" view. Chapter 8, "Constructive Approaches," sketches the views of Kant, Brouwer, Bishop, and Dummett, before turning to a useful survey of the main objections to constructivism. Chapter 9, "Proofs, Pictures, and Procedures in Wittgenstein," is a somewhat frenetic collection of issues (17 sub-sections in 23 pages), which ends with Brown massaging some of Wittgenstein's insights into another (rather un-Wittgensteinian) promotion of Platonism. (The picture "need not be accurate; it need only lead to the sense, the Platonic entity" (p. 153).) Chapter 10 surveys an interesting variety of philosophical problems related to "Computation, Proof and Conjecture." The final chapter, "Calling the Bluff," acts as a general conclusion summarizing many of the issues raised in the book; and re-examining the different facets of the "mathematical image" with which the book began.

Brown has three main aims for the book. The first is to introduce philosophers to the philosophy of mathematics, to show how integral issues about mathematics are to philosophy. The second is to provide an introduction that both covers traditional positions, such as Platonism and constructivism, and includes some new issues, such as computer assisted proofs and knot theory. Brown believes that we ought to pay more attention to these newer topics, for they are important to current mathematics in the same way that the older topics (Platonism, constructivism, nominalism) were important to former mathematics. His third aim is to argue for Platonism and for the viability of picture proofs in mathematics. How does the book fare?

On the first aim, the sheer accessibility of the book provides convincing evidence that the issues belong to basic mainstream philosophy. However, chapters on a few historical figures such as Plato, Aristotle, and Mill, would have shown how views about mathematics can be integral to one's general philosophical vision.

On the second aim, the attention to new issues is a bonus. This is part of the real excitement and accessibility of the book, for it helps students quickly grasp the point of philosophy of mathematics. One drawback is that at times the discussions are quite sketchy. Some sub-topics take up less than a page, giving the book a bit of a "sound-bites" feel. To supplement the speedy coverage it would have been useful to include suggestions for further study at the end of each chapter. In general, however, the attention to new topics does help to advance Brown's aim of urging the importance and excitement of these topics to current mathematics and its philosophy.

On the third aim, the argument for Platonism and picture proofs is rather less satisfactory. Brown claims quick victories for Platonism here and there without enough argument. For example, on the question of how to explain the application and usefulness of mathematics, Brown supports the representational account (mathematics provides representations of the world) and claims that this heavily favors Platonism (pp. 48–49). His view is somewhat structuralist, in that the representations provided by mathematics are "structurally similar models" of things in the world (p. 49); but he dismisses structuralism as a philosophy of mathematics. After a useful sketch of a few forms of structuralism, and a very brief overview of some of its outstanding challenges, Brown simply proclaims that "The real upshot of all this is that the support for a descriptive view of mathematics offered by structuralism cannot be upheld and the implicit challenge to the representational account can't be sustained. Platonism still has the upper hand." (p. 61)

Platonism solves some problems, yes. But each view solves some problems at the cost of others. That's the puzzle and the enticement of philosophy. The fact that Platonism "scores" well in some areas does not decisively vindicate its truth. And, to this reader, the scoring often seemed rather premature.

Furthermore, the argument for picture proofs appears to be invalid. [Cf. my Pictures, Proofs, and 'Mathematical Practice': Reply to James Robert Brown, ***The British Journal for the Philosophy of Science***, 50 (1999), pp. 425–429.] It begins with what Brown calls the common attitude, which boils down to the following conjunction: only proofs give us mathematical knowledge and proofs are verbal/symbolic derivations (p. 26). Pictures, by default on this account, cannot give us mathematical knowledge because they are not verbal/symbolic derivations. To support the ability of pictures to prove theorems, Brown argues against this conjunction, and his intended target is (rightly) the second conjunct.

But he does not actually argue against the second conjunct, which would involve arguing for a different conception of mathematical proof and showing that pictures can fall under this other conception. Instead he argues against the first conjunct. Brown provides examples of pictures that are both convincing and illuminating (pp. 27, 34–35). He argues that pictures are crucial for providing data against which a more formal theory (such as arithmetized analysis) can be 'tested', and that we should not have to "get along without them" (p. 29). And he urges that pictures are in good company: like inductive evidence, and the various reasons for believing axioms, there is more to mathematics and mathematical evidence than symbolic/verbal derivations (pp. 30–32, and chapters 10, 11).

I agree with each of these claims. The only problem with them is that they do not undermine the right conjunct for Brown. They reinforce the obvious truth that there is more to mathematics than theorem proving; and that pictures (and diagrams) are an important part of mathematics, e.g., as (perhaps indispensable) aids in following proofs. They do not show that pictures actually serve as proofs. So on Brown's third aim, that of convincing the reader of Platonism and the existence of picture proofs, I believe the book falls short. On the others it's a qualified success.

Despite these last criticisms, the book has significant and unique virtues. I highly recommend it, both for its useful overviews of topics for the professional and as an accessible, non-threatening book for the undergraduate.

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Ian Hacking. ***An introduction to probability and inductive logic.*** Cambridge University Press, 2000, xvii + 302 pp.

Judging from its title, this book is pitched as an introductory text on probability and inductive logic. The title is somewhat misleading, though, as inductive logic (in the traditional sense of that term) is discussed hardly at all. But, as an introduction to probability, the book is rather good, and as an introduction to statistical inference and decision theory, it is even better.

Hacking writes very clearly and engagingly throughout. A good supply of well-chosen exercises appear at the end of each chapter, and Hacking's solutions to the various problems in the book (both in the main text, and in a "solutions" section at the end of the book) are quite thought provoking and illuminating. The book begins with a set of seven "odd problems", each of which involves a somewhat surprising and/or counterintuitive aspect of probability. This is a very nice way to begin the book, as it grabs students' attention with puzzles, right from the start, and it places a strong emphasis on (probabilistic) *problem solving* — an emphasis which is sustained throughout the book. Hacking returns to and rethinks his "odd problems" (in interesting ways) as the book unfolds.

Chapter one briefly discusses basic deductive logic. This is helpful for students who have little or no deductive logic background, and it is intended to set the stage for Hacking's subsequent discussion of inductive logic in chapter two.

Chapter two is entitled "What is Inductive Logic?" I found this chapter somewhat disappointing. Hacking never makes clear precisely what inductive *logic* consists in. His "rough definition" (18) says that "Inductive logic analyzes risky arguments using probability ideas." Hacking also asserts (14) that probability is a "fundamental tool for inductive logic." This is all quite vague, and the examples in chapter two don't help very much. Some of the examples clearly involve probability (and various kinds of probabilities, at that), and some clearly do not. It is suggested that the scope of inductive logic is limited to arguments which in some way involve probability. But, it is unclear why the scope of inductive logic should be limited in this way. After all, the scope of deductive logic is not so limited. Indeed, it would be helpful if Hacking spent more time here clarifying the relationship between deductive logic and inductive logic (as Carnap so masterfully does in §43 of *Logical Foundations of Probability*). At the end of chapter two, readers may find themselves wondering (among other things) what makes Hacking's inductive logic *logical* (in the way deductive logic is). And, later on, readers may wonder what work the locution "inductive *logic*" is really doing in the book at all, as it appears scarcely beyond chapter two (the disparaging remarks Hacking makes about logical probability later in the book (131) will probably only add to the reader's perplexity — see below).

After a somewhat wobbly start in chapter two, Hacking begins to get into stride. In chapter 3, Hacking returns to some of his "odd problems" in a very clear and edifying discussion of the gambler's fallacy. The closing section "Two Ways to Go Wrong with a Model" (33) of chapter three provides crucial advice (and insight) for anyone who uses probability to model anything.

Chapters 4–7 cover the basic formal and technical machinery of probability theory. Hacking does a good job of explaining the mathematics of axiomatic probability calculus. His use of Venn Diagrams and "calculation trees" is explanatory and informative, and his proofs and illustrations are typically very simple and elegant. Moreover, Hacking cleverly uses his "odd problems" at various points to explain (and justify) important distinctions about conditional probability and Bayes' Theorem.

Chapters 8–10 provide a brief but refreshingly nutritious introduction to expected utility theory, which includes sober analyses of several paradoxical or troublesome examples like the St. Petersburg paradox (91–95), Pascal's wager (115–124), and the decision problems of Allais (109–111), each of which aim to undermine central tenets of the classical theory of rational choice. Hacking is able (in relatively few pages) not only to bring out the fundamental principles of classical decision theory, but also to motivate some of the most important and substantial objections to it. Hacking's plethora of examples, mini-dialogues, brief historical interludes, and other entertaining tid-bits make these chapters a joy to read for both student and teacher.

In chapters 11–15, Hacking turns to the various meanings, applications, and interpretations of probabilities and probability talk. His treatments of personalistic and frequency type probabilities are stimulating and interesting. I especially liked his discussions of Dutch Books, Bayesian learning, and conditionalization (on the personalistic side), and his discussion of limit theorems and stability (on the frequency side). But, his decision (131) to avoid the terms "subjective" and "objective" (which are traditionally used in this context) is not very well motivated, and seems to do little substantive work. And, Hacking's talk (131) of the "supposed logical relation" between (inductive) evidence and hypothesis indicates a less than enthusiastic attitude toward inductive logic *qua logic*. Notably absent here is any serious discussion of logical interpretations of probability (a là Keynes, Carnap, *et al*). This is especially puzzling, in light of the book's title, and the (apparently) "logical" set-up of chapters one and two. Despite this omission, Hacking manages in these chapters to effectively demarcate and explicate the kinds of probability (mainly, personalistic and frequency) that he discusses and applies most frequently in the remainder of the book.

Chapters 15–19 cover important issues in the foundations of statistical inference, often overlooked by textbooks in this genre. I think these chapters are some of the most useful and important in the book. Hacking provides a very accessible introduction to several of the main paradigms in modern statistics. Along the way, he offers sage advice to those who apply or interpret classical statistical methods such as hypothesis testing, confidence intervals, and p-values. These chapters should be required reading for anyone with a burgeoning interest in contemporary statistical science. My only complaint about these chapters involves Hacking's discussion of the relationship between Neyman-Pearson hypothesis testing and confidence intervals (240–41). Here, Hacking's usual clarity in exposition seems absent, and the connections made are sketchy, at best. Aside from this momentary (and minor) lapse, these chapters represent some of the most philosophically informed and enlightening introductory material on contemporary statistics that I have seen.

Chapters 20–22 focus on the philosophical problem of induction. Chapter 20 provides a brief historical overview of the (Humean) philosophical problem of induction. In chapter 21, Hacking characterizes Bayesian approaches to learning and induction as evasions of the traditional philosophical problem of induction. Chapter 22 reaches a similar conclusion about Neymanian "inductive behavior" approaches to induction. While Hacking's arguments here are provocative and interesting, they ignore some important material in the contemporary literature on the relationship between the philosophical problem of induction, probability, and inductive logic. I think these chapters would benefit by adding (for instance) some remarks on Goodman's new riddle of induction (a.k.a., "Grue") and the effect its introduction had on the traditional program of inductive logic (*e.g.*, Carnap's program). Moreover, it would be helpful if Hacking would comment on the role inductive *logic* has in this traditional philosophical debate. Hacking talks only about "learning" and "behavior" approaches to induction, and their failure to address the philosophical problem of induction head-on. What about *logical* approaches to induction like Carnap's (or Keynes')? Is Hacking suggesting that inductive logic (generally) is an evasion of the philosophical problem of induction?

To sum up: this book is very useful for getting students familiarized with the basics of probability theory, its various interpretations, and its applications to statistical inference and decision theory. Hacking's writing style is clear (although, sometimes a bit *too* concise). And, Hacking's mastery of (and flare for) the subject shines through in many places. The book also contains many well-placed historical interludes and references, which are essential for understanding the intellectual trajectory of these important concepts. My only significant word of caution is to readers with more of a logical bent (perhaps some readers of this *Bulletin*) who may prefer the more traditional introductory renditions of inductive logic and inductive/logical probability to be found in Skyrms' *Choice and Chance* or Kyburg's *Probability and Inductive Logic*. Nonetheless, I highly recommend this book to those who want a clear and thoughtful introduction to probability, with an emphasis on statistical inference and decision theory.

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Eckart Menzler-Trott. ***Gentzens Problem. Mathematische Logik im nationalsozialistischen Deutschland. Mit einem Essay von Jan von Plato.*** Birkhäuser Verlag, Basel, Boston, Berlin, 2001, xviii + 411 pp.

Gerhard Gentzen was one of the major figures in the development of modern logic during its stormy period in the 1930s. Born in Greifswald on the Baltic Sea in 1909 he finally was called as a lecturer to the Mathematics Department of the German University in Prague. Immediately after the war, in May 1945, he was imprisoned by the Czechs and died of starvation after two months.

The fact of Gentzen's tragic death was known to those working in the history of logic, reports by witnesses circulated and were finally published. What was missing up to now was a full scale scholarly biography of Gentzen combining an exposition of Gentzen's pioneering work in logic with an account of his far too short life.

The book under review is an important step on this way. It will be an indispensible source for everyone interested in Gentzen. Nevertheless it evokes mixed impressions especially in respect to its scholarly qualities. It is the work of an amateur in the best sense of the word, written in love for its subject, and with deep disgust for the circumstances which made Gentzen's tragic end possible. The fact that the author is neither a professional historian nor a logician is not necessarily a fault. Given the author's involved style, lacking objectivity in evaluating the circumstances of scientific practice in the National Socialist period in Germany may be accepted, the quoted sources often speak for themselves. Harder to accept, however, is the author's—although honest—political mission. He intends to show that mathematics and especially mathematical logic did not assist in the National Socialist industrial extinction of humans because of its special form of rationality (p. XIII). He thereby tries to answer questions which themselves make no sense, and implies that abstract entities like the scientific disciplines of physics, biology or chemistry are guilty for NS crimes. Even within mathematics he keeps the good guy–bad guy scheme. Those opposing formalism, metamathematics and mathematical logic are the bad guys, but they are, as the author attempts to show, no real mathematicians. With this, the complex mechanisms of an ideological infiltration of mathematics, logic and foundational studies are improperly simplified.

The book is organized in six chapters, five of which deal with periods in Gentzen's life: his youth and university studies (ch. 1), doctoral studies and first academic activities (ch. 2), military service and habilitation (ch. 3), lectureship 1942–1944 (ch. 5), imprisonment and death (ch. 6). Chapter 4 gives an excursus dealing with the struggle with what the author calls "German logic". It especially treats Heinrich Scholz's advancement of mathematical logic and foundational research against the opposition from the side of "German mathematics". Also included is an essay on "Gentzen and proof theory" by Jan von Plato (pp. 315–328), and, besides a collections of photographs, the reprint of three papers by Gentzen. These papers include the 1936 lecture on the concept of infinity in mathematics, the lecture on the concept of infinity and consistency in mathematics, given at the Descartes congress in Paris 1937, and the seminal 1937 lecture on the present situation in the foundations of mathematics which was published in the Journal ***Deutsche Mathematik*** in 1938.

In the biographical parts an impressive wealth of material is presented, besides published sources hitherto unpublished material, family documents, archival sources, interviews etc. A lot of these documents like evaluations, diplomas, certificates and correspondences are given at full length. This can be approved as documentary service, but, on the other hand, a lot of irrelevant material blows up the size of the book. This collage style is also used in passages on Gentzen's work. His contributions to logic are mostly characterized by full quotes of reviews published in the standard mathematical review media (but why are all the reviews Gentzen wrote for the ***Zentralblatt für Mathematik*** also reprinted in full text?) The author aims at presenting Gentzen as an independent thinker who cannot be reduced to an executor of Hilbert's program, as if such evaluation would be a depreciation at all. A reader interested in theoretical aspects of proof theory will be disappointed. He or she should pass over to von Plato's short essay (written in a clumsy German).

Chapter 4 is devoted to the bizarre disputes about NS "German mathematics" in its relation to the development of mathematical logic in Germany. While German mathematics fought against "Jewish abstract" mathematics, abstract Hilbertian formalism was accepted. Mathematical logic was the target of NS philosophers of science and mathematics like Hugo Dingler and Max Steck, but, nevertheless, Heinrich Scholz was able to establish his Institute for Mathematical Logic and Foundational Studies in Münster in 1943. The author partly tries to explain this schizophrenic situation in a psychological-hermeneutical way, e.g., when he weighs up Ludwig Bieberbach's efforts towards a racist typology of mathematics with his advancement of Hilbertian thought. Partly the author argues that the attacks against mathematical logic came from the exterior, from ideologists unrelated to the subject, thereby insinuating, however, that "real" mathematicians would never have acted this way. But was this really an exterior attack? Dingler had received his Ph.D. and his habilitation in mathematics, Steck was teaching geometry at the Technical University in Munich. These were professional mathematicians! The author even suspects everyone who later went back to theoretical ideas held by these ideologist (like Dingler's operationalism), obviously maintaining that every foundational theory held by a Fascist has also to be regarded as fascist.

An English translation of this book is in preparation. One can only hope that text and organization will be revised drastically. For the present edition the summarizing comment is: it's a pity!

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Giangiacomo Gerla. ***Fuzzy logic — Mathematical tools for approximate reasoning.*** Trends in Logic — Studia Logica Library 11. Kluwer Academic Publishers, 2001, xii + 269 pp.

The book under review is a very general investigation of fuzzy logic "in narrow sense", mathematically deep and formal. Mostly it deals with the real unit interval as the set of truth values but with an abstract (non-structural) set of formulas, in the style of pioneering work of Pavelka in late seventies of XX century. Here we present some basic notions of selected chapters.

Chapter 1. — Abstract logic in a lattice. An abstract logic is a triple $(L, \mathcal{D}, M)$ where L is a complete lattice, $\mathcal{D}\colon L \to L$ is a closure operation on L (non-decreasing, monotone w.r.t. $\wedge$ and $\vee$, idempotent) and $M \subseteq L - \{1_L\}$ called abstract semantics. $(L, \mathcal{D})$ is called a deduction system. For $x \in L$, the logical consequence $L_c\{x\} = \inf\{m \in M, x \leq m\}$.

Example: F is the set of formulas of classical logic, L is the power set of F. $\mathcal{D}(x)$ is the set of all consequences of X.

Chapter 2. — Abstract fuzzy logic. Let F be an abstract set, call its elements formulas. An abstract fuzzy logic is an abstract logic whose complete lattice is the set of all fuzzy subsets of F.

Chapter 3. — Extending a crisp logic. Let (F, D) be a crisp deduction system. For v being a fuzzy subset of F let $\mathcal{D}^*(v)(\alpha) = \sup\{\lambda \in [0,1] \mid \alpha \in \mathcal{D}(C(v,\lambda))\}$ where $C(v,\lambda)$ is the λ-cut of v, i.e., $\{x \in F, v(x) \geq \lambda\}$.

Chapter 4. — Approximate reasoning. This is "Pavelka-style": one has a fuzzy set of formulas called logical axioms, a system of fuzzy deduction rules saying "if α_1 is at least λ_1 true, … , α_n is at least λ_n true, then $v'(\alpha_1, \dots, \alpha_n)$ is $v''(\lambda_1, \dots, \lambda_n)$ true". This gives an obvious notion of a proof from a fuzzy set v of assumptions; each proof proves its conclusion in some truth degree. The corresponding "fuzzy Hilbert system" $(F, \mathcal{D}, M)$ satisfies

$$\mathcal{D}(v)|\alpha| = \text{the supremum of degrees of proofs of } \alpha \text{ from } v.$$

The "fuzzy semantics" M is axiomatizable if $\mathcal{D}$ equals to the logical consequence Lc given by M.

Chapter 5: Logic as management of constraints on the truth values, Chapter 6: Canonical extension of a crisp Hilbert logic, Chapter 7: graded consequence relations.

Chapter 8. — Truth-functional logic and fuzzy logic. The author deals with [0, 1]-valued propositional calculi, in which each connective has a truth-function; this defines the truth degree of any formula given the truth evaluation of propositional variables. He analyzes these logics using his apparatus and shows that the abstract fuzzy logic given by such a propositional logic is axiomatizable in his sense iff all truth functions are continuous. (This is a generalization of an old theorem of Pavelka.)

Chapter 9: Probabilistic fuzzy logic, Chapter 10: Fuzzy control and approximate reasoning.

Chapter 11. — Effectiveness in fuzzy logic. Here the author presents a fuzzification of main notions of recursion theory, notably: A fuzzy subset S of N (natural numbers) is recursively enumerable if there exists a recursive function h mapping $N \times N$ into the rational unit interval increasing in the second argument and such that $S(x) = \lim_n h(x, n)$ for each $x \in N$. Corresponding relations of arithmetical hierarchy, creative and productive sets are introduced.

These were only the selected main notions; the author proves many interesting and deep theorems in them. He also discusses relation of his notion to intuitive vague notion (e.g., the heap paradox). He takes distance from approaches to fuzzy logic in the style of (classical) truth-functional many-valued logic, e.g., he says (p. 152): We emphasize that traditional research in truth-functional logic is sharply different from the approach to approximative reasoning we consider in this book. Indeed, in the classical approach all deductive machinery is crisp and it is devoted to produce a crisp set of formulas that are considered "valid" for some reason. This sounds slightly pejorative and I feel obliged to stress that the truth-functional approach to fuzzy inference, as developed e.g., in my book ***Metamathematics of fuzzy logic*** (Kluwer 1998) is (i) fully justified by the praxis of "fuzzy logic in broad sense" and (ii) sufficiently deep having absolutely natural Tarski-style semantics (giving "some reasons" for definition of validity), leading to a very natural class of algebras of truth functions of connectives (BL-algebras), having "classical" completeness for both propositional and predicate logics with this general semantics and having rather interesting properties concerning arithmetical hierarchy with respect to the "standard" semantics given by continuous t-norms. Gerla recommends my book and similar books for reading and occasionally refers to their results. His distinction is: his logics "arise from fuzzification of notions having metalogical characters" whereas truth-functional fuzzy logics arise from 'fuzzy worlds' whose properties can be vague and therefore whose truth-value assignments can be graded. Hopefully he will agree with me that his non-truth-functional and the truth-functional approach to fuzziness are two well-founded approaches to fuzzy logic in the narrow sense and that their mutual influence is desirable.

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Philosophy of logic, An anthology, edited by Dale Jacquette, Blackwell philosophy anthologies, no. 14. Blackwell Publishers, Malden, Mass. and Oxford, 2002, xi + 372 pp.—therein:

DALE JACQUETTE. *Introduction: logic and philosophy of logic*. Pp. 1–6.

DALE JACQUETTE. *Introduction to part I*. Pp. 9–11.

ARTHUR PAP. *The laws of logic*. Pp. 13–20. A reprint of Chapter 6 of ***JSL*** XL 108(24).

KURT GÖDEL. *Russell's mathematical logic*. Pp. 21–34. A reprint of ***JSL*** XI 75.

LESLIE H. THARP. *Which logic is the right logic?* Pp. 35–45. (Reprinted from ***Synthese***, vol. 31 (1975), Pp. 1–21.)

KARL POPPER. *What can logic do for philosophy?* Pp. 46–52. A reprint of Pp. 141–154 of ***JSL*** XIX 290.

DALE JACQUETTE. *Introduction to part II*. Pp. 55–57.

DONALD DAVIDSON. *Truth and meaning*. Pp. 59–69. A reprint of ***JSL*** XL 276.

SAUL A. KRIPKE. *Outline of a theory of truth*. Pp. 70–85. A reprint of ***JSL*** L 1068.

HARTRY FIELD. *Tarski's theory of truth*. Pp. 86–102. A reprint of ***JSL*** XLVIII 208.

FRED SOMMERS. *Types and ontology*. Pp. 103–119. A reprint of ***JSL*** XXXII 406.

GEORGE BEALER. *Propositions*. Pp. 120–139. (Reprinted from ***Mind, n.s.*** vol. 107 (1998), Pp. 1–32.)

DALE JACQUETTE. *Introduction to part III*. Pp. 143–145.

CZESLAW LEJEWSKI. *Logic and existence*. Pp. 147–155. A reprint of ***JSL*** XL 102.

CHARLES PARSONS. *A plea for substitutional quantification*. Pp. 156–160. A reprint of ***JSL*** LIII 320.

RUTH BARCAN MARCUS. *Nominalism and the substitutional quantifier*. Pp. 161–168. A reprint of ***JSL*** LX 1005.

THOMAS BALDWIN. *Interpretations of quantifiers*. Pp. 169–182. (Reprinted from ***Mind, n.s.*** vol. 88 (1979), Pp. 215–240.)

JAAKKO HINTIKKA. *Language-games for quantifiers*. Pp. 183–198. (Reprinted from ***Studies in logical theory***, edited by Nicholas Rescher, American philosophical quarterly monograph series, no. 2, Basil Blackwell, Oxford 1968, Pp. 46–72.)

DALE JACQUETTE. *Introduction to part IV*. Pp. 201–204.

ROLF GEORGE. *Bolzano's concept of logical consequence*. Pp. 205–209. (Reprinted from ***The journal of philosophy***, vol. 83 (1986), Pp. 558–564.)

ALFRED TARSKI. *On the concept of logical consequence*. Pp. 210–215. A reprint of ***JSL*** XXXIV 100 (5). (English translation of ***JSL*** I 76 with added footnotes.)

ALAN ROSS ANDERSON AND NUEL D. BELNAP, JR.. *The pure calculus of entailment*. Pp. 216–236. A reprint of ***JSL*** XXX 240.

STEPHEN READ. *Formal and material consequence*. Pp. 237–246. (Reprinted from ***Journal of philosophical logic***, vol. 23 (1994), Pp. 247–265.)

JOHN ETCHEMENDY. *Tarski on truth and logical consequence*. Pp. 247–267. (Reprinted from ***The journal of symbolic logic***, vol. 53 (1988), Pp. 52–79.)

DALE JACQUETTE. *Introduction to part V*. Pp. 271–274.

JOHN E. NOLT. *What are possible worlds?* Pp. 275–284. (Reprinted from ***Mind, n.s.*** vol. 95 (1986), Pp. 432–445.)

W. V. QUINE. *Quantifiers and propositional attitudes*. Pp. 285–291. (Reprinted from ***The journal of philosophy***, vol. 53 (1956), Pp. 177–187.)

DAVID LEWIS. *Counterpart theory and quantified modal logic*. Pp. 292–300. (Reprinted from ***The journal of philosophy***, vol. 65 (1968), Pp. 113–126.)

DAGFIN FØLLESDAL. *Interpretation of quantifiers*. Pp. 301–307. (Reprinted from *Logic, methodology and philosophy of science III*, ***Proceedings of the Third International Congress for Logic, Methodology and Philosophy of Science***, Amsterdam, 1967, North-Holland Publishing Company, Amsterdam, 1968, Pp. 271–281.)

RUTH BARCAN MARCUS. *A backward look at Quine's animadversions on modalities*. Pp. 308–317. A reprint of ***JSL*** LX 1006 (4).

DAVID KAPLAN. *Quantifying in*. Pp. 318–337. A reprint of ***JSL*** XL 479.

GRAEME FORBES. *Substitutivity and the coherence of quantifying in*. Pp. 338–355. (Reprinted from ***The philosophical review***, vol. 105 (1996), Pp. 337–372.)

MICHAEL JUBIEN. *The intensionality of ontological commitment*. Pp. 356–361. (Reprinted from ***Nous***, vol. 6 (1972), Pp. 378–387.)

This volume contains twenty-seven previously published essays, many of them classics, divided into five sections. (The division is as follows: I. Classical Logic, essays 1 (Pap)–4 (Popper); II. Truth, Propositions, and Meaning, essays 5 (Davidson)–9 (Bealer); III. Quantifiers and Quantificational Theory, essays 10 (Lejewski)–14 (Hintikka); IV. Validity, Inference, and Entailment, essays 15 (George)–19 (Etchemendy); V. Modality, Intensionality, and Propositional Attitude, essays 20 (Nolt)–27 (Jubien).) The editor has provided a general introduction and a separate introduction to each section. In the Preface the editor says that he has " ... deliberately avoided republishing papers that are readily available in other anthologies, or that are more closely related to philosophy of language or philosophy of mathematics, regardless of their influence in contemporary work in logic." Yet at least thirteen of the twenty-seven articles already appear in anthologies, many of which are in print (as of March 2003), and nearly all of which will be in most university libraries.

The principal use for a volume such as this will be as a source book for courses that deal with the issues it covers. Yet in this age of ever broader and easier availability of scholarly articles via electronic means, and at a time when it is a simple matter for an instructor to assemble a collection of articles tailor made for the course that he or she intends to teach, the usefulness of such a volume as a source book for a course rests heavily on the particular selections made by the editor and on the introductory material provided. A volume of this kind can also be useful to an interested person who seeks to obtain, by independent study, an introduction to a subject and a guide through its basic literature. Here again usefulness depends heavily on selection and introductory material. In the reviewer's opinion the present volume is only moderately successful by the first of these measures and seriously deficient by the second.

The sections of the volume that are most successful with respect to the selection of papers included are III (Quantifiers and Quantificational Theory) and V (Modality, Intensionality, and Propositional Attitude). Section III is devoted mainly to the dispute over substitutional quantification, while Section V ranges broadly over issues concerning modal logic and the logic of propositional attitudes. Each contains important papers, some of which have become classics. The selection of papers appearing in the remaining sections is more problematic.

Section I (Classical Logic) contains an excerpt from Arthur Pap's ***An Introduction to the Philosophy of Science*** (The Free Press, New York, 1962) that argues effectively against linguistic conventionalism with respect to logical truth. It also contains Gödel's important essay on Russell. But Tharp's paper could just as well appear in section IV (Validity, Inference, and Entailment), since it is primarily concerned with whether there is a sharp line between logical and nonlogical terms. (It is also the most likely counterexample to the editor's claim in the Preface that "Although some of the papers are more technical than others, all ... can be read with good understanding by beginning students in philosophy who have completed a first course in symbolic logic".) Popper's paper, while interesting, is hardly a work of enduring importance. Its inclusion in a volume such as this would be more useful if the papers by William Kneale and A. J. Ayer that were part of the original symposium at which it was presented, or at least excerpts from them, were also included (see the review of all three in ***JSL*** XIX 290).

The inclusion of the papers by Davidson, Kripke, and Field in section II (Truth, Propositions, and Meaning) cannot be faulted, but the Sommers paper (on the type structure of natural languages) is an unexpected choice. Reasonable people can disagree about such matters, but in the reviewer's opinion section II would be improved by replacing the Sommers and Bealer papers with selections that present the rule of revision approach to truth and the Frege-Church semantics of sense and denotation.

The inclusion of Tarski's paper on logical consequence in section IV (Validity, Inference, and Entailment) is a must for a volume such as this. And George's paper provides insight into Bolzano's account of logical consequence, which, as Tarski points out in a footnote added after the first publication of his paper, anticipated his own work. Still there are selection problems with section IV. If one wants a single paper that provides an introduction to relevant or paraconsistent logics, there are better choices than the paper by Anderson and Belnap included here, which is needlessly complex for this purpose. Moreover, since it deals exclusively with a language whose sole logical term is a connective for entailment, it does not address questions that arise when quantifiers and truth-functional connectives are present, such as the correctness of disjunctive syllogism, which has been a key point of contention in the debate over relevant and paraconsistent logics. Etchemendy's criticisms of Tarski's work on logical consequence, and his own positive views on the matter (see ***The Concept of Logical Consequence***, Harvard University Press, 1990, reviewed in ***JSL*** LVII 254 and ***BSL*** VII 379), have attracted much attention and generated considerable controversy in recent years. Yet, although a paper by Etchemendy is included, and his frequently cited distinction between representational and interpretational semantics is discussed in Read's paper, the volume contains nothing that is critical of Etchemendy's work, in spite of the fact that it has been widely discussed and often criticized. The most serious problem with section IV, however, is the absence of anything on the view that a sharp line can be drawn between logical and nonlogical terms by taking the former to be those which designate operations that are invariant under permutations of a domain and under bijections between domains. This view, sometimes called the formal-structural account of logical consequence, derives from several sources, including Tarski's *What are logical notions?* (***History and Philosophy of Logic***, vol. 7 (1986), pp. 143–154, the text of a 1966 lecture, edited by John Corcoran.), and has been developed more recently by Gila Sher (***The Bounds of Logic: A Generalized Viewpoint***, Cambridge: MIT Press, 1991). (An interesting variation on this approach is found in Solomon Feferman, *Logic, logics, and logicism*, ***Notre Dame Journal of Formal Logic***, vol. 40 (1999), pp. 31–54.) It is the focus of much recent work and is of considerable interest since, if successful, it resolves the main unanswered question raised by Tarski in his 1936 paper on logical consequence (paper 16 in the volume under review).

The six introductory essays provided by the editor give the impression of having been written hastily and carelessly. Specifically: (1) at many points they are less clear than they could and should be, (2) they contain some outright errors, and (3) some important matters have been left out entirely. Point (1) is illustrated by the account of the liar paradox in the introduction to part II. After giving a standard account of the difficulties presented by the liar sentence the editor says, "Is the liar sentence a proposition? If not then the problem posed by the liar paradox simply disappears, since it is only propositions that according to classical logic are supposed to be exclusively true or false" (p. 55). Yet the term 'proposition' has not yet been defined or characterized in any way. There follows a long discussion in which 'proposition' is used many times and Tarski's hierarchical approach to truth definitions is discussed before finally, on the next page, we are told of " … [t]he traditional definition of a proposition as a true or false sentence or the abstract true or false meaning or thought expressed by a true or false sentence … ." Students struggling to understand these matters deserve better than this. And things are not helped by the editor's failure to distinguish the liar from the strengthened liar on the following page.

Another example of point (1) is the editor's use of the phrase 'class of a logic' in his account of Tharp's paper (p. 10), without giving any hint as to what it means. One can of course find out by reading Tharp's paper, but the reader of an introduction shouldn't have to go to such lengths to learn what could easily have been conveyed directly. Finally, consider the following sentence on David Lewis's counterpart theory from the introduction to section V: "Lewis follows G. W. Leibniz in arguing that strictly speaking different logically possible worlds do not contain identical objects, but only objects that are similar to as counterparts of one another to varying degrees with respect to particular categories of properties." (p. 272). No readers, least of all the "beginning students in philosophy who have completed a first course in symbolic logic", who are among those for whom the volume is intended (Preface, p. ix), will be helped by such exposition.

With respect to point (2) (outright errors) consider the account of logical consequence and logical truth given in the introduction to section IV. In the first paragraph the editor characterizes logical consequence as a modal relation, specifically, as the logical impossibility of true premises and a false conclusion (p. 201). In the next paragraph he presents "[a]n important theory of logical consequence [that] is attributed to Bernard Bolzano and later adapted by Tarski", namely, the view that logical consequence is truth preservation under all uniform substitutions of nonlogical terms throughout premises and conclusion. There is no attempt to reconcile these two accounts, nor is it even acknowledged that they are quite different and may not be coextensive. Furthermore, on the next page the editor compounds the confusion by claiming that the latter account is coextensive with Tarski's model-theoretic account. Specifically, switching from logical consequence to logical truth for a language L, and taking S' to be the sentential function that results from uniformly substituting variables of appropriate type for nonlogical symbols in sentence S, he says, " … S is said to be logically true if and only if S' is satisfaction-preserving on all sequences of objects in appropriate satisfaction domains for the variables in S'. This in turn means that S is logically true *if and only if* all the sentences obtained from sentence function S' by resubstituting names, predicates, connectives, and operators for corresponding variables in S' from the entire available satisfaction domains in each category produces only sentences true in L" (p. 202, emphasis added). Yet the substitutional account given in the second quoted sentence sounds exactly like Tarski's Condition (F), which Tarski explicitly denies is coextensive with his own model-theoretic account, on the grounds that the very objects, concepts, etc., that would provide a counterexample to the logical truth of S may not be expressed by any names, predicates, etc., of L. (See *On the concept of logical consequence*, ***Logic, Semantics, and Metamathematics***, second edition, edited by John Corcoran, Indianapolis: Hackett Publishing, Co. 1983, p. 417; p. 213 in the volume under review.) Furthermore, the editor's description of the substitutions in question as being of "names [etc.] for corresponding variables in S' from the entire available satisfaction domains in each category" will merely encourage a confusion that many students are already inclined to make, namely, between the objects that satisfy or fail to satisfy S' and the names of such objects that may be substituted for variables in S'.

With respect to point (3) (serious omissions) it is noteworthy that none of the introductory material mentions the formal-structural account of logical terms and logical consequence referred to earlier in this review. This is odd in view of the fact that the editor recognizes (pp. 202–203) that Tarski's 1936 account of logical consequence is in need of a criterion for distinguishing logical from nonlogical terms, that the formal-structural account purports to provide just such a criterion, and that Tarski himself contributed to and endorsed its development (see reference to Tarski's 1986 paper above). Inexplicably, Sher's book (see reference above), which presents the formal-structural approach in detail, is listed in the bibliography to section II (p. 57), but no mention of, or reference to, the formal-structural approach appears in section IV.

The problems with the introductory material mentioned above are not isolated instances. Numerous similar examples could be given. In view of these problems, and the problems having to do with the selection of essays mentioned earlier, it is the reviewer's opinion that this volume will be of limited usefulness.

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Robin Hirsch and Ian Hodkinson. ***Relation algebras by games.*** Elsevier, Amsterdam, 2002, xviii + 691 pp.

The first two sentences of this book assert, "Relation algebras are algebras arising from the study of binary relations. They form a part of the field of algebraic logic, and have applications in proof theory, modal logic, and computer science." This assessment reflects a major historical shift, for in the late nineteenth century "algebraic logic" was the name used for the *whole* of what we now call "mathematical logic" or "symbolic logic". The application of algebra to logic, initiated by G. Boole and highly extended and developed by C. S. Peirce, E. Schröder, and others, was one of the major contributors to the nineteenth century expansion of algebra from numbers to other sorts of objects (such as sets, relations, propositions, truth values, vectors, and so on), and was one of the principal topics in A. N. Whitehead's book, ***Universal algebra***, published in 1898. The Peirce-Schröder calculus of binary relations was still too new for inclusion in Whitehead's wide-ranging book. Indeed, the Peirce-Schröder calculus was founded by Peirce in his landmark papers of 1870, 1880, and 1883, and was highly elaborated in the third volume of Schröder's ***Algebra der Logik***, published in 1895. Löwenheim's theorem, which is perhaps the most historically significant theorem in the calculus of relations, came even later, in 1915. It asserts that if a relational equation is valid in all finite domains but fails in some infinite domain, then, in fact, it must fail in a countably infinite domain. During the last third of the 19th and the first third of the 20th centuries, efforts to provide solid (even absolute) foundations for Cantor's set theory (and mathematics in general) led to the development of first-order logic and to the theories of computability and provability. As the dust of the foundational debates settled, the relative viewpoint of the Peirce-Schröder calculus re-emerged, primarily through the work of Tarski, as the theory of models. This is expressed in the Chang-Keisler equation

$$\text{model theory} = \text{universal algebra} + \text{logic},$$

from their 1973 text on model theory. By the mid-twentieth century this development was also reversed, in the sense that algebras of logic were created (or recreated) from logic, including relation algebras and cylindric algebras by Tarski and polyadic algebras by Halmos. ***Relation algebras by games*** appropriately uses model theory to study relation algebras. A simple example illustrates this viewpoint. A representation of a relation algebra $\mathfrak{A}$ is a one-to-one homomorphism to an algebra of binary relations that happens to be a special kind of relation algebra. Similarly, a representation of a group, or a Boolean algebra, is a homomorphism to a special kind of group (containing endomorphisms of a vector space) or a special kind of Boolean algebra (containing sets). On the other hand, a model of a first-order theory T in a language L is a function that associates relations and functions with the relation symbols and function symbols of L. In particular, binary relation symbols are mapped to binary relations, just as is done by a representation of a relation algebra. This similarity has sometimes led to the exposition of models as homomorphisms. A model of a theory T is a homomorphism from a special algebra of formulas associated with T to an algebra of relations. This book goes the other way, and views a representation of a relation algebra $\mathfrak{A}$ as a model of a theory associated with $\mathfrak{A}$ in a first-order language with only binary relation symbols, one for each element of $\mathfrak{A}$. This device was first exploited by R. McKenzie in his 1966 dissertation. This circumlocution allows, for example, a contemporary statement of Löwenheim's theorem (due to Jónsson), that an equation true in the relation algebra $\mathfrak{A}$ of all binary relations on an infinite set of cardinality κ is also true in the relation algebra of all binary relations on any set of larger cardinality $\lambda > \kappa$, to be proved by applying the Löwenheim-Skolem-Tarski theorem to the theory associated with the algebra $\mathfrak{A}$. There are twenty-one chapters in six parts. Parts of eight chapters are modified versions of nine previously published papers (six by Hirsch and Hodkinson, one by Hirsch, one by Hodkinson, and one by Andréka and Hodkinson). There are more than 400 exercises ranging from drill to open problems. Chapter 21 has a list of twenty-one open problems, recalled from the body of the text, that the authors believe would "make a significant contribution to research in this field." The book is itself such a contribution, since it is built around the solutions to several open problems. The first third of the book is occupied by the five chapters of Part I. It contains a historical introduction, preliminaries from model theory, universal algebra, and Boolean algebras, two chapters on relation algebras (with basic definitions, elementary facts, and many examples), and brief surveys of relativized representations, cylindric algebras, and other approaches to algebraic logic. In Chapter 2, many basic definitions and facts from model theory are recalled, including the compactness theorem, the Löwenheim-Skolem-Tarski theorem, theorems on the existence of saturated models and ultraproducts, Los's theorem, and so on. The theory of Boolean algebras with operators is summarized up through the existence, uniqueness, and preservation theorems for completions and perfect extensions. This summary includes a survey of results on the interaction of atom structures, complex algebras, Sahlqvist equations, and modal logic. In Chapters 3 and 4, representations of a relation algebra are viewed as models of an appropriate first-order theory constructed from the algebra. This allows the authors to prove Monk's theorem, that the class of representable relation algebras is closed under perfect extensions, by using the existence of saturated models of the associated first-order theories. Using this result, they then prove Tarski's theorem, that the class of representable relation algebras is closed under homomorphisms, by constructing a representation of a homomorphic image of a representable algebra from a representation of its perfect extension. Examples of relation algebras include proper relation algebras (algebras of binary relations), group relation algebras (whose elements are subsets of a group), Lyndon algebras (whose elements are sets of points in a projective geometry), and Monk algebras (which are designed to utilize some basic results of Ramsey theory). Chapter 5 reviews the definitions, basic facts, and gives a brief survey of results about non-associative relation algebras, weakly associative relation algebras, semi-associative relation algebras, weakly representable relation algebras (wRRA), cylindric algebras, and the connections among these classes that arise through relativization and the formation of reducts. Chapter 6 is a short survey of other kinds of algebras of relations, including diagonal-free cylindric algebras, Halmos's polyadic algebras, and Pinter's substitution algebras. There is an informative discussion of the (as yet unresolved) finitization problem, and a list of representation results for relation algebras. The second third of the book is comprised of Part II (Chapters 7–11, on games, networks, and axiomatization) and Part III (Chapters 12 and 13, on approximations to representations). The general theme of Part II is that representability can be characterized by the existence of a winning strategy for a game played with networks. A network over some algebra is a (typically finite) graph whose edges are labeled with elements of the algebra. A network is essentially just a portion of a representation. Networks serve as positions in games. The object of a typical game on an algebra is, for the first player (male), to ferret out a difficulty that prevents the algebra from being representable (or having some other nice property), while the second player (female) attempts to build a representation (or some other appropriate structure) in response to challenges by the first player. These ideas are illustrated by a proof of Maddux's theorem that every weakly associative relation algebra has a relativized representation. An appropriate infinite game is defined and shown to be winnable by the second player, which implies that every countable weakly associative relation algebra has a relativized representation. This conclusion can then be extended to all weakly associative relation algebras by compactness or ultraproducts. There are many variations on the games. They can be played on networks in general, or on networks restricted in some way, such as having atoms as labels or bounded size (no more than p nodes, for some fixed bound $p < \omega$). Depending on the application, moves may consist of choosing an edge and two elements, choosing two nearly identical networks, dropping a node and its edges from a network, properly labeling a set of edges labeling of a network, or choosing one of two alternative networks. A given game can last for countably many moves (an infinite game), or it can be terminated after a fixed finite number of rounds (an r-round game for some finite length $r < \omega$). A key lemma is that if the second player can win all the finite-length games, then she can win the infinite game. According to the central result of Chapter 7, if the second player can win a certain game on a relation algebra, then that algebra is representable. The proof relies on the fact that the representable relation algebras form an equational class. The ability of the second player to win all finite-length games can be expressed as a sequence of first-order sentences, one for each finite-length game. The collection of such sentences characterizes representability. Each sentence in such a collection has a quantifier prefix of the form $(\forall\exists)^r$, where r is the number of rounds in the corresponding game. The sentence says, roughly, that *for every* move by the first player, *there is* a response available to the second player such that *for every* move by the first player, *etc*. By restricting the second player to choosing only one of two alternative moves, the existential quantifier can be reduced to a disjunction and the resulting sentence is universal. For discriminator varieties (the typical case in this book) a universal sentence is equivalent in all simple algebras to an equation. These techniques are applied to obtained axiomatizations for various classes, including representable relation algebras and representable cylindric algebras in Chapter 8, and some pseudo-elementary classes and pseudo-universal classes in Chapter 9. Chapter 10 is a more formal presentation of some of this material, including precise definitions of games and strategies in terms of trees labeled with variables and formulas. Chapter 11 is primarily about atomic relation algebras and Lyndon's conditions from his 1950 paper. Assume $\mathfrak{A}$ is an atomic relation algebra. Atomic networks are those networks whose labels are atoms of $\mathfrak{A}$. A new "atom-game" is introduced, played on atomic networks. The main results are that the atomic relation algebra $\mathfrak{A}$ satisfies the Lyndon conditions iff the second player wins all finite-length atom-games on $\mathfrak{A}$ iff $\mathfrak{A}$ is elementarily equivalent to a completely representable (hence also atomic) relation algebra. Furthermore, if $\mathfrak{A}$ is completely representable then the second player wins the infinite atom-game on $\mathfrak{A}$. The converse fails in general but holds if the number of atoms is countable. The exercises include examples of finite symmetric integral representable relation algebras with no finite representations, Monk's theorem that RRA, the class of representable relation algebra, is not finitely based, and Jónsson's proof that there is no equational axiomatization of RRA that uses only finitely many variables. The latter two results are both proved via Lyndon's algebras, obtained from projective lines, which were published in 1961. Jónsson's proof was found in 1988, but the result was stated by Tarski in a videotaped lecture in 1974. Perhaps this proof was at one time in the 1960's known to both Tarski and Jónsson. For $n \geq 3$, RA_n is the class of relation algebras of dimension n, and $\boldsymbol{S}\mathsf{RaCA}_n$ is the class of subalgebras of relation-algebraic reducts of cylindric algebras of dimension n. It has long been known that these classes are canonical varieties. RA_n is defined via the concept of relational basis. Every atomic algebra in $\boldsymbol{S}\mathsf{RaCA}_n$ has a relational basis. This implies that $\boldsymbol{S}\mathsf{RaCA}_n \subseteq \mathsf{RA}_n$. In spite of the names, RA_3 and $\boldsymbol{S}\mathsf{RaCA}_3$ are strictly larger classes than RA, the class of all relation algebras. For $n = 4$ and $n = \omega$ the situation is quite nice, and provides characterizations of RA and RRA, since $\mathsf{RA} = \mathsf{RA}_4 = \boldsymbol{S}\mathsf{RaCA}_4$ and $\mathsf{RRA} = \mathsf{RA}_\omega = \boldsymbol{S}\mathsf{RaCA}_\omega$. When $4 \leq n < \omega$, these classes form descending chains of varieties of relation algebras that converge to RRA, that is, $\mathsf{RA}_{n+1} \subset \mathsf{RA}_n$ and $\boldsymbol{S}\mathsf{RaCA}_{n+1} \subset \boldsymbol{S}\mathsf{RaCA}_n$, where all inclusions were known to be strict, and $\mathsf{RRA} = \bigcap_{3 \leq n < \omega} \mathsf{RA}_n = \bigcap_{3 \leq n < \omega} \boldsymbol{S}\mathsf{RaCA}_n$. In Chapters 12 and 13 these definitions and results are reviewed and extended. A variation on the atom-game is introduced, in which there is a fixed finite bound p on the number of nodes in the networks that appear in the game. The second player wins the infinite p-bounded atom-game iff the algebra has a p-dimensional relational basis. An extension of the concept of relational basis, called "hyperbasis", is used to characterize $\boldsymbol{S}\mathsf{RaCA}_n$ in a manner similar to the definition of RA_n. The classes RA_n and $\boldsymbol{S}\mathsf{RaCA}_n$ are also characterized in terms of relativized representations that have certain nice properties. The last third of the book consists of Part IV (Chapters 14–17), which contains various constructions of algebras, and Part V (Chapters 18 and 19) on decidability and the finite base property. In Chapter 14 a relational structure $\mathfrak{S}(G)$ is associated with every graph G, with these crucial properties. If G has infinite chromatic number (and is itself necessarily infinite), then $\mathfrak{Cm}\mathfrak{S}(G)$, the complex algebra of $\mathfrak{S}(G)$, is a representable relation algebra. On the other hand, if G has a finite chromatic number, and G itself is either infinite or sufficiently large compared to its chromatic number, then $\mathfrak{Cm}\mathfrak{S}(G) \notin \mathsf{RRA}$. This allows two significant theorems. If, for example, G is a countable chain (undirected edges, all nodes of degree 2), then $\mathfrak{Cm}\mathfrak{S}(G) \notin \mathsf{RRA}$, but it turns out that if $\mathfrak{A}$ is the subalgebra of $\mathfrak{Cm}\mathfrak{S}(G)$ generated by the atoms of $\mathfrak{Cm}\mathfrak{S}(G)$, then $\mathfrak{A}$ is representable. It follows that RRA is not closed under completions, for $\mathfrak{Cm}\mathfrak{S}(G)$ is the completion of $\mathfrak{A}$. This result settles a 30-year old problem of Monk that was first solved by Hodkinson using a different example. Then, using a result of Erdös that there are finite graphs with arbitrarily large girth and chromatic number, the authors construct a sequence of infinite graphs G_i with infinite chromatic number and increasing girth, say G_i has girth i. The ultraproduct of the graphs G_i is 2-colorable because it has infinite girth, so the corresponding structure has a nonrepresentable complex algebra. But the complex algebras of the $\mathfrak{S}(G_i)$ are in RRA. Thus the class of structures whose complex algebras are in RRA does not form an elementary class. For $i > 15$, the algebra $\mathfrak{Cm}\mathfrak{S}(G_i)$ satisfies only finitely many Lyndon conditions, so the second player will lose any sufficiently long finite-length atom-game played on $\mathfrak{Cm}\mathfrak{S}(G_i)$. This algebra cannot be completely representable (for it would otherwise satisfy all the Lyndon conditions) but it is, nevertheless, representable. It is complete, atomic, and representable, but has no complete representations. The earliest example of such an algebra appears in Lyndon's 1950 paper. Additional examples, from Maddux's 1978 dissertation, conclude Chapter 14. The main result of Chapter 15 is that $\boldsymbol{S}\mathsf{RaCA}_{n+1}$ is not finitely based relative to $\boldsymbol{S}\mathsf{RaCA}_n$ (and a similar statement for neat reducts of cylindric algebras). The proofs utilize variations on Monk algebras and techniques from Ramsey theory, and they produce other interesting results. For every finite $n \geq 5$, the inclusion $\boldsymbol{S}\mathsf{RaCA}_n \subseteq \mathsf{RA}_n$ is strict, and, in fact, $\boldsymbol{S}\mathsf{RaCA}_n$ is not finitely based relative to RA_n, nor is it finitely based relative to $\mathsf{RA}_n \cap \boldsymbol{S}\mathsf{RaCA}_{n-1}$. Furthermore, for every finite $n \geq 5$, RA_n is not included in $\boldsymbol{S}\mathsf{RaCA}_5$. Chapters 16 and 17 are devoted to the construction and application of the rainbow algebra $\mathfrak{A}_{A,B}$, which is built from two relational structures A and B for a language having only binary and unary relation symbols. The rainbow algebra $\mathfrak{A}_{A,B}$ is always complete and atomic, and it is finite whenever A and B are finite. The number of atoms in $\mathfrak{A}_{A,B}$ is approximately the sum of the squares of the cardinalities of A and B. The idea is to translate Ehrenfeucht-Fraïssé games on relational structures to games on algebras. It is a result from model theory that the second player wins the standard p-pebble r-round Ehrenfeucht-Fraïssé "forth" game from A to B just in case every positive existential sentence with p variables and quantifier depth r true in A is also true in B. The main result is that if the second player can win a (slightly modified) p-pebble finite-length Ehrenfeucht-Fraïssé game from A to B, then the second player can win a p-bounded finite-length atom-game on $\mathfrak{A}_{A,B}$. This construction is quite powerful. It provides alternate proofs of earlier theorems, a proof of Haiman's theorem that wRRA is not finitely based, and some significant new results. For example, the class of completely representable relation algebras is not elementary. Perhaps most important is that RA_{n+1} is not finitely based relative to RA_n whenever $4 \leq n < \omega$, and that RA_n and $\boldsymbol{S}\mathsf{RaCA}_n$ are not closed under completions whenever $6 \leq n$ (leaving open the case $n = 5$.) Chapter 18 is devoted to showing that if K is any class of algebras that contains RRA and is contained in either $\boldsymbol{S}\mathsf{RaCA}_5$ or wRRA, then there is no algorithm for determining whether a finite algebra belongs to K. In particular, the question "is a finite RA in RRA?" is undecidable. It follows readily from these results that if K is a variety, then the equational theory of K is also undecidable. The method is to encode each instance of the (undecidable) tiling problem as a relation algebra. Given a set of edge-labeled tiles τ, there is an algebra $\mathfrak{A}_\tau$ that is representable if and only if there is a tiling of plane using copies of tiles from τ so that adjacent tiles have matching labels on their common edge. Every weakly associative relation algebra has a relative representation, but if the algebra is finite then, in fact, such a representation can be constructed on a finite set. In Chapter 19, this and similar results for finite algebras in RA_n and subalgebras of relation-algebraic reducts of finite n-dimensional cylindric algebras, involving appropriate notions of relative representation, are derived fairly directly from recent deep results in model theory that are stated with references but without proof. Part VI, the Epilogue, has just two short chapters. One is a fifteen-page summary of the book, and the other is a list of twenty-one unsolved problems. There are 326 items in bibliography and ten pages of symbol indices, organized into ten tables according to subject. This book is a significant advance in the theory of relation algebras. Many of its main results solve difficult and long-standing problems. Its methods, techniques, and constructions are powerful tools for exploring the intricate and varied world of relation algebras. Its many open problems indicate fruitful directions for further research.

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GEORGE S. BOOLOS, JOHN P. BURGESS, and RICHARD C. JEFFREY. ***Computability and logic,*** Fourth edition. Cambridge University Press, Cambridge, 2002. xi + 356 pp.

John Burgess has prepared a much revised and extended fourth edition of Boolos's and Jeffrey's classic textbook ***Computability and logic***. (For a review of the first edition, see JSL XLII 585.) The book has long been a favorite among logicians for its accessible style and the wealth of results (in the later chapters) not usually found in logic textbooks. It has been particularly successful as a textbook for use in advanced logic courses taught in philosophy departments, partly due to its "non-threatening" style, and partly because it covers topics which are particularly of interest to philosophers, such as Turing machines, completeness and incompleteness. Its emphasis on computability and, in particular, its comparative treatment of several models of computation (Turing machines, abaci, and recursive functions) also makes it attractive to computer scientists. Burgess, in consultation with Jeffrey, has done an admirable job in updating and revising the material while keeping this target audience in mind. The highlights of the changes in the fourth edition are: (a) an expanded selection of problems with hints; (b) many added examples, especially in the first chapter on enumerability; (c) two new chapters (9 and 10) containing a survey of the syntax and semantics of first-order logic; (d) a new proof of the undecidability of first-order logic; (e) a new chapter (12) on the basics of models (e.g., isomorphisms); (f) a completely new approach to the proofs of compactness, Löwenheim-Skolem, completeness and Craig interpolation theorems; (g) a new proof of representability of recursive functions in Q; and (h) a new proof of Herbrand's theorem. Many sections have been rewritten or reorganized. A comparison between the third and fourth editions, as well as errata and addenda (which are incorporated into the second printing), are available on Burgess's website at `http://www.princeton.edu/~jburgess/addenda.htm`. The interested reader will find a complete list of changes there; I will focus here on the part that has undergone the most significant revisions: Chapters 9–14. Chapters 9 ("A précis of first-order logic: Syntax") and 10 ("A précis of first-order logic: Semantics") replace Chapter 9 of the third edition and introduce the language and semantics of first-order logic thoroughly, thus making the text more independent of a preceding introductory course to logic. Included also are proofs of unique readability and extensionality; definitions and basic results about metalogical notions such as implication and satisfiability are now collected in Chapter 10. The syntax of first-order logic is now given a precise definition in Chapter 9, with the notation used in the rest of the text introduced as informal abbreviations. Here, where pedantry would not be out of place, a surprising number of misprints will confuse the beginning student. Many should already be corrected in the second printing, however. Together with Chapter 12 ("Models"), these new chapters provide a much-needed reference point and review of the basics of first-order logic. Chapter 11, "The undecidability of first-order logic," contains a new reduction of the halting problem to the decision problem for first-order implication, due to Boolos. Whereas previous editions showed how to represent Turing machines in the first-order theory of one successor function plus some additional predicates, the new proof uses a binary successor relation **S**. The proof thus immediately shows the undecidability of first-order logic without function symbols (and at most binary predicate symbols). The second, and perhaps more interesting payoff is that the proof of Trakhtenbrot's Theorem (Exercise 11.7) is much easier, since the background theory is now formulated so that it has finite models. A not inconsiderable drawback is that the proof is now quite a bit more involved. It requires the introduction of numerals as conventions, and proofs that some simple facts follow from the background theory are now much more complicated if carried out in detail. The simplest example here is the proof that $\mathbf{p} \neq \mathbf{q}$ follows from the axioms if $p \neq q$. This now requires a proof that

$$\sim \exists x_1 \dots \exists x_p \exists y_1 \dots \exists y_q (\mathbf{S0}x_1 \ \ \& \ \ \mathbf{S}x_1x_2 \ \ \& \ \ \dots \ \ \& \ \ \mathbf{S}x_{p-1}x_p \ \ \& \\ \& \ \ \mathbf{S0}y_1 \ \ \& \ \ \mathbf{S}y_1y_2 \ \ \& \ \ \dots \ \ \& \ \ \mathbf{S}y_{q-1}y_q \ \ \& \ \ x_p = y_q).$$

so follows. Dealing with complicated abbreviations such as these will prove daunting for many a student for whom this is the first result covered after a crash review of the syntax and semantics of first-order logic. Owners of the first printing should also watch for confusing misprints in this dense section, in particular the last displayed formula on p. 129, which should read $\exists y(\mathbf{S}_{\pm 1}xy \ \ \& \ \ @uy \ \ \& \ \ (\mathbf{M}uy \leftrightarrow \mathbf{M}ty))$. Another significant change in the fourth edition is a new proof of the Löwenheim-Skolem, compactness, and completeness theorems. In contrast to the traditional path of deducing the first two from the last, Burgess proves two main lemmas: Lemma 13.6 states that if a collection S of sets of sentences satisfies the "satisfaction properties," then each set $\Gamma \in S$ can be extended to a set Γ^* (in an expanded language L^+) which has the "closure properties." Lemma 13.5 then states that each set of sentences Γ^* that has the closure properties has a term model in the language L^+. A set which has the closure properties is often called a Hintikka set (although not here)—it satisfies exactly the conditions needed for the extraction of a term model. The satisfaction properties are the conditions necessary for the proof that every satisfiable (finitely satisfiable, consistent) set can be extended to a Hintikka set. The above-mentioned results then easily follow by verifying that the collections of satisfiable sets of sentences, of finitely satisfiable sets of sentences, and of consistent sets of sentences each have the satisfaction properties. The presentation of the argument distracts from its elegance somewhat; the beginning reader will find it difficult to make out how the various lemmas fit together. For instance, it might have been better to first prove the Löwenheim-Skolem Theorem from Lemmas 13.3–6, as the need for term models is clear in this context, and derive the compactness theorem in a second step from those and Lemma 13.2. The proof of the crucial Lemma 13.6 in Section 13.4 does a good job of making the relation between the satisfaction properties and the closure properties and their role in the proofs clear; unfortunately the end of the proof is somewhat handwavy—unnecessarily so, since a concrete construction of the set satisfying the closure properties can be given without much difficulty. For the proof of the completeness theorem, the new edition uses a version of the sequent calculus as the proof system, which is briefly compared with Hilbert-style and natural deduction systems. Perhaps in a future edition this change will be taken as an opportunity to introduce new material on proof theory, which is sadly lacking from many textbooks at this level, yet of central interest for philosophers and computer scientists. The many revisions have resulted in a modernized and more streamlined text with many improvements small and large. Instructors will particularly appreciate the new examples, problems, and expanded explanations. They should be warned, however, that some central proofs are now harder—for the novice student, harder to follow; for the instructor, therefore, harder to teach.

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KAREL LAMBERT. ***Free logic: Selected essays.*** Cambridge University Press, Cambridge, 2003, xii + 191 pp.

In *Introduction to Mathematical Philosophy* Russell complained of the "impurity" of sentences like $\exists x.x = x$, which are valid in usual formulations of predicate logic but which he contends are true only if at least one object exists. Carnap, in *The Logical Syntax of*

Language §38a, sketches a method for constructing a logical system which, he says, does not make such assumptions. Neither Russell nor Carnap made provision for names (properly so-called) which do not refer to existent objects, preferring to explain away "Pegasus" and "The King of France" as not genuine names. It wasn't until the 1950s and 1960s that a sustained effort was made to provide suitable formulations of classical predicate logic innocent of such existential assumptions and which allowed for singular terms which do not refer to existing objects. An impressive cast of logicians contributed to this effort, including Henry Leonard, Hugues Leblanc, Theodore Halperin, Jaako Hintikka, Dana Scott, Bas van Fraassen, Robert Meyer, and many others.

Nobody contributed more to the development of and advocacy for what has come to be called "free logic" than the man who coined the name, Karel Lambert. This book gathers together a selection of his papers on the subject dating from between 1963 and 1997, rewritten and compressed to varying degrees, together with two papers written especially for this collection.

Much of the book is devoted to making the case that a satisfactory account of definite descriptions is only possible in free logic. A definite description $\imath x\varphi$ is *improper* if it is not the case that there is exactly one existing individual satisfying the description φ (i.e., $\exists!x\varphi$). It's pretty clear what the reference of *proper* descriptions must be, but there are a variety of views on what to do with improper descriptions. Lambert devotes much attention to careful exposition and criticism of theories of descriptions in non-free logics: a chapter is devoted to the differing versions of Russell's theory as found in "On Denoting" and in *Principia Mathematica*, on each of which description terms are not genuine singular terms and the appearance to the contrary is somehow explained away; another to careful formulation of a version of the Hilbert-Bernay's approach, on which $\imath x\varphi$ is a singular term in a language if and only if $\vdash \exists!x\varphi$ (and so well-formedness and provability are interdependent notions); and at various points in the book Fregean approaches, on which if $\exists!x\varphi$ is false, the referent of $\imath x\varphi$ is an arbitrarily chosen, existent individual, are investigated. Chapter 2 gathers together material from a series of papers from the 1960s in which Lambert worked out the first complete, consistent theory of definite descriptions in a free logic. Probably the most useful paper in the collection is Chapter 5, written especially for the collection, in which the extant *positive* free description theories in classical logic are partially ordered by deductive strength. (A *negative* free logic is one in which no atomic statement in which a non-referring singular term occurs is true. *Positive* free logics are ones in which some such statements are true.) The ordering depends on which of two sorts of principles are valid in a particular free description theory. One sort has to do with when something can truthfully be predicated of a description term (for instance, that $t = \imath x(x = t)$ for all t is a *stronger* condition than merely that $\exists x\varphi \rightarrow \varphi(\imath x\varphi)$ for all φ). The other sort governs identity conditions for description terms (for instance, are such terms *extensional* in the sense that $\forall x(\varphi \leftrightarrow \psi) \rightarrow \imath x\varphi = \imath x\psi$?).

Another theme is that free logic is a philosophically fruitful invention. Consider: the predicates "x rotates," "x is an existent thing that rotates" and "if x exists, then x rotates" are co-extensive; but whatever truth value is assigned to the sentence which results by putting the name of the non-existent planet "Vulcan" for x in the first, a sentence with a different truth value will result by similar substitution in at least one of the others. So substitution of coextensive general terms does not preserve truth. Lambert makes several uses of this simple argument, including arguing that it shows problems with Quine's preferred theory of predication. Lambert finds "common ancestry" for the self-contradictory statement "The round square both is and is not round" and the Russell sentence "the class of all classes not members of themselves both is and is not a member of itself" by showing that each follows, in non-free but not in free logic, from the plausible-seeming principle $\forall x(x = \imath y\varphi(y) \leftrightarrow \forall y(\varphi(y) \leftrightarrow y = x))$. And the volume includes a somewhat revised version of a 1969 attempt to use van Fraassen's method of supervaluations (which were invented as a semantics for free logics) to argue that quantum mechanics does not require a rejection of classical logic.

The volume will be of interest to enthusiasts of free logic, and to people interested in its history as a lot of attention is paid to priority questions. Moreover, careful reading will convince most readers that there are still philosophical lessons to be learned by the investigation of free logic. However, the collection could create a misleadingly narrow picture of free logic. Lambert gives a peculiar, seemingly territorial argument in which he denies the title "free logic" to a system due to Sören Stenlund: it "violates the spirit" of free logic in its treatment of names (p. 63), even though it incorporates mechanisms which track where existence assumptions are employed in reasoning. Also, all Lambert's investigations take place in classical logic, and he suggests that it is "useful to think of free logic as an extension of classical predicate logic" (p. 133). If one buys all this, free logics lose much of their current interest. When working in many sorted languages in which some sorts may be empty, it becomes important to keep track of where existence assumptions figure in reasoning, but many logics developed to do this are closer "in spirit" to Stenlund than to Lambert, I think. There is much philosophical insight in Dana Scott's *Identity and existence in intuitionistic logic* (***Applications of sheaves***, edited by M. P. Fourman, et al., Springer Lecture Notes in Mathematics vol. 753, 1979, pp. 661–696), which considers descriptions in a non-classical free logic. Indeed, the behaviour of term forming operators other than the description operator in non-classical free logic has turned out to be fertile philosophical ground. See the work of Neil Tennant (a recent example is *A general theory of abstraction operators.* ***The philosophical quarterly***, to appear (2004)) and John L. Bell (e.g., *Hilbert's ε-operator in intuitionistic type theories.* ***Mathematical logic quarterly***, vol. 39 (1993) pp. 323–337).

Finally, Lambert's work certainly deserved a more carefully produced volume. The book is riddled with dozens of typographical and typesetting errors, including at least ten on page 88.

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